A two-step indirect inference approach to estimate the long-run risk asset pricing model

J. Grammig • E.-M. Küchlin

centre for financial research
Look deeper

A two-step indirect inference approach to estimate the long-run risk asset pricing model

Joachim Grammig and Eva-Maria Küchlin [*]

May 27, 2017

Abstract

The long-run consumption risk model provides a theoretically appealing explanation for prominent asset pricing puzzles, but its intricate structure presents a challenge for econometric analysis. This paper proposes a two-step indirect inference approach that disentangles the estimation of the model's macroeconomic dynamics and the investor's preference parameters. A Monte Carlo study explores the feasibility and efficiency of the estimation strategy. We apply the method to recent U.S. data and provide a critical re-assessment of the long-run risk model's ability to reconcile the real economy and financial markets. This two-step indirect inference approach is potentially useful for the econometric analysis of other prominent consumption-based asset pricing models that are equally difficult to estimate.

Key words:	indirect inference estimation, asset pricing, long-run risk
JEL:	C58, G10, G12

---
[*]Joachim Grammig: University of Tübingen, Centre for Financial Research (CFR), Cologne joachim.grammig@uni-tuebingen.de, +49-7071-2976009. Eva-Maria Küchlin: University of Tübingen. eva-maria.kuechlin@uni-tuebingen.de, +49-7071-2972571. Address: University of Tübingen, Department of Econometrics, Mohlstrasse 36, D-72074 Tübingen, Germany.

1   Introduction

Allowing for long-run consumption risk in the pricing kernel, as advocated by Bansal and Yaron (2004), holds the promise of resolving prominent asset pricing puzzles and thus restoring the nexus between the real economy and financial markets. Numerical calibrations show that by taking long-run risk (LRR) into account, it becomes possible to explain the considerable U.S. postwar equity premium by means of a consumption-based asset pricing model that assumes plausible values for the representative agent's time preference, risk aversion, and propensity for intertemporal substitution.[1]

The LRR approach is theoretically appealing, but its econometric analysis is challenging. The model contains latent variables, such as a stochastic variance process and the model's keystone, a small predictable growth component. Using the efficient method of moments developed by Gallant and Tauchen (1996), the first econometric analysis of the LRR model was performed by Bansal, Gallant, and Tauchen (2007). However, even using a theoretically optimal estimation strategy, these authors had to calibrate several structural model parameters, which indicates that the identification of the structural parameters is not a matter of course. Some subsequent empirical studies report estimates of all LRR model parameters though, sometimes with remarkable precision (Constantinides and Ghosh, 2011; Hasseltoft, 2012; Bansal, Kiku, and Yaron, 2012b). Calvet and Czellar (2015) estimate a simplified version of the LRR model using an exactly identifying auxiliary model with an indirect inference estimation approach. They also report estimates of all model parameters, but their simplification, which greatly facilitates the model simulation, is not without implications (see Section 2.2). In a Monte Carlo experiment, Gram-

---

[1]See also Drechsler and Yaron (2011), who focus on the ability of the LRR model to explain size, value, and variance premia, and Bansal, Kiku, and Yaron (2012a), who compare the LRR approach with the habit model proposed by Campbell and Cochrane (1999).

mig and Küchlin (2015) find that some estimation strategies applied in previous studies have trouble recovering the true model parameters even with a very large sample.[2] Their evidence suggests that estimation problems and the need to fix some parameter values all originate from attempts to estimate the structural parameters in a single step, in which the LRR model's time series dynamics and equilibrium asset pricing implications are entangled.

This paper instead proposes a two-step indirect inference strategy to avoid the drawbacks of previous approaches. By recognizing the inherently recursive LRR model structure, the two steps separate the estimation of the macroeconomic dynamics from that of the investor preference parameters. Instead of using a single auxiliary model, which would confront the difficult task of capturing all important model features, the two estimation steps employ specific auxiliary parameters to account for the time series properties and asset pricing implications of the model, respectively. Our two-step estimation approach thus effectively implements Gourieroux, Monfort, and Renault's (1993) final suggestion to perform indirect inference estimation of different parts of a model by different criteria. Similar to the recommendations of Dridi, Guay, and Renault (2007), we advocate the use of tractable auxiliary models that reflect the LRR model's key features by well-chosen moment restrictions. A similar recursive structure is common to other prominent consumption-based asset pricing models, and the two-step indirect inference strategy offers an alternative for their often difficult econometric analysis.

The auxiliary parameters in the first estimation step are derived from the heterogeneous autoregressive (HAR) model proposed by Corsi (2009), which allows for the use of past information over long horizons in a parsimonious way. The investor preference parameters are estimated in the second step, for which we exploit the

---

[2]We focus on classical estimation approaches here. For Bayesian approaches towards estimating the LRR model see Aldrich and Gallant (2011) and Schorfheide, Song, and Yaron (2014).

LRR model's basic asset pricing implications. When reliable first-step estimates of the macro parameters are available, it is appropriate to use a few well-selected moment conditions that define the second-step auxiliary parameters. We derive the asymptotic properties of the two-step estimator; as an alternative, we also outline a bootstrap method as a useful robustness test.

A Monte Carlo study explores the feasibility of the two-step indirect inference approach, as well as the estimation precision that can be achieved with a sample size equivalent to what is currently available for empirical analysis. We find that the quality of the macro parameter estimates is crucial for ensuring precise preference parameter estimates. The empirical application in turn yields results that support the notion of a small persistent growth component, which is the crucial ingredient of the LRR framework. The point estimates of the parameters that describe the investor's subjective time preference (close to, but smaller than 1) and relative risk aversion (about 12) are economically reasonable. The estimate of the intertemporal elasticity of substitution (IES) is less than 1, although the data are also consistent with an IES>1. An IES greater than unity is the key condition for the ability of the LRR model to explain the prominent asset pricing puzzles. The confidence intervals indicate that estimation precision is inevitably limited by the relatively short low-frequency macroeconomic data series. The empirical evidence in favor of the LRR model is therefore less conclusive than suggested by some previous studies.

The remainder of the paper is organized as follows: Section 2 describes the LRR model structure. Section 3 presents the two-step indirect inference strategy. After we provide the results of the Monte Carlo study in Section 4, we describe the data in Section 5. Section 6 contains the empirical results. Section 7 concludes.

2 Theoretical framework

The outline of the LRR model structure that we present in this section focuses on the macroeconomic dynamics and asset pricing implications; it highlights numerical issues that become important when LRR model-implied data are simulated in the course of an indirect inference estimation.[3]

2.1 Time series dynamics, preferences, and asset pricing implications

The macroeconomy in the LRR model consists of two observable growth processes, log consumption growth g_t and log dividend growth $g_{d,t}$, which in turn are driven by two latent processes. Fluctuating expected growth rates are induced by a small predictable component x_t, and the stochastic variance process σ_t^2 accounts for fluctuating economic uncertainty:

$$g_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \tag{1}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1}, \tag{2}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1}, \tag{3}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}. \tag{4}$$

The i.i.d. innovations η, e, u, and w are assumed to be standard normally distributed and contemporaneously uncorrelated. For notational convenience, we collect the parameters in Equations (1)-(4) in the vector $\boldsymbol{\xi}^{\mathrm{M}} = (\mu_c, \mu_d, \rho, \varphi_e, \sigma, \phi, \varphi_d, \nu_1, \sigma_w)'$. As a special case, Bansal and Yaron (2004) (henceforth, BY) consider a model variant without fluctuating economic uncertainty, implying $\sigma_w{=}0$ and $\nu_1{=}0$, such that $\sigma_{t+1}^2 = \sigma^2$.

---

[3]Detailed derivations of the key model equations, which appear somewhat dispersed in prior literature, are provided in the Web Appendix.

The representative LRR investor has recursive Epstein-Zin-Weil preferences, as expressed by the utility function

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathbb{E}_t \left(U_{t+1}^{(1-\gamma)} \right) \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{5}$$

where C_t is aggregate consumption, and $\theta = \frac{(1-\gamma)}{\left(1-\frac{1}{\psi}\right)}$ (cf. Epstein and Zin, 1989). The subjective discount factor δ, the coefficient of relative risk aversion γ (RRA), and the intertemporal elasticity of substitution ψ are collected in the vector of preference parameters $\boldsymbol{\xi}^{\mathrm{P}} = (\delta, \gamma, \psi)'$. Utility maximization is performed under the budget constraint $W_{t+1} = (W_t - C_t)R_{a,t+1}$, where W denotes aggregate wealth. The gross return of the latent aggregate wealth portfolio, R_a, constitutes a claim to aggregate consumption. Epstein and Zin (1989) show that the first-order conditions of the maximization problem imply the following pricing equation for a gross return R_i,

$$\mathbb{E}_t \left[\delta^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1} \right] = 1, \tag{6}$$

where G denotes gross consumption growth.

Adapting the linear approximations suggested by Campbell and Shiller (1988), BY express the log return of the aggregate wealth portfolio r_a and the log return of the market portfolio r_m, which constitutes a claim to the dividend stream, as

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{7}$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}, \tag{8}$$

where z denotes the log price-consumption ratio, and z_m is the log price-dividend ratio. Moreover,

$$\kappa_1 = \frac{\exp(\bar{z})}{1 + \exp(\bar{z})}, \qquad\qquad \kappa_{1,m} = \frac{\exp(\bar{z}_m)}{1 + \exp(\bar{z}_m)}, \qquad (9)$$

$$\kappa_0 = \ln(1 + \exp(\bar{z})) - \kappa_1 \bar{z}, \text{ and } \quad \kappa_{0,m} = \ln(1 + \exp(\bar{z}_m)) - \kappa_{1,m} \bar{z}_m, \qquad (10)$$

where $\bar{z}$ and $\bar{z}_m$ are the time series means of z and z_m. The derivations of Equations (7)–(10) can be found in Section 1.2 of the Web Appendix.

2.2 Model simulation and solution

For the indirect inference estimation of the LRR model, we need to simulate model-implied data, which requires a model solution given $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$. To provide such a solution, we follow BY and write the latent log price-consumption ratio z and the observable log price-dividend ratio z_m as linear functions of the latent state variables:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \qquad (11)$$

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2. \qquad (12)$$

The A-parameters in Equations (11) and (12) can be obtained by pricing the gross returns of the wealth and market portfolios using Equation (6). The resulting expressions for the A-parameters depend on $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$ and on the κ-parameters in Equations (7) and (8), which in turn depend on $\bar{z}$ and $\bar{z}_m$.[4] Accordingly, the parameters in Equations (7), (8), (11), and (12) are endogenously determined by the solution of the model. In Appendix A.2 we explain how this solution can be

---

[4]The detailed expressions are provided in Equations (A-1)–(A-6) in Appendix A.1, which also contains the LRR model-implied equation for the log risk-free rate, r_f, and the equity premium, $\mathbb{E}_t [r_{m,t+1} - r_{f,t+1}]$.

obtained and how it is used for model simulation. Whether the model is solvable or not, and thus whether LRR model-implied data can be simulated in the first place, depends on the values of $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$. As pointed out by Grammig and Küchlin (2015), the LRR model is solvable for the parameter values calibrated by BY, whereas changes in the parameters within a plausible range can yield an insolvable model. The intricate nature of the admissible parameter space poses a challenge for the econometric analysis of the LRR model.[5]

3 A two-step indirect inference estimation strategy

3.1 Motivation and notation

This section details a two-step indirect inference strategy that separates the estimation of the macro parameters $\boldsymbol{\xi}^{\mathrm{M}}$ from that of the preference parameters $\boldsymbol{\xi}^{\mathrm{P}}$. We use a notation that draws on the seminal work by Gourieroux et al. (1993) and Smith (1993).

The LRR model, as outlined in the previous section, implies a vector stochastic process for consumption and dividend growth (macro variables) that depends only on $\boldsymbol{\xi}^{\mathrm{M}}$, as well as a vector stochastic process for the return of the market portfolio, the risk-free rate, and the price-dividend ratio (financial variables) that depends on both $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$. We denote the empirical time series of the macro variables by

---

[5]For their indirect inference estimation approach, Calvet and Czellar (2015) set the means of z_t and $z_{m,t}$, which should be endogenously determined, to fixed values $\bar{z}^*$ and $\bar{z}_m^*$. This choice circumvents the need to solve for the endogenous model parameters during the estimation process. However, the simplification comes at the cost of an inconsistency: When simulating the LRR model using $\bar{z}^*$ and $\bar{z}_m^*$, the means of the simulated z_t and $z_{m,t}$ series will be different from the fixed values. For example, using the LRR model parameter values calibrated by BY, and $\bar{z}^* = 6.96$ and $\bar{z}_m^* = 5.95$, as chosen by Calvet and Czellar (2015), to simulate LRR model-implied data series with T=100k, we obtain a sample mean of the log price-consumption ratio equal to 5.87 and a sample mean of the log price-dividend ratio equal to 5.19. These differences are large in economic terms.

$[\mathbf{y}^{\mathrm{M}}]^1_T = (\mathbf{y}^{\mathrm{M}}_1, \ldots, \mathbf{y}^{\mathrm{M}}_T)$, where $\mathbf{y}^{\mathrm{M}}_t = (g_t, g_{d,t})'$, and the empirical time series of the financial data by $[\mathbf{y}^{\mathrm{P}}]^1_T = (\mathbf{y}^{\mathrm{P}}_1, \ldots, \mathbf{y}^{\mathrm{P}}_T)$, where $\mathbf{y}^{\mathrm{P}}_t = (r_{m,t}, r_{f,t}, z_{m,t})'$.

The LRR model implicitly assumes that the macro and financial variables are observed at the decision frequency of the investor, which is typically higher than the empirical observation frequency of the data. In this case, it is necessary to perform a time aggregation of the model-implied processes.[6] We denote the LRR model-implied macro and financial series that are time-aggregated to the observation frequency by

$$[\tilde{\mathbf{y}}^{\mathrm{M}}(\boldsymbol{\xi}^{\mathrm{M}}, \mathbf{z}_0)]^1_T = [\tilde{\mathbf{y}}^{\mathrm{M}}_1(\boldsymbol{\xi}^{\mathrm{M}}, \mathbf{z}_0), \ldots, \tilde{\mathbf{y}}^{\mathrm{M}}_T(\boldsymbol{\xi}^{\mathrm{M}}, \mathbf{z}_0)] \tag{13}$$

and

$$[\tilde{\mathbf{y}}^{\mathrm{P}}(\boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)]^1_T = [\tilde{\mathbf{y}}^{\mathrm{P}}_1(\boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0), \ldots, \tilde{\mathbf{y}}^{\mathrm{P}}_T(\boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)], \tag{14}$$

where $\mathbf{z}_0 = (x_0, \sigma_0^2)'$ contains the initial values of the two state variables.

Assumption 1. *(i) There exists a unique set of parameters $\boldsymbol{\xi}^{\mathrm{M}}_0 \in \boldsymbol{\Xi}^{\mathrm{M}}$, such that $[\mathbf{y}^{\mathrm{M}}]^1_T$ and $[\tilde{\mathbf{y}}^{\mathrm{M}}(\boldsymbol{\xi}^{\mathrm{M}}_0, \mathbf{z}_0)]^1_T$ are drawn from the same distribution, and also a unique $\boldsymbol{\xi}^{\mathrm{P}}_0 \in \boldsymbol{\Xi}^{\mathrm{P}}$, such that $[\mathbf{y}^{\mathrm{P}}]^1_T$ and $[\tilde{\mathbf{y}}^{\mathrm{P}}(\boldsymbol{\xi}^{\mathrm{M}}_0, \boldsymbol{\xi}^{\mathrm{P}}_0, \mathbf{z}_0)]^1_T$ are drawn from the same distribution, and (ii) the vector processes $\{\mathbf{y}^{\mathrm{M}}_t\}$ and $\{\mathbf{y}^{\mathrm{P}}_t\}$ are stationary and ergodic for any $\boldsymbol{\xi}^{\mathrm{M}} \in \boldsymbol{\Xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}} \in \boldsymbol{\Xi}^{\mathrm{P}}$, respectively.*

The recursive LRR model structure suggests estimating the macro parameters $\boldsymbol{\xi}^{\mathrm{M}}$ and the preference parameters $\boldsymbol{\xi}^{\mathrm{P}}$ separately in two consecutive steps.[7] Consider, in particular, the macro dynamics in Equations (1)-(4), which only depend on $\boldsymbol{\xi}^{\mathrm{M}}$, and in which the presence of two latent processes poses a challenge for choosing an appropriate auxiliary model. The estimation of the auxiliary parameters must be

---

[6]The appropriate formulas are provided by Calvet and Czellar (2015) and given in Appendix A.3.

[7]A similar philosophy is pursued by Cecchetti, Lam, and Nelson (1993), who estimate by GMM the parameters of the endowment process in a macro asset pricing model in a first step, while computing confidence bounds for the investor's preference parameters in a second stage.

numerically tractable, but it must also capture the intricate time series properties induced by these latent processes. The estimation of the preference parameters $\boldsymbol{\xi}^{\mathrm{P}}$ imposes different requirements; the LRR model-implied properties of the market portfolio return and the risk-free rate need to be reflected by the auxiliary parameters. Entangling the information about these diverse aspects (i.e. time series dynamics, asset pricing implications, and investor preferences) does not seem prudent. As mentioned previously, Monte Carlo evidence and the discussion by Bansal et al. (2007) suggest that the joint estimation of all LRR model parameters may yield questionable results.

An indirect inference strategy that separates the estimation of $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$ can use specialized auxiliary models in each step, each of which is required only to capture the properties of the macro or financial data series, not both. The separate indirect inference estimation of the macro parameters also benefits from a simpler data simulation, because the solution for the endogenous LRR parameters is not required to generate model-implied macro data series.

3.2   First estimation step

3.2.1   First-step criterion and indirect estimation

The first step of the estimation strategy focuses solely on the macro parameters $\boldsymbol{\xi}^{\mathrm{M}}$. It is essentially a classical indirect inference approach using a GMM-type criterion function. We denote by $\boldsymbol{\theta}^{\mathrm{M}} \in \boldsymbol{\Theta}^{\mathrm{M}} \subset \mathcal{R}^{k^{\mathrm{M}}}$ the vector of first-step auxiliary parameters, where k^{M} must be at least as large as the number of structural macro parameters h^{M}. We define the first-step auxiliary parameters by a set of $g^{\mathrm{M}} \geq k^{\mathrm{M}}$ moment conditions on a random function $\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}})$, with $[\mathbf{y}^{\mathrm{M}}]_t^{t-l} = (\mathbf{y}_{t-l}^{\mathrm{M}}, \ldots, \mathbf{y}_t^{\mathrm{M}})$,

$$\mathbb{E}(\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}_0^{\mathrm{M}})) = 0, \tag{15}$$

where the expectation is taken with respect to G_0, the true distribution of the white noise innovations in Equations (1)-(4), and we make the identifying assumption:

Assumption 2. $\mathbb{E}(\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}})) \neq 0$ *for all* $\boldsymbol{\theta}^{\mathrm{M}} \neq \boldsymbol{\theta}_0^{\mathrm{M}} \in \boldsymbol{\Theta}^{\mathrm{M}}$.

The moment conditions in Equation (15) intentionally involve only the macro variables, not the financial variables of the LRR model. They should capture the key properties of the model-implied consumption and dividend growth processes. Using Equation (15) to define the auxiliary parameters offers flexibility in that a variety of motivations for moment conditions can be exploited. Moreover, it naturally suggests to obtain the estimate $\hat{\boldsymbol{\theta}}_T^{\mathrm{M}}$ of the first-step auxiliary parameters by maximizing a GMM-type criterion function,

$$\max_{\boldsymbol{\theta}^{\mathrm{M}} \in \boldsymbol{\Theta}^{\mathrm{M}}} \ Q_T^{\mathrm{M}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}}\right), \tag{16}$$

where

$$Q_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}}) = -\frac{1}{2}\mathbf{g}_T^{\mathrm{M}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}}\right)' \ \hat{\boldsymbol{\Omega}}_T^{\mathrm{M}} \ \mathbf{g}_T^{\mathrm{M}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}}\right), \tag{17}$$

with $\hat{\boldsymbol{\Omega}}_T^{\mathrm{M}}$ a positive semidefinite matrix that converges almost surely to a deterministic positive semidefinite matrix $\boldsymbol{\Omega}^{\mathrm{M}}$, and

$$\mathbf{g}_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}}) = \frac{1}{T}\sum_{t=1}^T \mathbf{u}_t^{\mathrm{M}}\left([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}}\right). \tag{18}$$

Following Hansen (1982) we assume:

Assumption 3. *(i)* $\boldsymbol{\Theta}^{\mathrm{M}}$ *is a compact subset of* $\mathcal{R}^{k^{\mathrm{M}}}$, *(ii)* $\mathbf{u}_t^{\mathrm{M}}(\cdot, \boldsymbol{\theta}^{\mathrm{M}})$ *is Borel measurable for each* $\boldsymbol{\theta}^{\mathrm{M}}$ *in* $\boldsymbol{\Theta}^{\mathrm{M}}$, *(iii)* $\mathbb{E}\left(\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}})\right)$ *exists and is finite for all* $\boldsymbol{\theta}^{\mathrm{M}}$ *in* $\boldsymbol{\Theta}^{\mathrm{M}}$, *and (iv)* $\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}})$ *is first-moment continuous at all* $\boldsymbol{\theta}^{\mathrm{M}} \in \boldsymbol{\Theta}^{\mathrm{M}}$.

Assumption 3, in conjunction with the stationarity and ergodicity Assumption 1, states sufficient conditions such that the criterion in Equation (17) converges almost surely uniformly to a non-stochastic limit criterion function that reads

$$Q_\infty^M (G_0, \boldsymbol{\xi}_0^M, \boldsymbol{\theta}^M) = -\frac{1}{2} \mathbb{E} \left[\mathbf{u}_t^M(\boldsymbol{\theta}^M) \right]' \ \boldsymbol{\Omega}^M \ \mathbb{E} \left[\mathbf{u}_t^M(\boldsymbol{\theta}^M) \right], \tag{19}$$

where $\mathbf{u}_t^M(\boldsymbol{\theta}^M)$ is a short-hand notation for $\mathbf{u}_t^M([\mathbf{y}^M]_t^{t-l}, \boldsymbol{\theta}^M)$. Moreover, Assumptions 1 and 2 imply that the limit criterion has a unique maximum at $\boldsymbol{\theta}_0^M$,

$$\boldsymbol{\theta}_0^M = \arg \max_{\boldsymbol{\theta}^M \in \ \boldsymbol{\Theta}^M} \ Q_\infty^M (G_0, \boldsymbol{\xi}_0^M, \boldsymbol{\theta}^M), \tag{20}$$

such that under Assumptions 1-3,

$$\hat{\boldsymbol{\theta}}_T^M = \arg \max_{\boldsymbol{\theta}^M \in \ \boldsymbol{\Theta}^M} \ Q_T^M([\mathbf{y}^M]_T^1, \boldsymbol{\theta}^M) \tag{21}$$

is a consistent estimator of $\boldsymbol{\theta}_0^M$. We refer to Singleton (2006) for a concise proof.

When T tends to infinity, we obtain the first-step binding function,

$$\mathbf{b}^M(G, \boldsymbol{\xi}^M) = \arg \max_{\boldsymbol{\theta}^M \in \ \boldsymbol{\Theta}^M} Q_\infty^M(G, \boldsymbol{\xi}^M, \boldsymbol{\theta}^M), \tag{22}$$

for which we demand, similar to Gourieroux et al. (1993):

Assumption 4. *(i)* $\mathbf{b}^M(G_0, .)$ *is one to one and (ii)* $\dfrac{\partial \mathbf{b}^M}{\partial \boldsymbol{\xi}^{M'}}(G_0, \boldsymbol{\xi}_0^M)$ *is of full column rank.*

Using simulated samples of macro data of length TH, $[\tilde{\mathbf{y}}^M(\boldsymbol{\xi}^M, \mathbf{z}_0)]_{TH}^1$, where H is an integer value, we can obtain an indirect estimator of $\boldsymbol{\xi}_0^M$ by

$$\hat{\boldsymbol{\xi}}_T^M = \arg \min_{\boldsymbol{\xi}^M \in \ \boldsymbol{\Xi}^M} \left[\hat{\boldsymbol{\theta}}_T^M - \tilde{\boldsymbol{\theta}}_{HT}^M(\boldsymbol{\xi}^M, \mathbf{z}_0) \right]' \widehat{\mathbf{W}}_T^M \left[\hat{\boldsymbol{\theta}}_T^M - \tilde{\boldsymbol{\theta}}_{HT}^M(\boldsymbol{\xi}^M, \mathbf{z}_0) \right], \tag{23}$$

11

where $\widehat{\mathbf{W}}_T^{\mathrm{M}}$ is a positive definite matrix that converges almost surely to a deterministic positive definite matrix $\mathbf{W}^{\mathrm{M}}$ and

$$\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}}(\boldsymbol{\xi}^{\mathrm{M}}, \mathbf{z}_0) = \arg \max_{\boldsymbol{\theta}^{\mathrm{M}} \in \boldsymbol{\Theta}^{\mathrm{M}}} Q_T^{\mathrm{M}} \left([\tilde{\mathbf{y}}^{\mathrm{M}}(\boldsymbol{\xi}^{\mathrm{M}}, \mathbf{z}_0)]_{TH}^1, \boldsymbol{\theta}^{\mathrm{M}} \right) \tag{24}$$

is a consistent functional estimator of $\mathbf{b}^{\mathrm{M}}(G_0, .)$. We refer to Gourieroux et al. (1993) to prove the following result:

Proposition 1. *Under Assumptions 1-4, the estimator $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ in Equation* (23) *is a consistent estimator of $\boldsymbol{\xi}_0^{\mathrm{M}}$.*

See Appendix A.7 for a proof of Proposition 1.

3.2.2 Choosing the first-step auxiliary parameters

The challenge in choosing the first-step auxiliary parameters and the corresponding moment conditions is to account for the predictable growth component x_t, which induces small but very persistent serial correlations in the growth series. These deviations from i.i.d. growth allow the asset pricing implications of the LRR model to unfold. A parsimonious way to capture the autocorrelation structure of a persistent process is the HAR model proposed by Corsi (2009). It has been used in the realized volatility literature to account for the long memory properties of squared returns by including different sampling frequencies in an autoregressive model. To obtain the first-step auxiliary parameters, we adopt the following HAR specification:[8]

$$\begin{bmatrix} g_t \\ g_{d,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \sum_{i=1}^{\tau} \boldsymbol{\Phi}_i L^i \begin{bmatrix} g_t \\ g_{d,t} \end{bmatrix} + \boldsymbol{\Phi}_{\tau+1} \begin{bmatrix} g_{t-1}^{f(h_1)} \\ g_{d,t-1}^{f(h_1)} \end{bmatrix} + \boldsymbol{\Phi}_{\tau+2} \begin{bmatrix} g_{t-1}^{f(h_2)} \\ g_{d,t-1}^{f(h_2)} \end{bmatrix} + \begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \end{bmatrix}. \tag{25}$$

---

[8]We are grateful to George Tauchen for suggesting the use of the HAR model to provide auxiliary parameters.

The parameter matrices $\boldsymbol{\Phi}_i$ and the constants c_1 and c_2 are defined by the orthogonality of the residuals $\zeta_{1,t}$ and $\zeta_{2,t}$ and the variables on the right-hand side of Equation (25). The data are on the base frequency, which could be quarterly (Hasseltoft, 2012) or annual (Constantinides and Ghosh, 2011). The superscripts $f(h_1)$ and $f(h_2)$ denote the lower frequencies that result from a time aggregation of the base frequency data over h_i periods. With a quarterly base frequency, we would use $h_1 = 4$ and $h_2 = 12$ to obtain annual and triannual aggregates. Compared with a standard vector-autoregressive process, the HAR specification can account for the long-run impact of shocks to consumption and dividend growth in a parsimonious way, because it replaces many required lagged growth rates by a few aggregates. The auxiliary parameters implied by the HAR model are collected in the vector

$$\boldsymbol{\theta}^{\text{HAR}} = \left(c_1, c_2, \text{vec}(\boldsymbol{\Phi}_1)', \ldots, \text{vec}(\boldsymbol{\Phi}_{\tau+2})', \sigma_{\zeta_1}, \sigma_{\zeta_2}, \sigma_{\zeta_1\zeta_2} \right)',$$

where σ_{ζ_1}, σ_{ζ_2}, and $\sigma_{\zeta_1\zeta_2}$ denote the standard deviations and the covariance of the residuals in Equation (25). We also augment the auxiliary parameter vector to include the means and standard deviations of the two growth processes as well as their time aggregates,

$$\boldsymbol{g}_t = \left(g_t, g_{d,t}, g_t^{f(h_1)}, g_{d,t}^{f(h_1)}, g_t^{f(h_2)}, g_{d,t}^{f(h_2)} \right)',$$

which we collect in the vectors $\boldsymbol{\mu_g} = \left(\mu_c, \mu_d, \mu_c^{f(h_1)}, \mu_d^{f(h_1)}, \mu_c^{f(h_2)}, \mu_d^{f(h_2)} \right)'$ and $\boldsymbol{\sigma_g} = \left(\sigma_c, \sigma_d, \sigma_c^{f(h_1)}, \sigma_d^{f(h_1)}, \sigma_c^{f(h_2)}, \sigma_d^{f(h_2)} \right)'$. The vector of first-step auxiliary parameters is then given by $\boldsymbol{\theta}^{\text{M}} = \left(\boldsymbol{\theta}^{\text{HAR}\prime}, \boldsymbol{\mu'_g}, \boldsymbol{\sigma'_g} \right)'$. The complete set of moment conditions that define $\boldsymbol{\theta}^{\text{M}}$ is provided in Equation (A-17) in Appendix A.4.1.

We also consider extending $\boldsymbol{\theta}^{\text{M}}$ with additional auxiliary parameters derived from the moment restrictions implied by an AR-ARCH specification for consumption

13

growth (see Appendix A.4.1). In both the basic and the extended setup, a numerical optimization is not required to obtain $\hat{\boldsymbol{\theta}}_T^{\mathrm{M}}$. Moreover, in our specification of the auxiliary parameters $g^{\mathrm{M}} = k^{\mathrm{M}}$, such that the values that maximize the criterion in Equation (16) are independent of the choice of $\hat{\boldsymbol{\Omega}}_T^{\mathrm{M}}$. The parameter values $\hat{\boldsymbol{\theta}}_T^{\mathrm{M}}$ that maximize the first-step criterion can be obtained by OLS and by computing sample moments.

While the rank condition in Assumption 4 can be examined using a large simulated sample size, the injectivity assumption is difficult to verify. The connections between auxiliary and structural parameters are obvious, though. The autoregressive parameter matrices $\boldsymbol{\Phi}$ should provide information about the persistence parameter ρ and the leverage ratio on expected consumption growth ϕ, while c_1, c_2, and $\boldsymbol{\mu_g}$ are linked to the unconditional expected values of log consumption and dividend growth, μ_c and μ_d. Moreover, σ_{ζ_1}, σ_{ζ_2}, $\sigma_{\zeta_1\zeta_2}$, and $\boldsymbol{\sigma_g}$ should contribute to the identification of the unconditional variance σ and the variance-scaling parameters φ_e and φ_d. The additional auxiliary parameters defined according to the AR-ARCH moments should be useful to identify the stochastic volatility (SV) parameters ν_1 and σ_w.

3.3   Second estimation step

3.3.1   Second step criterion and indirect estimation

The second estimation step focuses on the preference parameters $\boldsymbol{\xi}^{\mathrm{P}}$, taking the first-step estimates $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ as given. It uses auxiliary parameters to capture the key asset pricing implications of the LRR model. The second-step auxiliary parameters are collected in the vector $\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}} \subset \mathcal{R}^{k^{\mathrm{P}}}$, where k^{P} is at least as large as the number of preference parameters h^{P}. Similar to the first step, we define the auxiliary

parameters by a set of $g^{\mathrm{P}} \geq k^{\mathrm{P}}$ moment conditions on a random function of the macro and the financial data,

$$\mathbb{E}\left(\mathbf{u}_t^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}_0^{\mathrm{P}})\right) = 0, \tag{26}$$

and make the identifying assumption:

Assumption 5. $\mathbb{E}\left(\mathbf{u}_t^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}})\right) \neq 0$ *for all* $\boldsymbol{\theta}^{\mathrm{P}} \neq \boldsymbol{\theta}_0^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}$.

The moment conditions in Equation (26) can be motivated by various considerations, such as a simplified, possibly linearized asset pricing relation. They suggest that $\hat{\boldsymbol{\theta}}_T^{\mathrm{P}}$ can be obtained as a solution of

$$\max_{\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}} Q_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right), \tag{27}$$

where

$$Q_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right) = -\frac{1}{2}\mathbf{g}_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right)' \; \hat{\boldsymbol{\Omega}}_T^{\mathrm{P}} \; \mathbf{g}_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right), \tag{28}$$

with $\hat{\boldsymbol{\Omega}}_T^{\mathrm{P}}$ a positive semidefinite matrix that converges almost surely to a deterministic positive semidefinite matrix $\boldsymbol{\Omega}^{\mathrm{P}}$ and

$$\mathbf{g}_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right) = \frac{1}{T}\sum_{t=1}^{T} \mathbf{u}_t^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}}\right). \tag{29}$$

To assess the properties of $\hat{\boldsymbol{\theta}}_T^{\mathrm{P}}$, we proceed in a similar way as in the first step:

Assumption 6. *(i)* $\boldsymbol{\Theta}^{\mathrm{P}}$ *is a compact subset of* $\mathcal{R}^{k^{\mathrm{P}}}$, *(ii)* $\mathbf{u}_t^{\mathrm{P}}(\cdot, \cdot, \boldsymbol{\theta}^{\mathrm{P}})$ *is Borel measurable for each* $\boldsymbol{\theta}^{\mathrm{P}}$ *in* $\boldsymbol{\Theta}^{\mathrm{P}}$, *(iii)* $\mathbb{E}\left(\mathbf{u}_t^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}})\right)$ *exists and is finite for all* $\boldsymbol{\theta}^{\mathrm{P}}$ *in* $\boldsymbol{\Theta}^{\mathrm{P}}$, *and (iv)* $\mathbf{u}_t^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}})$ *is first-moment continuous at all* $\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}$.

Under Assumptions 1 and 6, the second-step criterion in Equation (27) converges almost surely uniformly to the non-stochastic limit function

$$Q_\infty^{\mathrm{P}}\left(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \boldsymbol{\theta}^{\mathrm{P}}\right) = -\frac{1}{2}\mathbb{E}\left(\mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}^{\mathrm{P}})\right)' \ \boldsymbol{\Omega}^{\mathrm{P}} \ \mathbb{E}\left(\mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}^{\mathrm{P}})\right), \tag{30}$$

where $\mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}^{\mathrm{P}})$ is a short-hand notation for $\mathbf{u}_t^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}}\right)$.

Under Assumption 5, the second-step limit function is uniquely maximized by $\boldsymbol{\theta}_0^{\mathrm{P}}$, such that under Assumptions 1, 5, and 6,

$$\hat{\boldsymbol{\theta}}_T^{\mathrm{P}} = \arg \max_{\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}} Q_T^{\mathrm{P}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right) \tag{31}$$

is a consistent estimator of $\boldsymbol{\theta}_0^{\mathrm{P}}$, referring to the same proof as in the first step. The second-step binding function

$$\mathbf{b}^{\mathrm{P}}(G, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) = \arg \max_{\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}} Q_\infty^{\mathrm{P}}(G, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \boldsymbol{\theta}^{\mathrm{P}}) \tag{32}$$

is assumed to have the following properties:

Assumption 7. *(i)* $\mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, .)$ *is one to one and (ii)* $\dfrac{\partial \mathbf{b}^{\mathrm{P}}}{\partial \boldsymbol{\xi}^{\mathrm{P}\prime}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}})$ *is of full column rank.*

The second-step indirect inference estimator of $\boldsymbol{\xi}_0^{\mathrm{P}}$ is given by

$$\hat{\boldsymbol{\xi}}_T^{\mathrm{P}} = \arg \min_{\boldsymbol{\xi}^{\mathrm{P}} \in \boldsymbol{\Xi}^{\mathrm{P}}} \left[\hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)\right]' \widehat{\mathbf{W}}_T^{\mathrm{P}} \left[\hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)\right], \tag{33}$$

where $\widehat{\mathbf{W}}_T^{\mathrm{P}}$ is a positive definite matrix that converges almost surely to a deterministic positive definite matrix $\mathbf{W}^{\mathrm{P}}$, and

$$\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0) = \arg \max_{\boldsymbol{\theta}^{\mathrm{P}} \in \boldsymbol{\Theta}^{\mathrm{P}}} Q_T^{\mathrm{P}}\left([\tilde{\mathbf{y}}^{\mathrm{M}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0)]_{TH}^1, [\tilde{\mathbf{y}}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)]_{TH}^1, \boldsymbol{\theta}^{\mathrm{P}}\right). \tag{34}$$

16

During the optimization, and while computing $\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}, \mathbf{z}_0)$, the first-step estimate $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ of the macro parameters remains unchanged. We can then prove:

Proposition 2. *Under Assumptions 1-7, the estimator $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$ in Equation (33) is a consistent estimator of $\boldsymbol{\xi}_0^{\mathrm{P}}$.*

See Appendix A.7 for a proof of Proposition 2.

3.3.2 Choosing the second-step auxiliary parameters

The need for tractable auxiliary parameter estimation is even more critical in the second step, for two reasons. First, as mentioned previously, the LRR model and its solution are already intricate. Second, the time aggregation from decision to observation frequency is a computer-intensive task, especially when the simulated sample size is chosen to be reasonably large. If the estimation of the auxiliary parameters on the simulated data were complicated and fragile, a comprehensive Monte Carlo study, bootstrap inference, and the robustness check to start the optimization on a grid of different starting values would become prohibitively time-consuming.

The second-step auxiliary parameters are therefore defined by a selected set of moment conditions that capture the basic asset pricing implications of the LRR model. Such a strategy is in line with the recommendations of Dridi et al. (2007), who delineate the connection between indirect inference estimation and calibration of DSGE models. They argue that if a model is misspecified, such that only some, but not all of its structural parameters have unknown true values that we want to estimate consistently and the rest are nuisance parameters, focusing on a well thought-out set of moment conditions is preferable to a sophisticated auxiliary model that tries to mimic the structural model as closely as possible. The promise of a more efficient estimation would be undone by misspecification.

The LRR model-implied equations for the risk-free rate and the market equity premium (see Equations (A-7) and (A-10) in Appendix A.1) guide our selection of the moment conditions that define the second-step auxiliary parameters. The mean of the log risk-free rate $\mathbb{E}(r_f) = \mu_{r_f}$ should convey information about the subjective time preference δ, the propensity for intertemporal substitution ψ, and also about precautionary savings due to risk aversion γ. The equity premium $\mu_{r_m^e} = \mathbb{E}(r_m - r_f)$, though a function of all three preference parameters, primarily should reflect the relative risk aversion. To disentangle risk aversion from intertemporal substitution, we exploit the contemporaneous relationship between the log price-dividend ratio and the log risk-free rate implied by the LRR model. Because it is predominantly determined by the IES, but largely unaffected by the RRA coefficient (see Appendix A.6), it facilitates the identification of ψ. We therefore include the intercept α and the slope coefficient β of a linear regression of $z_{m,t}$ on $r_{f,t}$ among the auxiliary parameters. Moreover, Equation (12) implies that $\mathbb{E}(z_m) = \mu_{z_m}$ depends on all preference parameters, but the standard deviation of z_m (σ_{z_m}) only depends on γ and ψ, so including μ_{z_m} and σ_{z_m} as auxiliary parameters provides separate information about risk aversion and time preference. We also include the standard deviations of the market excess return $(\sigma_{r_m^e})$ and the log risk-free rate (σ_{r_f}) in the set of second-step auxiliary parameters.

The moment conditions used to define $\boldsymbol{\theta}^{\mathrm{P}} = (\alpha, \beta, \mu_{r_m^e}, \mu_{r_f}, \mu_{z_m}, \sigma_{r_m^e}, \sigma_{r_f}, \sigma_{z_m})'$ are thus given by

$$\mathbb{E}(\mathbf{u}_t^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_t^{t-n}, [\mathbf{y}^{\mathrm{P}}]_t^{t-m}, \boldsymbol{\theta}^{\mathrm{P}})) = \mathbb{E} \begin{bmatrix} \zeta_{3,t} \\ \zeta_{3,t} \cdot r_{f,t} \\ r_{m,t}^e - \mu_{r_m^e} \\ r_{f,t} - \mu_{r_f} \\ z_{m,t} - \mu_{z_m} \\ [r_{m,t}^e]^2 - [\mu_{r_m^e}]^2 - [\sigma_{r_m^e}]^2 \\ [r_{f,t}]^2 - [\mu_{r_f}]^2 - [\sigma_{r_f}]^2 \\ [z_{m,t}]^2 - [\mu_{z_m}]^2 - [\sigma_{z_m}]^2 \end{bmatrix} = \mathbf{0}, \qquad (35)$$

where $\zeta_{3,t} = z_{m,t} - \alpha - \beta r_{f,t}$, and $r_{m,t}^e = r_{m,t} - r_{f,t}$. Assumption 5 asserts that Equation (35) must hold uniquely at $\boldsymbol{\theta}^{\mathrm{P}} = \boldsymbol{\theta}_0^{\mathrm{P}}$. The number of moment conditions (as in the first step) is equal to the number of auxiliary parameters, such that the values that maximize the criterion in Equation (27) are independent of the choice of $\hat{\boldsymbol{\Omega}}_T^{\mathrm{P}}$. Numerical optimization is not required, and the parameter values $\hat{\boldsymbol{\theta}}_T^{\mathrm{P}}$ that maximize the second-step criterion can be obtained by OLS and by computing sample moments.

3.4 Asymptotic distribution of the two-step indirect inference estimator

The two-step indirect inference approach outlined in Sections 3.2.1 and 3.3.1 implies that $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$ represent the solution of the following system of equations,

$$\begin{bmatrix} \dfrac{\partial \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}'}}{\partial \boldsymbol{\xi}^{\mathrm{M}}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0) & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}'}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \hat{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0) \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{W}}_T^{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{W}}_T^{\mathrm{P}} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0) \\ \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \hat{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0) \end{bmatrix} = \mathbf{0},$$
$$(36)$$

which is a starting point to derive the asymptotic distribution of the two-step indirect inference estimator. For that purpose, we make the following assumption:

Assumption 8. *A multivariate central limit theorem applies, such that, under Assumptions 2 and 5,*

$$\sqrt{T} \begin{bmatrix} \frac{1}{T} \sum_{t=1}^{T} \mathbf{u}_t^{\mathrm{M}}(\boldsymbol{\theta}_0^{\mathrm{M}}) \\ \frac{1}{T} \sum_{t=1}^{T} \mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}_0^{\mathrm{P}}) \end{bmatrix} \underset{d}{\to} \mathcal{N}(\mathbf{0}, \mathbf{S}), \tag{37}$$

with

$$\mathbf{S} = \boldsymbol{\Gamma}_0 + \sum_{j=1}^{+\infty} (\boldsymbol{\Gamma}_j + \boldsymbol{\Gamma}_j'), \tag{38}$$

$$\boldsymbol{\Gamma}_j = \mathbb{E} \begin{bmatrix} \mathbf{u}_t^{\mathrm{M}}(\boldsymbol{\theta}_0^{\mathrm{M}}) \mathbf{u}_{t-j}^{\mathrm{M}}(\boldsymbol{\theta}_0^{\mathrm{M}})' & \mathbf{u}_t^{\mathrm{M}}(\boldsymbol{\theta}_0^{\mathrm{M}}) \mathbf{u}_{t-j}^{\mathrm{P}}(\boldsymbol{\theta}_0^{\mathrm{P}})' \\ \mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}_0^{\mathrm{P}}) \mathbf{u}_{t-j}^{\mathrm{M}}(\boldsymbol{\theta}_0^{\mathrm{M}})' & \mathbf{u}_t^{\mathrm{P}}(\boldsymbol{\theta}_0^{\mathrm{P}}) \mathbf{u}_{t-j}^{\mathrm{P}}(\boldsymbol{\theta}_0^{\mathrm{P}})' \end{bmatrix}.$$

We can then prove the following proposition:

Proposition 3. *Under Assumptions 1-8, the two-step indirect inference estimator of $\boldsymbol{\xi}_0^{\mathrm{M}}$ and $\boldsymbol{\xi}_0^{\mathrm{P}}$ is asymptotically normal such that*

$$\sqrt{T} \begin{bmatrix} \hat{\boldsymbol{\xi}}_T^{\mathrm{M}} - \boldsymbol{\xi}_0^{\mathrm{M}} \\ \hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}} \end{bmatrix} \underset{d}{\to} \mathcal{N}(\mathbf{0}, \mathrm{Avar}(H, \mathbf{W}^{\mathrm{M}}, \mathbf{W}^{\mathrm{P}})), \tag{39}$$

with

$$\text{Avar}(H, \mathbf{W}^{\text{M}}, \mathbf{W}^{\text{P}}) =$$

$$\left(1 + \frac{1}{H}\right) \begin{bmatrix} \mathbf{A}(\mathbf{W}^{\text{M}})\mathbf{A}_0^{\text{M}} & \mathbf{0} \\ \mathbf{B}(\mathbf{W}^{\text{P}})\mathbf{C}(\mathbf{W}^{\text{M}})\mathbf{A}_0^{\text{P}} & \mathbf{B}(\mathbf{W}^{\text{P}})\mathbf{A}_0^{\text{P}} \end{bmatrix} \mathbf{S} \begin{bmatrix} \mathbf{A}_0^{\text{M}\prime}\mathbf{A}(\mathbf{W}^{\text{M}})' & \mathbf{A}_0^{\text{P}\prime}\mathbf{C}(\mathbf{W}^{\text{M}})'\mathbf{B}(\mathbf{W}^{\text{P}})' \\ \mathbf{0} & \mathbf{A}_0^{\text{P}\prime}\mathbf{B}(\mathbf{W}^{\text{P}})' \end{bmatrix}, \tag{40}$$

$$\mathbf{A}(\mathbf{W}^{\text{M}}) = \left(\frac{\partial \mathbf{b}^{\text{M}\prime}}{\partial \boldsymbol{\xi}^{\text{M}}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\mathbf{W}^{\text{M}}\frac{\partial \mathbf{b}^{\text{M}}}{\partial \boldsymbol{\xi}^{\text{M}\prime}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\right)^{-1} \frac{\partial \mathbf{b}^{\text{M}\prime}}{\partial \boldsymbol{\xi}^{\text{M}}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\mathbf{W}^{\text{M}}, \tag{41}$$

$$\mathbf{B}(\mathbf{W}^{\text{P}}) = \left(\frac{\partial \mathbf{b}^{\text{P}\prime}}{\partial \boldsymbol{\xi}^{\text{P}}}(G_0, \boldsymbol{\xi}_0^{\text{M}}, \boldsymbol{\xi}_0^{\text{P}})\mathbf{W}^{\text{P}}\frac{\partial \mathbf{b}^{\text{P}}}{\partial \boldsymbol{\xi}^{\text{P}\prime}}(G_0, \boldsymbol{\xi}_0^{\text{M}}, \boldsymbol{\xi}_0^{\text{P}})\right)^{-1} \frac{\partial \mathbf{b}^{\text{P}\prime}}{\partial \boldsymbol{\xi}^{\text{P}}}(G_0, \boldsymbol{\xi}_0^{\text{M}}, \boldsymbol{\xi}_0^{\text{P}})\mathbf{W}^{\text{P}}, \tag{42}$$

$$\mathbf{C}(\mathbf{W}^{\text{M}}) = \frac{\partial \mathbf{b}^{\text{P}}}{\partial \boldsymbol{\xi}^{\text{M}\prime}}(G_0, \boldsymbol{\xi}_0^{\text{M}}, \boldsymbol{\xi}_0^{\text{P}})\mathbf{A}(\mathbf{W}^{\text{M}}), \tag{43}$$

$$\mathbf{A}_0^{\text{M}} = \left(\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{M}}(\boldsymbol{\theta}_0^{\text{M}})'}{\partial \boldsymbol{\theta}^{\text{M}}}\right]\boldsymbol{\Omega}^{\text{M}}\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{M}}(\boldsymbol{\theta}_0^{\text{M}})}{\partial \boldsymbol{\theta}^{\text{M}\prime}}\right]\right)^{-1}\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{M}}(\boldsymbol{\theta}_0^{\text{M}})'}{\partial \boldsymbol{\theta}^{\text{M}}}\right]\boldsymbol{\Omega}^{\text{M}}, \tag{44}$$

$$\mathbf{A}_0^{\text{P}} = \left(\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{P}}(\boldsymbol{\theta}_0^{\text{P}})'}{\partial \boldsymbol{\theta}^{\text{P}}}\right]\boldsymbol{\Omega}^{\text{P}}\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{P}}(\boldsymbol{\theta}_0^{\text{P}})}{\partial \boldsymbol{\theta}^{\text{P}\prime}}\right]\right)^{-1}\mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{P}}(\boldsymbol{\theta}_0^{\text{P}})'}{\partial \boldsymbol{\theta}^{\text{P}}}\right]\boldsymbol{\Omega}^{\text{P}}. \tag{45}$$

If $g^{\text{M}} = k^{\text{M}}$, then $\mathbf{A}_0^{\text{M}} = \mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{M}}(\boldsymbol{\theta}_0^{\text{M}})}{\partial \boldsymbol{\theta}^{\text{M}\prime}}\right]^{-1}$ and if $g^{\text{P}} = k^{\text{P}}$, then $\mathbf{A}_0^{\text{P}} = \mathbb{E}\left[\frac{\partial \mathbf{u}_t^{\text{P}}(\boldsymbol{\theta}_0^{\text{P}})}{\partial \boldsymbol{\theta}^{\text{P}\prime}}\right]^{-1}$.

A proof of Proposition 3 is given in Appendix A.7.

Asymptotically optimal weighting matrices can be provided for each step. Using the partitioning

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}^{\text{M}} & \mathbf{S}^{\text{MP}\prime} \\ (g^{\text{M}} \times g^{\text{M}}) & (g^{\text{M}} \times g^{\text{P}}) \\ \mathbf{S}^{\text{MP}} & \mathbf{S}^{\text{P}} \\ (g^{\text{P}} \times g^{\text{M}}) & (g^{\text{P}} \times g^{\text{P}}) \end{bmatrix}, \tag{46}$$

we can prove:

Proposition 4. *Under Assumptions 1-8, the asymptotically optimal weighting matrix for the first indirect infernce estimation step is given by*

$$\mathbf{W}^{\text{M}*} = (\mathbf{A}_0^{\text{M}}\mathbf{S}^{\text{M}}\mathbf{A}_0^{\text{M}\prime})^{-1}, \tag{47}$$

and the asymptotically optimal weighting for the second-step indirect inference estimation, given the choice of the first-step weighting, is given by

$$\mathbf{W}^{\mathrm{P}*} =$$

$$\left(\mathbf{A}_0^{\mathrm{P}}\mathbf{S}^{\mathrm{P}}\mathbf{A}_0^{\mathrm{P}\prime} + \mathbf{C}(\mathbf{W}^{\mathrm{M}})(\mathbf{W}^{\mathrm{M}*})^{-1}\mathbf{C}(\mathbf{W}^{\mathrm{M}})' + \mathbf{C}(\mathbf{W}^{\mathrm{M}})\mathbf{A}_0^{\mathrm{M}}\mathbf{S}^{\mathrm{MP}\prime}\mathbf{A}_0^{\mathrm{P}\prime} + \mathbf{A}_0^{\mathrm{P}}\mathbf{S}^{\mathrm{MP}}\mathbf{A}_0^{\mathrm{M}\prime}\mathbf{C}(\mathbf{W}^{\mathrm{M}})'\right)^{-1}.$$

$$(48)$$

A proof of Proposition 4 is given in Appendix A.7.

As an alternative to using the asymptotic results, the LRR model structure suggests a bootstrap simulation to obtain parameter standard errors and confidence intervals. The procedure can be characterized as a parametric residual bootstrap and is described in Appendix A.8.

3.5 One-step estimation revisited

Consider attempting a one-step indirect inference estimation of $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$ based on stacked, but otherwise unchanged, auxiliary parameters $\boldsymbol{\theta}^{\mathrm{M}}$ and $\boldsymbol{\theta}^{\mathrm{P}}$, defined by the same moment conditions as before. Here, we denote the auxiliary parameter estimates obtained in a single step by $\overline{\boldsymbol{\theta}}_T^{\mathrm{M}}$ and $\overline{\boldsymbol{\theta}}_T^{\mathrm{P}}$ and the resulting structural parameter estimates obtained by minimizing a one-step indirect inference criterion by $\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\overline{\boldsymbol{\xi}}_T^{\mathrm{P}}$. Using a properly partitioned weighing matrix $\widehat{\mathbf{W}}$, the first-order conditions of this optimization read

$$\begin{bmatrix} \dfrac{\partial \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{M}}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0) & \dfrac{\partial \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{M}}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \overline{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0) \\[2mm] \mathbf{0} & \dfrac{\partial \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \overline{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0) \end{bmatrix} \begin{bmatrix} \underset{(k^{\mathrm{M}}\times k^{\mathrm{M}})}{\widehat{\mathbf{W}}_{11}} & \underset{(k^{\mathrm{M}}\times k^{\mathrm{P}})}{\widehat{\mathbf{W}}_{12}} \\[2mm] \underset{(k^{\mathrm{P}}\times k^{\mathrm{M}})}{\widehat{\mathbf{W}}_{21}} & \underset{(k^{\mathrm{P}}\times k^{\mathrm{P}})}{\widehat{\mathbf{W}}_{22}} \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{\theta}}_T^{\mathrm{M}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0) \\[2mm] \overline{\boldsymbol{\theta}}_T^{\mathrm{P}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \overline{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0) \end{bmatrix} = \mathbf{0}.$$

$$(49)$$

Comparing Equation (49) with Equation (36), we observe that in both cases, linear combinations of the differences of the auxiliary parameter estimates are set to zero. Yet, while the weights of the linear combinations in Equation (49) lead to an inevitable interference of the auxiliary parameter matches $\overline{\boldsymbol{\theta}}_T^{\mathrm{P}} - \tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\overline{\boldsymbol{\xi}}_T^{\mathrm{M}}, \overline{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0)$ with the estimation of the macro parameters $\boldsymbol{\xi}^{\mathrm{M}}$, even if we were to use a block-diagonal weighting matrix, such that $\widehat{\mathbf{W}}_{21} = \mathbf{0}$ and $\widehat{\mathbf{W}}_{12} = \mathbf{0}$, the weights of the linear combinations in Equation (36) prevent the second-step auxiliary model from interfering with the estimation of the macro parameters. The two-step indirect inference strategy thus accounts for the caveat that the entanglement of macro and financial moment matches in a one-step generalized or simulated method of moments estimation of the LRR model needs to be avoided, because it yields unreliable parameter estimates. Our experiences with one-step indirect inference estimation strategies lead to the same conclusion.

4 Monte Carlo study

4.1 Design

The Monte Carlo study explores the viability of the two-step indirect inference strategy and the estimation precision that can be expected when using empirically available sample sizes. For that purpose, we generate 400 independent LRR model-implied data series of g, g_d, r_m, r_f, and z_m, using as true parameter values those calibrated by BY (see Table 1), and we perform the estimation on the simulated data. We assume that data and decision frequency are identical, such that time aggregation is not required. BY's calibration corresponds to a monthly decision frequency. The lengths of the simulated data series are $T=275$, 1k, and 100k.

[Insert Table 1 here]

23

As mentioned previously, an analytical validation of the estimation strategy is not possible. The T=100k study provides a substitute check on whether the estimation strategy is viable, such that the true parameters can be recovered as T grows large. Then T=1k represents a large but not implausible sample size for an empirical study that could use monthly data, and T=275 corresponds to the number of observations that are currently available at the quarterly frequency.

In the simulated economy, the investor has a positive time preference, such that δ is close to but smaller than 1. The risk aversion parameter γ=10 lies at the upper bound of economic plausibility (cf. Mehra and Prescott (1985)). The intertemporal elasticity of substitution is larger than 1 (ψ=1.5), which is a necessary precondition for the LRR model to be able to resolve the equity premium and risk-free rate puzzles. Growth expectations are very persistent (ρ=0.979), which is also pivotal for the asset pricing implications of the LRR model. The predictable growth component x_t is indeed small, as a result of scaling consumption volatility by φ_e=0.044. Consumption growth expectations are leveraged into dividend growth expectations, according to ϕ=3. The expected values of consumption and dividend growth are identical, but dividend volatility is considerably larger than consumption volatility ($\varphi_d = 4.5$). Moreover, the stochastic variance process is highly persistent (ν_1=0.987), but not very volatile (σ_w=2.3 $\cdot$ 10^{-6}). Panel (a) of Figure 1 illustrates that the fluctuation of macroeconomic uncertainty is indeed moderate; Panel (b) shows that the model-implied theoretical autocorrelations of squared consumption and dividend growth, as induced by fluctuating economic uncertainty, are persistent but very small. The $\pm 2\sqrt{T}$ confidence bounds for a sample size of T=1k illustrate why the persistent but small autocorrelation of the stochastic variance may be impossible to detect in small samples.

[Insert Figure 1 here]

Indirect inference estimation uses $H=10$ for $T=100$k and $T=1$k, as suggested by Smith (1993). For the $T=275$ study we use $H=100$, because initial estimations revealed that the stability of the numerical optimization benefits from a larger simulated sample size. To ensure robust, yet rapid optimization, we use the Nelder-Mead (1965) algorithm. Conventional gradient-based optimization methods perform poorly. To safeguard against false convergence close to favorably chosen starting values, we start the optimizations with values that are distant from the true parameter values. This safety measure helps to avoid reporting overly optimistic results, but it makes the optimization more difficult.[9] The estimates of ρ, μ_c, and μ_d are restricted to values between 0 and 1 by means of a logit transformation and the estimates for σ, ϕ, φ_e, and φ_d are restricted to positive values by an exponential transformation of the unrestricted parameters.

4.2 Monte Carlo results: first estimation step

In initial experiments, we attempted to estimate all macro parameters $\boldsymbol{\xi}^{\mathrm{M}}$, as described in Section 3.2.1, using the simulated consumption and dividend growth data series with $T=100$k. We found that the macro parameters can be reliably recovered, except the SV parameters ν_1 and σ_w, for which we obtain vastly different estimates when using different initial parameter values. Alternative weighting matrices (identity, optimal, and others) and augmenting the auxiliary parameter vector as described in Section A.4.1 all yielded the same result.

---

[9]As a result, the numerical optimization cannot be accomplished for some replications, in particular for small T. In these cases, the Nelder-Mead algorithm either exceeds the maximum number of iterations, or terminates at implausible values (i.e. more than ten times larger than the true value in absolute terms). We consider these cases failed optimization attempts and exclude them from the tables and plots that summarize the simulation study results. In the second estimation step, we classify an optimization as failed if the LRR model is not solvable at the parameter values at which the optimization terminates. In an empirical study, these problematic data could receive special treatment, such as increasing the number of iterations or using alternative optimization algorithms. Such an expensive approach is not tenable in a Monte Carlo study.

It is not surprising that the simultaneous estimation of all parameters that characterize fluctuating expected growth and economic uncertainty proves extremely difficult, even when using a large sample. Figure 1 suggests that the two intertwined latent components should be hard to detect in the observable growth series. Essentially, the estimation of ν_1 and σ_w must rely on the information available from the small autocorrelations of the squared growth series (see Panel (b) in Figure 1).[10]

In a more comprehensive simulation experiment, we fix the values of the SV parameters ν_1 and σ_w to their true values, and then estimate σ along with the other macro parameters. The auxiliary parameter vector $\boldsymbol{\theta}^{\mathrm{M}}$ (base version) is constructed as described in Section 3.2.1, with the following customization: In the HAR model in Equation (25), we account for consumption and dividend growth on the annual and the triannual levels by choosing $h_1 = 12$ and $h_2 = 36$. The first few monthly lags should be particularly informative for estimating the persistence parameter ρ, so we set $\tau = 6$. Initial experiments indicate that exactly matching the means and standard deviations of consumption and dividend growth enhances the precision of the estimates of μ_c and μ_d and the variance-scalers φ_e and φ_d. The matches rely on a diagonal weighting matrix $\mathbf{W}^{\mathrm{M}}$ with values of 1 on the main diagonal, except for the entries that correspond to the first two elements of $\boldsymbol{\mu_g}$ and $\boldsymbol{\sigma_g}$, which receive a greater weight (10^4). Using an estimate of the optimal first-step weighting matrix does not provide an improvement in small samples.

As a benchmark, we also consider a GMM estimation of $\boldsymbol{\xi}^{\mathrm{M}}$ that relies on moment matches inspired by Constantinides and Ghosh (2011). For that purpose, we exploit the possibility to express the population moments of log consumption and dividend growth implied by the LRR model as functions of $\boldsymbol{\xi}^{\mathrm{M}}$. The just-identified GMM

---

[10]Extending the first-step auxiliary parameter vector in various directions does not alleviate the problem, as documented in Section 2 in the Web Appendix. The issue also persists with increasing values of σ_w.

strategy is based on the seven moments given in Appendix A.4.1. Equations (A-22)-(A-28) show that these moments do not depend on ν_1 and σ_w (only higher moments of consumption and dividend growth do). Therefore, they are useful only to estimate the other macro parameters. In principle, we can thus estimate the remaining seven macro parameters regardless of the true values of ν_1 and σ_w. This insight suggests a modified indirect inference estimation strategy,

If the volatility of economic uncertainty is small (as in BY's calibration), the simulated growth series resulting from either σ_t^2 or $\mathbb{E}(\sigma_t^2) = \sigma^2$ will be similar. The model that is simulated for indirect inference estimation is then effectively BY's special case without fluctuating economic uncertainty (see Section 2.1), with parameter vector $\boldsymbol{\xi}^{\text{M}*} = (\mu_c, \mu_d, \rho, \varphi_e, \sigma, \phi, \varphi_d, 0, 0)'$. In this modified estimation strategy, ν_1 and σ_w are not estimated, although the data-generating process does exhibit stochastic volatility.

[Insert Table 2 here]

[Insert Figure 2 here]

Table 2 contains the medians and root mean squared errors (RMSEs) of the resulting estimates. Panels A and B show the indirect inference results; Panel C reports the GMM estimation results. Figure 2 illustrates the results using kernel estimates.

The T=100k results indicate the viability of the two indirect inference estimation strategies. The biases and RMSEs shrink, there are no estimation failures, and the bell-shaped kernel estimates center closely around the true parameter values. Comparing Panel A and Panel B of Table 2, we observe that replacing σ_t^2 by σ^2 (assuming the true values for ν_1 and σ_w are known) when simulating the model-implied data does not impair the quality of the macro parameter estimates. This conclusion holds for all simulated sample sizes.

27

The estimation precision is different across parameters. Not surprisingly, the estimates of the variance-scaler φ_e and the leverage parameter ϕ are less precise. However, compared with the GMM results in Panel C, the indirect inference RMSEs are notably smaller. A considerably smaller RMSE also results for the persistence parameter ρ. Figure 3 shows that the distribution of the indirect inference estimate is much more closely centered around the true value than that of its GMM counterpart.

[Insert Figure 3 here]

Precise estimation becomes more difficult with the currently available sample size, as indicated by the increase in the RMSE and the wider distribution of the estimates around the true parameters. Efficiency varies across parameters, similar to the way it does in the large sample. As we might expect, the critical parameters φ_e and ϕ prove most difficult to estimate precisely. However, the optimization of the indirect inference objective function yields reliable results in that the algorithm converges to the same minimum, irrespective of the starting values.[11]

4.3 Monte Carlo results: second estimation step

The second-step estimation based on the simulated data is performed as described in Section 3.3.1, using the identity matrix for $\widehat{\mathbf{W}}_T^{\mathrm{P}}$. Alternative weighting schemes, including the asymptotically optimal second-step weighting matrix, do not offer an improvement in smaller samples.[12] To evaluate the performance of the second

---

[11]The GMM estimation strategy does not provide such robustness. Varying the starting values yields different results for smaller samples. We accordingly refrain from reporting the GMM results for smaller sample sizes.

[12]To find starting values for the optimization, we conduct an initial grid search that mimics the recommended procedure in an empirical application. Generating LRR model-implied data during the second estimation step entails solving for the endogenous parameters in Equations (7) – (12). As we point out in Section 2.2, the solution may not exist, which would cause the estimation to break down if the optimization algorithm were to probe inadmissible parameter combinations. A constrained indirect inference estimation, as proposed by Calzorari, Fiorentini, and Sentana (2004), cannot be employed, because the constraint would be imposed not on the auxiliary model

estimation step independently of the precision of the first-step input, we first perform the estimation of $\boldsymbol{\xi}^{\mathrm{P}}$ assuming that $\boldsymbol{\xi}^{\mathrm{M}}$ is known.

[Insert Table 3 here]

Panel B in Table 3 reports the medians, RMSEs, and 95% confidence bounds of the resulting preference parameter estimates. The $T=100$k study again serves as a check on the viability of the estimation approach, which is corroborated by the shrinking RMSEs, tight confidence bounds around the true parameters, and absence of estimation failures. The preference parameters can be efficiently estimated for the smaller sample sizes. Although the second-step auxiliary parameters are defined by only a few basic asset pricing relations, and despite the more intricate data simulation in the second step, the indirect inference strategy works well. Panel A of Table 3 shows that replacing the conditional variance σ_t^2 by its unconditional expectation σ^2 when simulating model-implied data (i.e., estimating a model without fluctuating macroeconomic uncertainty) does not impair the estimation of $\boldsymbol{\xi}^{\mathrm{P}}$. This conclusion is based on BY's calibrated model economy, but it should extend further as well. We use unconditional moments of the equity premium and the risk-free rate to estimate the investor's subjective time preference, risk aversion, and IES. It is plausible that knowledge of the dynamics of the conditional variance process does not substantially improve the precision of the preference parameter estimation.

To assess the quality of the second-step estimation when $\boldsymbol{\xi}^{\mathrm{M}}$ is unknown, we estimate $\boldsymbol{\xi}^{\mathrm{P}}$ using the first-step estimates of the macro parameters; σ_t^2 is replaced by $\hat{\sigma}^2$ when simulating data in the course of indirect inference estimation. The results are contained in Panel C of Table 3. The $T=100$k results corroborate our conjecture

parameters but on the structural model parameters. Moreover, it is impossible to formulate explicit constraints that would ensure that only eligible (structural) parameter combinations are used. To obtain a stable optimization procedure, we add 10^3 to the objective function whenever the optimization algorithm probes structural parameter values that imply an unsolvable model.

that the two-step estimation strategy can recover the true parameters as RMSEs decrease and confidence bounds narrow, and Figure 4 shows that the bell-shaped kernel estimates center closely around the true values. Compared with the case in which the macro parameters are known, estimation precision decreases considerably. For smaller sample sizes, the subjective discount factor can still be estimated accurately, whereas the RRA and IES estimates become less precise. Table 3 shows that the RMSEs are influenced by a small number of large estimates that produce the right-skewed kernel estimates for $\hat{\gamma}$ and $\hat{\psi}$ depicted in Figure 4. The masses of the distributions remain centered around the true values.

[Insert Figure 4 here]

5 Data

The empirical application of the two-step indirect inference strategy is based on quarterly U.S. data from 1947Q2 to 2014Q4. The construction of the data base closely follows Beeler and Campbell (2012). Consumption growth is computed from real personal consumption per capita of nondurable goods and services, as obtained from the Bureau of Economic Analysis. The market portfolio return, dividend growth, and the price-dividend ratio are calculated for the CRSP value-weighted market portfolio. Conversions into real terms are performed using the consumer price index from the Bureau of Labor Statistics. To calculate a risk-free rate proxy, we use the three-month nominal T-Bill yield from the CRSP database. Following Beeler and Campbell (2012), we approximate the ex ante risk-free rate by using a forecast for the ex post real rate, where the predictors are the quarterly T-Bill yield and the average of quarterly log inflation over the past year. Figure 5 shows time series plots of these data.

[Insert Figure 5 here]

Dividend payments occur irregularly, such that the quarterly dividend growth series depicted in Panel (b) of Figure 5 is erratic. The time series exhibits a strong negative first-order autocorrelation that cannot be accounted for by the dividend growth process in Equation (3). To address this problem, we follow Hasseltoft (2012) and take the average of the current period's log dividend growth and that of the previous three quarters to obtain a smoothed dividend growth series, as depicted in Panel (d) of Figure 5. We provide descriptive statistics in Table 4 and make the data available in the Web Appendix.

[Insert Table 4 here]

6 Empirical Results

The application of the two-step indirect inference strategy assumes a monthly decision frequency; the time aggregation to the quarterly frequency follows the description in Section 3.1, and the auxiliary parameters are defined as in Sections 3.2.2 (using the basic version of the first-step auxiliary parameter vector) and 3.3.2. We rely on the estimation strategy to replace σ_t^2 by $\sigma^2 = \mathbb{E}(\sigma_t^2)$ when simulating LRR model-implied data, such that we effectively estimate BY's special case without fluctuating macroeconomic uncertainty. The Monte Carlo results suggest that the estimation of the remaining macro and preference parameters is not impaired, even if the data-generating process exhibits some stochastic volatility. We include annual and triannual aggregates in the HAR model in Equation (25) by setting $h_1{=}4$ and $h_2{=}12$ and use $HT{=}100\text{k}$ to improve simulation accuracy. Otherwise, the estimation setup is the same as in the Monte Carlo study. Table 5 contains the point estimates,

along with the bounds of the 95% bootstrap confidence intervals and the standard errors based on the asymptotic and bootstrapped distributions.[13]

[Insert Table 5 here]

The first-step estimates of the macro parameters are consistent with the LRR paradigm; they support the notion of a small persistent growth component. The lower bound of the 95% confidence interval for φ_e is distinctly greater than 0 (φ_e=0 would imply i.i.d. growth processes), and the 95% confidence interval for the difference ρ-φ_e does not include 0 (φ_e=ρ would imply an AR(1) consumption growth process). The estimate $\hat{\rho}$=0.991 indicates strong persistence in the growth expectations. With an estimated base volatility of $\hat{\sigma}\hat{\varphi}_e\sqrt{12}$=0.053%, the growth component is small compared to the estimated base volatility of the consumption growth innovations, $\hat{\sigma}\sqrt{12}$=0.83%, and to the estimated base volatility of dividend growth innovations, $\hat{\sigma}\hat{\varphi}_d\sqrt{12}$=2.54%. Moreover, the estimate $\hat{\phi}$=5.14 indicates that the effect of expected consumption growth on dividend growth is leveraged, as predicted. The estimates $\hat{\mu}_c$ and $\hat{\mu}_d$ imply plausible mean growth rates of 2.0% p.a. and 2.3% p.a. for consumption and dividends, respectively.

Estimation accuracy and its variation across macro parameters is in line with the Monte Carlo results. As in Cochrane (2005), the bootstrap standard errors are larger than their asymptotic counterparts, suggesting that the latter may encourage an overly optimistic view. The 95% confidence bands contain plausible parameter values, but the intervals are rather wide. In light of the Monte Carlo results, we believe that the bootstrapped distribution provides a realistic view of the estimation precision. Shephard and Harvey (1990) note that it is very difficult to distinguish

---

[13]Considering the aforementioned numerical difficulties, it is worth noting that we obtain the same estimates and the same minima of the first- and second-step indirect inference objective functions using very different starting values. The selection criteria for successful bootstrap replications that are included in the calculation of the confidence bounds correspond to those applied in the Monte Carlo study.

between a purely i.i.d. process and one that incorporates a small persistent component.

We also should not expect the estimation of macro parameters to improve by relying on the LRR model's asset pricing implications. These implications are required to disentangle risk aversion from intertemporal substitution, and consumption and dividend growth are independent of investor preferences. Most importantly, using asset pricing relations to identify the macro parameters would forestall the two-step estimation strategy, which is designed to disentangle the estimation of the macro and preference parameters.

The second-step estimate for the subjective discount factor implies positive time preferences ($\hat{\delta} = 0.99998$, which is a plausible value at a monthly decision frequency), and $\hat{\gamma} = 11.8$ indicates a reasonable relative risk aversion. These estimates are comparable to BY's calibration. As might be expected from the Monte Carlo study, the RRA and IES estimates have large standard errors (bootstrap and asymptotic), and the 95% confidence intervals are wide. The subjective discount factor in turn can be estimated more precisely. As Table 7 shows, $\hat{\delta}$ and $\hat{\gamma}$ are comparable to the estimates reported by Bansal et al. (2007), although they report a narrower confidence band for the RRA coefficient. However, Bansal et al. (2007) also resort to fixing the value of the IES. They report that the efficient method of moments objective function is flat in ψ, and so, instead of estimating the IES, they calibrate $\psi = 2$, which is a crucial choice. If the IES is greater than 1, the intertemporal substitution effect dominates the wealth effect, which is the key condition for the LRR asset pricing implications to unfold.[14] Table 5 shows that our IES point estimate is smaller than 1 ($\hat{\psi} = 0.29$), but the 95% confidence interval also includes values larger than unity.

---

[14]See Equation (5) in Bansal and Yaron (2004). With relative risk aversion and IES both greater than 1, we have $\gamma > 1/\psi$, i.e. a preference for early resolution of uncertainty.

In contrast to theoretical considerations, estimates of the IES in previous literature tend to be below 1 and quite small (cf. Havránek, 2015; Thimme, 2017). As noted by Beeler and Campbell (2012), the IES can be represented by the slope of a regression of log consumption growth on the log risk-free rate and a constant. Using the empirical data, the OLS estimate of the IES amounts to $\hat{\psi}_{\text{OLS}} = 0.23$, which is similar to the indirect inference estimate but much smaller than the IESs calibrated by Bansal and Yaron (2004) and Bansal et al. (2007). To show that OLS yields a reasonable (albeit biased) IES estimate, we run the regression on simulated LRR model data, using BY's calibration as true parameter values. With a sample size of $T=100$k, we obtain $\hat{\psi}_{\text{OLS}} = 1.446$ on a monthly level, and $\hat{\psi}_{\text{OLS}} = 1.443$ for quarterly aggregates. Both estimates are close to the true $\psi = 1.5$.

[Insert Table 6 here]

The consequences of an IES estimate smaller than unity are also reflected in Table 6, which reports the model-implied first and second moments of the macro and financial variables, computed using the point estimates from Table 5. The sample moments of consumption and dividend growth are well matched, yet some sample and model-implied moments of the financial variables differ notably. With an IES < 1, the LRR model cannot account for the empirically observed small average T-Bill return and the high equity risk premium. The bootstrap standard errors of the model-implied moments are large, such that their confidence intervals overlap the sample moments, but they also fit into the general picture in which the small sample size limits estimation precision.

[Insert Table 7 here]

As Table 7 shows, some results reported in previous literature are more favorable for the LRR paradigm, in that the reported IES values are greater than 1. However,

some of these values are conveniently calibrated, and others result from one-step estimation strategies that should be considered with caution, as we have argued. It is doubtful that the identification problems addressed by Bansal et al. (2007) can be easily resolved by a one-step estimation approach.

7 Conclusion

The long-run consumption risk asset pricing model constitutes a leading paradigm for financial economics, but the estimation of its deep parameters is challenging. For example, Bansal et al. (2007) had to fix the values of several model parameters as a result of identification problems. Following a suggestion by Gourieroux et al. (1993) to use different criteria to estimate different parts of a model, we propose a two-step indirect inference strategy that exploits the recursive structure of the LRR model, in which dividend and consumption growth processes determine the model-implied asset pricing relations, but not vice versa. The first-step auxiliary parameter vector should capture the time series properties of the macroeconomic growth processes. The second-step auxiliary parameters are defined by a parsimonious set of moment conditions that help to identify the three dimensions of investor preferences: subjective time preference, propensity for intertemporal substitution, and risk aversion. We derive the asymptotic properties of the two-step estimator and outline a bootstrap procedure that exploits the parametric nature of the LRR model.

The discussion by Bansal et al. (2007) emphasizes that identification issues should be a major concern for any econometric analysis of LRR asset pricing models. Unfortunately, analytical checks are not possible, so we perform a Monte Carlo study to assess the viability of our estimation approach. We find that the investor preference parameters can be precisely estimated, provided that high-quality macro parameter

estimates are available. The parameters of the stochastic variance process prove difficult to estimate, so instead of relying on estimates of weakly-identified parameters, we propose replacing the stochastic variance by its expected value when simulating data in the course of an indirect inference estimation. The Monte Carlo results suggest that this estimation strategy does not impair the estimation of the other model parameters.

In an empirical application, we find support for the LRR paradigm; there is evidence for the existence of a small persistent growth component. We also obtain a plausible and precisely estimated subjective time preference parameter and an economically reasonable RRA estimate. An estimate of the IES below unity is a less favorable result for the LRR paradigm. In calibration studies, the IES is always greater than 1, because the ability of the LRR model to explain the prominent asset pricing puzzles requires that the intertemporal substitution effect dominates the wealth effect, which is the case when $\psi > 1$. The available data series are relatively short, which leads to wide confidence bounds. As a result, the confidence interval for the IES includes values greater than 1, so the LRR model is at least broadly consistent with the empirical data. The evidence in favor of the long-run risk asset pricing paradigm, however, is not as conclusive as implied by some previous studies.

The Monte Carlo results show that, provided good macro parameter estimates are available, the preference parameters can be efficiently estimated by indirect inference. For the first estimation step, and in particular for the estimation of the parameters of the latent growth component, it would be desirable to enhance estimation precision. Efforts to improve the accuracy of the preference parameter estimates therefore should focus on increasing the estimation precision of the macroeconomic parameters.

Other seminal consumption-based asset pricing models, which account for a reference level of consumption (Campbell and Cochrane, 1999) or rare disaster risk (Barro, 2006), resemble the LRR model structure in that they also feature macroeconomic processes with latent variables and asset pricing relations that demand the use of simulation-based estimation methods. The two-step indirect inference strategy may also prove useful for the econometric analysis of these models, which are notoriously difficult to estimate. Moreover, the approach presented herein could be combined with the calibration-estimation framework outlined by Dridi et al. (2007), who focus on the implications of a misspecified structural model, while estimating the true and pseudo-true parameters in one step, according to a single binding function. We suggest using two auxiliary models to identify different parts of a structural model that proves difficult to estimate. Both approaches agree in their suggestion to use simple, yet well thought-out auxiliary models that can capture the economically important features of the structural model, instead of striving for efficiency with a sophisticated auxiliary model that mimics the structural model as precisely as possible. It would be interesting to connect the two approaches more closely. We leave these topics for further research.

A   Appendix

A.1   LRR model details

As outlined in the main text, the following expressions result from pricing the gross returns of the aggregate wealth portfolio and that of the market portfolio, using Equation (6):

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \tag{A-1}$$

$$A_2 = \frac{1}{2} \frac{\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta A_1 \kappa_1 \varphi_e)^2}{\theta(1 - \kappa_1 \nu_1)}, \tag{A-2}$$

$$A_0 = \frac{1}{1 - \kappa_1} \left[\ln \delta + \left(1 - \frac{1}{\psi}\right) \mu_c + \kappa_0 + \kappa_1 A_2 \sigma^2 (1 - \nu_1) + \frac{\theta}{2} (\kappa_1 A_2 \sigma_w)^2 \right], \tag{A-3}$$

$$A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho}, \tag{A-4}$$

$$A_{2,m} = \frac{(1 - \theta)(1 - \kappa_1 \nu_1) A_2}{(1 - \kappa_{1,m} \nu_1)},$$
$$+ \frac{\frac{1}{2}[(-\frac{\theta}{\psi} + \theta - 1)^2 + ((\kappa_{1,m} A_{1,m} \varphi_e) - ((1 - \theta)\kappa_1 A_1 \varphi_e))^2 + \varphi_d^2]}{(1 - \kappa_{1,m} \nu_1)}, \tag{A-5}$$

$$A_{0,m} = \frac{1}{(1 - \kappa_{1,m})} \left[\theta \ln \delta - \frac{\theta}{\psi} \mu_c + (\theta - 1) \left[\kappa_0 + \kappa_1 A_0 + \kappa_1 A_2 (1 - \nu_1)\sigma^2 - A_0 + \mu_c \right], \right.$$
$$\left. + \kappa_{0,m} + \kappa_{1,m} A_{2,m} \sigma^2 (1 - \nu_1) + \mu_d + \frac{1}{2} \left[(\theta - 1)\kappa_1 A_2 + \kappa_{1,m} A_{2,m} \right]^2 \sigma_w^2 \right]. \tag{A-6}$$

A detailed derivation of Equations (A-1)–(A-6) can be found in Sections 1.3 and 1.4 of the Web Appendix. The expression for the log risk-free rate is given by:

$$r_{f,t} = -\theta \ln(\delta) + \frac{\theta}{\psi} (\mu_c + x_t) + (1 - \theta)\mathbb{E}_t(r_{a,t+1}) - \frac{1}{2}\mathrm{Var}_t(m_{t+1}), \tag{A-7}$$

where m_t is the logarithm of the stochastic discount factor M_t, and

$$\mathbb{E}_t(r_{a,t+1}) = \kappa_0 + \kappa_1 \left[A_0 + A_1 \rho x_t + A_2(\sigma^2 + \nu_1(\sigma_t^2 - \sigma^2)) \right] \qquad \text{(A-8)}$$

$$- A_0 - A_1 x_t - A_2 \sigma_t^2 + \mu_c + x_t,$$

$$\text{Var}_t(m_{t+1}) = \left(\frac{\theta}{\psi} + 1 - \theta \right)^2 \sigma_t^2 + [(1-\theta)\kappa_1 A_1 \varphi_e]^2 \sigma_t^2 \qquad \text{(A-9)}$$

$$+ [(1-\theta)\kappa_1 A_2]^2 \sigma_w^2.$$

The detailed derivation is provided in Section 1.5 of the Web Appendix.

The expression for the equity premium is given by:

$$\mathbb{E}_t(r_{m,t+1} - r_{f,t+1}) = \lambda_{m,e}\kappa_{1,m}A_{1,m}\varphi_e\sigma_t^2 + \lambda_{m,w}\kappa_{1,m}A_{2,m}\sigma_w^2$$

$$- \frac{1}{2} \left[\varphi_d^2 \sigma_t^2 + (\kappa_{1,m}A_{1,m}\varphi_e)^2\sigma_t^2 + (\kappa_{1,m}A_{2,m})^2\sigma_w^2 \right], \qquad \text{(A-10)}$$

$$\text{where} \quad \lambda_{m,\eta} = -\frac{\theta}{\psi} + \theta - 1,$$

$$\lambda_{m,e} = (1-\theta)\kappa_1 A_1 \varphi_e,$$

$$\text{and} \quad \lambda_{m,w} = (1-\theta)\kappa_1 A_2.$$

The detailed derivation is provided in Section 1.6 of the Web Appendix.

A.2 Simulation of LRR model-implied data

A simulation of LRR model-implied data is required for the indirect inference estimation, bootstrap inference, and the Monte Carlo study. The LRR model is inherently recursive. Taking the values for the structural parameters in $\boldsymbol{\xi}^{\text{M}}$ and $\boldsymbol{\xi}^{\text{P}}$ as given, we first simulate data for the macro variables x_t, σ_t^2, g_t, and $g_{d,t}$, and then generate time series of the financial variables z_t, $r_{a,t}$, $z_{m,t}$, $r_{m,t}$, and $r_{f,t}$ in a second step. For

the estimation of the macro parameters only the first step is required, whereas for the estimation of the preference parameters the full model must be simulated.

To simulate the macro variables, we independently draw $4 \times (\mathcal{T} + L)$ standard normally distributed random variables to obtain realizations of the innovations $\{\eta_t\}_{t=1}^{\mathcal{T}+L}$, $\{e_t\}_{t=1}^{\mathcal{T}+L}$, $\{u_t\}_{t=1}^{\mathcal{T}+L}$, and $\{w_t\}_{t=1}^{\mathcal{T}+L}$ in Equations (1)–(4). Here, $\mathcal{T}$ is the desired number of observations in the simulated data set. For the data simulation within the optimizations in Equations (23) and (33), $\mathcal{T}$ is chosen to provide the required HT observations. If the decision frequency is equal to the observation frequency, then $\mathcal{T} = HT$; if the decision frequency is higher than the observation frequency, $\mathcal{T}$ is chosen to ensure the required HT observations are obtained after a proper time aggregation. In turn, L is the number of observations of a "swing-in" period, which we discard to mitigate the impact of the choice of starting values. Our default value is $L = 100$. When generating data for the latent processes x_t and σ_t^2, we use the unconditional expectations as starting values for the forward iterations of Equations (2) and (4), such that $x_0 = 0$ and $\sigma_0^2 = \sigma^2$. Incidental negative values of σ_t^2 are replaced by 0. We can then generate series for g_t and $g_{d,t}$ using Equations (1) and (3).

On the basis of the simulated macro series, we simulate data for the financial variables by solving for the endogenous κ- and A-parameters in Equations (9), (10), and (A-1)–(A-3). Solving the model amounts to finding the means $\bar{z}$ and $\bar{z}_m$, such that Equations (9)–(12) and (A-1)–(A-6) are fulfilled. This end can be achieved by numerically solving for the means of z and z_m, such that the functions

$$f_1(\bar{z}, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) = \bar{z} - A_0(\bar{z}, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) - A_2(\bar{z}, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}})\sigma^2 \quad \text{and} \tag{A-11}$$

$$f_2(\bar{z}, \bar{z}_m, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) = \bar{z}_m - A_{0,m}(\bar{z}, \bar{z}_m, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) - A_{2,m}(\bar{z}, \bar{z}_m, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}})\sigma^2 \tag{A-12}$$

are equal to zero, holding $\boldsymbol{\xi}^{\mathrm{M}}$ and $\boldsymbol{\xi}^{\mathrm{P}}$ fixed. The endogenous parameters are thus implied by the roots of the functions f_1 and f_2.

Numerically solving the equation $f_1(\bar{z}, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) = 0$ for the mean of the log price-consumption ratio ($\bar{z}$) yields values for κ_1 and κ_0, as well as for A_1, A_2, and A_0, computed in that order. We can then determine z_t and $r_{a,t}$ using Equations (11) and (7). For the simulation of the time series of $z_{m,t}$ and $r_{m,t}$, we obtain values for the endogenous parameters $\kappa_{1,m}$ and $\kappa_{0,m}$, as well as $A_{1,m}$, $A_{2,m}$, and $A_{0,m}$, by numerically solving the equation $f_2(\bar{z}, \bar{z}_m, \boldsymbol{\xi}^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) = 0$ for the mean of the log price-dividend ratio ($\bar{z}_m$). We compute the time series of $z_{m,t}$ and $r_{m,t}$ using Equations (12) and (8). A series of LRR model-implied log risk-free rates $r_{f,t}$, in turn, is obtained from Equation (A-7). The simulated time series take the assumed decision frequency of the LRR investor, and it may be necessary to aggregate the data to a lower frequency using the formulas in Appendix A.3.

A.3 Time aggregation

The formulas for the time aggregation of the LRR model variables over h periods, provided by Calvet and Czellar (2015), are as follows:

$$g_t^{f(h)} = \ln \frac{\sum_{i=(t-1)h+1}^{th} \exp\left[\sum_{j=(t-1)h+1}^{i} g_j\right]}{1 + \sum_{i=(t-2)h+2}^{(t-1)h} \exp\left[-\sum_{j=i}^{(t-1)h} g_j\right]}, \tag{A-13}$$

$$g_{d,t}^{f(h)} = \ln \frac{\sum_{i=(t-1)h+1}^{th} \exp\left[\sum_{j=(t-1)h+1}^{i} g_{d,j}\right]}{1 + \sum_{i=(t-2)h+2}^{(t-1)h} \exp\left[-\sum_{j=i}^{(t-1)h} g_{d,j}\right]}, \tag{A-14}$$

$$z_{m,t}^{f(h)} = z_{m,th} + \sum_{i=(t-1)h+1}^{th} g_{d,i} - \ln\left[\sum_{i=(t-1)h+1}^{th} \exp\left(\sum_{j=(t-1)h+1}^{i} g_{d,j}\right)\right], \tag{A-15}$$

$$r_{m,t}^{f(h)} = \sum_{i=(t-1)h+1}^{th} r_{m,i} \quad \text{and} \quad r_{f,t}^{f(h)} = \sum_{i=(t-1)h+1}^{th} r_{f,i}. \tag{A-16}$$

A.4 Moment conditions defining the first-step auxiliary parameters

A.4.1 Basic set of moment conditions

The first-step auxiliary parameter vector $\boldsymbol{\theta}^{\mathrm{M}} = (\boldsymbol{\theta}^{\mathrm{HAR}\prime}, \boldsymbol{\mu_g}', \boldsymbol{\sigma_g}')'$ is defined by the moment conditions:

$$
\mathbb{E}(\mathbf{u}_t^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_t^{t-l}, \boldsymbol{\theta}^{\mathrm{M}})) = \mathbb{E}
\begin{bmatrix}
\mathrm{vec}\left[
\begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \end{bmatrix} \otimes
\begin{bmatrix}
1 \\ g_{t-1} \\ \vdots \\ g_{t-\tau} \\ g_{d,t-1} \\ \vdots \\ g_{d,t-\tau} \\ g_{t-1}^{f(h_1)} \\ g_{d,t-1}^{f(h_1)} \\ g_{t-1}^{f(h_2)} \\ g_{d,t-1}^{f(h_2)}
\end{bmatrix}
\right] \\
\zeta_{1,t}^2 - [\sigma_{\zeta_1}]^2 \\
\zeta_{2,t}^2 - [\sigma_{\zeta_2}]^2 \\
\zeta_{1,t} \cdot \zeta_{2,t} - \sigma_{\zeta_1 \zeta_2} \\
g_t - \mu_c \\
g_{d,t} - \mu_d \\
g_t^{f(h_1)} - \mu_c^{f(h_1)} \\
g_{d,t}^{f(h_1)} - \mu_d^{f(h_1)} \\
g_t^{f(h_2)} - \mu_c^{f(h_2)} \\
g_{d,t}^{f(h_2)} - \mu_d^{f(h_2)} \\
[g_t]^2 - [\mu_c]^2 - [\sigma_c]^2 \\
[g_{d,t}]^2 - [\mu_d]^2 - [\sigma_d]^2 \\
[g_t^{f(h_1)}]^2 - [\mu_c^{f(h_1)}]^2 - [\sigma_c^{f(h_1)}]^2 \\
[g_{d,t}^{f(h_1)}]^2 - [\mu_d^{f(h_1)}]^2 - [\sigma_d^{f(h_1)}]^2 \\
[g_t^{f(h_2)}]^2 - [\mu_c^{f(h_2)}]^2 - [\sigma_c^{f(h_2)}]^2 \\
[g_{d,t}^{f(h_2)}]^2 - [\mu_d^{f(h_2)}]^2 - [\sigma_d^{f(h_2)}]^2
\end{bmatrix}
= \mathbf{0}, \qquad (\text{A-17})
$$

where

$$
\begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \end{bmatrix} = \begin{bmatrix} g_t \\ g_{d,t} \end{bmatrix} - \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \sum_{i=1}^{\tau} \mathbf{\Phi}_i L^i \begin{bmatrix} g_t \\ g_{d,t} \end{bmatrix} - \mathbf{\Phi}_{\tau+1} \begin{bmatrix} g_{t-1}^{f(h_1)} \\ g_{d,t-1}^{f(h_1)} \end{bmatrix} - \mathbf{\Phi}_{\tau+2} \begin{bmatrix} g_{t-1}^{f(h_2)} \\ g_{d,t-1}^{f(h_2)} \end{bmatrix} , \quad (\text{A-18})
$$

which are assumed to hold uniquely at $\boldsymbol{\theta}^{\mathrm{M}} = \boldsymbol{\theta}_0^{\mathrm{M}}$.

A.4.2 Extended first-step auxiliary parameter vector

We also consider extending $\boldsymbol{\theta}^{\mathrm{M}}$ by a set of auxiliary parameters, defined by additional moment conditions implied by an AR-ARCH specification for consumption growth:

$$
\mathbb{E}(\mathbf{u}_t^+([\mathbf{y}^{\mathrm{M}}]_t^{t-\tilde{\tau}}, a_1, a_2, \psi_0, \psi_1, \ldots, \psi_{\tilde{\tau}}, \sigma_v)) = \mathbb{E} \begin{bmatrix} \omega_t \\ \omega_t g_{t-1} \\ v_t \\ v_t \omega_{t-1}^2 \\ \vdots \\ v_t \omega_{t-\tilde{\tau}}^2 \\ v_t^2 - [\sigma_v]^2 \end{bmatrix} = \mathbf{0}, \qquad (\text{A-19})
$$

where

$$
\omega_t = g_t - a_1 - a_2 g_{t-1}, \tag{A-20}
$$

$$
v_t = \omega_t^2 - \left(\psi_0 + \sum_{i=1}^{\tilde{\tau}} \psi_i L^i \omega_t^2 \right). \tag{A-21}
$$

The extended first-step auxiliary parameter vector $\boldsymbol{\theta}^{\mathrm{M}}$ then also includes ψ_0, ψ_1, $\psi_2, \ldots, \psi_{\tilde{\tau}}$, and σ_v. One of our referees suggested using a GARCH specification for consumption growth (as in Bansal, Khatchatrian, and Yaron (2005)) as an auxiliary model. As Baillie and Chung (2001) show, matching the autocovariances of a

squared process can serve to estimate the parameters of a GARCH(1,1) model, so our approach is compatible with that suggestion.

A.5   Theoretical moments of log consumption and dividend growth in the LRR model

The LRR model implies the following theoretical moments, which we use for the GMM estimation as an alternative to the first-step indirect inference estimation:

$$\mathbb{E}(g_t) = \mu_c, \tag{A-22}$$

$$\mathbb{E}(g_{d,t}) = \mu_d, \tag{A-23}$$

$$\mathbb{E}(g_t^2) = \mu_c^2 + \frac{\varphi_e^2 \sigma^2}{1 - \rho^2} + \sigma^2, \tag{A-24}$$

$$\mathbb{E}(g_{d,t}^2) = \mu_d^2 + \phi^2 \frac{\varphi_e^2 \sigma^2}{1 - \rho^2} + \varphi_d^2 \sigma^2, \tag{A-25}$$

$$\mathbb{E}(g_{d,t} g_t) = \mu_c \mu_d + \phi \frac{\varphi_e^2 \sigma^2}{1 - \rho^2}, \tag{A-26}$$

$$\mathbb{E}(g_{t+1} g_t) = \mu_c^2 + \rho \frac{\varphi_e^2 \sigma^2}{1 - \rho^2}, \tag{A-27}$$

$$\mathbb{E}(g_{t+2} g_t) = \mu_c^2 + \rho^2 \frac{\varphi_e^2 \sigma^2}{1 - \rho^2}. \tag{A-28}$$

A.6   Identification of the IES

The expression for the log risk-free rate in Equation (A-7) can be written as:

$$r_{f,t} = A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2, \tag{A-29}$$

where $A_{0,f}$ collects all terms from the right-hand side of Equation (A-7) that do not depend on either of the two state variables, and $A_{2,f}$ collects all terms of Equation (A-7) that depend on σ_t^2. It can be shown that $A_{0,f}$ and $A_{2,f}$ depend on all

three preference parameters, whereas $A_{1,f}$, which collects all terms of the right-hand side of Equation (A-7) related to x_t, depends only on ψ:

$$A_{1,f} = \left[1 - \theta + \frac{\theta}{\psi} - (1-\theta)A_1(1-\kappa_1\rho)\right] = \left[1 - \theta + \frac{\theta}{\psi} - (1-\theta)\left(1 - \frac{1}{\psi}\right)\right] = \frac{1}{\psi}.$$
$$(A\text{-}30)$$

Using the expression for $z_{m,t}$ in Equation (12), the contemporaneous covariance of $z_{m,t}$ and $r_{f,t}$ is given by

$$\mathrm{Cov}(z_{m,t}, r_{f,t}) = A_{1,m}A_{1,f}\mathrm{Var}(x_t) + A_{2,m}A_{2,f}\mathrm{Var}(\sigma_t^2), \qquad (A\text{-}31)$$

where $A_{1,m}$ is given in Equation (A-4), and $A_{2,m}$ is given in Equation (A-5). Equation (A-4) shows that of the three preference parameters, only ψ affects $A_{1,m}$.

For economically plausible parameter values, such as the BY calibration, the expression for $\mathrm{Var}(x_t)$ is several orders of magnitude greater than $\mathrm{Var}(\sigma_t^2)$. The covariance of $z_{m,t}$ and $r_{f,t}$ is thus dominated by the term $A_{1,m}A_{1,f}\mathrm{Var}(x_t)$, which depends only on ψ, but not on δ or γ. The influence of the subjective discount factor and the RRA coefficient on the covariance of $z_{m,t}$ and $r_{f,t}$ is negligible. The identification of the IES thus is facilitated by the slope parameter of a contemporaneous regression of $z_{m,t}$ on $r_{f,t}$, which is why we include it in the second-step auxiliary parameter vector.

A.7 Proofs of Propositions

Proof of Proposition 1. We draw on a result in Gourieroux et al. (1993) and note that the limit of the optimization problem in Equation (23) is, under Assumptions 1-4, given by

$$\min_{\boldsymbol{\xi}^{\mathrm{M}} \in \boldsymbol{\Xi}^{\mathrm{M}}} \left[\mathbf{b}^{\mathrm{M}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}) - \mathbf{b}^{\mathrm{M}}(G_0, \boldsymbol{\xi}^{\mathrm{M}}) \right]' \mathbf{W}^{\mathrm{M}} \left[\mathbf{b}^{\mathrm{M}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}) - \mathbf{b}^{\mathrm{M}}(G_0, \boldsymbol{\xi}^{\mathrm{M}}) \right], \qquad \text{(A-32)}$$

from which the consistency of $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ stated in Proposition 1 follows. □

Proof of Proposition 2. Because $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ is a consistent estimator of $\boldsymbol{\xi}_0^{\mathrm{M}}$, we observe that the limit of the optimization problem in Equation (33), under Assumptions 1-7, is given by

$$\min_{\boldsymbol{\xi}^{\mathrm{P}} \in \boldsymbol{\Xi}^{\mathrm{P}}} \left[\mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) - \mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) \right]' \mathbf{W}^{\mathrm{P}} \left[\mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) - \mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}^{\mathrm{P}}) \right],$$

$$\text{(A-33)}$$

from which the consistency of $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$ stated in Proposition 2 follows. □

Proof of Proposition 3. To derive the asymptotic joint distribution of $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$, and for a proof of Proposition 3, we consider alternative versions of the first- and second-step estimators in Equations (23) and (33).[15] Instead of estimating the auxiliary parameters using one simulated sample with HT observations, these alternative estimators average the auxiliary parameter estimates using H independent

---

[15]Our line of reasoning and notation draw on Gourieroux et al. (1993).

paths of simulated data, each of length T. In particular, the alternative first-step estimator defines $\hat{\boldsymbol{\xi}}_T^{\text{M}}$ as a solution of

$$\min_{\boldsymbol{\xi}^{\text{M}} \in \boldsymbol{\Xi}^{\text{M}}} \left[\hat{\boldsymbol{\theta}}_T^{\text{M}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{M},h}(\boldsymbol{\xi}^{\text{M}}, \mathbf{z}_0^h) \right]' \widehat{\mathbf{W}}_T^{\text{M}} \left[\hat{\boldsymbol{\theta}}_T^{\text{M}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{M},h}(\boldsymbol{\xi}^{\text{M}}, \mathbf{z}_0^h) \right], \qquad \text{(A-34)}$$

where

$$\tilde{\boldsymbol{\theta}}_T^{\text{M},h}(\boldsymbol{\xi}^{\text{M}}, \mathbf{z}_0^h) = \arg\max_{\boldsymbol{\theta}^{\text{M}} \in \boldsymbol{\Theta}^{\text{M}}} Q_T^{\text{M}} \left([\tilde{\mathbf{y}}^{\text{M},h}(\boldsymbol{\xi}^{\text{M}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}^{\text{M}} \right), \qquad \text{(A-35)}$$

while $\hat{\boldsymbol{\xi}}_T^{\text{P}}$ is alternatively defined as the solution of

$$\min_{\boldsymbol{\xi}^{\text{P}} \in \boldsymbol{\Xi}^{\text{P}}} \left[\hat{\boldsymbol{\theta}}_T^{\text{P}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{P},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \boldsymbol{\xi}^{\text{P}}, \mathbf{z}_0^h) \right]' \widehat{\mathbf{W}}_T^{\text{P}} \left[\hat{\boldsymbol{\theta}}_T^{\text{P}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{P},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \boldsymbol{\xi}^{\text{P}}, \mathbf{z}_0^h) \right], \quad \text{(A-36)}$$

where

$$\tilde{\boldsymbol{\theta}}_T^{\text{P},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \boldsymbol{\xi}^{\text{P}}, \mathbf{z}_0^h) = \arg\max_{\boldsymbol{\theta}^{\text{P}} \in \boldsymbol{\Theta}^{\text{P}}} Q_T^{\text{P}} \left([\tilde{\mathbf{y}}^{\text{M},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\text{P},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \boldsymbol{\xi}^{\text{P}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}^{\text{P}} \right). \quad \text{(A-37)}$$

The first-order conditions for the optimization problems in Equations (A-34) and (A-36) imply that

$$\left[\begin{array}{cc} \frac{1}{H} \sum_{h=1}^{H} \dfrac{\partial \tilde{\boldsymbol{\theta}}_T^{\text{M},h\prime}}{\partial \boldsymbol{\xi}^{\text{M}}}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \mathbf{z}_0^h) & \mathbf{0} \\[2ex] \mathbf{0} & \frac{1}{H} \sum_{h=1}^{H} \dfrac{\partial \tilde{\boldsymbol{\theta}}_T^{\text{P},h\prime}}{\partial \boldsymbol{\xi}^{\text{P}}}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \hat{\boldsymbol{\xi}}_T^{\text{P}}, \mathbf{z}_0^h) \end{array} \right] \times$$

$$\left[\begin{array}{cc} \widehat{\mathbf{W}}_T^{\text{M}} & \mathbf{0} \\[1ex] \mathbf{0} & \widehat{\mathbf{W}}_T^{\text{P}} \end{array} \right] \times \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_T^{\text{M}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{M},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \mathbf{z}_0^h) \\[1ex] \hat{\boldsymbol{\theta}}_T^{\text{P}} - \frac{1}{H} \sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\text{P},h}(\hat{\boldsymbol{\xi}}_T^{\text{M}}, \hat{\boldsymbol{\xi}}_T^{\text{P}}, \mathbf{z}_0^h) \end{array} \right] = \mathbf{0}. \qquad \text{(A-38)}$$

Using the expansion

$$
\begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0^h) \\[2mm] \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \hat{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix} \simeq \begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) \\[2mm] \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix} +
$$

$$
\begin{bmatrix} \frac{1}{H}\sum_{h=1}^{H} \frac{\partial \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}}{\partial \boldsymbol{\xi}^{\mathrm{M}\prime}}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) & \mathbf{0} \\[2mm] \frac{1}{H}\sum_{h=1}^{H} \frac{\partial \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}}{\partial \boldsymbol{\xi}^{\mathrm{M}\prime}}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) & \frac{1}{H}\sum_{h=1}^{H} \frac{\partial \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}}{\partial \boldsymbol{\xi}^{\mathrm{P}\prime}}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\xi}}_T^{\mathrm{M}} - \boldsymbol{\xi}_0^{\mathrm{M}} \\[2mm] \hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}} \end{bmatrix} \qquad \text{(A-39)}
$$

we obtain asymptotically

$$
\sqrt{T}\begin{bmatrix} \hat{\boldsymbol{\xi}}_T^{\mathrm{M}} - \boldsymbol{\xi}_0^{\mathrm{M}} \\[2mm] \hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}} \end{bmatrix} \simeq
$$

$$
\begin{bmatrix} \frac{\partial \mathbf{b}^{\mathrm{M}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{M}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}})\mathbf{W}^{\mathrm{M}}\frac{\partial \mathbf{b}^{\mathrm{M}}}{\partial \boldsymbol{\xi}^{\mathrm{M}\prime}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}) & \mathbf{0} \\[2mm] \frac{\partial \mathbf{b}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}})\mathbf{W}^{\mathrm{P}}\frac{\partial \mathbf{b}^{\mathrm{P}}}{\partial \boldsymbol{\xi}^{\mathrm{M}\prime}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) & \frac{\partial \mathbf{b}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}})\mathbf{W}^{\mathrm{P}}\frac{\partial \mathbf{b}^{\mathrm{P}}}{\partial \boldsymbol{\xi}^{\mathrm{P}\prime}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) \end{bmatrix}^{-1} \times
$$

$$
\begin{bmatrix} \frac{\partial \mathbf{b}^{\mathrm{M}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{M}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}})\mathbf{W}^{\mathrm{M}} & \mathbf{0} \\[2mm] \mathbf{0} & \frac{\partial \mathbf{b}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}})\mathbf{W}^{\mathrm{P}} \end{bmatrix} \sqrt{T}\begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) \\[2mm] \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix}.
$$

$$
\text{(A-40)}
$$

Partitioned inversion then yields:

$$
\sqrt{T}\begin{bmatrix} \hat{\boldsymbol{\xi}}_T^{\mathrm{M}} - \boldsymbol{\xi}_0^{\mathrm{M}} \\[2mm] \hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}} \end{bmatrix} \simeq \begin{bmatrix} \mathbf{A}(\mathbf{W}^{\mathrm{M}}) & \mathbf{0} \\[2mm] \mathbf{B}(\mathbf{W}^{\mathrm{P}})\mathbf{C}(\mathbf{W}^{\mathrm{M}}) & \mathbf{B}(\mathbf{W}^{\mathrm{P}}) \end{bmatrix} \sqrt{T}\begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) \\[2mm] \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \frac{1}{H}\sum_{h=1}^{H} \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix},
$$

$$
\text{(A-41)}
$$

with $\mathbf{A}(\mathbf{W}^{\mathrm{M}})$, $\mathbf{B}(\mathbf{W}^{\mathrm{P}})$, and $\mathbf{C}(\mathbf{W}^{\mathrm{M}})$ as defined in Equations (41), (42), and (43).

The maximization of $Q_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}^{\mathrm{M}})$ in Equation (16) and the maximization of $Q_T^{\mathrm{P}}([\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}^{\mathrm{P}})$ in Equation (27) imply the following first-order conditions:

$$- \frac{\partial \mathbf{g}_T^{\mathrm{M}\prime}}{\partial \boldsymbol{\theta}^{\mathrm{M}}}([\mathbf{y}^{\mathrm{M}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{M}}) \; \hat{\boldsymbol{\Omega}}_T^{\mathrm{M}} \; \mathbf{g}_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{M}}) = \mathbf{0} \tag{A-42}$$

$$- \frac{\partial \mathbf{g}_T^{\mathrm{P}\prime}}{\partial \boldsymbol{\theta}^{\mathrm{P}}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{P}}) \; \hat{\boldsymbol{\Omega}}_T^{\mathrm{P}} \; \mathbf{g}_T^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{P}}) = \mathbf{0}. \tag{A-43}$$

Using the expansions

$$\mathbf{g}_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{M}}) \simeq \mathbf{g}_T^{\mathrm{M}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{M}}\right) + \frac{\partial \mathbf{g}_T^{\mathrm{M}}}{\partial \boldsymbol{\theta}^{\mathrm{M}\prime}}\left([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{M}}\right)(\hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \boldsymbol{\theta}_0^{\mathrm{M}}) \tag{A-44}$$

$$\mathbf{g}_T^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \hat{\boldsymbol{\theta}}_T^{\mathrm{P}}) \simeq \mathbf{g}_T^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{P}}) + \frac{\partial \mathbf{g}_T^{\mathrm{P}}}{\partial \boldsymbol{\theta}^{\mathrm{P}\prime}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{P}})(\hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \boldsymbol{\theta}_0^{\mathrm{P}}), \tag{A-45}$$

we obtain asymptotically

$$\sqrt{T} \begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \boldsymbol{\theta}_0^{\mathrm{M}} \\ \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \boldsymbol{\theta}_0^{\mathrm{P}} \end{bmatrix} \simeq - \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}} \end{bmatrix} \sqrt{T} \begin{bmatrix} \mathbf{g}_T^{\mathrm{M}}([\mathbf{y}^{\mathrm{M}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{M}}) \\ \mathbf{g}_T^{\mathrm{P}}([\mathbf{y}^{\mathrm{M}}]_T^1, [\mathbf{y}^{\mathrm{P}}]_T^1, \boldsymbol{\theta}_0^{\mathrm{P}}) \end{bmatrix}, \tag{A-46}$$

where $\mathbf{A}_0^{\mathrm{M}}$ and $\mathbf{A}_0^{\mathrm{P}}$ are defined as in Equations (44) and (45). Under Assumption 8, we obtain

$$\sqrt{T} \begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \boldsymbol{\theta}_0^{\mathrm{M}} \\ \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \boldsymbol{\theta}_0^{\mathrm{P}} \end{bmatrix} \xrightarrow{d} \mathcal{N} \left(\mathbf{0}, \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}} \end{bmatrix} \mathbf{S} \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}\prime} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}\prime} \end{bmatrix} \right), \tag{A-47}$$

where $\mathbf{S}$ is defined in Equation (38). Similarly, the maximization of $Q_T^{\mathrm{M}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}^{\mathrm{M}}\right)$ implies the first-order conditions

$$-\frac{\partial \mathbf{g}_T^{\mathrm{M}\prime}}{\partial \boldsymbol{\theta}^{\mathrm{M}}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)\right) \times \hat{\boldsymbol{\Omega}}_T^{\mathrm{M}}$$
$$\times \mathbf{g}_T^{\mathrm{M}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)\right) = \mathbf{0}, \qquad (\text{A-48})$$

and the maximization of $Q_T^{\mathrm{P}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}^{\mathrm{P}}\right)$ implies

$$-\frac{\partial \mathbf{g}_T^{\mathrm{P}\prime}}{\partial \boldsymbol{\theta}^{\mathrm{P}}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)\right) \times \hat{\boldsymbol{\Omega}}_T^{\mathrm{P}}$$
$$\times \mathbf{g}_T^{\mathrm{P}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)\right) = \mathbf{0}. \qquad (\text{A-49})$$

Expansions similar to those in Equations (A-44) and (A-45) then yield, under Assumption 1:

$$\sqrt{T}\begin{bmatrix} \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) - \boldsymbol{\theta}_0^{\mathrm{M}} \\ \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) - \boldsymbol{\theta}_0^{\mathrm{P}} \end{bmatrix} \simeq - \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}} \end{bmatrix} \times$$
$$\sqrt{T}\begin{bmatrix} \mathbf{g}_T^{\mathrm{M}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}_0^{\mathrm{M}}\right) \\ \mathbf{g}_T^{\mathrm{P}}\left([\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}_0^{\mathrm{P}}\right) \end{bmatrix}.$$
$$(\text{A-50})$$

Because each of the simulated samples and the "simulation of the nature" that produces $[\mathbf{y}^{\mathrm{M}}]_T^1$ and $[\mathbf{y}^{\mathrm{P}}]_T^1$ are independent, we obtain:

$$\sqrt{T}\begin{bmatrix} \hat{\boldsymbol{\theta}}_T^{\mathrm{M}} - \frac{1}{H}\sum_{h=1}^H \tilde{\boldsymbol{\theta}}_T^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h) \\ \hat{\boldsymbol{\theta}}_T^{\mathrm{P}} - \frac{1}{H}\sum_{h=1}^H \tilde{\boldsymbol{\theta}}_T^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h) \end{bmatrix} \xrightarrow{d} \mathcal{N}\left(\mathbf{0}, \left(1 + \frac{1}{H}\right)\begin{bmatrix} \mathbf{A}_0^{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}} \end{bmatrix} \mathbf{S} \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}\prime} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}\prime} \end{bmatrix}\right),$$
$$(\text{A-51})$$

and thus can conclude that

$$\sqrt{T}\begin{bmatrix}\hat{\boldsymbol{\xi}}_T^{\mathrm{M}} - \boldsymbol{\xi}_0^{\mathrm{M}} \\ \hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}}\end{bmatrix}$$

converges in distribution to a mean-zero vector that is normally distributed with the variance-covariance matrix

$$\left(1 + \frac{1}{H}\right)\begin{bmatrix} \mathbf{A}(\mathbf{W}^{\mathrm{M}})\mathbf{A}_0^{\mathrm{M}} & \mathbf{0} \\ \mathbf{B}(\mathbf{W}^{\mathrm{P}})\mathbf{C}(\mathbf{W}^{\mathrm{M}})\mathbf{A}_0^{\mathrm{P}} & \mathbf{B}(\mathbf{W}^{\mathrm{P}})\mathbf{A}_0^{\mathrm{P}} \end{bmatrix} \mathbf{S} \begin{bmatrix} \mathbf{A}_0^{\mathrm{M}\prime}\mathbf{A}(\mathbf{W}^{\mathrm{M}})' & \mathbf{A}_0^{\mathrm{P}\prime}\mathbf{C}(\mathbf{W}^{\mathrm{M}})'\mathbf{B}(\mathbf{W}^{\mathrm{P}})' \\ \mathbf{0} & \mathbf{A}_0^{\mathrm{P}\prime}\mathbf{B}(\mathbf{W}^{\mathrm{P}})' \end{bmatrix},$$

$$(\text{A-52})$$

which is the result stated in Proposition 3.

Using the original two-step estimator and starting the expansions from Equation (36) yields the same asymptotic distribution, because

$$\sqrt{T}(\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0) - \boldsymbol{\theta}_0^{\mathrm{M}}) \simeq -\mathbf{A}_0^{\mathrm{M}}\sum_{h=1}^{H}\frac{\sqrt{T}}{H}\mathbf{g}_T^{\mathrm{M}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}_0^{\mathrm{M}}\right) \qquad (\text{A-53})$$

and

$$\sqrt{T}(\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0) - \boldsymbol{\theta}_0^{\mathrm{P}}) \simeq -\mathbf{A}_0^{\mathrm{P}}\sum_{h=1}^{H}\frac{\sqrt{T}}{H}\mathbf{g}_T^{\mathrm{P}}\left([\tilde{\mathbf{y}}^{\mathrm{M},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \mathbf{z}_0^h)]_T^1, [\tilde{\mathbf{y}}^{\mathrm{P},h}(\boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}, \mathbf{z}_0^h)]_T^1, \boldsymbol{\theta}_0^{\mathrm{P}}\right),$$

$$(\text{A-54})$$

where $\mathbf{z}_0^1 = \mathbf{z}_0 = (x_0, \sigma_0^2)'$ and $\mathbf{z}_0^h = (\tilde{x}_{T(h-1)}, \tilde{\sigma}_{T(h-1)}^2)'$. The original and alternative versions of the two-step indirect inference estimator thus have the same asymptotic properties. $\qquad\square$

Proof of Proposition 4. To prove the first part of Proposition 4 (asymptotically optimal weighting in the first step), we note that Equations (A-41) and (A-51) imply that

$$\sqrt{T}(\hat{\boldsymbol{\xi}}_T^{\text{M}} - \boldsymbol{\xi}_0^{\text{M}}) \underset{d}{\rightarrow} \mathbf{A}(\mathbf{W}^{\text{M}})\mathbf{X}_1 \sim \mathcal{N}(\mathbf{0}, \text{Avar}^{\text{M}}(\mathbf{W}^{\text{M}})), \tag{A-55}$$

where $\mathbf{X}_1 \sim \mathcal{N}(\mathbf{0}, \left(1 + \frac{1}{H}\right) \mathbf{A}_0^{\text{M}} \mathbf{S}^{\text{M}} \mathbf{A}_0^{\text{M}\prime})$, with $\mathbf{S}^{\text{M}}$ defined as in Equation (46), such that

$$\text{Avar}^{\text{M}}(\mathbf{W}^{\text{M}}) = \left(1 + \frac{1}{H}\right) \mathbf{A}(\mathbf{W}^{\text{M}})\mathbf{A}_0^{\text{M}} \mathbf{S}^{\text{M}} \mathbf{A}_0^{\text{M}\prime} \mathbf{A}(\mathbf{W}^{\text{M}})'. \tag{A-56}$$

To prove the first part of Proposition 4, we seek to show that

$$\text{Avar}^{\text{M}}(\mathbf{W}^{\text{M}}) - \text{Avar}^{\text{M}}(\mathbf{W}^{\text{M}*}), \tag{A-57}$$

where

$$\mathbf{W}^{\text{M}*} = (\mathbf{A}_0^{\text{M}} \mathbf{S}^{\text{M}} \mathbf{A}_0^{\text{M}\prime})^{-1} \tag{A-58}$$

is a positive semidefinite matrix. Recognizing the structure by which

$$\mathbf{A}(\mathbf{W}^{\text{M}}) = \left(\frac{\partial \mathbf{b}^{\text{M}\prime}}{\partial \boldsymbol{\xi}^{\text{M}}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\mathbf{W}^{\text{M}}\frac{\partial \mathbf{b}^{\text{M}}}{\partial \boldsymbol{\xi}^{\text{M}\prime}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\right)^{-1} \frac{\partial \mathbf{b}^{\text{M}\prime}}{\partial \boldsymbol{\xi}^{\text{M}}}(G_0, \boldsymbol{\xi}_0^{\text{M}})\mathbf{W}^{\text{M}}, \tag{A-59}$$

positive semidefiniteness can be shown by adapting the proof provided by Hall (2005), p. 88-90. The result is essentially a special case of Gourieroux et al.'s (1993) results with a GMM-type criterion function.

To prove the second part of Proposition 4 (optimal weighting in the second step, given the weighting in the first), we turn to Equations (A-41) and (A-51), which imply

$$\sqrt{T}(\hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}}) \xrightarrow{d} \mathbf{B}(\mathbf{W}^{\mathrm{P}})(\mathbf{C}(\mathbf{W}^{\mathrm{M}})\mathbf{X}_1 + \mathbf{X}_2), \tag{A-60}$$

where $\mathbf{X}_2 \sim \mathcal{N}(\mathbf{0}, \left(1 + \frac{1}{H}\right) \mathbf{A}_0^{\mathrm{P}} \mathbf{S}^{\mathrm{P}} \mathbf{A}_0^{\mathrm{P}\prime})$, with $\mathbf{S}^{\mathrm{P}}$ defined in Equation (46). Equation (A-51) implies that $\mathbb{E}(\mathbf{X}_1 \mathbf{X}_2') = \left(1 + \frac{1}{H}\right) \mathbf{A}_0^{\mathrm{M}} \mathbf{S}^{\mathrm{MP}\prime} \mathbf{A}_0^{\mathrm{P}\prime}$, where $\mathbf{S}^{\mathrm{MP}}$ is defined in Equation (46), and

$$\mathbf{C}(\mathbf{W}^{\mathrm{M}})\mathbf{X}_1 + \mathbf{X}_2 \sim \mathcal{N}\left(\mathbf{0}, \left(1 + \frac{1}{H}\right) \boldsymbol{\Sigma}(\mathbf{W}^{\mathrm{M}})\right), \tag{A-61}$$

where

$$\boldsymbol{\Sigma}(\mathbf{W}^{\mathrm{M}}) = \mathbf{A}_0^{\mathrm{P}} \mathbf{S}^{\mathrm{P}} \mathbf{A}_0^{\mathrm{P}\prime} + \mathbf{C}(\mathbf{W}^{\mathrm{M}})(\mathbf{A}_0^{\mathrm{M}} \mathbf{S}^{\mathrm{M}} \mathbf{A}_0^{\mathrm{M}\prime}) \mathbf{C}(\mathbf{W}^{\mathrm{M}})'$$
$$+ \mathbf{C}(\mathbf{W}^{\mathrm{M}}) \mathbf{A}_0^{\mathrm{M}} \mathbf{S}^{\mathrm{MP}\prime} \mathbf{A}_0^{\mathrm{P}\prime} + \mathbf{A}_0^{\mathrm{P}} \mathbf{S}^{\mathrm{MP}} \mathbf{A}_0^{\mathrm{M}\prime} \mathbf{C}(\mathbf{W}^{\mathrm{M}})'. \tag{A-62}$$

We can therefore conclude that

$$\sqrt{T}(\hat{\boldsymbol{\xi}}_T^{\mathrm{P}} - \boldsymbol{\xi}_0^{\mathrm{P}}) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathrm{Avar}^{\mathrm{P}}(\mathbf{W}^{\mathrm{M}}, \mathbf{W}^{\mathrm{P}})), \tag{A-63}$$

where

$$\mathrm{Avar}^{\mathrm{P}}(\mathbf{W}^{\mathrm{M}}, \mathbf{W}^{\mathrm{P}}) = \left(1 + \frac{1}{H}\right) \mathbf{B}(\mathbf{W}^{\mathrm{P}}) \boldsymbol{\Sigma}(\mathbf{W}^{\mathrm{M}}) \mathbf{B}(\mathbf{W}^{\mathrm{P}})'. \tag{A-64}$$

To prove the second part of Proposition 4, we must show that

$$\mathrm{Avar}^{\mathrm{P}}(\mathbf{W}^{\mathrm{M}}, \mathbf{W}^{\mathrm{P}*}) - \mathrm{Avar}^{\mathrm{P}}(\mathbf{W}^{\mathrm{M}}, \mathbf{W}^{\mathrm{P}}), \tag{A-65}$$

where $\mathbf{W}^{\mathrm{P}*} = (\mathbf{\Sigma}(\mathbf{W}^{\mathrm{M}}))^{-1}$, is a positive semidefinite matrix. Observing the structure of

$$\mathbf{B}(\mathbf{W}^{\mathrm{P}}) = \left(\frac{\partial \mathbf{b}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) \mathbf{W}^{\mathrm{P}} \frac{\partial \mathbf{b}^{\mathrm{P}}}{\partial \boldsymbol{\xi}^{\mathrm{P}\prime}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) \right)^{-1} \frac{\partial \mathbf{b}^{\mathrm{P}\prime}}{\partial \boldsymbol{\xi}^{\mathrm{P}}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}}) \mathbf{W}^{\mathrm{P}}, \quad \text{(A-66)}$$

we can again establish the positive semidefiniteness by adapting the aforementioned proof provided by Hall (2005). $\qquad\square$

A.8 Bootstrap inference

As an alternative to using the asymptotic results, the parametric structure of the LRR model suggests the use of a bootstrap simulation. After performing the two-step estimation on the empirical data, which yields the estimates $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$, we independently draw $4 \times (T^* + L)$ standard normally distributed random variables to obtain realizations of the i.i.d. innovations $\{\eta_t\}_{t=1}^{T^*+L}$, $\{e_t\}_{t=1}^{T^*+L}$, $\{u_t\}_{t=1}^{T^*+L}$, and $\{w_t\}_{t=1}^{T^*+L}$ in Equations (1)–(4). The appropriate time series length T^* is determined by the number of observations and the sampling frequency of the empirical data, as well as the assumed decision frequency of the investor. For example, the data used for our empirical application comprise $T{=}271$ quarterly observations. We assume a monthly decision frequency, such that $T^*{=}813$. The simulated innovations are used to generate time series of length $T^*{+}L$ for the LRR model-implied macro and financial variables, as described in Appendix A.2. For that purpose, $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$ serve as "true" parameters. If the empirical data frequency is lower than the decision frequency, the simulated time series are time-aggregated to match the empirical data frequency. The first L observations are discarded to mitigate the effect of the choice of starting values; we use $L = 100$ as a default.

The two indirect inference estimation steps are then performed on the bootstrap sample. Data simulation and estimation are repeated R independent times, with new i.i.d. draws of standard normally distributed innovations, simulation of the LRR model variables, and the two-step estimation performed on the simulated samples. The resulting sequences of estimates $\{\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}(r)\}_{r=1}^R$ and $\{\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}(r)\}_{r=1}^R$ serve to compute the parameter standard errors and construct confidence intervals. The latter are obtained by the percentile method, which amounts to using the appropriate quantiles of the bootstrap distribution as upper and lower bounds (Efron and Tibshirani, 1993).

To assess its validity, we have to check the conditions in which the bootstrap is consistent, such that the bootstrap estimator of the distribution function (cdf) of the statistic of interest (here, one of the parameter estimates in $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ or $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$) is uniformly close to the statistic's asymptotic cdf for large T. The formal definition and conditions for consistency of the bootstrap are provided by Horowitz (2001).[16] Briefly, consistency requires that the cdf of the probability distribution from which the data are sampled and its bootstrap estimator are uniformly close to each other when T is large, and that suitable continuity conditions regarding the asymptotic cdf of the statistic of interest hold.

Although the conditions for consistency cannot be checked formally in the present application, we argue that the proposed procedure is not subject to the issues that are known to provoke failure of the bootstrap. As Horowitz (2001) notes, failures of the bootstrap are associated with heavy-tailed or dependent data, or else true parameters that lie on the boundary of the parameter space. The i.i.d. draws of innovations from the standard normal distribution, along with economically plausible LRR model parameters, preclude heavy-tailed data. The parametric residual boot-

---

[16]See Horowitz's (2001) Definition 2.1 and Theorem 2.1, as originally formulated by Beran and Ducharme (1991).

strap also avoids drawing directly from the macro and financial data series, which may exhibit considerable serial dependence. Provided that the parameter estimates are consistent, the bootstrap estimate thus should constitute a good approximation of the true cdf of the data for large T. Furhtermore, violations of the continuity assumption regarding the asymptotic cdfs of the parameter estimates are not indicated; the intricate parameter space should not affect the validity of the bootstrap. However, we must assume that the LRR model is solvable in the neighborhood of the true parameters; in other words, we must rule out that a true parameter lies on the boundaries of the admissible parameter space. In the present application, the bootstrap does not provide asymptotic refinement, because the statistics of interest (i.e. the elements of $\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\xi}}_T^{\mathrm{P}}$) are not pivotal.

A.9 Web Appendix

<http://tinyurl.com/GS2017-Web-Appendix> contains a pdf with detailed derivations of the LRR model equations, additional results and discussions, and the Matlab code to perform the two-step indirect inference estimation, as well as the data used for the empirical analysis.

References

Aldrich, E. M., Gallant, A. R., 2011. Habit, long-run risks, prospect? A statistical inquiry. Journal of Financial Econometrics 9 (4), 589–618.

Baillie, R. T., Chung, H., 2001. Estimation of GARCH models from the autocorrelations of the squares of a process. Journal of Time Series Analysis 22 (6), 631–650.

Bansal, R., Gallant, A. R., Tauchen, G., 2007. Rational pessimism, rational exuberance, and asset pricing models. Review of Economic Studies 74 (4), 1005–1033.

Bansal, R., Khatchatrian, V., Yaron, A., 2005. Interpretable asset markets? European Economic Review 49 (3), 531–560.

Bansal, R., Kiku, D., Yaron, A., 2012a. An empirical evaluation of the long-run risks model for asset prices. Critical Finance Review 1 (1), 183–221.

Bansal, R., Kiku, D., Yaron, A., 2012b. Risks for the long run: Estimation with time aggregation. Working Paper 18305, National Bureau of Economic Research.

Bansal, R., Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. Journal of Finance 59 (4), 1481–1509.

Barro, R. J., 2006. Rare disasters and asset markets in the twentieth century. Quarterly Journal of Economics 121 (3), 823–866.

Beeler, J., Campbell, J. Y., 2012. The long-run risks model and aggregate asset prices: An empirical assessment. Critical Finance Review 1 (1), 141–182.

Beran, J., Ducharme, G., 1991. Asymptotic theory for bootstrap methods in statistics, Les Publications CRM, Centre de recherches mathémathiques, Université de Montreal.

Calvet, L. E., Czellar, V., 2015. Through the looking glass: Indirect inference via simple equilibria. Journal of Econometrics 185 (2), 343–358.

Calzorari, G., Fiorentini, G., Sentana, E., 2004. Constrained indirect estimation. Review of Economic Studies 71 (4), 945–973.

Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107 (2), 205–251.

Campbell, J. Y., Shiller, R. J., 1988. The dividend-price ratio and expectations of future dividends and discount factors. Review of Financial Studies 1 (3), 195–228.

Cecchetti, S. G., Lam, P.-S., Nelson, M. C., 1993. The equity premium and the risk-free rate: Matching the moments. Journal of Monetary Economics 31 (1), 21–45.

Chen, S. X., 1999. Beta kernel estimators for density functions. Computational Statistics & Data Analysis 31 (2), 131–145.

Cochrane, J. H., 2005. The risk and return of venture capital. Journal of Financial Economics 75 (1), 3 – 52.

Constantinides, G. M., Ghosh, A., 2011. Asset pricing tests with long-run risks in consumption growth. Review of Asset Pricing Studies 1 (1), 96–136.

Corsi, F., 2009. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics 7 (2), 174–196.

Drechsler, I., Yaron, A., 2011. What's vol got to do with it. Review of Financial Studies 24 (1), 1–45.

Dridi, R., Guay, A., Renault, E., 2007. Indirect inference and calibration of dynamic stochastic general equilibrium models. Journal of Econometrics 136 (2), 397 – 430.

Efron, B., Tibshirani, R. J., 1993. An introduction to the bootstrap. Chapman & Hall.

Epstein, L. G., Zin, S. E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57 (4), 937–969.

Gallant, A. R., Tauchen, G., 1996. Which moments to match? Econometric Theory 12, 657–681.

Gourieroux, C., Monfort, A., Renault, E., 1993. Indirect inference. Journal of Applied Econometrics 8 (S1), S85–S118.

Grammig, J., Küchlin, E.-M., 2015. Give me strong moments and time: Combining GMM and SMM to estimate the long-run risk asset pricing model, Working Paper 14-05, Centre for Financial Research, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2508125.

Hall, A., 2005. Generalized Method of Moments. Advanced Texts in Econometrics. Oxford University Press.

Hansen, L. P., 1982. Large sample properties of generalized method of moments estimators. Econometrica 50 (4), 1029–1054.

Hasseltoft, H., 2012. Stocks, bonds, and long-run consumption risks. Journal of Financial and Quantitative Analysis 47, 309–332.

Havránek, T., 2015. Measuring intertemporal substitution: The importance of method choices and selective reporting. Journal of the European Economic Association 13 (6), 1180–1204.

Horowitz, J. L., 2001. The Bootstrap. In: Handbook of Econometrics. Vol. 5. Elsevier, pp. 3159–3228.

Mehra, R., Prescott, E. C., 1985. The equity premium – A puzzle. Journal of Monetary Economics 15, 145–161.

Nelder, J. A., Mead, R., 1965. A simplex method for function minimization. Computer Journal 7 (4), 308–313.

Schorfheide, F., Song, D., Yaron, A., 2014. Identifying long-run risks: A Bayesian mixed-frequency approach. Working paper, National Bureau of Economic Research.

Shephard, N. G., Harvey, A., 1990. On the probability of estimating a deterministic component in the local level model. Journal of Time Series Analysis 11, 339–347.

Silverman, B. W., 1986. Density estimation for statistics and data analysis. Chapman & Hall.

Singleton, K. J., 2006. Empirical Dynamic Asset Pricing. Princeton University Press.

Smith, A. A., 1993. Estimating nonlinear time-series models using simulated vector autoregressions. Journal of Applied Econometrics 8 (S1), 63–84.

Thimme, J., 2017. Intertemporal substitution in consumption: A literature review. Journal of Economic Surveys 31, 226?257.

Yogo, M., 2006. A consumption-based explanation of expected stock returns. Journal of Finance 61 (2), 539–580.

Table 1: LRR model parameter values as calibrated by Bansal and Yaron (2004)

μ_c	μ_d	ρ	φ_e	ν_1	σ_w	σ	ϕ	φ_d	δ	γ	ψ
0.0015	0.0015	0.979	0.044	0.987	$2.3 \cdot 10^{-6}$	0.0078	3	4.5	0.998	10	1.5

Table 2: Monte Carlo results: first-step estimates

	μ_c	μ_d	ρ	φ_e	σ	ϕ	φ_d	$\tilde{R}$
true parameter	0.0015	0.0015	0.979	0.044	0.0078	3	4.5	
Panel A: Indirect inference, σ_t^2 predicted by σ^2								
T=275	*0.0016*	*0.0023*	*0.946*	*0.0745*	*0.0075*	*3.38*	*4.57*	245
	0.0013	0.0034	0.265	0.1198	0.0013	3.07	1.95	
T=1k	*0.0015*	*0.0018*	*0.965*	*0.0555*	*0.0080*	*2.97*	*4.36*	348
	0.0006	0.0017	0.218	0.0642	0.0005	1.85	0.24	
T=100k	*0.0015*	*0.0015*	*0.980*	*0.0430*	*0.0078*	*2.95*	*4.50*	400
	0.0001	0.0002	0.004	0.0046	0.0000	0.17	0.02	
Panel B: Indirect inference, ν_1 and σ_w known								
T=275	*0.0016*	*0.0023*	*0.938*	*0.0736*	*0.0076*	*3.45*	*4.54*	252
	0.0012	0.0034	0.328	0.1176	0.0015	3.69	1.06	
T=1k	*0.0015*	*0.0017*	*0.967*	*0.0496*	*0.0079*	*3.00*	*4.33*	347
	0.0006	0.0016	0.274	0.0696	0.0005	2.48	0.27	
T=100k	*0.0015*	*0.0015*	*0.981*	*0.0429*	*0.0078*	*2.94*	*4.50*	400
	0.0001	0.0002	0.004	0.0046	0.0000	0.17	0.02	
Panel C: GMM								
T=100k	*0.0015*	*0.0015*	*0.958*	*0.0620*	*0.0078*	*2.83*	*4.51*	397
	0.0001	0.0002	0.110	0.0559	0.0000	1.62	0.04	

Note: The table reports the medians (in italics) and the RMSEs (normal font) of the first-step macro parameter estimates obtained by the Monte Carlo study. The last column contains the number of successfully estimated replications $\tilde{R}$.

Table 3: Monte Carlo results: second-step estimates

	Panel A $\boldsymbol{\xi}^{\mathrm{M*}}$ known		Panel B $\boldsymbol{\xi}^{\mathrm{M}}$ known		Panel C $\boldsymbol{\xi}^{\mathrm{M*}}$ estimated	
δ=0.998						
T=275	*0.9979*	0.0004	*0.9980*	0.0004	*0.9973*	0.0033
	[0.9970	0.9987]	[0.9970	0.9988]	[0.9930	1.0001]
T=1k	*0.9980*	0.0002	*0.9980*	0.0002	*0.9975*	0.0014
	[0.9976	0.9985]	[0.9976	0.9985]	[0.9945	1.0000]
T=100k	*0.9980*	0.0000	*0.9980*	0.0000	*0.9980*	0.0002
	[0.9979	0.9980]	[0.9980	0.9980]	[0.9977	0.9984]
γ=10						
T=275	*10.1*	2.0	*9.8*	1.8	*13.7*	18.9
	[5.8	14.2]	[5.8	13.2]	[4.0	71.6]
T=1k	*10.6*	1.2	*10.2*	1.0	*12.9*	14.6
	[8.4	12.7]	[8.3	12.0]	[5.6	55.2]
T=100k	*10.3*	0.3	*10.0*	0.1	*10.0*	1.1
	[10.1	10.6]	[9.8	10.2]	[8.3	12.5]
ψ=1.5						
T=275	*1.51*	0.02	*1.51*	0.02	*2.41*	3.81
	[1.46	1.55]	[1.46	1.55]	[0.66	11.43]
T=1k	*1.51*	0.02	*1.51*	0.02	*2.12*	2.66
	[1.48	1.54]	[1.48	1.54]	[0.72	10.59]
T=100k	*1.51*	0.01	*1.51*	0.01	*1.48*	0.16
	[1.51	1.51]	[1.51	1.51]	[1.20	1.85]
Successful replications						
T=275	400		400		153	
T=1k	400		400		289	
T=100k	400		400		400	

Note: The table reports the medians (in italics) and the RMSEs (normal font) along with the 95% confidence bounds (in brackets) of the second-step indirect inference parameter estimates. $\boldsymbol{\xi}^{\mathrm{M*}} = (\mu_c, \mu_d, \rho, \varphi_e, \sigma, \phi, \varphi_d, 0, 0)'$.

Table 4: Data descriptives

	mean	std. dev.	AC(1)
log consumption growth g	0.0048	0.0051	0.3116
log dividend growth g_d	0.0066	0.0247	0.4443
log market return r_m	0.0176	0.0825	0.0840
log risk-free rate r_f	0.0017	0.0045	0.9138
log price-dividend ratio z_m	3.4979	0.4217	0.9804

Note: The data are on a quarterly frequency and range from 1947Q2 to 2014Q4. AC(1) is the first-order autocorrelation. A four-quarter moving average of the raw log dividend growth time series is used to obtain g_d.

Table 5: Estimation results

	μ_c	μ_d	ρ	φ_e	σ	ϕ	φ_d	δ	γ	ψ
estimate	0.0017	0.0019	0.991	0.0643	0.0024	5.14	3.06	0.99998	11.8	0.29
s.e.$_\mathrm{b}$	0.0007	0.0028	0.095	0.0639	0.0008	2.18	3.45	0.00446	27.1	0.24
s.e.$_\mathrm{a}$	*0.0002*	*0.0009*	*0.010*	*0.0292*	*0.0005*	*1.24*	*0.80*	*0.02603*	*15.8*	*0.18*
low.	0.0011	0.0000	0.757	0.0220	0.0002	2.72	1.68	0.98399	2.2	0.22
upp.	0.0033	0.0088	1.000	0.2687	0.0029	8.96	16.29	1.00036	110.3	1.20

Note: The table reports two-step indirect inference estimates, along with bootstrap standard errors (denoted by s.e.$_\mathrm{b}$) and standard errors based on the asymptotic results (denoted by s.e.$_\mathrm{a}$, in italics). The computation of the asymptotic standard errors uses a consistent estimate of the asymptotic variance-covariance matrix in Equation (40). The derivatives of the binding functions in Equations (41), (42) and (43) are computed numerically by replacing $\mathbf{b}^{\mathrm{M}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}})$ by $\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{M}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \mathbf{z}_0)$ and $\mathbf{b}^{\mathrm{P}}(G_0, \boldsymbol{\xi}_0^{\mathrm{M}}, \boldsymbol{\xi}_0^{\mathrm{P}})$ by $\tilde{\boldsymbol{\theta}}_{HT}^{\mathrm{P}}(\hat{\boldsymbol{\xi}}_T^{\mathrm{M}}, \hat{\boldsymbol{\xi}}_T^{\mathrm{P}}, \mathbf{z}_0)$. In Equations (44) and (45), expected values are replaced by sample means and $\boldsymbol{\theta}_0^{\mathrm{M}}$ and $\boldsymbol{\theta}_0^{\mathrm{P}}$ are replaced by their consistent estimates $\hat{\boldsymbol{\theta}}_T^{\mathrm{M}}$ and $\hat{\boldsymbol{\theta}}_T^{\mathrm{P}}$. The estimation of $\mathbf{S}$ in Equation (38) uses a Bartlett kernel-based estimate with bandwidth equal to 10. Bootstrapped upper and lower 95% confidence bounds are obtained by the percentile method.

Table 6: Sample and model-implied means and standard deviations

	data	**model-implied**	s.e.$_b$
$\mathbb{E}(g)$	0.0048	0.0050	0.0037
$\mathbb{E}(g_d)$	0.0066	0.0056	0.0140
$\mathbb{E}(z_m)$	3.4979	3.6305	0.5112
$\mathbb{E}(r_m)$	0.0176	0.0322	0.0149
$\mathbb{E}(r_f)$	0.0017	0.0262	0.0146
$\sigma(g)$	0.0051	0.0049	0.0017
$\sigma(g_d)$	0.0247	0.0208	0.0059
$\sigma(z_m)$	0.4217	0.1153	0.2947
$\sigma(r_m)$	0.0825	0.0315	0.0198
$\sigma(r_f)$	0.0045	0.0120	0.0078

Note: The second column reports the empirical sample means and standard deviations of the observable LRR model variables. The counterparts implied by the point estimates in Table 5 are reported in the third column. The fourth column reports the bootstrap standard errors of the model-implied estimates. All quantities computed reflect a quarterly frequency. To obtain the model-implied means and standard deviations, the parameter estimates are used to simulate LRR model-implied data for 10^6 months. The monthly series are then time-aggregated to the quarterly frequency, at which the respective means and standard deviations are computed. The bootstrap standard errors are obtained by the standard deviations of the model-implied means and standard deviations across the bootstrap replications.

Table 7: Comparison of preference parameter estimates

	$\hat{\delta}$	$\hat{\gamma}$	$\hat{\psi}$	T/Freq.
Two-step ind. inference	0.99998 [0.98399 1.00036]	11.8 [2.2 110.3]	0.29 [0.22 1.20]	271/Q
Yogo (2006)	0.9000 [0.7922 1.0078]	191.4 [93.7 289.2]	0.024 [0.006 0.042]	204/Q
BGT (2007)	0.9996 [0.9989 1.0002]	7.1 [-0.3 14.6]	2 (c)	73/Y
CG (2011)	0.968 [0.8563 1.0797]	9.3 [-0.1 18.8]	1.41 [-4.35 7.17]	79/Y
Hasseltoft (2012)	0.9992 (c)	6.8 [3.6 9.9]	2.51 [1.06 3.96]	223/Q
BKY (2012)	0.9989 [0.9969 1.0009]	7.4 [4.4 10.5]	2.05 [0.40 3.70]	80/Y
CC (2015)	1.0081 [1.0034 1.0129]	27.1 n.a.	0.20 [0.04 0.36]	247/Q
BY (2004)	0.9980 (c)	10 (c)	1.5 (c)	70/Y

Note: The table reports the point estimates of the preference parameters and the bounds of the 95% confidence intervals (in brackets). We compare the two-step indirect inference results with the results reported in studies by Yogo (2006), Bansal, Gallant, and Tauchen (2007) (BGT), Constantinides and Ghosh (2011) (CG), Hasseltoft (2012), Bansal, Kiku, and Yaron (2012b) (BKY), Calvet and Czellar (2015) (CC), as well as with the calibrated values by Bansal and Yaron (2004) (BY). Calibrated parameters are indicated with (c). The confidence bounds for the other studies are computed using the reported standard errors. Sample size and data frequency appear in the last column.

(a)

(b)
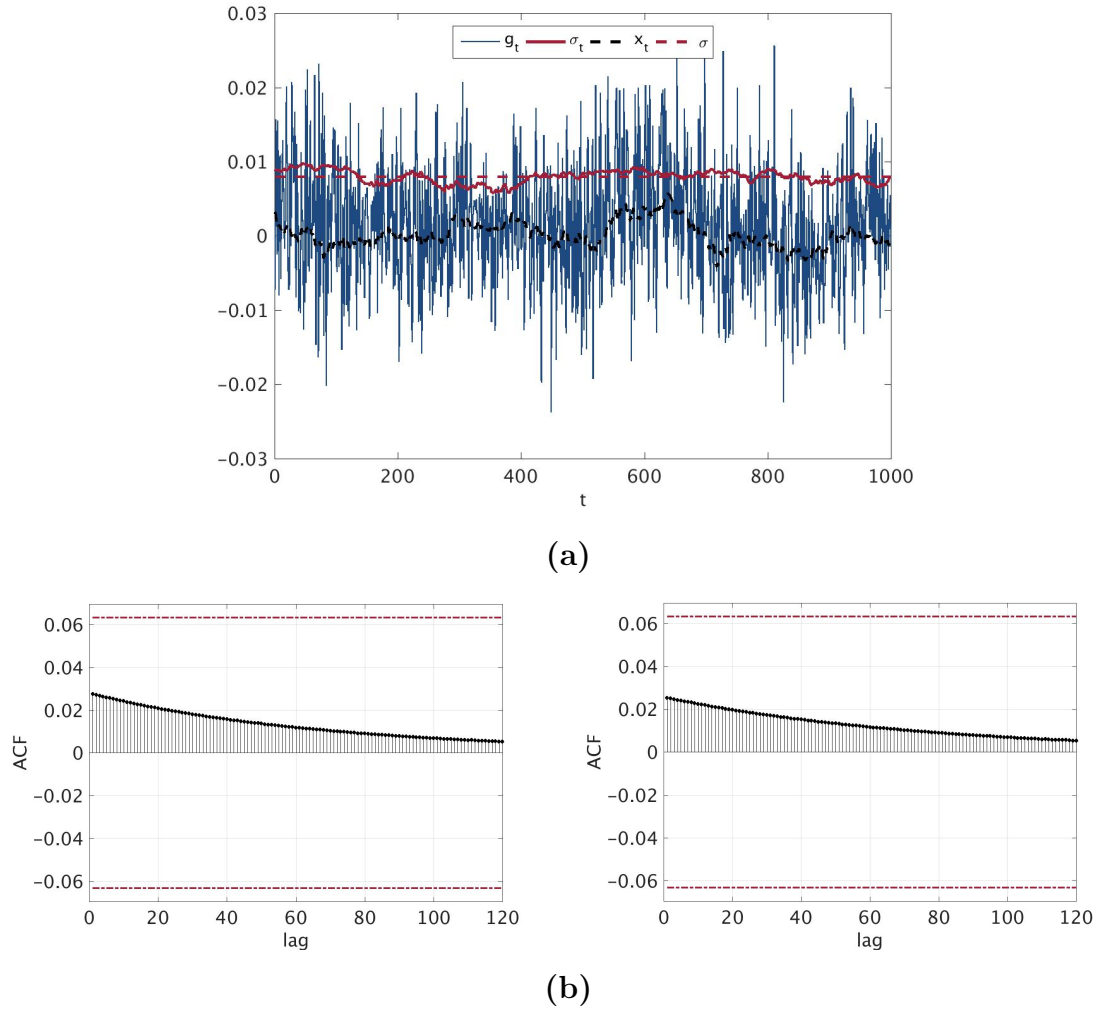
Figure 1: *Fluctuating macroeconomic uncertainty and growth expectations in BY's calibrated economy.* Panel (a) displays simulated data of length T=1k for log consumption growth g_t, stochastic volatility σ_t and the predictable growth component x_t using the parameter values from Table 1. Panel (a) also shows the unconditional volatility $\sigma = \sqrt{\mathbb{E}(\sigma_t^2)}$ from Table 1. Panel (b) presents the calibration-implied theoretical autocorrelations of squared consumption (left) and dividend growth (right), along with the $\pm 2\sqrt{T}$ confidence bounds for T=1k.
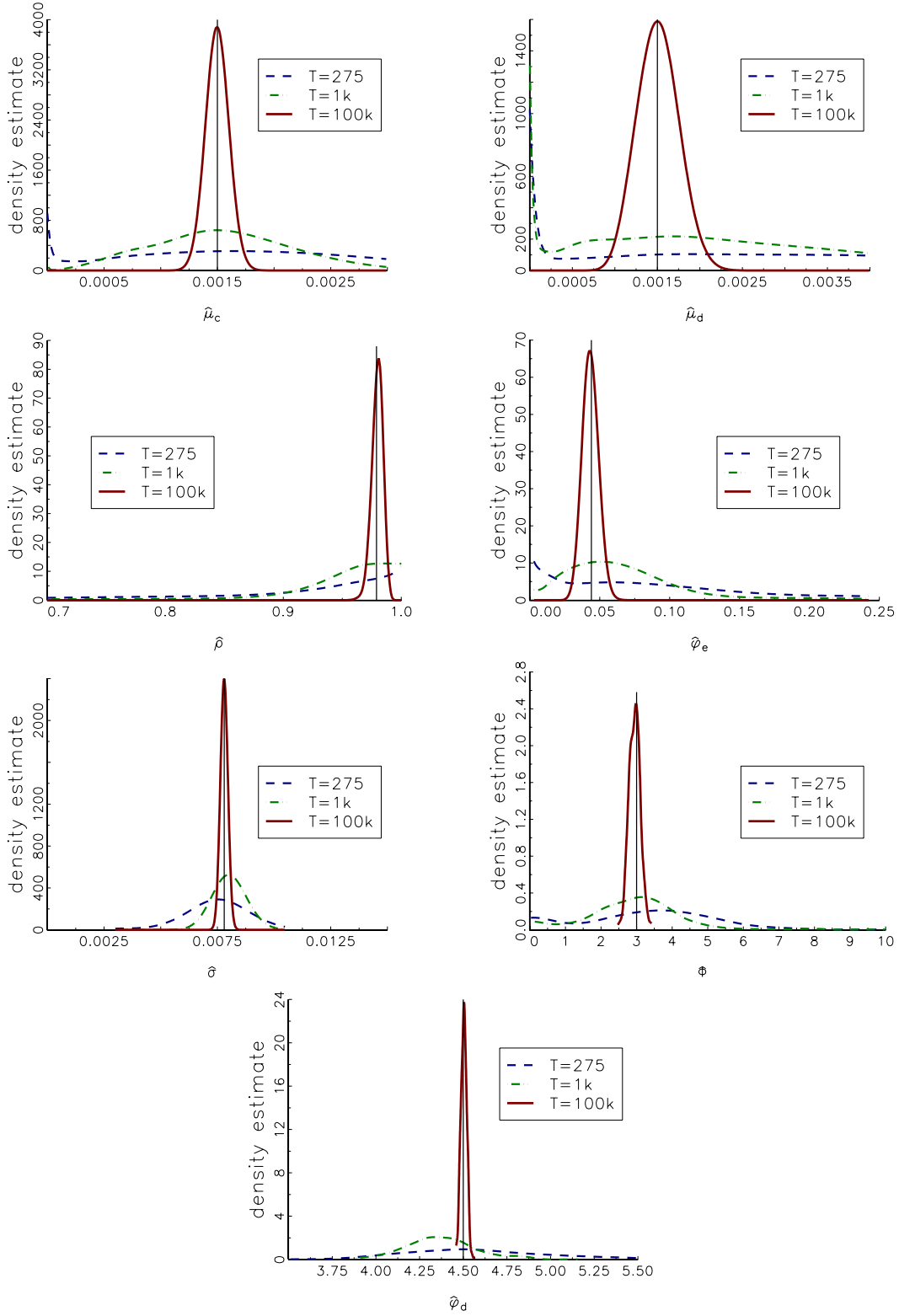
Figure 2: *Monte Carlo results: distribution of first-step indirect inference estimates.* This figure shows kernel estimates across simulated sample sizes. The beta kernel proposed by Chen (1999) is used together with the bandwidth selector by Silverman (1986), adjusted for variable kernels. The vertical lines indicate the positions of the true parameter value.

69
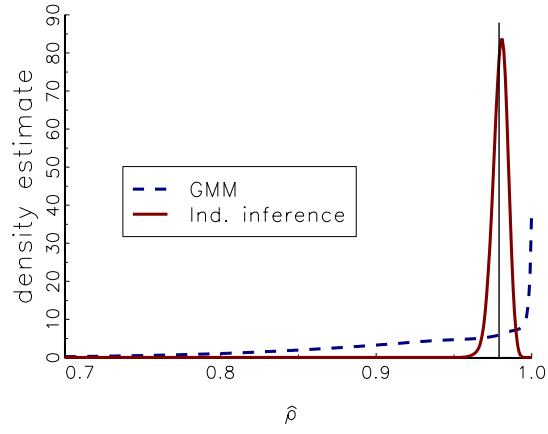
Figure 3: *Monte Carlo results: asymptotic efficiency indirect inference vs. GMM.* This figure shows the kernel estimates of the LRR parameter estimate $\hat{\rho}$ implied by the indirect inference estimation strategy and GMM. The simulated sample size is $T = 100$k. The beta kernel proposed by Chen (1999) is used together with the bandwidth selector by Silverman (1986), adjusted for variable kernels. The vertical line indicates the position of the true parameter value.
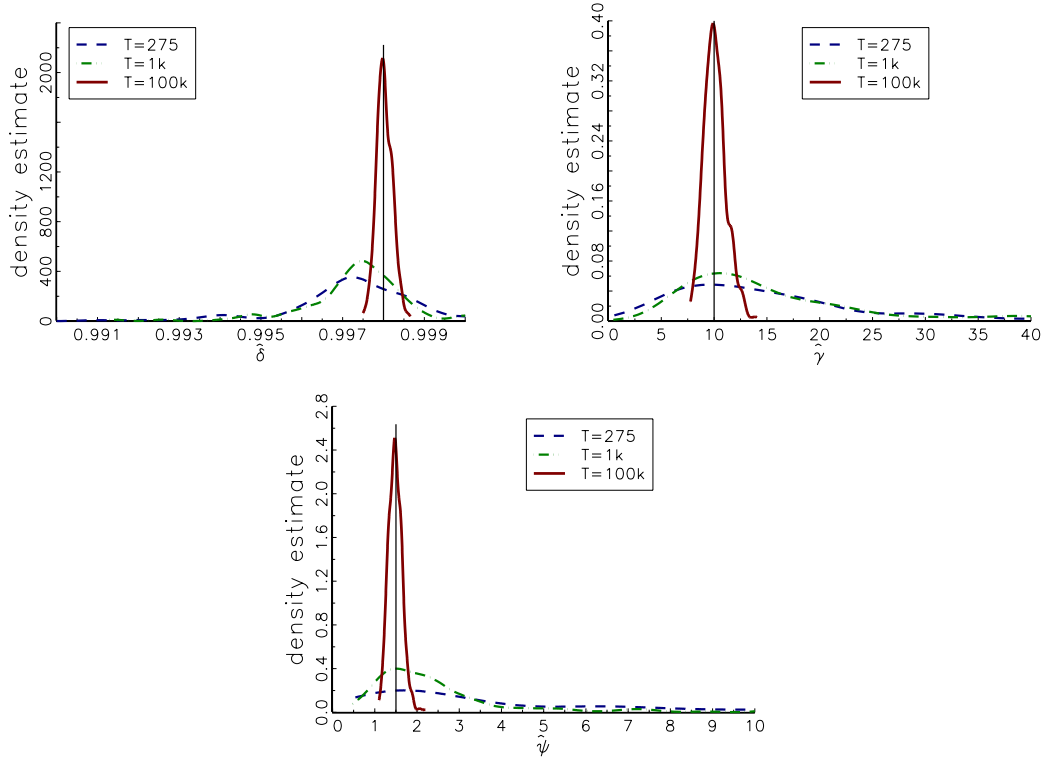
Figure 4: *Monte Carlo results: distribution of second-step indirect inference estimates.* This figure displays kernel estimates for $\hat{\delta}$, $\hat{\gamma}$, and $\hat{\psi}$ obtained in the second estimation step. σ_t^2 is replaced by $\hat{\sigma}^2$ when simulating data in the course of indirect inference estimation. The vertical lines indicate the positions of the true parameter values.
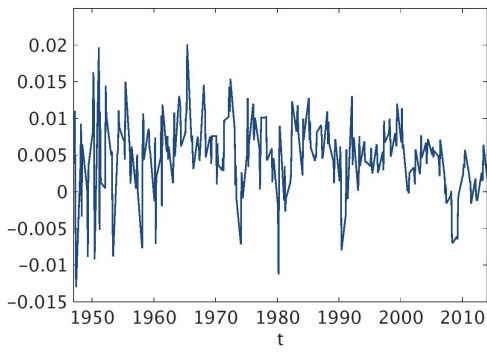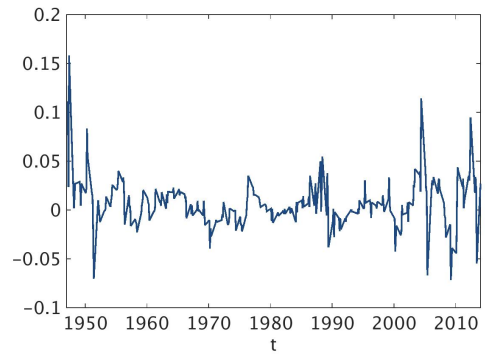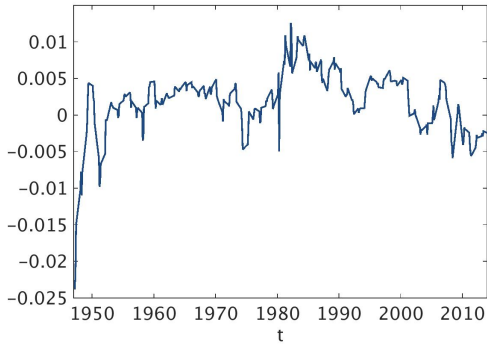
(a) log consumption growth g_t
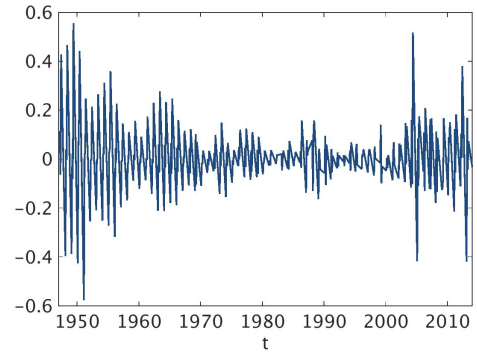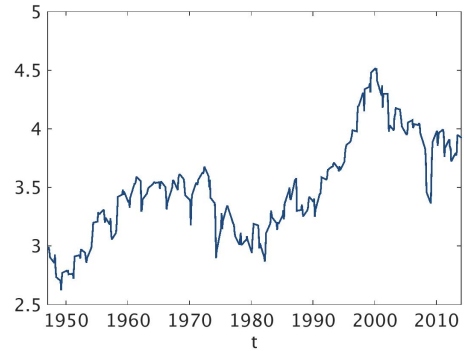
(b) log dividend growth (raw)
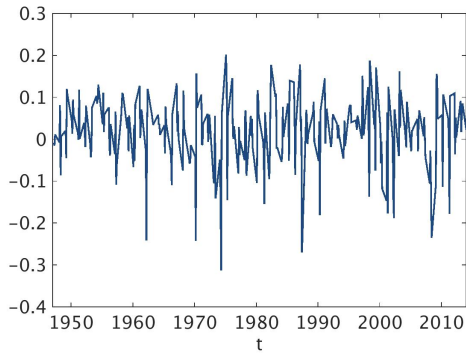
(c) log market return $r_{m,t}$

(d) log dividend growth moving avg. $g_{d,t}$

(e) log risk-free rate $r_{f,t}$

(f) log price-dividend ratio $z_{m,t}$

Figure 5: *Empirical data series.* This figure displays the time series used in the empirical application. The sample period is 1947Q2 to 2014Q4.

72

cfr working paper series

centre for financial research
cologne

cfr working papers are available for download from www.cfr-cologne.de.

Hardcopies can be ordered from: centre for financial research (cfr),
Albertus Magnus Platz, 50923 Koeln, Germany.

2017

No.	Author(s)	Title
17-01	J.Grammig, E.-M. Küchlin	A two-step indirect inference approach to estimate the long-run risk asset pricing model

2016

No.	Author(s)	Title
16-12	A.Betzer, M. Ibel, H.S. Lee, P. Limbach, J.M. Salas	Are Generalists Beneficial to Corporate Shareholders? Evidence from Sudden Deaths
16-11	P. Limbach, M. Schmid, M. Scholz-Daneshgari	Do CEOs Matter? Corporate Performance and the CEO Life Cycle
16-10	V. Agarwal, R. Vashishtha, M. Venkatachalam	Mutual fund transparency and corporate myopia
16-09	M.-A. Göricke	Do Generalists Profit from the Fund Families' Specialists? Evidence from Mutual Fund Families Offering Sector Funds
16-08	S. Kanne, O. Korn, M.Uhrig-Homburg	Stock Illiquidity, Option Prices and Option Returns
16-07	S. Jaspersen	Market Power in the Portfolio: Product Market Competition and Mutual Fund Performance
16-06	O. Korn, M.-O. Rieger	Hedging With Regret
16-05	E. Theissen, C. Westheide	Call of Duty: Designated Market Maker Participation in Call Auctions
16-04	P. Gomber, S. Sagade, E. Theissen, M.C. Weber, C. Westheide	Spoilt for Choice: Order Routing Decisions in Fragmented Equity Markets
16-03	T.Martin, F. Sonnenburg	Managerial Ownership Changes and Mutual Fund Performance
16-02	A.Gargano, A. G. Rossi, R. Wermers	The Freedom of Information Act and the Race Towards Information Acquisition
16-01	G. Cici, S. Gibson, C. Rosenfeld	Cross-Company Effects of Common Ownership: Dealings Between Borrowers and Lenders With a Common Blockholder

2015

No.	Author(s)	Title
15-17	O. Korn, L. Kuntz	Low-beta Strategies
15-16	D. Blake, A.G. Rossi, A. Timmermann, I. Tonks, R. Wermers	Network Centrality and Pension Fund Performance
15-15	S. Jank, E. Smajbegovic	Dissecting Short-Sale Performance: Evidence from Large Position Disclosures
15-14	M. Doumet, P. Limbach, E. Theissen	Ich bin dann mal weg: Werteffekte von Delistings deutscher Aktiengesellschaften nach dem Frosta-Urteil
15-13	G. Borisova, P.K. Yadav	Government Ownership, Informed Trading and Private Information
15-12	V. Agarwal, G.O. Aragon, Z. Shi	Funding Liquidity Risk of Funds of Hedge Funds: Evidence from their Holdings
15-11	L. Ederington, W. Guan, P.K. Yadav	Dealer spreads in the corporate Bond Market: Agent vs. Market-Making Roles
15-10	J.R. Black, D. Stock, P.K. Yadav	The Pricing of Different Dimensions of Liquidity: Evidence from Government Guaranteed Bank Bonds
15-09	V. Agarwal, H. Zhao	Interfund lending in mutual fund families: Role of internal capital markets
15-08	V. Agarwal, T. C. Green, H. Ren	Alpha or Beta in the Eye of the Beholder: What drives Hedge Fund Flows?
15-07	V. Agarwal, S. Ruenzi, F. Weigert	Tail risk in hedge funds: A unique view from portfolio holdings
15-06	C. Lan, F. Moneta, R. Wermers	Mutual Fund Investment Horizon and Performance
15-05	L.K. Dahm, C. Sorhage	Milk or Wine: Mutual Funds' (Dis)economies of Life
15-04	A. Kempf, D. Mayston, M. Gehde-Trapp, P. K. Yadav	Resiliency: A Dynamic View of Liquidity
15-03	V. Agarwal, Y. E. Arisoy, N. Y. Naik	Volatility of Aggregate Volatility and Hedge Funds Returns
15-02	G. Cici, S. Jaspersen, A. Kempf	Speed of Information Diffusion within Fund Families
15-01	M. Baltzer, S. Jank, E. Smajlbegovic	Who trades on momentum?

2014

No.	Author(s)	Title
14-15	M. Baltzer, S. Jank, E.Smajlbegovic	Who Trades on Monumentum?
14-14	G. Cici, L. K. Dahm, A. Kempf	Trading Efficiency of Fund Families: Impact on Fund Performance and Investment Behavior

No.	Author(s)	Title
14-13	V. Agarwal, Y. Lu, S. Ray	Under one roof: A study of simultaneously managed hedge funds and funds of hedge funds
14-12	P. Limbach, F. Sonnenburg	Does CEO Fitness Matter?
14-11	G. Cici, M. Gehde-Trapp, M. Göricke, A. Kempf	What They Did in their Previous Life: The Investment Value of Mutual Fund Managers' Experience outside the Financial Sector
14-10	O. Korn, P. Krischak, E. Theissen	Illiquidity Transmission from Spot to Futures Markets
14-09	E. Theissen, L. S. Zehnder	Estimation of Trading Costs: Trade Indicator Models Revisited
14-08	C. Fink, E. Theissen	Dividend Taxation and DAX Futures Prices
14-07	F. Brinkmann, O. Korn	Risk-adjusted Option-implied Moments
14-06	J. Grammig, J. Sönksen	Consumption-Based Asset Pricing with Rare Disaster Risk
14-05	J. Grammig, E. Schaub	Give me strong moments and time – Combining GMM and SMM to estimate long-run risk asset pricing
14-04	C. Sorhage	Outsourcing of Mutual Funds' Non-core Competencies
14-03	D. Hess, P. Immenkötter	How Much Is Too Much? Debt Capacity And Financial Flexibility
14-02	C. Andres, M. Doumet, E. Fernau, E. Theissen	The Lintner model revisited: Dividends versus total payouts
14-01	N.F. Carline, S. C. Linn, P. K. Yadav	Corporate Governance and the Nature of Takeover Resistance

2013

No.	Author(s)	Title
13-11	R. Baule, O. Korn, S. Saßning	Which Beta is Best? On the Information Content of Option-implied Betas
13-10	V. Agarwal, L. Ma, K. Mullally	Managerial Multitasking in the Mutual Fund Industry
13-09	M. J. Kamstra, L.A. Kramer, M.D. Levi, R. Wermers	Seasonal Asset Allocation: Evidence from Mutual Fund Flows
13-08	F. Brinkmann, A. Kempf, O. Korn	Forward-Looking Measures of Higher-Order Dependencies with an Application to Portfolio Selection
13-07	G. Cici, S. Gibson, Y. Gunduz, J.J. Merrick, Jr.	Market Transparency and the Marking Precision of Bond Mutual Fund Managers
13-06	S. Bethke, M. Gehde-Trapp, A. Kempf	Investor Sentiment, Flight-to-Quality, and Corporate Bond Comovement
13-05	P. Schuster, M. Trapp, M. Uhrig-Homburg	The Term Structure of Bond Liquidity
13-04	V. Agarwal, K. Mullally, Y. Tang, B. Yang	Mandatory Portfolio Disclosure, Stock Liquidity, and Mutual Fund Performance

13-03	V. Agarwal, V. Nanda, S.Ray	Institutional Investment and Intermediation in the Hedge Fund Industry
13-02	C. Andres, A. Betzer, M. Doumet, E. Theissen	Open Market Share Repurchases in Germany: A Conditional Event Study Approach
13-01	J. Gaul, E. Theissen	A Partially Linear Approach to Modelling the Dynamics of Spot and Futures Price

2012

No.	Author(s)	Title
12-12	M. Gehde-Trapp, Y. Gündüz, J. Nasev	The liquidity premium in CDS transaction prices: Do frictions matter?
12-11	Y. Wu, R. Wermers, J. Zechner	Governance and Shareholder Value in Delegated Portfolio Management: The Case of Closed-End Funds
12-10	M. Trapp, C. Wewel	Transatlantic Systemic Risk
12-09	G. Cici, A. Kempf, C. Sorhage	Do Financial Advisors Provide Tangible Benefits for Investors? Evidence from Tax-Motivated Mutual Fund Flows
12-08	S. Jank	Changes in the composition of publicly traded firms: Implications for the dividend-price ratio and return predictability
12-07	G. Cici, C. Rosenfeld	A Study of Analyst-Run Mutual Funds: The Abilities and Roles of Buy-Side Analysts
12-06	A. Kempf, A. Pütz, F. Sonnenburg	Fund Manager Duality: Impact on Performance and Investment Behavior
12-05	L. Schmidt, A. Timmermann, R. Wermers	Runs on Money Market Mutual Funds
12-04	R. Wermers	A matter of style: The causes and consequences of style drift in institutional portfolios
12-03	C. Andres, A. Betzer, I. van den Bongard, C. Haesner, E. Theissen	Dividend Announcements Reconsidered: Dividend Changes versus Dividend Surprises
12-02	C. Andres, E. Fernau, E. Theissen	Should I Stay or Should I Go? Former CEOs as Monitors
12-01	L. Andreu, A. Pütz	Choosing two business degrees versus choosing one: What does it tell about mutual fund managers' investment behavior?

2011

No.	Author(s)	Title
11-16	V. Agarwal, J.-P. Gómez, R. Priestley	Management Compensation and Market Timing under Portfolio Constraints
11-15	T. Dimpfl, S. Jank	Can Internet Search Queries Help to Predict Stock Market Volatility?
11-14	P. Gomber, U. Schweickert, E. Theissen	Liquidity Dynamics in an Electronic Open Limit Order Book: An Event Study Approach
11-13	D. Hess, S. Orbe	Irrationality or Efficiency of Macroeconomic Survey Forecasts? Implications from the Anchoring Bias Test

11-12	D. Hess, P. Immenkötter	Optimal Leverage, its Benefits, and the Business Cycle
11-11	N. Heinrichs, D. Hess, C. Homburg, M. Lorenz, S. Sievers	Extended Dividend, Cash Flow and Residual Income Valuation Models – Accounting for Deviations from Ideal Conditions
11-10	A. Kempf, O. Korn, S. Saßning	Portfolio Optimization using Forward - Looking Information
11-09	V. Agarwal, S. Ray	Determinants and Implications of Fee Changes in the Hedge Fund Industry
11-08	G. Cici, L.-F. Palacios	On the Use of Options by Mutual Funds: Do They Know What They Are Doing?
11-07	V. Agarwal, G. D. Gay, L. Ling	Performance inconsistency in mutual funds: An investigation of window-dressing behavior
11-06	N. Hautsch, D. Hess, D. Veredas	The Impact of Macroeconomic News on Quote Adjustments, Noise, and Informational Volatility
11-05	G. Cici	The Prevalence of the Disposition Effect in Mutual Funds' Trades
11-04	S. Jank	Mutual Fund Flows, Expected Returns and the Real Economy
11-03	G.Fellner, E.Theissen	Short Sale Constraints, Divergence of Opinion and Asset Value: Evidence from the Laboratory
11-02	S.Jank	Are There Disadvantaged Clienteles in Mutual Funds?
11-01	V. Agarwal, C. Meneghetti	The Role of Hedge Funds as Primary Lenders

2010

No.	Author(s)	Title
10-20	G. Cici, S. Gibson, J.J. Merrick Jr.	Missing the Marks? Dispersion in Corporate Bond Valuations Across Mutual Funds
10-19	J. Hengelbrock, E. Theissen, C. Westheide	Market Response to Investor Sentiment
10-18	G. Cici, S. Gibson	The Performance of Corporate-Bond Mutual Funds: Evidence Based on Security-Level Holdings
10-17	D. Hess, D. Kreutzmann, O. Pucker	Projected Earnings Accuracy and the Profitability of Stock Recommendations
10-16	S. Jank, M. Wedow	Sturm und Drang in Money Market Funds: When Money Market Funds Cease to Be Narrow
10-15	G. Cici, A. Kempf, A. Puetz	The Valuation of Hedge Funds' Equity Positions
10-14	J. Grammig, S. Jank	Creative Destruction and Asset Prices
10-13	S. Jank, M. Wedow	Purchase and Redemption Decisions of Mutual Fund Investors and the Role of Fund Families
10-12	S. Artmann, P. Finter, A. Kempf, S. Koch, E. Theissen	The Cross-Section of German Stock Returns: New Data and New Evidence
10-11	M. Chesney, A. Kempf	The Value of Tradeability
10-10	S. Frey, P. Herbst	The Influence of Buy-side Analysts on Mutual Fund Trading

10-09	V. Agarwal, W. Jiang, Y. Tang, B. Yang	Uncovering Hedge Fund Skill from the Portfolio Holdings They Hide
10-08	V. Agarwal, V. Fos, W. Jiang	Inferring Reporting Biases in Hedge Fund Databases from Hedge Fund Equity Holdings
10-07	V. Agarwal, G. Bakshi, J. Huij	Do Higher-Moment Equity Risks Explain Hedge Fund Returns?
10-06	J. Grammig, F. J. Peter	Tell-Tale Tails: A data driven approach to estimate unique market information shares
10-05	K. Drachter, A. Kempf	Höhe, Struktur und Determinanten der Managervergütung-Eine Analyse der Fondsbranche in Deutschland
10-04	J. Fang, A. Kempf, M. Trapp	Fund Manager Allocation
10-03	P. Finter, A. Niessen-Ruenzi, S. Ruenzi	The Impact of Investor Sentiment on the German Stock Market
10-02	D. Hunter, E. Kandel, S. Kandel, R. Wermers	Mutual Fund Performance Evaluation with Active Peer Benchmarks
10-01	S. Artmann, P. Finter, A. Kempf	Determinants of Expected Stock Returns: Large Sample Evidence from the German Market

2009

No.	Author(s)	Title
09-17	E. Theissen	Price Discovery in Spot and Futures Markets: A Reconsideration
09-16	M. Trapp	Trading the Bond-CDS Basis – The Role of Credit Risk and Liquidity
09-15	A. Betzer, J. Gider, D.Metzger, E. Theissen	Strategic Trading and Trade Reporting by Corporate Insiders
09-14	A. Kempf, O. Korn, M. Uhrig-Homburg	The Term Structure of Illiquidity Premia
09-13	W. Bühler, M. Trapp	Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets
09-12	W. Bühler, M. Trapp	Explaining the Bond-CDS Basis – The Role of Credit Risk and Liquidity
09-11	S. J. Taylor, P. K. Yadav, Y. Zhang	Cross-sectional analysis of risk-neutral skewness
09-10	A. Kempf, C. Merkle, A. Niessen-Ruenzi	Low Risk and High Return – Affective Attitudes and Stock Market Expectations
09-09	V. Fotak, V. Raman, P. K. Yadav	Naked Short Selling: The Emperor`s New Clothes?
09-08	F. Bardong, S.M. Bartram, P.K. Yadav	Informed Trading, Information Asymmetry and Pricing of Information Risk: Empirical Evidence from the NYSE
09-07	S. J. Taylor , P. K. Yadav, Y. Zhang	The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks
09-06	S. Frey, P. Sandas	The Impact of Iceberg Orders in Limit Order Books

09-05	H. Beltran-Lopez, P. Giot, J. Grammig	Commonalities in the Order Book
09-04	J. Fang, S. Ruenzi	Rapid Trading bei deutschen Aktienfonds: Evidenz aus einer großen deutschen Fondsgesellschaft
09-03	A. Banegas, B. Gillen, A. Timmermann, R. Wermers	The Cross-Section of Conditional Mutual Fund Performance in European Stock Markets
09-02	J. Grammig, A. Schrimpf, M. Schuppli	Long-Horizon Consumption Risk and the Cross-Section of Returns: New Tests and International Evidence
09-01	O. Korn, P. Koziol	The Term Structure of Currency Hedge Ratios

2008

No.	Author(s)	Title
08-12	U. Bonenkamp, C. Homburg, A. Kempf	Fundamental Information in Technical Trading Strategies
08-11	O. Korn	Risk Management with Default-risky Forwards
08-10	J. Grammig, F.J. Peter	International Price Discovery in the Presence of Market Microstructure Effects
08-09	C. M. Kuhnen, A. Niessen	Public Opinion and Executive Compensation
08-08	A. Pütz, S. Ruenzi	Overconfidence among Professional Investors: Evidence from Mutual Fund Managers
08-07	P. Osthoff	What matters to SRI investors?
08-06	A. Betzer, E. Theissen	Sooner Or Later: Delays in Trade Reporting by Corporate Insiders
08-05	P. Linge, E. Theissen	Determinanten der Aktionärspräsenz auf Hauptversammlungen deutscher Aktiengesellschaften
08-04	N. Hautsch, D. Hess, C. Müller	Price Adjustment to News with Uncertain Precision
08-03	D. Hess, H. Huang, A. Niessen	How Do Commodity Futures Respond to Macroeconomic News?
08-02	R. Chakrabarti, W. Megginson, P. Yadav	Corporate Governance in India
08-01	C. Andres, E. Theissen	Setting a Fox to Keep the Geese - Does the Comply-or-Explain Principle Work?

2007

No.	Author(s)	Title
07-16	M. Bär, A. Niessen, S. Ruenzi	The Impact of Work Group Diversity on Performance: Large Sample Evidence from the Mutual Fund Industry
07-15	A. Niessen, S. Ruenzi	Political Connectedness and Firm Performance: Evidence From Germany
07-14	O. Korn	Hedging Price Risk when Payment Dates are Uncertain
07-13	A.Kempf, P. Osthoff	SRI Funds: Nomen est Omen
07-12	J. Grammig, E. Theissen, O. Wuensche	Time and Price Impact of a Trade: A Structural Approach

07-11	V. Agarwal, J. R. Kale	On the Relative Performance of Multi-Strategy and Funds of Hedge Funds
07-10	M. Kasch-Haroutounian, E. Theissen	Competition Between Exchanges: Euronext versus Xetra
07-09	V. Agarwal, N. D. Daniel, N. Y. Naik	Do hedge funds manage their reported returns?
07-08	N. C. Brown, K. D. Wei, R. Wermers	Analyst Recommendations, Mutual Fund Herding, and Overreaction in Stock Prices
07-07	A. Betzer, E. Theissen	Insider Trading and Corporate Governance: The Case of Germany
07-06	V. Agarwal, L. Wang	Transaction Costs and Value Premium
07-05	J. Grammig, A. Schrimpf	Asset Pricing with a Reference Level of Consumption: New Evidence from the Cross-Section of Stock Returns
07-04	V. Agarwal, N.M. Boyson, N.Y. Naik	Hedge Funds for retail investors? An examination of hedged mutual funds
07-03	D. Hess, A. Niessen	The Early News Catches the Attention: On the Relative Price Impact of Similar Economic Indicators
07-02	A. Kempf, S. Ruenzi, T. Thiele	Employment Risk, Compensation Incentives and Managerial Risk Taking - Evidence from the Mutual Fund Industry -
07-01	M. Hagemeister, A. Kempf	CAPM und erwartete Renditen: Eine Untersuchung auf Basis der Erwartung von Marktteilnehmern

2006

No.	Author(s)	Title
06-13	S. Čeljo-Hörhager, A. Niessen	How do Self-fulfilling Prophecies affect Financial Ratings? - An experimental study
06-12	R. Wermers, Y. Wu, J. Zechner	Portfolio Performance, Discount Dynamics, and the Turnover of Closed-End Fund Managers
06-11	U. v. Lilienfeld-Toal, S. Ruenzi	Why Managers Hold Shares of Their Firm: An Empirical Analysis
06-10	A. Kempf, P. Osthoff	The Effect of Socially Responsible Investing on Portfolio Performance
06-09	R. Wermers, T. Yao, J. Zhao	Extracting Stock Selection Information from Mutual Fund holdings: An Efficient Aggregation Approach
06-08	M. Hoffmann, B. Kempa	The Poole Analysis in the New Open Economy Macroeconomic Framework
06-07	K. Drachter, A. Kempf, M. Wagner	Decision Processes in German Mutual Fund Companies: Evidence from a Telephone Survey
06-06	J.P. Krahnen, F.A. Schmid, E. Theissen	Investment Performance and Market Share: A Study of the German Mutual Fund Industry
06-05	S. Ber, S. Ruenzi	On the Usability of Synthetic Measures of Mutual Fund Net-Flows
06-04	A. Kempf, D. Mayston	Liquidity Commonality Beyond Best Prices
06-03	O. Korn, C. Koziol	Bond Portfolio Optimization: A Risk-Return Approach
06-02	O. Scaillet, L. Barras, R. Wermers	False Discoveries in Mutual Fund Performance: Measuring Luck in Estimated Alphas
06-01	A. Niessen, S. Ruenzi	Sex Matters: Gender Differences in a Professional Setting

2005

No.	Author(s)	Title
05-16	E. Theissen	An Analysis of Private Investors´ Stock Market Return Forecasts
05-15	T. Foucault, S. Moinas, E. Theissen	Does Anonymity Matter in Electronic Limit Order Markets
05-14	R. Kosowski, A. Timmermann, R. Wermers, H. White	Can Mutual Fund „Stars" Really Pick Stocks? New Evidence from a Bootstrap Analysis
05-13	D. Avramov, R. Wermers	Investing in Mutual Funds when Returns are Predictable
05-12	K. Griese, A. Kempf	Liquiditätsdynamik am deutschen Aktienmarkt
05-11	S. Ber, A. Kempf, S. Ruenzi	Determinanten der Mittelzuflüsse bei deutschen Aktienfonds
05-10	M. Bär, A. Kempf, S. Ruenzi	Is a Team Different From the Sum of Its Parts? Evidence from Mutual Fund Managers
05-09	M. Hoffmann	Saving, Investment and the Net Foreign Asset Position
05-08	S. Ruenzi	Mutual Fund Growth in Standard and Specialist Market Segments
05-07	A. Kempf, S. Ruenzi	Status Quo Bias and the Number of Alternatives - An Empirical Illustration from the Mutual Fund Industry
05-06	J. Grammig, E. Theissen	Is Best Really Better? Internalization of Orders in an Open Limit Order Book
05-05	H. Beltran-Lopez, J. Grammig, A.J. Menkveld	Limit order books and trade informativeness
05-04	M. Hoffmann	Compensating Wages under different Exchange rate Regimes
05-03	M. Hoffmann	Fixed versus Flexible Exchange Rates: Evidence from Developing Countries
05-02	A. Kempf, C. Memmel	Estimating the Global Minimum Variance Portfolio
05-01	S. Frey, J. Grammig	Liquidity supply and adverse selection in a pure limit order book market

2004

No.	Author(s)	Title
04-10	N. Hautsch, D. Hess	Bayesian Learning in Financial Markets – Testing for the Relevance of Information Precision in Price Discovery
04-09	A. Kempf, K. Kreuzberg	Portfolio Disclosure, Portfolio Selection and Mutual Fund Performance Evaluation
04-08	N.F. Carline, S.C. Linn, P.K. Yadav	Operating performance changes associated with corporate mergers and the role of corporate governance
04-07	J.J. Merrick, Jr., N.Y. Naik, P.K. Yadav	Strategic Trading Behaviour and Price Distortion in a Manipulated Market: Anatomy of a Squeeze
04-06	N.Y. Naik, P.K. Yadav	Trading Costs of Public Investors with Obligatory and Voluntary Market-Making: Evidence from Market Reforms
04-05	A. Kempf, S. Ruenzi	Family Matters: Rankings Within Fund Families and Fund Inflows
04-04	V. Agarwal, N.D. Daniel, N.Y. Naik	Role of Managerial Incentives and Discretion in Hedge Fund Performance

centre for financial research
cfr/university of cologne
Albertus-Magnus-Platz
D-50923 cologne
Fon +49(0)221-470-6995
Fax +49(0)221-470-3992
Kempf@cfr-cologne.de
www.cfr-cologne.de