Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets

Wolfgang Bühler
University of Mannheim
Chair of Finance
D-68131 Mannheim
E-Mail: w.buehler@uni-mannheim.de

Monika Trapp
Finance Department
Faculty of Management
University of Cologne
E-Mail: mtrapp@wiso.uni-koeln.de

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ABSTRACT

We develop a reduced-form model that allows us to decompose bond spreads and CDS premia into a pure credit risk component, a pure liquidity component, and a component measuring the relation between credit risk and liquidity. CDS liquidity has important consequences for the bond credit risk and liquidity components. Besides the credit risk link, we document a liquidity link between the bond and the CDS market. Liquidity in both markets dries up as credit risk increases, and higher bond market liquidity leads to lower CDS market liquidity. Ignoring CDS liquidity results in partly negative liquidity premia, particularly when CDS liquidity is low.
Introduction

Financial crises underscore the vital relation between credit risk and liquidity, and between liquidity in different markets. As investors re-assess the credit risk of many types of securities in their portfolio, trading of these securities dries up, and prices across a broad range of instruments and markets plummet. Quantifying credit risk from these prices is virtually impossible, since the credit risk and liquidity impact are only jointly observable.

However, decomposing corporate bond yield spreads into their credit risk and liquidity components is important for multiple reasons. First, policymakers and banks must approach the external supervision and the internal management of these risks differently. The stochastic properties of the economic factors causing these risks, as well as their price impact, must be specified empirically via a decomposition study. Second, when analyzing the impact of new information regarding an issuer’s credit risk on prices of fixed-income instruments, we must know to which extent liquidity changes also affect these prices. Third, if a firm attempts to reduce its cost of debt, it may find it cheaper to improve the liquidity of its traded bonds, compared to reducing its credit risk.

In the last decade, a second market for credit risk has developed through credit default swaps (CDS). This development has facilitated the estimation of the credit risk component in bond spreads. Several studies use the CDS mid premium as a measure of pure credit risk and attribute the residual bond spread to liquidity effects. Hence, these studies assume that CDS markets are perfectly liquid, or that liquidity does not affect CDS mid premia. Contrary to these studies, and in line with typical arguments in market microstructure theory, we argue that the liquidity of CDS contracts affects CDS bid and ask quotes. To the best of our knowledge, our paper is the first to explicitly model bond prices and CDS bid and ask quotes as a function of interest rates, credit risk, and liquidity. This more general view has two important consequences. First, the CDS mid premium does not necessarily coincide with the pure credit risk premium, but may also contain a liquidity component. Second, by deriving directly comparable liquidity premia in bond spreads and CDS premia, we can study liquidity spillovers between the CDS market and the underlying bond market.

Our paper thus ties into the theoretical and empirical literature on the link between asset markets and markets for derivatives written on these assets. Both in an equilibrium context and in the simpler intensity-based reduced-form setting, this link depends on the price impact of common fundamental factors and of market-specific factors. While common factors, such as firm-specific credit risk in our
setting, obviously result in joint price changes, it is not clear whether market-specific factors also
generate a relation between the two markets. In our model, we explicitly consider three channels
through which bond spreads and CDS premia can be related. First, as the fundamental variable, a
pure firm-specific credit risk factor directly affects bond spreads and CDS premia. Second, we allow
credit risk to affect instrument-specific liquidity, and vice versa. Finally, we allow for a pure liquidity
link between bonds and CDS, which is independent of the credit risk link.

By exploring the idea that bond prices and CDS bid and ask quotes contain information about
credit risk and liquidity, we study three basic questions. First, how can credit risk and liquidity effects
be modelled parsimoniously for the corporate bond market and the derivative CDS market? Here,
we specifically consider possible arbitrage relationships between these two markets and the market for
risk-free bonds. Second, how are credit risk and liquidity premia in the bond and the CDS market
related? Third, can we document a liquidity-driven relation between the markets in excess of the credit
risk connection, which we can consistently explain by investor behavior?

In the theoretical part of our analysis, we propose a reduced-form credit-risk model that differs from
the literature by treating liquidity effects in the bond and CDS market differently. Bond illiquidity
results in price discounts and yield surcharges, as in Longstaff, Mithal, and Neis (2005). In the CDS
market, liquidity has a twofold effect. First, bond-specific liquidity affects CDS premia since a poten-
tially illiquid bond is delivered if default occurs. Hence, CDS premia account for bond liquidity as a
source of bond price variation. In addition to this direct liquidity spillover, we include a CDS-specific
liquidity with a more intricate effect. We circumvent the question of systematic liquidity premia in
CDS mid premia by modeling ask and bid premia instead. From these, we infer a pure credit risk CDS
premium. Our measures of pure liquidity, and of the correlation between credit risk and liquidity, arise
as the difference between this liquidity-free CDS premium and the mid premium. Our model allows
us to consistently interpret the empirical relationship between time-varying bond and CDS liquidity
premia via demand relations for credit risk between the bond and the CDS market.

In the empirical part of our analysis, we separate bond spreads and CDS premia into their pure
credit risk, pure liquidity, and correlation-induced components. We use Euro-denominated bonds and
CDS contracts between mid-2001 and mid-2007 for firms covering a broad range of sectors and rating
classes. We then analyze the time-series relation between the credit risk, liquidity, and correlation
premia for the two markets.
Our most important empirical findings are threefold. First, adding a CDS-specific liquidity component to the model results in consistently positive credit risk and liquidity premia in corporate bond markets, while neglecting CDS-specific liquidity partly results in negative bond liquidity premia. We thus find a possible explanation for the puzzling result of Longstaff, Mithal, and Neis (2005), who partly estimate negative non-default components in high-risk corporate bond yields. Overall, we attribute 60% of the bond spread to credit risk, 35% to liquidity, and 5% to the correlation between credit risk and liquidity. These results stand in sharp contrast to Elton, Gruber, Agrawal, and Mann (2001) and Huang and Huang (2003) who report that the non-credit risk component accounts for the largest percentage of the bond spread.

For CDS, the credit risk component constitutes 95% of the observed mid premium, the pure liquidity component 4%, and the correlation component 1%. We interpret these on average positive liquidity premia consistently by a demand pressure for credit protection in the CDS market, a finding that supports the cross-sectional results of Chen, Cheng, and Wu (2005) and Meng and ap Gwilym (2007). However, these studies do not quantify a CDS liquidity premium. Our results indicate a higher CDS liquidity than the regression analyses of Bongaerts, De Jong, and Driessen (2008), who estimate a lower bound at 5 bp and an upper bound at 38 bp for the liquidity premium in expected CDS portfolio returns (compared to a credit risk premium of 42 bp), and Tang and Yan (2007) who obtain a surprisingly high liquidity premium of 13.2 bp (11% of the observed mid quote). This latter value is of a similar magnitude as the Treasury bond liquidity premium reported by Longstaff (2004), and the average non-default bond spread component of Longstaff, Mithal, and Neis (2005). As we use a pure liquidity variable, our result does not depend on the choice of a specific liquidity proxy such as the bid-ask spread, that is likely to be affected by credit risk. We thus believe it to be more plausible.

Second, our model allows us to determine the relation between credit risk and liquidity premia in the bond and the CDS market. We find that bond liquidity dries up as credit risk increases. This empirical result supports the theoretical prediction by Ericsson and Renault (2006) that liquidity shocks to the bond holder are positively correlated with default risk. In the CDS market, the liquidity dynamics imply widening bid-ask spreads as credit risk increases, with the ask quote reacting more sensitively than the bid. This analysis complements the cross-sectional evidence by Dunbar (2008), who calibrates a reduced-form model with credit and liquidity risk factors to CDS premia only, and of Chen, Fabozzi, and Sverdlove (2007), who calibrate a similar reduced-form model to CDS ask quotes or mid quotes.
only and deduce liquidity premia in bond prices. A delicate result of the latter study are the on average negative bond liquidity premia for all investment grade rating classes when using CDS ask premia.

Third, we substantiate the empirical evidence of Nashikkar, Subrahmanyam, and Mahanti (2007) on the relation between bond and CDS liquidity. Our model allows us to determine directly comparable pure liquidity premia for bonds and CDS. We obtain a significant relationship between these premia. Specifically, we demonstrate that higher bond liquidity premia lead to decreasing CDS liquidity premia. This finding can be consistently interpreted if investors use CDS primarily to hedge bond exposures.

The remainder of the paper is structured as follows. We introduce our reduced-form model in Section I and derive credit risk, liquidity, and correlation premia in Section II. Section III presents the empirical results of the model calibration, and a detailed analysis of the estimated time-varying premia. Section IV summarizes and concludes.

I. The Credit Risk and Liquidity Model

A. Specification of the Risk Structure

We first specify the underlying risk structure of the model. We assume a standard Duffie and Singleton (1997) framework in which default-free zero coupon bonds, default-risky coupon-bearing bonds and CDS are traded. The liquidity of these instruments can differ, and we choose the default-free zero coupon bonds as “liquidity numéraire” with a liquidity discount factor equal to 1. We thus avoid specifying a perfectly liquid instrument compared to which each illiquid instrument trades at a discount.

The default-free term structure of interest rates is driven by one risk factor, the instantaneous default-free interest rate \( r(t) \). The credit risk for a specific bond issuer is characterized by the stochastic default-risk hazard rate \( \lambda(t) \), which is reflected in CDS premia and corporate bond prices. The process \( \gamma^b(t) \) defines the liquidity intensity in the bond market. It determines the fraction of a bond’s price due to liquidity deviations from the liquidity numéraire. In the CDS market, we use two liquidity intensities \( \gamma^\text{ask}(t) \) and \( \gamma^\text{bid}(t) \) to describe the individual liquidity effects for ask and bid premia. Modeling liquidity effects via these intensities captures the intuition that investors are exposed to instrument-specific shocks forcing them to sell the instrument at a possibly unfavorable price.
The risk factors result in the discount factor

\[
\tilde{D}(t, \tau) = \exp \left( - \int_t^\tau r(s) \, ds \right)
\]

for interest rates, the risk-neutral survival probability

\[
\tilde{P}(t, \tau) = \exp \left( - \int_t^\tau \lambda(s) \, ds \right),
\]

and the liquidity discount factor for bonds (\(l = b\)) and CDS (\(l = \text{ask, bid}\)):

\[
\tilde{L}^l(t, \tau) = \exp \left( - \int_t^\tau \gamma^l(s) \, ds \right).
\]

We assume that \(r\) evolves independently from the default and liquidity intensities. The model can easily be generalized to capture correlation effects between \(r\) and the other risk factors. As credit risk and liquidity may be related, we allow the default and liquidity intensities to be correlated. By orthogonalizing the intensities, we obtain four latent factors \(x, y_b, y_{\text{ask}},\) and \(y_{\text{bid}}\). These variables capture the pure credit risk and the pure, instrument-specific liquidity. We model \(x\) as a mean-reverting square root process, and \(y_b, y_{\text{ask}},\) and \(y_{\text{bid}}\) as arithmetic Brownian motions. The following four-factor model describes the relation between the intensities and the latent factors:

\[
\begin{pmatrix}
    d\lambda(t) \\
    d\gamma^b(t) \\
    d\gamma^\text{ask}(t) \\
    d\gamma^\text{bid}(t)
\end{pmatrix}
= \begin{pmatrix}
    1 & g_b & g_{\text{ask}} & g_{\text{bid}} \\
    f_b & 1 & \omega_{b,\text{ask}} & \omega_{b,\text{bid}} \\
    f_{\text{ask}} & \omega_{b,\text{ask}} & 1 & \omega_{\text{ask},\text{bid}} \\
    f_{\text{bid}} & \omega_{b,\text{bid}} & \omega_{\text{ask},\text{bid}} & 1
\end{pmatrix}
\begin{pmatrix}
    dx(t) \\
    dy^b(t) \\
    dy^\text{ask}(t) \\
    dy^\text{bid}(t)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
    1 & g_b & g_{\text{ask}} & g_{\text{bid}} \\
    f_b & 1 & \omega_{b,\text{ask}} & \omega_{b,\text{bid}} \\
    f_{\text{ask}} & \omega_{b,\text{ask}} & 1 & \omega_{\text{ask},\text{bid}} \\
    f_{\text{bid}} & \omega_{b,\text{bid}} & \omega_{\text{ask},\text{bid}} & 1
\end{pmatrix}
\begin{pmatrix}
    \alpha - \beta x(t) \\
    \mu^b \\
    \mu_{\text{ask}} \\
    \mu_{\text{bid}}
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
    \sigma \sqrt{x(t)} dW^b_x(t) \\
    \eta^b dW^b_{y^b}(t) \\
    \eta_{\text{ask}} dW^b_{y_{\text{ask}}}(t) \\
    \eta_{\text{bid}} dW^b_{y_{\text{bid}}}(t)
\end{pmatrix},
\]

with parameters \(\alpha, \beta, \mu^l, f_l, g_l,\) and with \(\sigma > 0, \eta^l > 0.\) \(W_x\) and \(W^l\) are independent Brownian motions, \(l \in \{b, \text{ask, bid}\}.\) We assume that the matrix of the factor sensitivities has full rank to ensure parameter identification.

The factor structure in equation (4) enables us to capture the following effects: first, the latent pure credit risk factor \(x\) may affect liquidity intensities, and thus liquidity premia, through a non-zero value of \(f_l.\) Economically, a positive relation is plausible if investors attach a higher importance to trading
securities that are close to a default event. Second, the latent pure bond or CDS liquidity factor affect credit risk premia if \( g_t \neq 0 \). This effect can prevail if firms with less liquid bonds a higher probability of financial distress, as they may find issuing new debt more difficult. Analogously, lower CDS liquidity may complicate hedging bond exposures, such that bond investors demand higher compensation for bearing default risk.

Third, a non-zero correlation between the liquidity intensities can be caused by two mechanisms. On the one hand, there can be an indirect link through the impact of \( x \) via \( f_l \) such that correlation is due to joint dependence on the pure credit risk factor. More interestingly, the coefficients \( \omega_{l,k} \) imply a direct link between the liquidity intensities through the latent risk factors \( y^l \) and \( y^k \). A correlation between the liquidity intensities not due to \( x \) allows us to determine whether liquidity effects are transmitted from one market into the other.

The bond liquidity intensity can be related to the CDS ask and bid liquidity intensities \( (\omega_{b,\text{ask}} \neq 0, \omega_{b,\text{bid}} \neq 0) \) since long (short) credit risk positions can be incurred either by buying (short-selling) a bond or by selling (buying) credit protection in a CDS contract on the ask (bid) side. Liquidity changes in one market result in characteristic liquidity reactions in the other market, depending on whether investors hedge exposures across the two markets, or whether they take on positions alternately in either market. Consider the case where investors mainly use CDS for hedging purposes. Then, a liquidity shock which causes a lower bond transaction volume also leads to a lower transaction volume in the CDS market. To partly offset this decrease, rational CDS traders increase bid quotes, and decrease ask quotes. As a consequence, the CDS bid-ask spread decreases after a liquidity shock in the bond market. Note that this result does not imply that the total transaction volume in the CDS market increases. If investors use bonds and CDS as substitutes by taking on and selling off credit risk exposures alternately, a lower bond transaction volume results in a higher transaction volume in the CDS market. As a reaction, CDS traders increase ask quotes and decrease bid quotes.

B. Bond Market

We represent the value of a default-risky and potentially illiquid coupon-bearing bond as the expectation under the risk-neutral measure. If default occurs at time \( \tau \), the bondholder recovers a fixed fraction \( R \) of the face value \( F \). Rather than assuming continuous payments, we choose a discrete-time structure that matches the bond’s actual payment dates. Default can occur at any time, but recovery
takes place on the first trading day following the default event. Hence, the time-t price $CB(t)$ of a coupon-bearing bond with a fixed coupon $c$ paid at times $t_1, \ldots, t_n$, notional $F$, and recovery at times $\theta_j$ ($t \leq \theta_1 < \ldots < \theta_N \leq t_n$) is given by

$$CB(t) = c \cdot \sum_{i=1}^{n} D(t, t_i) E_t \left[ \tilde{P}(t, t_i) \tilde{L}^b(t, t_i) \right] + F \cdot D(t, t_n) E_t \left[ \tilde{P}(t, t_n) \tilde{L}^b(t, t_n) \right]$$

$$+ R \cdot F \cdot \sum_{j=1}^{N} D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^b(t, \theta_j) \right].$$

$E_t$ is the conditional expectation under the risk-neutral measure, and $\theta_0 := t$. Since $r$ is independent of the other risk factors, we can compute $D(t, \tau) := E_t \left[ \tilde{D}(t, \tau) \right]$ separately from the joint expectation of the default risk factor and the liquidity factor. $\Delta \tilde{P}(t, \theta_j) := \tilde{P}(t, \theta_{j-1}) - \tilde{P}(t, \theta_j)$ denotes the probability of surviving from $t$ until $\theta_{j-1}$ and then defaulting between $\theta_{j-1}$ and $\theta_j$ conditional on the current date $t$. Equation (6) can be interpreted as the expected present value of all future bond cash-flows: the first term gives the expected present value of the coupon payments. The second term equals the expected present value of the principal payment. The last term denotes the expected present value of the recovery rate payment.

C. CDS Market

We model the following basic form of a CDS contract. At inception, the protection buyer and seller agree on the CDS premium $s$. The premia are quoted annualized and in basis points (bp) per unit of face value of the underlying asset. Premium payments are made in arrears on fixed payment dates. In case of a credit event before the maturity of the CDS, the contract automatically terminates. The buyer pays the premium accrued since the last payment date to the seller, delivers the bond on which the CDS contract is written, and obtains the face value.

It is not obvious whether liquidity should be included in a model for CDS premia, and if so, how this should be done. After all, a CDS is a derivative, and thus not exposed to illiquidity effects caused by fixed supply or shorting costs, which result in systematically lower mid bond prices. For the CDS market, both empirical studies and theoretical models, see e.g. Schueler and Galletto (2003) or Longstaff, Mithal, and Neis (2005), generally assume that the mid premium reflects a price free of liquidity risk. Undoubtedly, however, bid and ask quotes also reflect liquidity aspects of a CDS. We model the ask and bid side of a CDS contract separately, since traders can be exposed to different
volumes for the supply than for the demand for credit protection. Hence, liquidity effects can have a
different impact on bid and ask quotes.\(^3\)

The value of the fixed leg of a CDS contract at time \(t\) with fixed in-arrear premium payment \(s_{\text{ask}}\) at times \(T_1, \ldots, T_m\), maturity \(T_m\), and stochastic settlement times \(\theta_j \ (t \leq \theta_1 < \ldots < \theta_M \leq T_m)\) in case of a credit event equals

\[
CDS_{\text{fix}}(t) = s_{\text{ask}} \left( \sum_{i=1}^{m} D(t, T_i) E_t \left[ \tilde{P}(t, T_{i-1}) \tilde{L}^{\text{ask}}(t, T_i) \right] + \sum_{j=1}^{M} D(t, \theta_j) \delta_j E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{\text{ask}}(t, \theta_j) \right] \right). \tag{7}
\]

In the second term of equation (7), \(\delta_j\) accounts for the premium fraction accrued between the last premium payment and the settlement time \(\theta_j\). \(\tilde{L}^{\text{ask}}\) is defined as \(\tilde{L}^{b}\) with the bond liquidity intensity \(\gamma^{b}\) replaced by the CDS ask liquidity intensity \(\gamma^{\text{ask}}\). Equation (7) reflects that the payment of all ask premia \(s_{\text{ask}}\) is discounted for the default probability, as a payment at time \(T_{i-1}\) only occurs with probability \(\tilde{P}(t, T_{i-1})\). The CDS-specific liquidity discount factor for the ask premium \(\tilde{L}^{\text{ask}}(t, T_i)\) accounts for the possibility that part of the ask premium is due to the fact that the protection seller demands an additional premium because of illiquidity.

The value of the floating leg is given by

\[
CDS_{\text{float}}(t) = F \sum_{j=1}^{M} D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \right] - RD(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{b}(t, \theta_j) \right]. \tag{8}
\]

The first term in equation (8) equals the expected discounted present value of the face value \(F\), the second the expected discounted present value of the defaulted bond that the protection seller obtains on selling the delivered bond.\(^4\) This second term is identical to the third term in the bond pricing equation (6), and thus also contains the discounting factor for bond liquidity. Bond liquidity directly affects the floating leg of the CDS contract both in the case of physical delivery and cash settlement. A less liquid bond has a lower post-default price, and the CDS premium is higher to compensate the protection seller for this lower value. The effect pertains even if the CDS market is perfectly liquid.

From equation (7) and (8) we obtain

\[
s_{\text{ask}}(t) = \frac{F \sum_j D(t, \theta_j) E_t \left[ \left( 1 - R \tilde{L}^{b}(t, \theta_j) \right) \Delta \tilde{P}(t, \theta_j) \right]} {\sum_t D(t, T_i) E_t \left[ \tilde{P}(t, T_{i-1}) \tilde{L}^{\text{ask}}(t, T_i) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{\text{ask}}(t, \theta_j) \right]}. \tag{9}
\]
The closed-form solution for the CDS bid premium is identical to that for the ask premium with the only exception that $\tilde{L}^{\text{ask}}$ is replaced by $\tilde{L}^{\text{bid}}$:

$$s^{\text{bid}}(t) = \frac{F \sum_j D(t, \theta_j) E_t \left[ \left( 1 - R \tilde{L}^{b}(t, \theta_j) \right) \Delta \tilde{P}(t, \theta_j) \right]}{\sum_i D(t, T_i) E_t \left[ \tilde{P}(t, T_i - 1) \tilde{L}^{\text{bid}}(t, T_i) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{\text{bid}}(t, \theta_j) \right]}.$$ (10)

Our model does not necessarily satisfy the no-arbitrage relation derived by Duffie (1999) that a portfolio consisting of a defaultable bond and a CDS on this bond earns the risk-free rate. A violation of this condition does not imply arbitrage in our model, as in addition to credit risk, there is also liquidity risk. Consider first a long position in a five-year corporate bond which is protected against default by a CDS of the same maturity. Even though this position is default-risk free, it is not risk-free as both the bond and the CDS are subject to liquidity risk. In a “liquidity event”, investors have to unwind either the bond or the CDS position before the maturity of the contracts. Hence, they are exposed to price risk due to a liquidity shock. Therefore, a yield to maturity of this position different from the risk-free yield does not imply arbitrage. The same argument holds for short-selling the bond and selling protection at the CDS bid quote - the position is not risk-free, and should thus have a different yield than the default-free, reference-liquid bond. A similar argument applies if the CDS bid quote is above the ask quote. Again, positions where investors simultaneously buy and sell credit risk protection only constitute an arbitrage opportunity if these positions can be held until maturity. If, however, a liquidity shock on the long or short credit risk component the position can force investors to unwind one of these components at an uncertain price, the position does not yield a non-negative profit with certainty.\(^5\)

In practice, CDS contracts can be written on multiple reference obligations, include multiple credit events, allow the protection buyer to choose which asset to deliver upon default or to specify an auction process for cash settlement instead of physical delivery, and the payments are subject to counterparty risk. In our setting, we abstract from these features to keep the model tractable. However, the impact of the two most important unmodelled risk factors, counterparty risk and the delivery option, on the bid and ask quotes is straightforward. Since a default of the protection buyer only results in the loss of the CDS premia accrued since the last payment date, CDS traders should only react with small ask quote increases to higher counterparty risk. Bid quotes, on the other hand, should be affected more strongly since the protection seller’s default renders the protection worthless. Especially when
anticipating a high default correlation of protection seller and underlying reference, CDS traders should set lower bid quotes to reflect this risk. Thus, the mid premium with counterparty risk is lower than if this risk is neglected. The delivery option also ought to affect CDS bid negatively, and ask quotes positively, but to the same extent: As CDS traders have a long position in the delivery option after a bid-induced trade, and a short position in the delivery option after a ask-induced trade, the delivery option results in lower bid and higher ask quotes. We discuss the consequences of these effects on the CDS liquidity premium in Section III.

Due to the factor model structure in equation (4) and the pairwise independence of $x$ and $y^l$, the expected values of $\tilde{P}(t\tau_i) \cdot \tilde{L}^l(t, \tau_i)$ and $\tilde{P}(t, \tau_i) \cdot \tilde{L}^l(t, \tau_{i+1})$ in equations (6), (9), and (10) can be represented by analytical functions which result in an affine term-structure model. Substituting these functions in equations (6), (9), and (10) yields the analytical solutions for the bond price, the CDS ask premium, and for the CDS bid premium.

II. Measures for Credit Risk, Liquidity, and Correlation Premia

The model developed in Section I allows us to disaggregate the full bond spread $bs$ into a pure credit risk component $bd$, a pure liquidity component $bl$, and a correlation-induced component $bc$. By an analogous procedure based on CDS bid and ask quotes, we can compute a pure credit risk component $sd$, a pure liquidity component $sl$, and a correlation-induced component $sc$. The rationale for this decomposition is most obvious for the bond. The pure credit risk premium $bd$ equals the bond spread that applies if credit risk is the only priced factor. In this case, the latent factor $y^b$ is identical to 0, the factor sensitivities $f$ and $g$ become irrelevant, and the credit risk intensity $\lambda$ and the latent factor $x$ coincide. The liquidity premium $bl$ equals the bond spread that applies if liquidity is also priced, and the correlation premium $bc$ measures the bond spread incurred because the credit risk and liquidity intensities are related.

Assuming a perfectly liquid bond and CDS market, the bond spread is directly comparable to the CDS premium if the maturity of both instruments is identical and the bond price equals its face value. The second condition is important to avoid the issues discussed by Duffie (1999) and Duffie and Liu (2001) who show that yield spreads on non-par fixed-coupon bonds cannot be directly compared to CDS premia.
We proceed in four steps to determine bond credit risk, liquidity, and correlation premia.\textsuperscript{7} Conditionally on a traded CDS contract, we sequentially construct four bonds with the same maturity and payment dates as the CDS. First, we determine the coupon of a default-free liquidity numéraire bond such that it trades at par. This coupon equals its yield to maturity. Second, we consider a par bond which is subject to credit risk, but not to liquidity risk. In this case, only the latent credit risk factor $x$ affects the bond’s price. Since it is default-risky, it must have a higher coupon, and thus a higher yield, than the default-free bond, to be priced at par. Its yield spread over the default-free bond is our pure credit risk premium $bd$. In the third step, we consider a bond which is subject to independent credit risk and liquidity such that $x$ and $y^b$, $y^{\text{ask}}$, and $y^{\text{bid}}$ affect its pricing. To be priced at par, it must have a different coupon, and thus a different yield, than the bond subject only to credit risk. The difference between the two bonds’ yields defines the pure liquidity premium $bl$. Since we measure liquidity with regard to the reference-liquid bond, $bl$ can be either positive or negative. Finally, we consider a bond with correlated credit risk and liquidity intensities. Hence, its price additionally must account for the factor sensitivities $f$ and $g$. The difference between this bond’s yield and the yield of the bond in the third step is the correlation premium $bc$. If the credit risk and liquidity intensities $\lambda$ and $\gamma^b$ are positively related, $bc$ is positive. If they are negatively related, $bc$ is negative due to a risk reduction effect. Overall, these three components must sum up to the full bond spread $bs$ of a synthetical par bond with maturity and payment dates identical to the CDS.\textsuperscript{8}

We define the credit risk, liquidity, and correlation components of a CDS analogously to those in the bond market. First, we compute the pure credit risk premium $sd$ assuming that CDS liquidity has no impact. Equations (9) and (10) illustrate that in this case, $sd$ is determined by the default-free interest rates, the default probability, and the bond liquidity. Thus, $bd$ and $sd$ differ because of the additional effect of the bond liquidity on the CDS. Since CDS liquidity has no impact, the bid and the ask quote are identical, and $sd$ equals the theoretical mid premium without CDS liquidity. This makes $sd$ directly comparable to $bd$, which refers to the mid bond price.\textsuperscript{9}

In a CDS market whose liquidity differs from the liquidity numéraire, the ask and bid premia differ from the pure credit risk premium $sd$. In line with the market microstructure literature, it seems apparent to select the size of the bid-ask spread as a measure of illiquidity. This approach is not appropriate in our context for two reasons. First, a comparison of equations (9) and (10) shows that the bid-ask spread is also affected by pure credit risk. Second, the bid-ask spread, even if taken relatively to $sd$, is not comparable to our liquidity measure $bl$ in the bond market.
We proceed analogously to the bond market and define the CDS liquidity premium $sl$ as the difference between the theoretical mid premium for uncorrelated credit risk and liquidity intensities, and the pure credit risk premium $sd$. This definition of $sl$ corresponds fully to the definition of the bond liquidity premium $bl$ as the change in the mid bond spread is due to the impact of pure instrument-specific liquidity.

In addition to this formal analogy, $sl$ allows for an inventory-related interpretation: If a trader enters into a number of ask-initiated CDS trades, she move her ask quotes, and possibly her bid quotes, upwards to balance her portfolio. Since the pure credit risk premium $sd$ remains at its initial value while $s^{\text{ask}}$ and $s^{\text{bid}}$ increase, $sl$ increases as well. An analogous argument holds for bid-initiated trades, where $sl$ decreases. Our measure of CDS liquidity is thus economically consistent with the measure of bond liquidity premia: if more investors want to sell credit risk by selling bonds — which can be interpreted as buying credit protection — the bond liquidity premium increases and vice versa.$^{10}$

Finally, the CDS correlation premium $sc$ equals the difference between the mid premium that includes the impact of the factor sensitivities $f$ and $g$ and the theoretical, correlation-free mid premium. The three premium components then sum up to the CDS mid premium. In contrast to the bond for which we determine synthetical, unobservable par bond spreads, this theoretical full CDS mid premium equals the observed CDS mid premium.

### III. Empirical Analysis

#### A. Data

We exclusively focus on Euro-denominated data since more Euro-denominated than US-Dollar denominated CDS contracts are traded in the early phase of our research interval: Between June 1, 2001 and September 30, 2001, we observe CDS ask and bid quotes on 119 Euro-denominated contracts versus 16 US-Dollar denominated contracts. For the current term structure of the default-free interest rates, we use the estimates provided by the Deutsche Bundesbank on a daily basis. These estimates are determined by the Nelson-Siegel-Svensson method from prices of German Government Bonds which represent the benchmark bonds in the Euro area for most maturities. From this term structure of interest rates, we compute prices of default-free zero-coupon bonds which we assume to have the reference liquidity discount factor of 1. The recovery rate is assumed to equal 40%.$^{11}$
Daily CDS ask and bid closing premia for senior unsecured debt were made available to us by a large US-domiciled international bank. The research period runs from June 1, 2001 to June 30, 2007, covering 1,548 trading days. We restrict ourselves to using Euro-denominated CDS premia with a reference maturity of 5 years to obtain a sample with homogenous CDS liquidity. According to the time conventions in the CDS market, we obtain the true CDS maturities by adding the distance between the quoting day and the next reference date to the quoted 5-year maturity.

Bond data are obtained from Bloomberg. Since historic bond bid and ask quotes are unavailable, we use daily average mid quotes. We collect all mid quotes of Euro-denominated straight bonds for firms which had at least 2 bonds outstanding at some point-in-time during the observation interval. We drop a firm if bond prices and CDS quotes are not available for a period of at least 20 consecutive trading days.

For each remaining firm, we collect a rating history from Bloomberg for the period where we observe bond prices and CDS premia. Both the Standard&Poor’s (S&P) rating and the Moody’s rating are used and converted to a linear score. If the numerical rating by the two agencies differs, we take the average of the two ratings, and round up if necessary. If no rating can be found for at least 20 consecutive trading days, we drop the firm from our sample.

The above procedure leaves us with a set of 155 firms from 8 industry sectors. A detailed overview is given in Table I.

Table I shows that the majority of firms has an average investment grade rating; only 9 lie in the subinvestment grade range. The largest industry group are “Financials” with 54 firms, which are also among the top-rated ones. No firm had an average rating below B, albeit some firms were rated CCC for a short period of time. Overall, Table I demonstrates that our sample is skewed towards financial and investment grade firms.

To present the time-series of bond spreads and CDS premia, we compute the average bond spread and CDS mid premium for each rating class at every observation date as follows. First, we identify the rating for a particular firm on each day. We then compute the bond spread for each bond of that particular firm as the difference between its yield and that of a synthetical default-free bond with identical coupon and maturity. Next, we interpolate the resulting bond spreads to match the CDS
maturity. We proceed by taking averages of the interpolated bond spreads and the observed CDS mid premia for all firms with an average investment, respectively subinvestment grade rating. The resulting time series for the investment and subinvestment grade are depicted in Figure 1.

As Figure 1 shows, mean investment grade bond spreads consistently exceed mean CDS mid premia. Overall, the mean investment grade bond spread has a time-series average of 89.4 bp, fluctuating between 33.5 bp and 178.9 bp. Mean investment grade CDS premia fluctuate between 15.9 bp and 143.8 bp with a time-series average of 45.4 bp. The two time series give a first impression of the differences between the bond and the CDS market. Mean subinvestment grade bond spreads fluctuate between 87.6 bp and 1,320.3 bp. Mean subinvestment grade CDS mid premia are partly above and partly below the bond spreads, but the time-series average of 341.3 bp lies below the average of 369.6 bp for the bond spread.

We calibrate the model to the observed data in three steps. For each firm, we choose initial values for the parameters \((\alpha, \beta, \sigma, \mu^b, \eta^b, \mu^{ask}, \eta^{ask}, \mu^{bid}, \eta^{bid})\) that describe the drift and diffusion of the latent factors, and the factor sensitivities \(f = (f_b, f_{ask}, f_{bid})\), \(g = (g_b, g_{ask}, g_{bid})\), and \(\omega = (\omega_{b,ask}, \omega_{b,bid}, \omega_{ask,bid})\).

In the first step, we estimate the time series of the latent variables by minimizing the sum of squared differences between the model-implied and the observed values. In the second step, we update the factor sensitivities from the time series of the latent variables. In the third step, we iterate across the drift and diffusion parameters of the latent factors. We follow this three-step procedure until convergence is achieved. Across the entire sample, we obtain an almost perfect fit to the CDS ask and bid quotes with a mean error below 0.01 bp and a mean squared error of 0.01 bp due to our model’s flexibility. For the bonds, we obtain a mean yield spread error of 0.1 bp, and a mean absolute error of 4.5 bp.

B. Credit Risk, Liquidity, and Correlation Premia: Cross-Sectional Results

B.1. Factor Sensitivities

We first present the empirical estimates for the relation between credit risk and liquidity, and between bond and CDS liquidity, via the factor matrix in equation (4).
As the estimates for the factor sensitivities in Table II show, pure credit risk has an impact on the bond liquidity intensity and the CDS liquidity intensities, but pure liquidity does not affect credit risk. The latent factor $x$ affects the bond liquidity intensity $\gamma^b$ significantly for 140 out of 155 firms. 138 of these estimates for $f_b$ are positive, and the negative ones are obtained for one utility and one financial firm with AAA, respectively AA, rating. The positive mean factor sensitivity estimate of 0.16 suggests that bond liquidity dries up as credit risk increases; we quantify the impact on the premia components below. The impact of $x$ on the CDS ask intensity $\gamma^{ask}$, measured by $f_{ask}$, is significant for 138 and positive for 137 firms with a mean estimate of 0.37. Increasing pure credit risk thus causes a twofold price increase for buying protection, first because the ask has to compensate the protection seller for a higher pure credit risk, and second because of liquidity. The CDS bid intensity $\gamma^{bid}$ is significantly affected by $x$ for only 66 firms with a negative estimate for $f_{bid}$ for 37 firms. The mean estimate of -0.07, however, differs significantly from 0 at the 1% level. As equation (10) shows, the negative value implies that higher pure credit risk increases CDS bid quotes less strongly than the pure credit risk premia $bd$ and $sd$. We conclude that higher credit risk moves both ask and bid quotes away from pure credit risk premia, such that higher credit risk causes higher bid-ask spreads. Bid quotes, however, remain relatively closer to the pure credit risk premia.

The impact of the latent factors $y^b$, $y^{ask}$, and $y^{bid}$ on the default intensity $\lambda$ is negligible. We obtain one significant coefficient estimate for $g_b$, three for $g_{ask}$ – out of which two are positive – and two for $g_{bid}$ with a positive and a negative one. Hence, liquidity changes hardly affect credit risk premia.

The liquidity spillover between bonds and CDS is captured by $\omega_{b,ask}$ and $\omega_{b,bid}$. The coefficient estimate for $\omega_{b,ask}$ is significant for 123 firms and negative for 118 with a mean value of -0.02. The estimate for the CDS bid liquidity factor sensitivity $\omega_{b,bid}$ is significant for 85 firms, positive for 80, and has a mean value of 0.01. These values are consistent with a hedging effect between the bond and the CDS market: Consider a negative bond liquidity shock, i.e. fewer bond trades, and a higher value of $y^b$ such that bond prices decrease. If investors hedge long and short credit risk exposures in the bond, which they now take on to a lesser degree, via positions the CDS market, the bond liquidity shock also results in fewer transactions for competitive CDS traders. To partly offset this decrease in hedging-initiated transactions, they increase bid quotes, and decrease ask quotes. The estimate for $\omega_{ask,bid}$ is significant for 131 firms and positive for 116 firms. The negative mean of -0.38 implies that liquidity shocks in the CDS market lead to opposite changes of bid and ask quotes. This finding agrees with an overall decreasing CDS bid-ask spreads as the market matures.
Comparing the results for the investment and the subinvestment grade, we observe a similar result as for the entire sample. Only the absolute value of the coefficient estimates tends to be larger in the subinvestment grade, which points to a stronger relation than between investment grade bond and CDS.

B.2. Premia Components

We now analyze the bond spread and CDS premium components. Table III displays the results.

Table III demonstrates that credit risk, liquidity, and correlation premia increase as the rating deteriorates. The pure credit risk premium in bond spreads $bd$ has an average of 6.1 bp for the AAA rating class, which approximately doubles for each rating downgrade in the investment grade range. The subinvestment grade sector exhibits values of $bd$ at least five times as large.\footnote{13}

Concerning the liquidity premia $bl$, the increase from investment to subinvestment grade is less strong, although we still obtain strictly positive estimates for $bl$ for each firm. The average correlation premia $bc$ increase in the rating up to the CCC rating class, and are strictly positive except for the AAA rating class. This negative average is driven by the negative estimates of the parameter $f_b$. On average, $bd$ accounts for 60% of the bond spread, $bl$ for 35%,\footnote{14} and $bc$ for 5%. These results are approximately in line with the bond spread proportion due to credit risk by Longstaff, Mithal, and Neis (2005), but stand in sharp contrast to the studies by Elton, Gruber, Agrawal, and Mann (2001) and Huang and Huang (2003) who report that the non-default component accounts for the largest percentage of the bond spread.

The CDS pure credit risk premia $sd$ exceed $bd$ by a relatively small amount. The difference is due to the liquidity price discount for deliverable bonds if default occurs. The minimal difference between $bd$ and $sd$ is attained for the AAA rating class with on average 0.1 bp, and the maximal one for the B class with on average 5.0 bp. This relation is consistent with the increasing average level of the bond pure liquidity premia $bl$.

The final results of Table III concern the CDS pure liquidity premia $sl$ and the correlation premia $sc$. As explained in Section II, non-zero values of $sl$ arise because the bid and ask quote have a different distance to the pure credit risk premium. If our estimate of $sd$ is closer to the bid quote, $sl$ has a
positive value and vice versa. On average, $sl$ is positive, which we interpret as a sign that transactions in the CDS market are mainly ask-initiated.\textsuperscript{15}

If the rating deteriorates, the asymmetry between the bid and the ask quote relative to the pure credit risk premium $sd$ increases, and results in higher liquidity premia. Taken relative to pure credit risk premia, however, pure liquidity premia are smaller for the subinvestment grade, and 19.15\% of CDS liquidity premia are in effect negative. In section C, we attribute these negative liquidity premia to unusual market events. As for the bond market, relative liquidity premia decrease in a particularly pronounced way for the transition from the investment grade to the subinvestment grade. In contrast to the bond market, however, CDS liquidity premia are much smaller for all rating classes. Their average size across all rating classes equals 1.9 bp compared to 26.4 bp in the bond market.\textsuperscript{16}

The average correlation premium $sc$ is almost negligible for the investment grade and grows by more than a factor of 10 for the subinvestment grade. This observation suggests that changes in credit risk result in a strong decline of liquidity in the subinvestment grade. The negative minima of $sc$ are due to the fact that for some firms, the sensitivity of the CDS bid liquidity intensity to the latent credit risk factor is larger than that of the CDS ask liquidity intensity. The CDS bid quote can thus increase more strongly than the ask quote. Concerning the decomposition of the full CDS premia, we observe that on average 95\% of the observed mid premium is due to pure credit risk, 4\% to pure liquidity, and 1\% to correlation.

In our estimation, we measure credit risk and liquidity with respect to government bond yields as the benchmark curve. This choice has the advantage that it allows for a plausibility check of our model. As government bonds are usually the most liquid bonds, the liquidity premium $bl$ should be positive for each bond in the sample. As Table III shows, this is the case. Obviously, a different benchmark choice could result in a different decomposition of bond spreads and CDS premia. To determine how robust our results are, we use a second benchmark curve, based on daily EURIBOR rates with maturities from 1 week to 12 months, and Euro interest rate swap rates from Bloomberg with maturities from 1 year to 10 years. As a result, the average bond spread decreases by 8.9 bp. 63\% of the bond spread is now attributed to pure credit risk, 32\% to pure liquidity, and 5\% to correlation. The corresponding fractions for the CDS market are 91\%, 8\%, and 1\%. We take the relative stability of our results as an indication that the proportions we attribute to the different risk factors appropriately reflect the true values.
C. Credit Risk, Liquidity, and Correlation Premia: Time-Series Results

C.1. Comparison of Bond Spread and CDS Premia Components over Time

The estimated credit risk, liquidity, and correlation premia components are depicted in Figure 2. For ease of presentation, only the averages of the investment and subinvestment grade premia are given.

Panels A and B of Figure 2 show that the pure credit risk premia $bd$ and $sd$ are almost identical. For the investment grade, there are two distinct spikes in late 2001 and late 2002 at the Enron and WorldCom defaults. The reaction of the subinvestment grade to the Enron default is almost negligible which may be due to the fact that there are only 2 subinvestment grade firms in our sample between June 2001 and February 2002. Overall, we observe the well-known decline of the pure credit risk premia time series. The end of the observation interval coincides with the beginning of the financial crisis.

The bond liquidity premia $bl$ exhibit a different behavior across the investment and subinvestment grade as we observe from Panel A of Figure 2. During high credit risk periods, liquidity premia are volatile and flatten out at a higher level during the latter part of the observation interval for the investment grade. In the subinvestment grade, $bl$ is highest shortly after the high-risk periods and decreases to a lower level towards the end of the observation interval. For CDS liquidity premia, we observe a trend towards 0 as the CDS market matures. Overall, $sl$ is higher in the investment grade when credit risk is high, but becomes mostly negative in the subinvestment grade when credit risk is high. This finding suggests that the ask-initiated transactions are partly replaced by bid-initiated transactions for the subinvestment grade, pointing at a high number of investors who attempt to take on credit risk synthetically in the CDS market.

Due to the insignificant estimates for the sensitivity of the credit risk intensity $\lambda$ to the liquidity risk factor $y$, the correlation premia $bc$ and $sc$ are closely associated with credit risk premia. Comparing $bl$ and $bc$, Panel A of Figure 2 shows that pure liquidity premia lie below correlation premia during high-risk periods and above during low-risk periods for the investment grade. In the subinvestment grade, we observe a similar result during high risk periods, e.g. at the WorldCom default in 2002. Overall, however, $bl$ tends to be higher than $bc$ in the lower rating classes. We interpret this as an indication that liquidity is reduced disproportionately in high credit risk phases, in particular for the...
investment grade. This agrees with the flight to quality and the flight to liquidity effects which are theoretically derived by Vayanos (2004) and documented empirically by Beber, Brandt, and Kavajecz (2009). The CDS correlation premia are, in contrast, almost negligible, i.e. CDS liquidity is mostly independent of credit risk.

We conclude this section by two comments. The first refers to a specification test of our model, the second to an extrapolation of our results to the financial crisis. In our specification test, we proceed as follows. First, we compute the changes of the average pure credit risk and the average pure liquidity premia $\Delta bd_t$, $\Delta sd_t$, $\Delta bl_t$, and $\Delta sl_t$, for the investment and the subinvestment grade. We perform an augmented Dickey-Fuller test for these four time series, and reject a unit root for all. Second, we perform an OLS regression of the premia changes on the change of the Dow Jones VSTOXX level, and the changes of the European Financial Market Liquidity Indicator (FML), which was made available to us through the European Central Bank. Higher values of FML indicate higher market-wide liquidity. We find that $\Delta bd$ and $\Delta sd$ are significantly positively dependent on the VSTOXX change, and independent of the FML change, both for the investment and the subinvestment grade. $\Delta bl$ and $\Delta sl$ are independent of the VSTOXX change, and except for $\Delta bl$ in the subinvestment grade, negatively dependent on the change in FML. These results support the disaggregation obtained through our model.

Our model in principle allows an out-of-sample test for the period of the financial crisis. We refrain from re-calibrating our model for the period after June 2007 as the financial crisis deserves a separate study. Instead, we report some summary statistics and extrapolate our results from the period around the WorldCom default to the period from July 2007 to February 2009. For this period, bond spreads and CDS quotes for 116 firms of our sample are available. The average bond spread increased to 125 bp, or by 30 %, the CDS mid premium to 127 bp, or by 80%. If we identify the WorldCom crisis by the time interval from June to December 2002, bond spreads increased to 136 bp, and average CDS mid premia changed to 111 bp. These values illustrate that the recent crisis affected the CDS market more strongly.

During the WorldCom crisis, the pure credit risk premium $bd$ amounted to 68%, the pure liquidity premium $bl$ to 12%, and the correlation component $bc$ to 20%. Extrapolating these percentage shares to the financial crisis, we obtain average pure credit risk, pure liquidity, and correlation premia of 85 bp, 15 bp, and 25 bp. For CDS mid premia, we find percentages of 92%, 6% and 2% for the pure credit
risk, pure liquidity, and correlation premia. These percentages translate into premia of 117 bp, 8 bp, and 2 bp.

These results illustrate two properties of the financial crisis. First, a large fraction of the total bond spread is due to the correlation between credit risk and liquidity. While the pure credit risk premium only doubles, the bond correlation premium increases more than sixfold. Hence, we observe a much higher correlation between credit risk and liquidity during the financial crisis. Second, the fraction of the bond spread due to pure credit risk increases from 60% for the entire observation interval. In the CDS market, we find the reverse relation; the pure credit risk component only constitutes 92% during the crisis, compared to 95% for the entire sample. Even though the above proportions do not suggest that the CDS market is less liquid than the bond market, we can conclude that CDS liquidity dries up more strongly than bond market liquidity during crises. This finding agrees with the model-independent observation that CDS mid premia increase more strongly than bond yield spreads.

C.2. Dynamic Interaction between Bond and CDS Market

To study the dynamic interaction between the bond and the CDS market, we perform a time-series analysis of the premia across the different markets. Since pure credit risk premia and pure liquidity premia are by construction independent of one another, as are correlation premia and pure liquidity premia, we focus on the pairwise relation of these premia across the two markets.

We estimate three standard Johansen vector error correction models (VECM) to study the long-run equilibrium relationship between the premia \( bd \) and \( sd \), \( bl \) and \( sl \), and \( bc \) and \( sc \), across the markets and the reactions of the premia changes to short-run deviations from this relationship. The explanatory variables are the error correction terms \((bd_{t-1} + \beta_d sd_{t-1}\) for the first, \(bl_{t-1} + \beta_l sl_{t-1}\) for the second, and \(bc_{t-1} + \beta_c sc_{t-1}\) for the third system of equations\), and the lagged premium changes. We consider time lags up to 5 trading days to capture a weekly time interval, which we found to be sufficient for all firms via both Akaike’s Information criterion and Schwartz’s criterion. The results are displayed in Table IV.

As Panel A of Table IV indicates, the pure credit risk premia \( bd \) and \( sd \) are cointegrated with an estimated coefficient of \(-1\). This value suggests that there is a one-to-one relation between the two
credit risk premia, and that the effect of the bond liquidity on sd is almost negligible. The estimates of the error correction terms imply that both bond and CDS pure credit risk premia react to deviations from this one-to-one relation, but that bond premia are much more sensitive.

The positive cointegration coefficient estimate for bl and sl implies that the pure liquidity premia move in opposite directions. This finding is in line with the liquidity spillover discussed in Section B.1. As we argued, a bond liquidity shock results in higher CDS bid quotes and lower CDS ask quotes. In market micro structure terms, this smaller bid-ask spread corresponds to higher CDS liquidity compared to the case if the CDS trader does not adjust bid and ask quotes as a reaction to the bond liquidity shock. Since the CDS ask quote decreases more strongly than the CDS bid quote increases, our measure of the CDS liquidity premium, sl, also decreases. Therefore, the liquidity effect we capture fully corresponds to the market micro structure intuition.

The error correction coefficient estimate for the bond is only significant at the 10% level while the error correction term for the CDS is significant at the 1% level, suggesting that CDS liquidity adjusts to bond liquidity. This result — only CDS liquidity premia react to bond liquidity premia — is also supported by the coefficients of the lagged premium changes: the lagged bond premium change significantly affects the CDS premium change, and due to the lower level of the CDS liquidity premium, the bond’s impact is also economically significant. The lagged CDS premium change, on the other hand, has no significant impact on the bond premium change.

The bond and CDS correlation premia bc and sc exhibit a negative estimate of the cointegration coefficient, pointing at a comovement of the credit-risk related part of the liquidity premia in both markets. We partly attribute its large negative value to the relation in the investment grade below. Otherwise, the time-series behavior of correlation premia resembles that of the credit risk premia.

The comovement of the correlation premia has an important implication for the disaggregation of the liquidity premium into the pure liquidity and the credit risk-induced correlation component. If we model credit risk and liquidity intensities independently, correlation premia become subsumed in liquidity premia since the credit risk intensity $\lambda$ is not affected by the liquidity factors $y^b$, $y^{ask}$, and $y^{bid}$. This matters for the dynamics of the liquidity premia: As Table IV shows, pure liquidity premia move in opposite directions while correlation premia exhibit a high degree of co-movement with a much larger cointegration coefficient. Thus, subsuming the pure liquidity component and the credit risk-induced correlation component would obscure the actual countermovement.
As discussed above, the time-series behavior of the premia components differs between the investment and the subinvestment grade. The results for the investment grade in Panel B of Table IV show that the size of the coefficient estimates and the explanatory power decrease compared to the entire sample. Overall, the investment grade exhibits a lower connection between bonds and CDS. Economically, this finding implies that the premia for investment grade firms may be affected by market-specific conditions in excess of the firm-specific ones.

Panel C of Table IV shows the results for the subinvestment grade. Overall, the coefficient estimates imply that subinvestment grade bond and CDS premia are more closely interconnected than investment grade premia. For credit risk premia, the cointegration coefficient estimate is slightly lower than in the investment grade due to the (now higher) effect of bond liquidity on \(sd\). The error correction terms, on the other hand, are significantly higher than for the investment grade, signifying a stronger relation between the two markets. The cointegration coefficient estimate for liquidity premia increases to 17.29, and the error correction estimate for CDS liquidity premia decreases to -0.26, suggesting that CDS liquidity is more sensitive to bond liquidity.

D. The Effect of Excluding CDS Illiquidity

We find strictly positive bond liquidity premia \(bl\), and interpret this result as ex-post support for our modelling CDS markets as not perfectly liquid. To substantiate this insight, we propose a modification of our model which allows us to check whether neglecting CDS liquidity results in an omitted variables issue. In this version, we assume that there are no liquidity components in the CDS market, i.e. \(\gamma^l = \mu^l = \eta^l = 0\), \(l = \text{ask, bid}\). As a consequence, the CDS mid quote equals the pure credit risk premium \(sd\), and we calibrate the model to CDS mid quotes.

Table V displays the mean, standard deviation, minimum, and maximum of the re-estimated bond spread components.

Insert Table V about here.

Overall, we find at least one negative value of the bond pure liquidity premium \(bl\) for 120 firms. In total, 8,938 bond liquidity intensity estimates (this corresponds to 6.8% of the firm-day observations) are negative.
A comparison of the estimates in Table V to the original estimates in Table III shows several striking results. On average, the pure bond credit risk premia \( bd \) is higher by 1.9 bp if CDS liquidity is ignored. This difference is due to the fact that both \( sd \) and \( bd \) must now match CDS mid quotes. It thus almost exactly matches the mean CDS liquidity premium in Table III. The bond liquidity premia are on average lower by 1.7 bp. Although the average effect of CDS liquidity on the bond market is small, the individual consequences can be substantial. We find at least one negative value of the bond pure liquidity premium \( bl \) for 120 firms. In total, 8,938 bond liquidity intensity estimates (this corresponds to 6.8% of the firm-day observations) are negative. For each rating class, the minimal value of \( bl \) is negative with the smallest value of -204.4 bp in the BB rating class.

Our results show that neglecting stochastic CDS liquidity yields overestimates of bond liquidity and results in bond price surcharges instead of discounts. Simultaneously, default risk and default probabilities are overestimated when the bond liquidity premia become negative. Hence, we conclude that liquidity aspects in the CDS market matter for the identification of bond credit risk, liquidity, and correlation. Since neglecting CDS liquidity attributes yield differences between the bond and the CDS market directly to bond liquidity, the effect is especially pronounced when the bond liquidity is high relative to the CDS liquidity.

IV. Summary and Conclusion

We develop a reduced-form model to decompose bond spreads and CDS quotes into three components: a pure credit risk, a pure liquidity, and a correlation component that covers the comovement of credit risk and liquidity. To the best of our knowledge, our paper is the first to use information available in CDS bid and ask quotes about the firm’s credit risk and the liquidity of a CDS contract. We combine market microstructure aspects with no-arbitrage pricing principles to study the interaction between credit risk and liquidity, and the liquidity spillover effects between credit derivatives and the underlying bonds.

Our empirical results cover the cross-sectional behavior of credit risk and liquidity, and the dynamic links of these components on the state space and the premia level. Cross-sectionally, we find that CDS liquidity effects can have a large impact on the liquidity component in the bond market. The dynamic analysis of the state variables reveals a liquidity spillover between the bond and the CDS market. The asymmetric consequences of a bond liquidity shock on CDS bid and ask quotes shows that CDS
contracts are primarily used to hedge credit risk, and not as a substitute for a bond position. On the premium level, our vector error correction analysis shows that bond liquidity shocks affect the CDS market, but not vice versa.

Our results have several implications for a firm’s cost of debt, and the risk management of a bank’s trading book. In the investment grade sectors, bond credit risk and liquidity premia have roughly the same size. Measures to reduce the credit premium, such as improving the debt/equity ratio or a smoothing of earnings and cash flows, have possibly larger implicit costs than measures to improve the liquidity of newly issued bonds. Liquidity premia can be reduced by establishing market makers who guarantee a small maximal bid-ask spread. This institution was established in European bond markets some years ago.

We further find that default risk and liquidity are positively, but not perfectly, correlated. Results which are not reported in the paper show that the strength of the liquidity link between the bond and the CDS market depends on whether market-wide credit risk increases or decreases. Hence, hedging bond market credit risk exposures via CDS can increase the exposure to liquidity changes.
References


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Notes

1 March, June, September, and December 20th have evolved as the standard dates. If a contract is entered into on a non-standard date, the time until the next standard date is added to the quoted maturity of the contract.


3 To explore whether the asymmetry in modeling only bond mid quotes versus CDS bid and ask quotes affects the liquidity relations implied by our model, we perform the following analysis. First, we extend the model in equation (4) to two latent bond liquidity factors, one for ask and bid quotes each. Second, we specify two factor matrices, where one implies a bond and CDS liquidity comovement, and the other a countermovement. Third, we simulate a 1-year time series for the latent, independent factors, transform these into the correlated intensity time series, and determine bond and CDS bid and ask quotes. Last, we compute bond mid quotes as the average of the bid and ask, and estimate the implied liquidity relation. We repeat this procedure 100 times, and find that we correctly identify comovement in 98, and countermovement in 97 cases. These results justify our choice of modeling only bond mid quotes as opposed to CDS bid and ask quotes.

4 Typically, no single bond matches the CDS maturity. As long as one bond matures later than the CDS, we assume that an issuer default triggers the credit event. If the CDS maturity exceeds that of all bonds, there is no deliverable asset after the maturity of the last bond, and the protection becomes worthless. We capture this effect by assuming that a credit event cannot occur if no bond is outstanding.

5 In the empirical part of our study, Section III, we obtain no estimates that translate into negative bid-ask spreads. To quantify how likely negative bid-ask spreads are, we perform a simulation study. We simulate 10,000 5-year time series for the intensity time series, using the average intensities as starting values. We then compute the implied bid and ask premia for a 5-year CDS contract from inception until maturity, and for a constant 5-year maturity CDS contract. We only obtain only two negative bid-ask spreads for the CDS contract with decreasing maturity immediately before the maturity date.

6 The explicit analytical representations are available in an online supplement.

7 The formal definition of the credit risk, liquidity, and correlation premia are available in an online supplement.

8 Our model does not include an assumption on the position of the pure credit risk premium $b_d$ relative to the total bond spread $b_s$. Both $b_l$ and $b_c$ could theoretically be negative, such that $b_d$ exceeds $b_s$. Such a relation implies that the default-risky bond is more liquid than the default-free liquidity numéraire bond, and that credit risk and liquidity are negatively related. In Section III.B.B.2, we show that empirically $b_d$ consistently lies below $b_s$.

9 As for the bond, we make no assumption on the position of $s_d$ relative to the observed CDS mid, bid, and ask quote. There is not reasons why $s_d$ should be consistently smaller or larger than any of the CDS premia. It can be reasonable that CDS traders increase both bid and ask quotes beyond the unknown pure credit risk premium after a number of
ask-induced transactions to rebalance their portfolios. Similarly, \( sd \) could exceed the bid and ask quote. Empirically, we show in Section III.B.B.2 that \( sd \) on average lies below the CDS mid quote.

\( \text{Our model allows us to further disaggregate the pure liquidity premia } bl \text{ and } sl \text{ into a liquidity risk and a liquidity level component.} \) We define the pure liquidity risk component as the theoretical liquidity premium we would observe if the current value of the latent pure liquidity factor \( y^l_t \) were equal to zero, but the drift and diffusion parameters \( \mu^l \) and \( \eta^l \) differ from zero. The liquidity level component is the difference between the total pure liquidity premium and the liquidity risk component. These definitions reflect the idea that the liquidity risk premium arises from a change of the latent pure liquidity factor from its current value \( y^l_t \).

\( \text{It is not obvious whether varying the recovery rate leads to corresponding changes in the default probability, keeping expected loss constant, or whether the liquidity estimates is also affected. We thus also choose recovery rates of 30\% and 50\%, and find similar proportions of credit risk, liquidity, and correlation premia.} \)

\( \text{Details on the calibration procedure are available in an online supplement.} \)

\( \text{It is surprising that } bd \text{ is smaller for the CCC rating class. However, this finding agrees with the observation that bond yield spreads and mid CDS premia are also lower in the CCC than in the B rating class. Ericsson, Reneby, and Wang (2005) also report that average mid CDS premia for the BB rating class are larger than for the B and CCC rating class.} \)

\( \text{In terms of the pure liquidity level and the pure liquidity risk premium, we find that on average 0.95 \text{ bp (4\% of } bl) \text{ are due to liquidity risk. This percentage is lower than the value documented by Acharya and Pedersen (2005), who find that 24\% of the liquidity component in annual expected stock returns is due to liquidity risk. As bonds tend to be more illiquid than stocks, we find it reasonable that the bond liquidity risk component constitutes a smaller fraction.} \)

\( \text{This important result cannot be due to our excluding the delivery option and counterparty risk from our model. The delivery option, as we argued in Section I.C, should increase observed ask quotes and decrease observed bid quotes by the same amount, leaving the mid quotes unaffected. This symmetric effect suggests that the delivery option has no impact on the CDS liquidity premium } sl \text{, but is subsumed in our estimate of the pure credit risk premium } sd. \text{ A preliminary study in which we model the delivery option explicitly confirms this argument. With regard to counterparty risk, we argue in three steps that excluding it can only lead to downwards biased estimates of the CDS liquidity premium. First, we consider perfectly liquid CDS contracts without counterparty risk. In this case, the bid, ask, and mid quotes coincide, and are equal to the pure credit risk premium } sd. \text{ Second, we include counterparty risk. As argued in Section I.C, this additional risk decreases bid quotes and has no impact on ask quotes. As a consequence, the mid quotes decreases, the ask quote still equals } sd, \text{ and the counterparty risk premium } scr := s^{\text{mid}} - sd \text{ is negative. Third, assuming that } sd \text{ and } sc \text{ are unaffected by counterparty risk, the decomposition of the observed mid quote } s^{\text{mid}} = sd + sc + \hat{sl} + scr \text{ will result in a larger liquidity premium } \hat{sl} \text{ as } scr \text{ is negative.} \)

\( \text{Further separating } sl \text{ into the liquidity level and the liquidity risk component, we find that on average 12\% of } sl \text{ are due to liquidity risk. As for the bond, the fraction is lower in the investment grade than in the subinvestment grade with on average 6\% compared to 89\% in the subinvestment grade. In contrast with Bongaerts, De Jong, and Driessen (2008),} \)
who find that liquidity risk premia in expected CDS portfolio returns are negligible, these percentages point at a large liquidity risk impact for CDS on reference entities with a low rating.

17The large absolute difference between $bd$ and $sd$ of 32 bp during the financial crisis and 9 bp during the WorldCom crisis compared to 0.4 bp for the entire sample implies that the value of $sd$ we would obtain if we recalibrated the model would be even smaller.
Figure 1. Average Bond Spreads and CDS Mid Premia Time Series

The figure depicts average bond spreads and CDS mid premia between June 1, 2001 and June 30, 2007. Bond spreads are computed as a bond’s yield-to-maturity computed from the mid price less the yield-to-maturity of a bond with identical coupon and maturity. This price is determined using the Nelson-Siegel-Svensson term structure of interest rates for German Government Bonds as provided by the Deutsche Bundesbank. Bond spreads are subsequently interpolated to obtain a synthetic 5-year maturity. Averages are taken across all firms which are rated investment grade or subinvestment grade, respectively, on a given date. Average bond spreads are denoted in black, CDS mid premia in grey. The solid line is used to depict the investment grade, the dashed line to depict the subinvestment grade time series.
Figure 2. Estimated Credit Risk, Liquidity, and Correlation Premia Time Series

The figure depicts the model-implied premia components for the investment and subinvestment grade between June 1, 2001 and June 30, 2007. Bond premia components refer to a synthetical 5-year par bond, the CDS premia components to a 5-year contract. Credit risk premia (solid black line) reflect the impact of the credit risk factor (and the bond liquidity for the CDS). Liquidity premia (dotted grey line) reflect the impact of the instrument-specific pure liquidity factor. Correlation premia (dashed grey line) reflect the cross-impact of the credit risk factor on the liquidity intensities. Averages are computed across the investment and subinvestment grade on each date. All values are in basis points.

Panel A: Bond Premia
Panel B: CDS Premia

Investment Grade Premia CDS

Subinvestment Grade Premia CDS
Table I: **Firms by Rating Class and Industry Sector**

The table presents the number of firms in each rating class and industry group. We first compute a firm’s average numerical rating across all days when there are at least two bond price quotes, and a CDS bid and ask quote. We then map the numerical value to the S&P rating and use this as the column heading. "Since the lowest average firm rating is B, the table does not contain a CCC column, even though there are 171 firm-day observations with a CCC-rating." The last columns and rows show the number of mid bond prices and mid CDS premia for each industry group and rating class in our sample between June 1, 2001 and June 30, 2007.

<table>
<thead>
<tr>
<th>Industry Group</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>All</th>
<th># Obs. Bonds</th>
<th># Obs. CDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Materials</td>
<td>-</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>16</td>
<td>33,393</td>
<td>13,079</td>
</tr>
<tr>
<td>Communication</td>
<td>-</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td>-</td>
<td>19</td>
<td>73,211</td>
<td>20,481</td>
</tr>
<tr>
<td>Cycl. Cons. Goods</td>
<td>-</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>2</td>
<td>-</td>
<td>16</td>
<td>47,497</td>
<td>15,634</td>
</tr>
<tr>
<td>Noncycl. Cons. Goods</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>8</td>
<td>1</td>
<td>-</td>
<td>14</td>
<td>40,519</td>
<td>12,319</td>
</tr>
<tr>
<td>Diversified</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>6,536</td>
<td>3,096</td>
</tr>
<tr>
<td>Financial</td>
<td>-</td>
<td>22</td>
<td>28</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>54</td>
<td>175,870</td>
<td>38,046</td>
</tr>
<tr>
<td>Industrial</td>
<td>-</td>
<td>4</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9</td>
<td>40,624</td>
<td>9,531</td>
</tr>
<tr>
<td>Utility</td>
<td>1</td>
<td>5</td>
<td>13</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>23</td>
<td>79,604</td>
<td>19,036</td>
</tr>
<tr>
<td>All</td>
<td>1</td>
<td>32</td>
<td>66</td>
<td>47</td>
<td>8</td>
<td>1</td>
<td>155</td>
<td>497,254</td>
<td>131,222</td>
</tr>
</tbody>
</table>

# Obs. Bonds: 3,552 106,206 116,359 248,343 21,238 1,556 497,254
# Obs. CDS: 1,085 27,015 53,203 41,338 7,842 739 131,222
**Table II**

**Factor Sensitivities**

The table presents the estimates for the factor sensitivities. $f_b$, $f_{ask}$, and $f_{bid}$ measure the impact of the latent credit risk factor $x$ on the bond, CDS ask, and CDS bid liquidity intensities $\gamma^b$, $\gamma^{ask}$, and $\gamma^{bid}$. $g_b$, $g_{ask}$, and $g_{bid}$ measure the impact of the latent bond, CDS ask, and CDS bid liquidity factors $y^b$, $y^{ask}$, and $y^{bid}$ on the default intensity $\lambda$. $\omega_{b,ask}$, $\omega_{b,bid}$, and $\omega_{ask,bid}$ measure the cross-impact of the latent bond, CDS ask, and CDS bid liquidity factors on the bond, CDS ask, and CDS bid liquidity intensities $\gamma^b$, $\gamma^{ask}$, and $\gamma^{bid}$. The first row of each panel gives the number of firms for which the sensitivity estimate is significantly different from 0, the second row the number of estimates significantly larger than 0, the third row the number of estimates significantly smaller than 0, where we determine significance via block-bootstrapping with 1,000 resamplings. The fourth and fifth row present the mean estimate and the standard deviation. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms.

<table>
<thead>
<tr>
<th></th>
<th>$f_b$</th>
<th>$f_{ask}$</th>
<th>$f_{bid}$</th>
<th>$g_b$</th>
<th>$g_{ask}$</th>
<th>$g_{bid}$</th>
<th>$\omega_{b,ask}$</th>
<th>$\omega_{b,bid}$</th>
<th>$\omega_{ask,bid}$</th>
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</thead>
<tbody>
<tr>
<td><strong># Firms</strong></td>
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<td>138</td>
<td>66</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>123</td>
<td>85</td>
<td>131</td>
</tr>
<tr>
<td><strong># &gt; 0</strong></td>
<td>138</td>
<td>137</td>
<td>29</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>80</td>
<td>15</td>
</tr>
<tr>
<td><strong># &lt; 0</strong></td>
<td>2</td>
<td>1</td>
<td>37</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>118</td>
<td>5</td>
<td>116</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.16***</td>
<td>0.37***</td>
<td>-0.07***</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>-0.02***</td>
<td>0.01***</td>
<td>-0.38***</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>-</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>$f_b$</th>
<th>$f_{ask}$</th>
<th>$f_{bid}$</th>
<th>$g_b$</th>
<th>$g_{ask}$</th>
<th>$g_{bid}$</th>
<th>$\omega_{b,ask}$</th>
<th>$\omega_{b,bid}$</th>
<th>$\omega_{ask,bid}$</th>
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<tbody>
<tr>
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<td>132</td>
<td>62</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>116</td>
<td>81</td>
<td>124</td>
</tr>
<tr>
<td><strong># &gt; 0</strong></td>
<td>132</td>
<td>132</td>
<td>25</td>
<td>1</td>
<td>2</td>
<td>-</td>
<td>1</td>
<td>77</td>
<td>13</td>
</tr>
<tr>
<td><strong># &lt; 0</strong></td>
<td>2</td>
<td>-</td>
<td>37</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>115</td>
<td>4</td>
<td>111</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.15***</td>
<td>0.37***</td>
<td>-0.06***</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>-0.02***</td>
<td>0.01***</td>
<td>-0.38***</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$f_b$</th>
<th>$f_{ask}$</th>
<th>$f_{bid}$</th>
<th>$g_b$</th>
<th>$g_{ask}$</th>
<th>$g_{bid}$</th>
<th>$\omega_{b,ask}$</th>
<th>$\omega_{b,bid}$</th>
<th>$\omega_{ask,bid}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong># Firms</strong></td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td><strong># &gt; 0</strong></td>
<td>6</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td><strong># &lt; 0</strong></td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.19***</td>
<td>0.46***</td>
<td>-0.07***</td>
<td>-</td>
<td>-</td>
<td>0.00</td>
<td>-0.06***</td>
<td>0.03***</td>
<td>-0.21***</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table III
Estimated Credit Risk, Liquidity, and Correlation Premia

The table presents the mean, standard deviation, minimum, and maximum of the model-implied premia components. Contrary to Table I, the column headings denote the actual, not the average, firm-day rating. We thus also report results for the CCC rating class. \(bd\) gives the pure credit risk, \(bl\) the pure liquidity, and \(bc\) the correlation component in the yield spread of a synthetical 5-year par bond. \(sd\) gives the pure credit risk, \(sl\) the pure liquidity, and \(sc\) the correlation component in the mid premium for a 5-year CDS contract. The standard deviation, minimum, and maximum are determined both over time and across observations within the rating class on each date. All values are in basis points.

<table>
<thead>
<tr>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bond Pure Credit Risk Premium ((bd))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.11</td>
<td>13.05</td>
<td>28.46</td>
<td>55.18</td>
<td>246.54</td>
<td>345.04</td>
<td>256.62</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.46</td>
<td>10.60</td>
<td>28.35</td>
<td>64.67</td>
<td>237.99</td>
<td>163.16</td>
<td>33.71</td>
</tr>
<tr>
<td>Min.</td>
<td>0.96</td>
<td>1.38</td>
<td>2.96</td>
<td>2.96</td>
<td>33.86</td>
<td>32.60</td>
<td>113.59</td>
</tr>
<tr>
<td>Max</td>
<td>52.21</td>
<td>260.85</td>
<td>352.11</td>
<td>1,214.39</td>
<td>1,807.09</td>
<td>1,126.95</td>
<td>386.33</td>
</tr>
<tr>
<td><strong>Bond Pure Liquidity Premium ((bl))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.67</td>
<td>12.16</td>
<td>24.92</td>
<td>32.63</td>
<td>50.65</td>
<td>61.85</td>
<td>3.09</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>3.12</td>
<td>30.12</td>
<td>43.95</td>
<td>55.77</td>
<td>54.77</td>
<td>58.19</td>
<td>3.48</td>
</tr>
<tr>
<td>Min.</td>
<td>0.35</td>
<td>1.41</td>
<td>3.02</td>
<td>3.09</td>
<td>1.51</td>
<td>1.48</td>
<td>1.50</td>
</tr>
<tr>
<td>Max</td>
<td>30.78</td>
<td>567.08</td>
<td>495.96</td>
<td>349.34</td>
<td>451.59</td>
<td>296.79</td>
<td>10.44</td>
</tr>
<tr>
<td><strong>Bond Correlation Premium ((bc))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.02</td>
<td>0.25</td>
<td>2.93</td>
<td>8.25</td>
<td>16.89</td>
<td>19.51</td>
<td>1.96</td>
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<td>16.01</td>
<td>44.38</td>
<td>14.17</td>
<td>1.98</td>
</tr>
<tr>
<td>Min.</td>
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<td>-0.61</td>
<td>1.01</td>
<td>1.08</td>
<td>1.16</td>
<td>0.72</td>
<td>0.38</td>
</tr>
<tr>
<td>Max</td>
<td>0.54</td>
<td>97.21</td>
<td>120.79</td>
<td>251.51</td>
<td>353.46</td>
<td>133.98</td>
<td>5.09</td>
</tr>
<tr>
<td><strong>CDS Pure Credit Risk Premium ((sd))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.18</td>
<td>13.45</td>
<td>28.98</td>
<td>56.33</td>
<td>249.52</td>
<td>349.97</td>
<td>258.83</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.42</td>
<td>12.88</td>
<td>28.35</td>
<td>65.59</td>
<td>242.82</td>
<td>167.45</td>
<td>34.96</td>
</tr>
<tr>
<td>Min.</td>
<td>5.12</td>
<td>4.24</td>
<td>4.65</td>
<td>4.58</td>
<td>34.92</td>
<td>33.97</td>
<td>115.92</td>
</tr>
<tr>
<td>Max</td>
<td>52.91</td>
<td>279.05</td>
<td>356.83</td>
<td>1,281.93</td>
<td>1,948.43</td>
<td>1,175.58</td>
<td>397.29</td>
</tr>
<tr>
<td><strong>CDS Pure Liquidity Premium ((sl))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.16</td>
<td>1.64</td>
<td>1.79</td>
<td>2.21</td>
<td>4.31</td>
<td>9.00</td>
<td>8.77</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.30</td>
<td>6.47</td>
<td>2.32</td>
<td>8.23</td>
<td>40.46</td>
<td>43.38</td>
<td>12.05</td>
</tr>
<tr>
<td>Min.</td>
<td>-0.61</td>
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<td>0.92</td>
<td>0.82</td>
<td>-153.82</td>
<td>-152.59</td>
<td>-6.86</td>
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<tr>
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<td>8.97</td>
<td>27.66</td>
<td>123.06</td>
<td>194.25</td>
<td>96.38</td>
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<td><strong>CDS Correlation Premium ((sc))</strong></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.14</td>
<td>0.46</td>
<td>0.19</td>
<td>0.45</td>
<td>5.41</td>
<td>8.48</td>
<td>5.57</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.34</td>
<td>0.83</td>
<td>0.49</td>
<td>1.49</td>
<td>10.53</td>
<td>7.96</td>
<td>1.30</td>
</tr>
<tr>
<td>Min.</td>
<td>-1.26</td>
<td>-0.89</td>
<td>-6.68</td>
<td>-1.24</td>
<td>-3.19</td>
<td>-1.42</td>
<td>-1.38</td>
</tr>
<tr>
<td>Max</td>
<td>0.49</td>
<td>4.86</td>
<td>5.35</td>
<td>51.83</td>
<td>98.85</td>
<td>43.77</td>
<td>18.17</td>
</tr>
</tbody>
</table>
Table IV

The Dynamic Relationship of Credit Risk, Liquidity, and Correlation Premia

The table presents the coefficient estimates for the vector error correction model. \( \Delta bd \) and \( \Delta sd \) are the pure credit risk, \( \Delta bl \) and \( \Delta sl \) the pure liquidity, \( \Delta bc \) and \( \Delta sc \) the correlation premium components for the bond and the CDS. The dependent variables are the premium changes, the explanatory variables are the vector error correction terms and the lagged premia changes. The first row of each panel displays the number of firms for which a) the augmented Dickey-Fuller test cannot reject a unit root in the premia time series at the 10% significance level, b) the augmented Dickey-Fuller test can reject a unit root in the first differences at the 10% level, c) the Johansen test cannot reject cointegration of the time series at the 10% level, d) the augmented Dickey-Fuller can reject a unit root in the residuals of the VECM at the 10% level. The second row displays the average cointegration coefficient across these firms. The third row gives the average error correction coefficient where the bond (column \( \Delta bd \), \( \Delta bl \), and \( \Delta bc \)) and the CDS premium change (column \( \Delta sd \), \( \Delta sl \), and \( \Delta sc \)) are the dependent variables. The fourth row gives the average coefficient for the five lagged bond premium changes, the fifth row the average coefficient for the five lagged CDS premium changes. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms. Coefficients are given for premia in basis points, the adjusted \( R^2 \) is in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>Pure Credit Risk Premia</th>
<th>Pure Liquidity Premia</th>
<th>Correlation Premia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta bd )</td>
<td>( \Delta sd )</td>
<td>( \Delta bl )</td>
</tr>
<tr>
<td># Firms</td>
<td>147</td>
<td>143</td>
<td>140</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-1.00***</td>
<td>9.38***</td>
<td>-62.92***</td>
</tr>
<tr>
<td>Error Corr. Coef.</td>
<td>-0.89***</td>
<td>-0.19***</td>
<td>-0.05*</td>
</tr>
<tr>
<td>Lagged ( \Delta ) Bond</td>
<td>-0.77***</td>
<td>0.70***</td>
<td>-0.32***</td>
</tr>
<tr>
<td>Lagged ( \Delta ) CDS</td>
<td>0.65***</td>
<td>-0.85***</td>
<td>-0.03</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>15.35</td>
<td>13.22</td>
<td>26.65</td>
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</table>

Panel A: All

<table>
<thead>
<tr>
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<th>Panel B: Investment Grade</th>
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<tr>
<td></td>
<td># Firms</td>
</tr>
<tr>
<td></td>
<td>138</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-1.00***</td>
</tr>
<tr>
<td>Error Corr. Coef.</td>
<td>-0.77***</td>
</tr>
<tr>
<td>Lagged ( \Delta ) Bond</td>
<td>-0.40***</td>
</tr>
<tr>
<td>Lagged ( \Delta ) CDS</td>
<td>0.32***</td>
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<tr>
<td>Adj. ( R^2 )</td>
<td>8.75</td>
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</table>

Panel B: Investment Grade

<table>
<thead>
<tr>
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<th>Panel C: Subinvestment Grade</th>
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<tr>
<td></td>
<td># Firms</td>
</tr>
<tr>
<td></td>
<td>9</td>
</tr>
<tr>
<td>Coint. Coef.</td>
<td>-0.98***</td>
</tr>
<tr>
<td>Error Corr. Coef.</td>
<td>-1.35***</td>
</tr>
<tr>
<td>Lagged ( \Delta ) Bond</td>
<td>-0.87***</td>
</tr>
<tr>
<td>Lagged ( \Delta ) CDS</td>
<td>0.23***</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>21.99</td>
</tr>
</tbody>
</table>
Table V
Credit Risk, Liquidity, and Correlation Premia Without CDS Liquidity

The table presents the mean, standard deviation, minimum, and maximum of the model-implied premia components for each rating class when stochastic liquidity in the CDS market is ignored. As in Table III, the column headings denote the actual, not the average, firm-day rating. We thus also report results for the CCC rating class. \( bd \) gives the pure credit risk, \( bl \) the pure liquidity, and \( bc \) the correlation component in the yield spread of a synthetical 5-year par bond. \( sd \) gives the mid premium for a 5-year CDS contract which only reflects credit risk.

The standard deviation, minimum, and maximum are determined both over time and across observations within the rating class on each date. All values are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>All</th>
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<tbody>
<tr>
<td><strong>Bond Pure Credit Risk Premium ((bd))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.18</td>
<td>13.72</td>
<td>28.05</td>
<td>54.89</td>
<td>249.92</td>
<td>348.59</td>
<td>272.20</td>
<td>46.26</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.41</td>
<td>18.30</td>
<td>28.36</td>
<td>64.67</td>
<td>238.09</td>
<td>163.18</td>
<td>33.73</td>
<td>82.61</td>
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<tr>
<td>Min.</td>
<td>4.04</td>
<td>3.38</td>
<td>3.95</td>
<td>4.04</td>
<td>33.60</td>
<td>33.98</td>
<td>209.11</td>
<td>3.38</td>
</tr>
<tr>
<td>Max</td>
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<td>255.31</td>
<td>351.75</td>
<td>1,214.12</td>
<td>1,807.87</td>
<td>1,126.66</td>
<td>386.34</td>
<td>1,806.87</td>
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<tr>
<td><strong>Bond Pure Liquidity Premium ((bl))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.11</td>
<td>11.82</td>
<td>25.63</td>
<td>31.11</td>
<td>49.12</td>
<td>56.63</td>
<td>1.58</td>
<td>24.65</td>
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<tr>
<td>Std. Dev.</td>
<td>1.70</td>
<td>36.47</td>
<td>52.32</td>
<td>58.23</td>
<td>60.46</td>
<td>63.38</td>
<td>12.05</td>
<td>50.74</td>
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<tr>
<td>Min.</td>
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<td>-120.48</td>
<td>-18.62</td>
<td>-200.82</td>
<td>-204.39</td>
<td>-150.95</td>
<td>-16.68</td>
<td>-204.39</td>
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<tr>
<td>Max</td>
<td>30.34</td>
<td>109.36</td>
<td>485.57</td>
<td>328.30</td>
<td>452.84</td>
<td>281.89</td>
<td>105.82</td>
<td>485.57</td>
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<tr>
<td><strong>Bond Correlation Premium ((bc))</strong></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>Mean</td>
<td>-0.31</td>
<td>-0.08</td>
<td>2.62</td>
<td>10.05</td>
<td>15.04</td>
<td>21.20</td>
<td>1.80</td>
<td>4.40</td>
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<tr>
<td>Std. Dev.</td>
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<td>3.86</td>
<td>6.55</td>
<td>15.23</td>
<td>24.31</td>
<td>14.16</td>
<td>0.87</td>
<td>10.05</td>
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<tr>
<td>Min.</td>
<td>-1.35</td>
<td>-1.25</td>
<td>-1.00</td>
<td>-1.10</td>
<td>-1.83</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.83</td>
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<tr>
<td>Max</td>
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<td>193.46</td>
<td>522.07</td>
<td>325.42</td>
<td>112.67</td>
<td>8.62</td>
<td>522.07</td>
</tr>
<tr>
<td><strong>CDS Pure Credit Risk Premium ((sd))</strong></td>
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<td></td>
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<tr>
<td>Mean</td>
<td>6.20</td>
<td>15.57</td>
<td>31.04</td>
<td>58.99</td>
<td>259.22</td>
<td>367.48</td>
<td>273.55</td>
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<tr>
<td>Std. Dev.</td>
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<td>29.59</td>
<td>68.36</td>
<td>237.40</td>
<td>164.85</td>
<td>30.06</td>
<td>85.87</td>
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<tr>
<td>Min.</td>
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<td>4.77</td>
<td>4.80</td>
<td>36.71</td>
<td>37.11</td>
<td>211.53</td>
<td>4.18</td>
</tr>
<tr>
<td>Max</td>
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<td>256.11</td>
<td>355.75</td>
<td>1,302.80</td>
<td>1,849.63</td>
<td>1,158.31</td>
<td>400.66</td>
<td>1,849.63</td>
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