Shock Propagation Through Cross-Learning with Costly Price Acquisition*

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Abstract

This paper shows that cross-learning from other firms' stock prices leads to the propagation of unrelated shocks. Located in a circular network, neighboring firms share a productivity shock. Stock prices are informative about future productivity and managers learn from them to improve investment efficiency. With costly price acquisition, each firm only learns from its closest neighbors and is thus exposed to movements in their prices reflecting their own cross-learning from more distant firms. This non-local noise is then reflected in each firm's stock price and transmitted further to other firms, particularly in uncertain times and highly correlated networks.

Keywords: cross-learning, feedback effects, information acquisition, managerial learning.

JEL Classification: D83, D85, G14, G31.

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1 Introduction

Informative stock prices can guide real decisions because they aggregate the private information of a large number of market participants. The idea that there exists such a "feedback effect" from the financial market to firm decisions has received empirical and theoretical support from the existing literature.[1] In modern economies firms are highly interconnected. For example, they might be linked to each other through a production network or operate in overlapping markets. As a result, firms should be able to improve their decisions by learning from the prices of other firms as well. For instance, consider a firm that has to decide whether to increase its production of a given good and that observes an unusually high stock price of a firm operating in the same market. The firm does not know whether this price move is driven by fundamentals (like higher future demand) or other factors (like positive market sentiment). Overall, the firm should, however, interpret this increase as a positive signal about future demand and scale up investment.

In this paper, I analyze a setting in which multiple firms are located in a circular network. Firms are pairwise connected through their exposure to a common productivity shock. Hence each firm's productivity shock is correlated with that of its left and right neighbor but uncorrelated with that of all other firms. Firm managers are imperfectly informed about these shocks and have an incentive to learn additional information about them to invest more efficiently. Because stock prices reflect informed traders' private information about local productivity shocks, each manager can improve his knowledge by relying, in part, on the stock prices of the firm's neighbors when deciding on firm investment.

As a first result, I show that in this setup firm managers can get the most precise signal about their firm's productivity shock by combining the prices of all firms in the network.

[1]See Bond et al. (2012) for a comprehensive survey of the feedback literature.

Therefore, even though only the stock price of each firm's direct neighbor reflects actual information about its future productivity shock, the prices of other, unconnected firms are useful signals as well. Intuitively, these prices are necessary to correctly interpret movements in the stock price of the direct neighbor and their observation allows the manager to ignore price movements that are solely due to this firm's learning from its neighbor. In this benchmark equilibrium, firm investment and stock prices are correlated for neighboring firms that share a common productivity shock, but uncorrelated for all other firm pairs. Thus, a given location-specific shock affects the two firms that are directly exposed, but not other firms, such that there is no shock propagation under costless price acquisition.

In the main model, I introduce an informational friction by assuming that the collection of price signals is costly. Therefore, firm managers have to weigh the benefit of collecting more price signals (more efficient investment) against its cost (higher information acquisition cost). In equilibrium, firm managers thus only observe a subset of price signals and cannot perfectly filter out all non-local noise. One might argue that in reality price acquisition costs should be negligible because price data is freely available to all market participants. However, this cost should be interpreted in a broader sense. It requires a significant amount of time and resources by firms to properly analyze these prices or as Vives and Yang (2016) put it, "data can be viewed as information only after it has been analyzed." That is to say, it is relatively easy to observe the level of prices, but it requires a lot of in-depth analysis and background knowledge to map this number into an informative signal about a specific firm's future fundamentals or payoffs.

In the model, financial markets are populated by informed insiders who trade claims to the local firm's terminal payoff. These traders receive a private signal about the local shock and trade based on this information. The equilibrium stock price therefore reflects the aggregated private information about this shock together with a local noisy supply shock and serves as an endogenous public signal. Benevolent firm managers can then combine these signals with their private information to improve the efficiency of their capital investment. This investment decision and two productivity shocks (the "fundamentals") determine the firm's terminal value and therefore final cash flow. While information about the first component (the "local" shock) is perfectly known by the local manager, he only observes the prior distribution for the second component (the "non-local" shock). The manager can, however, learn additional information about the non-local shock from stock prices of other firms.

In equilibrium, each firm's stock price reflects the insiders' expectation about both components of the final payoff, the composite productivity shock and future firm investment. Moreover, the price is also affected by a firm-specific random supply shock that adds non-fundamental noise. Each firm's investment decision depends on an endogenously chosen vector of stock prices that helps the manager to invest more efficiently. Importantly, I show that the equilibrium prices and investment decisions differ from those in the benchmark equilibrium along several dimensions if each firm manager faces a price acquisition cost.

Most importantly, if firms can only learn from a subset of stock prices, their investment decision and stock price are no longer only exposed to local shocks, but depend on fundamental and financial shocks from multiple remote locations. Intuitively, each firm cannot filter out all non-local noise from the stock price of its direct neighbor such that its manager's conditional expectation, and so the firm's investment decision, is affected by this noise. Since this firm's stock price, in turn, reflects the expected investment decision, the remote shock is further transmitted through the entire network of firms. Interestingly, I show that this propagation mechanism is stronger in times of high fundamental uncertainty and high pairwise correlation of fundamentals, when managers' have a particularly high incentive to rely on price signals. A novel implication of the model is that the propagation of shocks is non-monotonic. Thus, a given shock can affect a certain group of nearby firms, then skip several locations, and affect more distant firms again. Moreover, I show that a larger number of observed stock prices always allows firms to invest more efficiently because each manager's expectation of his firm's future productivity shock becomes more precise. However, as a byproduct, the sensitivity of stock prices and investment decisions to unrelated, non-local shocks also increases.

Next, I allow each firm to choose the number of observed prices competitively. Interestingly, I show that each firm's private incentive to acquire more price information is always higher than the social incentive. Intuitively, each firm does not internalize that it renders its own price more noisy for its backward neighbor by collecting more price signals from its forward neighbors. As a result, the equilibrium number of observed prices is always inefficiently high. In a numerical exercise, I explicitly compute the number of observed prices and other key variables in equilibrium and compare it to the social optimum that maximizes all firms' ex ante value collectively.

Overall, the main contribution of this paper is to show the equilibrium implications of firms' cross-learning when the collection or interpretation of price data is costly. Stepping away from the frictionless benchmark highlights several novel mechanisms that can help to understand a variety of stylized facts regarding the impact and propagation of idiosyncratic macroeconomic and financial shocks. First, there is a substantial empirical literature concluding that reward for risk reflects, to some extent, local factors that should be diversifiable.[2] In this paper, I show that local, firm-specific shocks can be transmitted through the entire network of firms as long as each firm partially bases its investment decision on the stock price of another firm (but not on all prices). These shocks, thus, do not wash out quickly as the number of firms increases and cannot be diversified away easily trough investment in an index fund for instance. Moreover, I show that the rate of decay for idiosyncratic shocks depends crucially on the information environment in the financial market. For example, higher prior uncertainty or less noisy supply increase the propagation intensity and market-wide effect of firm-level shocks. Second, episodes of high uncertainty (e.g. due to the arrival of novel technologies) are often associated with "exuberant" joint movements in asset prices and real economic activity.[3] In these episodes, when firms have a high incentive to learn from their neighbors' prices, the propagation of shocks through the network of firms is strongest. As a result, a local fundamental (productivity) or non-fundamental (liquidity) shock can be transformed into a (quasi) systematic shock that affects directly and indirectly connected firms, such that a large positive shock to a specific firm can lead to above-average ("exuberant") investment and a rally in stock prices for many firms. Taken together, this paper helps to understand the real effects of financial markets with many connected firms in general. In particular, it shows that cross-learning amplifies the degree of interconnectedness in an economy because fundamentally independent firms *appear* correlated even though they do not directly learn from each other. From a technical perspective, I provide a novel setup with multiple interconnected firms and learning from several prices. A specific functional form for the noisy supply of assets and the traders' objective function, that have both been used in other contexts before, allow me to keep the model tractable.

[2]See e.g. Bekaert and Harvey (1995) for empirical results or Garleanu et al. (2015) for a summary of this literature and a unifying theory.
[3]See e.g. Angeletos et al. (2012) or Huang and Zeng (2015) for models along these lines.

This paper builds on the idea in Hayek (1945) that efficient markets aggregate private and public information into prices. The extent to which prevailing prices are informative about the future value of a firm is important for both traders and real decision makers, such as firm managers, central bankers, or politicians. The more information these agents can extract from stock prices, the more they can improve on their economic decisions, such as trading, corporate investment, and policy interventions.[4] This key insight of informative prices led to the large literature on noisy rational expectations equilibria following Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985). A recent literature builds further on this insight and models an informational feedback effect from the financial market to firm decisions. For instance, in Subrahmanyam and Titman (2001) and Goldstein et al. (2013), informative signals originating from the financial market influence a single firm's investment decision: firm managers are imperfectly informed about future productivity and can learn some additional information from stock prices.[5]

This paper is most closely related to Foucault and Fresard (2014), Huang and Zeng (2015), and Dessaint et al. (2016). These three papers also consider setups in which a firm can learn additional information from the stock prices of other firms in the economy. There are two key differences with respect to these papers. First, the circular structure in my paper implies that there exist firms in the economy which are not directly linked to each other, but only indirectly because both have a common neighbor, for example. Consequently, this novel framework allows me to study the impact and importance of idiosyncratic shocks on economic aggregates and the equilibrium decisions of *unrelated*

[4]See Luo (2005), Chen et al. (2007), Bakke and Whited (2010), Edmans et al. (2012), and Edmans et al. (2017) for empirical evidence of a feedback effect from the stock market to real decisions.

[5]See also Subrahmanyam and Titman (2013), Goldstein and Yang (2014), David et al. (2016), Goldstein and Yang (2015a), and Hassan and Mertens (2017) for models with a feedback effect from informative stock prices to a single firm's investment decision.

firms. Second, firms endogenously determine the precision and extent of their composite price signal. I show that this information acquisition decision is closely related to the propagation of shocks throughout the network.

Furthermore, this paper is related to the literature on (information-based) financial contagion such as Admati (1985), Kodres and Pritsker (2002), Pasquariello (2007), and Caballero and Simsek (2013).[6] Prior work in this literature also emphasizes how local shocks can be transmitted across assets or firms and that this transmission can lead to aggregate fluctuations. The main contribution relative to this literature is to derive a novel contagion mechanism. In this paper, contagion arises through a feedback effect between stock prices and investment. Therefore, local shocks are transmitted through a *different* learning channel: prices in one location affect investment in another location because they convey information. This impact on investment is then reflected in the local price which in turn affects investment at another location. Interestingly, this contagion mechanism does not rely on the fact that one agent learns from unrelated prices and thus transmits distant shocks directly, as e.g. in Admati (1985) or Kodres and Pritsker (2002). In the main model, traders and managers only learn from a subset of prices but non-local shocks from other firms are still transmitted throughout the network.

The remainder of this paper is organized as follows: Section 2 sets up the model. Section 3 solves for the equilibrium in a benchmark economy where all firms observe all prices. Section 4 shows the equilibrium in the main model where each firm only observe a subset of prices. Section 5 endogenizes the number of observed prices and Section 6 concludes.

[6]See also Gabaix (2011), Acemoglu et al. (2015), Barrot and Sauvagnat (2016), and Bigio and La'O (2016).

2 The Model

2.1 Setup

There is a large number of firms (or "locations"), indexed by $i \in \mathcal{N} = \{1, \ldots, N\}$. Each firm is run by a benevolent manager who decides on its capital investment. Together with a random productivity shock, this decision determines the firm's terminal output. Each firm has access to a linear production technology $Y_i = e^{\theta_i} K_i$, where K_i denotes capital investment and θ_i is a cross-correlated productivity shock described in greater detail below. Claims to this output are traded in a secondary financial market. Three time periods exist. In $t = 0$, firm managers acquire a set of n costly price signals from the other firms' stock prices to improve their investment decision. In $t = 1$, the financial market is active and stock prices are determined. In $t = 2$, the firm managers make an investment decision, the terminal payoffs are realized and traders get paid.

Productivity Shocks and Information Sets

The N firms are located in a circular network as depicted in Figure 1. I choose this network structure primarily because it allows for (i) the presence of connected and unconnected firms and (ii) a tractable solution. Neighboring firms are exposed to a common shock such that their fundamentals (θ_i) are correlated. For instance, the fundamental shock for firm $i = 1$ is correlated with that of its forward neighbor ($i = 2$) and that of its backward neighbor ($i = N$), but uncorrelated with that of all other firms in the network.

Definition 1 *Let $x_i^{(j)} \equiv x_k$, such that $x_i^{(j)}$ denotes the realization of the generic random variable x at location k, where k is $j \in \mathcal{J}$ firms clockwise away from firm i. Let $\mathcal{J} = \{1, \ldots, N-1\}$ denote the set of neighbors for each firm.*

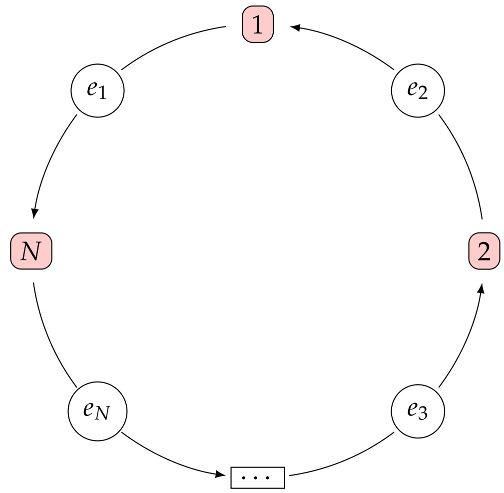

Figure 1: Circular network with N firms.

I assume that θ_i, the productivity shock for firm i, equals the sum of two components:[7]

$$\theta_i \equiv e_i + \rho e_i^{(1)} \quad \text{with} \quad \rho \in [-1, 1]. \tag{1}$$

The two components are independent, i.e. $e_i \overset{iid}{\sim} N\left(0, \sigma_e^2\right)$ for all $i \in \mathcal{N}$. Intuitively, each firm's future productivity shock is mainly determined by the "local" shock e_i, which also affects the productivity of the firm's backward neighbor. Vice versa, θ_i also depends on the local shock of the firm's forward neighbor, $e_i^{(1)}$. The constant ρ determines the strength and sign of this effect, and thus the degree of fundamental entanglement in the economy. Consequently, this structure for θ_i implies that each firm shares a common shock with its forward and backward neighbor. This cross-exposure in productivity shocks is important to give firm managers an incentive to "cross-learn," i.e. to learn from prices of other firms in the network.

Each firm manager is perfectly informed about the local component e_i, but uninformed about all other shocks, including $e_i^{(1)}$ which also affects his firm's future productivity.

[7]Several papers in the finance literature assume that the fundamental value is affected by more than one shock. See Goldstein and Yang (2015b) or Kondor (2012) for recent examples. Note that the shocks of neighboring firms can be positively or negatively correlated.

This information structure captures the fact that firm managers are most likely precisely informed about local factors that determine future productivity (e_i). However, there might be other, non-local factors, such as future product demand for a peer firm, that are also relevant for future profitability. Because information about these non-local factors is reflected in the stock prices of other firms, firm managers have an incentive to partially base their investment decisions on these prices.

There are several possible reasons why two firms might be exposed to a common fundamental shock. The main interpretation in this paper is that each firm sells its final product in the local market and the local market of its forward neighbor. Because it sells its output primarily in the local market, total demand is mostly determined by local factors, captured by e_i. Firm managers are precisely informed about this pool of uncertainty and can adjust firm investment and thus output accordingly. However, firm managers are less precisely informed about demand conditions (captured by $e_i^{(1)}$) in the local market of its forward neighbor. The manager therefore tries to update his belief based on the local stock price in this location. Alternatively, two firms could rely on a common supplier or, more generally, be connected to each other in a production network. To keep the model as simple as possible and to focus on the effects of cross-learning, I treat the underlying economic reason for the correlation in fundamentals as exogenous.

In addition to their private signals, firm managers use the stock prices of other firms to update their prior about the non-local productivity shock. A key feature of the model is the assumption that acquiring these price signals is costly. In particular, at $t = 0$ each firm manager has to pay a cost $C(n_i)$ for observing the stock prices of the next n_i firms in the network.[8]

[8] I show below that it is optimal for firms to focus on the prices of the next (in a clockwise direction) firms, i.e. they do not have an incentive to "skip" firms in the information acquisition decision.

Trading and Firm Investment

Claims to each firm's final payoff Y_i are traded by a unit continuum of identical "insiders" in each location, indexed by $j \in [0,1]$. These agents are risk-neutral and trade competitively based on the same information set as the local manager, $\mathcal{I}_i^M = \{e_i, P_i^{(1)}, \ldots, P_i^{(n_i)}\}$. In particular, insider j in location i chooses his asset holdings in the local stock to maximize a quadratic objective function:

$$\max_{z_{ij}} E\left[z_{ij}\left(Y_i - P_i\right)|\mathcal{I}_i^M\right] - \frac{1}{2}z_{ij}^2. \tag{2}$$

Intuitively, each insider maximizes his expected trading profit minus a quadratic trading cost. This specific objective function ensures that each trader's demand remains finite and has been used in the existing literature, like Banerjee et al. (2017) and Vives (2011).

To prevent the price from fully revealing the insiders' private information, I assume that each asset is in noisy supply $L\left(x_i, P_i\right)$ with $x_i \overset{iid}{\sim} N\left(0, \sigma_x^2\right)$. To get a closed-form solution for the equilibrium stock prices and investment decisions, I assume a particular form for this noisy supply curve: $L\left(x_i, P_i\right) = \left(e^{-x_i} - 1\right)P_i$, similar to that used in Goldstein et al. (2013) or Huang and Zeng (2015).[9]

The market clearing condition for firm i, then requires that aggregate demand equals the noisy supply: $\int_0^1 z_{ij}dj = L\left(x_i, P_i\right)$, which implies that the equilibrium price for firm i is given by:

$$P_i = E\left[Y_i|\mathcal{I}_i^M\right]e^{x_i}. \tag{3}$$

Intuitively, each firm's stock price is equal to the expected payoff (under the insiders' information set) disturbed by an independent noisy supply shock.

Firm managers are also risk-neutral and choose capital investment (K_i) to maximize the

[9]These two papers use the functional form $L(x_i, P_i) = 1 - 2\Phi\left(x_i - \log P_i\right)$ to obtain tractability.

expected firm value, V_i.[10] Following the existing literature such as Goldstein et al. (2013), I assume that the firm's terminal value is equal to output net of a quadratic investment cost and the information acquisition cost:[11]

$$V_i \equiv Y_i - \frac{1}{2}K_i^2 - C(n_i). \tag{4}$$

Due to the curvature of each manager's objective function, they effectively act risk-aversely when choosing K_i and therefore have an incentive to collect the most precise signal (subject to the information acquisition cost).

2.2 Optimal Trading and Firm Investment

For a given set of observed stock prices, determined at $t = 0$, each firm manager chooses K_i to maximize the expected firm value conditional on the information set $\mathcal{I}_i^M$. As a result, the optimal log capital investment decision is given by:[12]

$$k_i = E\left[\theta_i|\mathcal{I}_i^M\right] + \frac{1}{2}\mathrm{Var}\left(\theta_i|\mathcal{I}_i^M\right). \tag{5}$$

Intuitively, each manager increases firm investment if the conditional expectation about his firm's future productivity increases. Thus, k_i depends positively on the manager's private information about the local shock e_i and his conditional expectation about the non-local component. Given that this expectation depends on the set of observed stock prices, it represents the feedback channel in this setup.

From equations (3) and (5), it follows that the equilibrium log price for firm i can be written as:

$$p_i = 2E\left[\theta_i|\mathcal{I}_i^M\right] + \mathrm{Var}\left(\theta_i|\mathcal{I}_i^M\right) + x_i. \tag{6}$$

[10] Instead of explicitly modeling the managers' contracts, I take this step as given and assume that they act benevolently.

[11] Following the existing feedback literature, I assume that the investment and acquisition cost are private, such that the asset is only a claim to Y_i.

[12] Throughout, I denote log prices and investment decision by lower case letters: $p_i \equiv \log P_i$ and $k_i \equiv \log K_i$.

Intuitively, each firm's stock price depends on a constant (the conditional variance term), the insiders' expectation of the firm's future fundamental, and a random supply shock.

2.3 Discussion

Before proceeding, I discuss the two main assumptions of the baseline model. First, I assume that it is costly for firm managers to acquire price signals. I model this friction through the information acquisition cost $C(n_i)$ that makes it costly for firms to observe the stock prices of their peer firms. One might argue that price data is easily available to all market participants, such that firm managers should be able to collect as many price signals as possible to get the most precise estimate about their firms future shock. However, this argument neglects the fact that it requires a significant amount of sophistication or attention to interpret these signals correctly. Therefore, the main idea is similar to Vives and Yang (2016) who argue that prices can only be considered informative after they have been analyzed (by traders) or to the literature on optimal attention allocation (or inattention), such as Abel et al. (2013) and Kacperczyk et al. (2016). Second, I assume that local insiders are the only informed agents who trade firm i's stock. This assumption implies that p_i only reflects information about the local shock e_i such that the firm's backward neighbor can use this price to improve his knowledge about his firm's future productivity. If this price also reflected information about the non-local shock $e_i^{(1)}$, each firm manager would have an incentive to directly learn from his own price as well which would make the solution more complicated. This assumption is, however, not crucial for the underlying economic mechanism which only requires that each firm manager has an incentive to infer some information from the prices of his peer firms as well.

3 Full Information Benchmark Equilibrium

In this section, I solve for the cross-learning equilibrium with freely observable prices, i.e. $C(\cdot) = 0$, which serves as a benchmark for the main results. Thus, all traders and managers observe the N equilibrium stock prices and use the entire price vector $\vec{p} = [p_1, \ldots, p_N]'$ (together with private information) in their equilibrium decisions.

Definition 2 *A symmetric, benchmark equilibrium consists of a log-price function for each firm* $p(e_i, x_i, \vec{p}) : \mathbb{R}^{N+2} \to \mathbb{R}$ *and a log-investment function for each firm* $k(e_i, \vec{p}) : \mathbb{R}^{N+1} \to \mathbb{R}$ *such that:*

(a) insiders maximize their expected utility,

(b) each firm manager maximizes the expected firm value by choosing K_i*, and*

(c) the stock market clears for each firm.

As a first step, I can rewrite the equilibrium condition for firm i's stock price in equation (6) as:

$$p_i = \pi_0 + 2e_i + 2\rho E[e_i^{(1)} | \vec{p}] + x_i \tag{7}$$

where π_0 is a constant that subsumes the conditional variance term.

Given that each informed trader receives an informative signal about the local fundamental e_i, the equilibrium log-price p_i reflects this information to the firm's backward neighbor that tries to infer information about this component. Furthermore, p_i is also affected by stock prices of other firms and the noisy supply shock, x_i.

Lemma 1 *In the benchmark economy, the vector of stock prices can be combined to get an unbiased signal* $z_i(\vec{p}) = e_i + \frac{1}{2}x_i$ *about the fundamental shock of firm* $i \in \mathcal{N}$.

Proof: *See Appendix A.2.1.*

Lemma 1 shows that each firm manager can combine the N stock prices to retrieve the local traders' private signal about e_i plus ($\frac{1}{2}$ times) the local supply shock. Interestingly, this signal cannot be inferred from the local price p_i alone because this price also contains non-local information through the manager's cross-learning. Consider for a moment that each manager only learned from its forward neighbor's price $p_i^{(1)}$. Then, the manager of firm i's backward neighbor needs to observe $p_i^{(1)}$, i.e. the price of a firm two locations away to wipe out this source of uncertainty from p_i. However, if firm i's manager also observes the prices of its two forward neighbors, its backward neighbor has to observe three prices, and so on. Continuing this logic shows that the optimal price signal $z_i(\vec{p})$ depends on all N stock prices. Therefore, if each firm is able to condition its decision on all prices, it optimally uses all prices to get the most precise signal about the non-local component in its composite productivity shock.

Proposition 1 (Benchmark Equilibrium) *There exists a unique symmetric log-linear benchmark equilibrium in which the log-investment decision and the log-price for firm i is given by:*

$$
\begin{aligned}
k_i &= a_0 + e_i + a_1 \rho \left(e_i^{(1)} + \frac{1}{2} x_i^{(1)} \right) \\
p_i &= 2 \left(a_0 + e_i + a_1 \rho \left(e_i^{(1)} + \frac{1}{2} x_i^{(1)} \right) \right) + x_i
\end{aligned}
$$

where $i \in \mathcal{N}$ and the expressions for all coefficients are provided in Appendix A.2.2.

Proof: *See Appendix A.2.2.*

Proposition 1 shows that in the benchmark economy, firm investment is affected by the local productivity shock and that of the firm's forward neighbor, given that the firms' fundamentals are correlated ($\rho \neq 0$). Because each firm manager is able to partially recover the informed trader's private signal about $e_i^{(1)}$ from the vector of prices, managerial expectations and thus firm investment also depend on this signal and on the local supply shock $x_i^{(1)}$. Note, however, that other non-local shocks do not affect firm investment k_i since managers are able to filter out these shocks perfectly when constructing their price signal $z_i(\vec{p})$. Similarly, each firm's price is affected by local fundamental and non-fundamental shocks, as well as both shocks originating from its forward neighbor's location.

As a result, if all prices are observed, shocks from unrelated firms are not transmitted throughout the network. Thus, somewhat paradoxically, even though each firm observes and uses all N prices (Lemma 1), only price shocks from its forward neighbor are reflected in its investment decision (Proposition 1). Intuitively, precisely the fact that all prices are observed allows firm managers to filter out unrelated noise in prices and to recover the informed trader's private signal about the non-local component of their productivity shock $e_i^{(1)}$.

4 Cross-Learning Equilibrium with Costly Price Acquisition

In this section, I solve for the equilibrium in the main model for a fixed choice of $n_i = n \in \mathcal{J} = \{1, \dots, N-1\}$ for all firms and endogenize this number in Section 5. Therefore, the manager and insiders in location i only observe price signals for the next n firms and can no longer use the optimal price signal in Lemma 1.

Definition 3 *A symmetric, cross-learning equilibrium with costly price acquisition consists of a log-price function for each firm $p(e_i, x_i, p_i^{(1)}, \dots, p_i^{(n)}) : \mathbb{R}^{n+2} \to \mathbb{R}$, and a log-investment function for each firm $k(e_i, p_i^{(1)}, \dots, p_i^{(n)}) : \mathbb{R}^{n+1} \to \mathbb{R}$ such that:*

(a) insiders maximize their expected utility,

(b) each firm manager maximizes the expected firm value by choosing K_i and n_i, and

(c) the stock market clears for each firm.

As before, I can simplify equation (6) to get:

$$p_i = \pi_0^* + 2e_i + 2\rho E[e_i^{(1)}|p_i^{(1)}, \ldots, p_i^{(n)}] + x_i \tag{8}$$

where the conditional expectation now only depends on the n observed prices and π_0^* subsumes the conditional variance term.[13]

As in the benchmark model, p_i reflects information about the local productivity shock e_i through the local insiders' perfect knowledge about this shock. As before, this price also reflects local noisy supply, x_i. However, in sharp contrast to the benchmark model, equation (8) cannot be easily inverted by the manager of firm i's backward neighbor to back out this unbiased signal. Intuitively, the conditional expectation $E[e_i^{(1)}|p_i^{(1)}, \ldots, p_i^{(n)}]$ cannot be filtered out by the manager due to its reliance on $p_i^{(n)}$. Since, this price is not observed by the firm's backward neighbor, it creates an additional source of uncertainty in the price signal. Therefore, all shocks affecting this price are now transmitted from a firm's forward neighbor to its backward neighbor (and further to other firms). Intuitively, there are now three sources of uncertainty in each stock price: (i) local fundamental variation (e_i), (ii) local non-fundamental variation (x_i), (iii) non-local variation induced by $p_i^{(n)}$.

Proposition 2 (Main Equilibrium for a fixed n) *There exists a unique symmetric log-linear cross-learning equilibrium with a fixed choice of $n_i = n \in \mathcal{J}$ in which the log-investment decision and the log-price is given by:*

$$\begin{aligned} k_i &= b_0 + e_i + b_1\rho\left(e_i^{(1)} + \frac{1}{2}x_i^{(1)} + b_n\rho p_i^{(n+1)}\right) \\ p_i &= 2\left(b_0 + e_i + b_1\rho\left(e_i^{(1)} + \frac{1}{2}x_i^{(1)} + b_n\rho p_i^{(n+1)}\right)\right) + x_i \end{aligned}$$

where $i \in \mathcal{N}$ and the expressions for all coefficients are provided in Appendix A.2.3.

Proof: *See Appendix A.2.3.*

[13] As I show in Section 5, there always exists a symmetric information acquisition equilibrium such that $n_i = n \forall i \in \mathcal{N}$.

Proposition 2 shows firm i's equilibrium investment and stock price in the economy with costly price acquisition. As before in the benchmark equilibrium, both equilibrium quantities depend on the local shocks and those of the firm's forward neighbor. However, now shocks to the stock price of the firm $n + 1$ locations away also impact k_i and p_i. Intuitively, firm i would like to use the price of its forward neighbor $p_i^{(1)}$ to learn about $e_i^{(1)}$. Importantly, firm i would like to filter out as much non-fundamental variation from this price signal as possible. As a result, firm i uses all n observed prices to do this. However, as firm $i + 1$ does the same with its forward neighbor (firm $i + 2$), firm i can only imperfectly subtract non-fundamental variation from $p_i^{(1)}$. More specifically, firm i is unable to control for movements in this price that are due to changes in $p_i^{(n+1)}$, which affects firm $i + 1$'s investment decision, but is unobserved by firm i. Therefore, all shocks that affect this remote stock price are reflected in firm i's investment decision and stock price.

As a result, the fact that firms can no longer condition on the entire set of prices leads to the propagation of non-local shocks. Importantly, the price $p_i^{(n+1)}$ that affects k_i and p_i is itself exposed to shocks from three different locations. (i) its local shocks $e_i^{(n+1)}$ and $x_i^{(n+1)}$; (ii) the local shocks of its forward neighbor $e_i^{(n+2)}$ and $x_i^{(n+2)}$; and (iii) shocks to the stock price $n + 1$ locations apart that cannot be filtered out from its forward neighbor's price, $p_i^{(2n+2)}$. Continuing this logic forward it follows that shocks from multiple remote locations affect investment decisions and stock prices.

Paradoxically, the fact that firms do *not* perfectly observe all prices leads to the propagation of shocks. Thus, shocks from distant locations have an impact on a given firm's investment decision precisely when this firm does not observe its price. Intuitively, if it could observe its price, it would be able to filter out this source of unrelated noise.

This mechanism is therefore fundamentally different from alternative models of learning-based financial contagion such as Admati (1985) or Kodres and Pritsker (2002), in which all asset prices are observed and agents' learning from unrelated prices leads to non-local shock exposure.

4.1 Shock Propagation

Based on the preceding analysis, I now analyze the propagation of shocks through firms' cross-learning in the main model, again taking the chosen number of observed prices as given. Proposition 2 shows that firm i's investment decision depends on the manager's knowledge of the local shock as well as a feedback signal from the stock market. The latter signal helps the manager to improve his knowledge about the non-local shock that affects his firm's composite productivity shock, but it also transmits non-local shocks as I formally show below.

Corollary 1 *If all firms have independent productivity shocks ($\rho = 0$), only local shocks affect firm investment (k_i) and stock prices (p_i).*

$$\frac{\partial k_i}{\partial e_i^{(j)}} = \frac{\partial k_i}{\partial x_i^{(j)}} = \frac{\partial p_i}{\partial e_i^{(j)}} = \frac{\partial p_i}{\partial x_i^{(j)}} = 0$$

for all $i \in \mathcal{N}$ and $j \in \mathcal{J}$.

Proof: *See Appendix A.2.4.*

Corollary 1 shows that if all firms' productivity shocks are independent, only local shocks affect firm investment and prices. Intuitively, in this case firm managers do not have an incentive to learn from their neighbors' stock prices and only use their private signal about e_i when choosing firm investment. Similarly, even if $\rho \neq 0$ but managers ignore the informational content of prices, non-local shocks cannot affect k_i and p_i. Therefore, firms' cross-learning is a necessary condition for shock propagation.

Corollary 2 *If neighboring firms have correlated productivity shocks ($\rho \neq 0$), firm investment (k_i) and stock prices (p_i) are affected by non-local shocks.*

$$\frac{\partial k_i}{\partial e_i^{(j)}} = \frac{1}{2}\frac{\partial p_i}{\partial e_i^{(j)}} = \begin{cases} (-1)^{D(j)}(\rho b_1)^j & j \in \mathcal{J}^{inf} \\ 0 & j \in \mathcal{J} \setminus \mathcal{J}^{inf} \end{cases}$$

and

$$\frac{\partial k_i}{\partial x_i^{(j)}} = \frac{1}{2}\frac{\partial p_i}{\partial x_i^{(j)}} = \begin{cases} \frac{1}{2}(-1)^{D(j)}(\rho b_1)^j & j \in \mathcal{J}^{inf} \\ 0 & j \in \mathcal{J} \setminus \mathcal{J}^{inf} \end{cases}$$

for all $i \in \mathcal{N}$. The expression for $b_1 > 0$ is given in the proof of Proposition 2 and the set of infecting firms is defined as: $\mathcal{J}^{inf} = \{1, n+1, n+2, \ldots, \kappa(n+1), \kappa(n+1)+1\}$ where κ is the largest positive integer such that $\mathcal{J}^{inf} \subseteq \mathcal{J}$. The indicator variable $D(j)$ determines the sign of the sensitivities and is defined in Appendix A.2.5.

Proof: *See Appendix A.2.5.*

The results in Corollary 2 show that fundamental ($e_i^{(j)}$) and non-fundamental ($x_i^{(j)}$) shocks from distant locations affect each firm's stock price and capital investment. Hence, this result stands in stark contrast to the benchmark equilibrium in which only local shocks affect each firm's equilibrium variables because all non-fundamental variation can be filtered out of the forward neighbor's price signal. It can be seen that the sensitivity to these non-local shocks is proportional to $(\rho b_1)^j$ and thus depends on three factors: (i) the level of entanglement between neighboring firms (ρ), (ii) each firm's weight on the optimal price signal (b_1 - see Proposition 2), and (iii) the distance to the "infecting" firm (j). Intuitively, each firm manager bases his expectation about future productivity on the price of its forward neighbor and filters out as much unrelated variation from this price as possible. However, as shown in Proposition 2, firm i cannot filter out variation in $p_i^{(1)}$ coming from the firm $n+1$ locations away because that firm's price is too costly

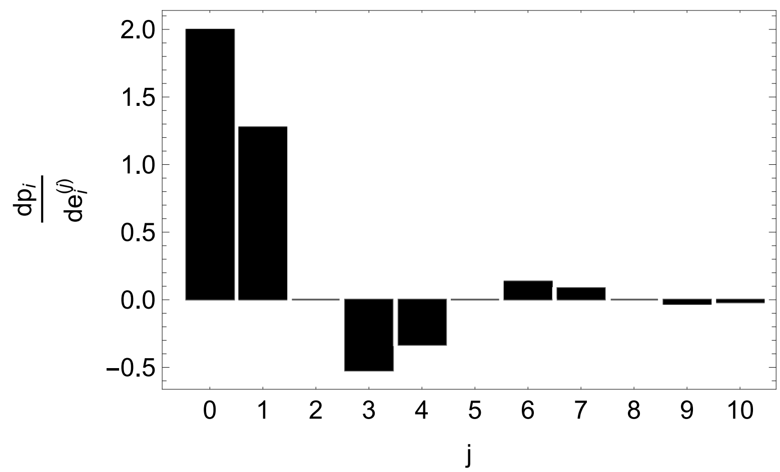

Figure 2: This figure plots the sensitivity of each firm's stock price to fundamental shocks, $e_i^{(j)}$, when each firm observes the stock prices of the next two firms ($n = 2$). Other parameters: $N = 11, \rho = 1, \sigma_e = 1, \sigma_x = 1$.

to acquire. Thus, all shocks that affect this price $\left(p_i^{n+1}\right)$ are transmitted to firm i's price and investment. Consequently, fundamental and non-fundamental shocks in location $i+n+1$ are transmitted to firm i. Since the distant firm is also affected by this transmission mechanism, shocks to (the even more distant price) $p_i^{2(n+1)}$ are also transmitted to firm i. The impact of each shock originating in a *infecting location* is proportional to the weight that each firm assigns to the feedback signal from the stock market, ρb_1 (see Proposition 2), raised to the power of j because $j-1$ other firms have already attached a Bayesian weight to this signal.

Figure 2 show the sensitivity of firm i's stock price with respect to $e_i^{(j)}$ for a specific set of parameters, when each firm can only observe the stock prices of the next two firms in the network such that $n = 2$. The figure confirms the results in Corollary 2: (i) shocks from non-related firms are transmitted to firm i's stock price (and investment), (ii) the absolute impact of these shocks is stronger for locations that are more closely located, (iii) the sign of the sensitivities alternates and (iv) some locations can be skipped (e.g. $j \in \{2, 5, 7\}$ in Figure 2).

Corollary 3 *The absolute sensitivity of firm i's stock price and firm investment with respect to non-local shocks from infecting locations $\mathcal{J}^{inf}$ (defined in Corollary 2) is higher if:*

a. σ_e *increases*

b. σ_x *decreases*

c. $|\rho|$ *increases.*

Proof: *See Appendix A.2.6.*

Corollary 3 shows how the sensitivity of investment and prices to non-local shocks depend on the three key parameters of the economy: (a) the volatility of fundamentals σ_e, (b) the volatility of the supply shock σ_x, and (c) the interdependence of productivity shocks ρ. A higher ex ante variance of the fundamentals implies that managers have a more diffuse prior about the non-local component of their productivity shock. Therefore, they have a higher incentive to learn information about this shock from other firms' prices such that they place a higher weight on these signals. As a result, shocks to these non-local prices are reflected to a larger extent in the firm's equilibrium variables such that the propagation of non-local shocks is stronger in more uncertain times featuring higher shock volatility. Similarly, lower supply risk renders the neighbor's stock prices more informative about the non-local component such that firm managers place a larger Bayesian weight on stock prices which again leads to stronger shock propagation. Lastly, if two neighboring firms are more strongly entangled (high $|\rho|$), the non-local shock plays a more important role for the firm's overall investment decision such that firm managers have a higher incentive to place a larger weight on other firms' prices. Figure 3 plots ρb_1, a measure of each firm's absolute sensitivity to non-local shocks, against $\tau_e \equiv \sigma_e^{-2}$ and $\tau_x \equiv \sigma_x^{-2}$. The plots confirm the analytical results in Corollary 3 that shock propagation

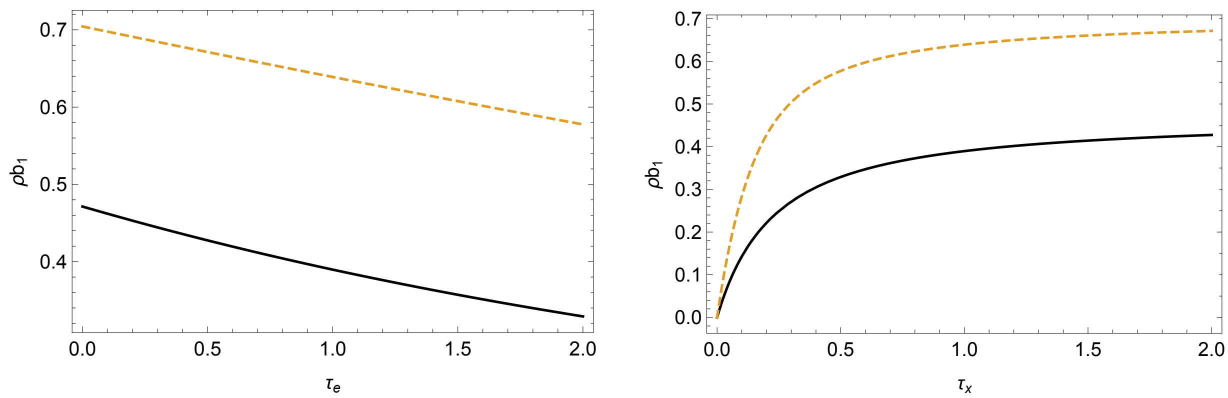

Figure 3: These two figures plot ρb_1 (a measure of each firm's absolute sensitivity to non-local shocks), against τ_e (left plot) and τ_x (right plot). Other parameters: $n = 2$, $\tau_x = 1$ (left), and $\tau_e = 1$ (right). The solid line corresponds to $\rho = \frac{1}{2}$, the dashed line to $\rho = 1$.

increases (decreases) in τ_x (τ_e) and that it is stronger for a higher degree of entanglement between neighboring firms.

4.2 The impact of n

In this section, I discuss the impact of n, the number of observed stock prices, on the key results in the main model. While this section analyzes exogenous changes in this number, the next section endogenizes n.

Increasing the number of observed prices allows each firm to collect more price signals about the non-local component in its productivity shock. Thus, each firm manager becomes better informed about $e_i^{(1)}$ and can invest more efficiently. Moreover, a higher value of n also incentivizes the managers to rely more heavily on this price signal, i.e. to choose a higher value of b_1 in Proposition 2. As shown in Corollary 2, this increase in b_1 increases the impact of non-local shocks originating from infecting firms. At the same time, the proportion of infecting firms in the economy decreases with more observed prices, as firms are able to filter out more non-local variation in their neighbor's stock price.

Corollary 4 *Increasing the number of observed stock prices n always increases*

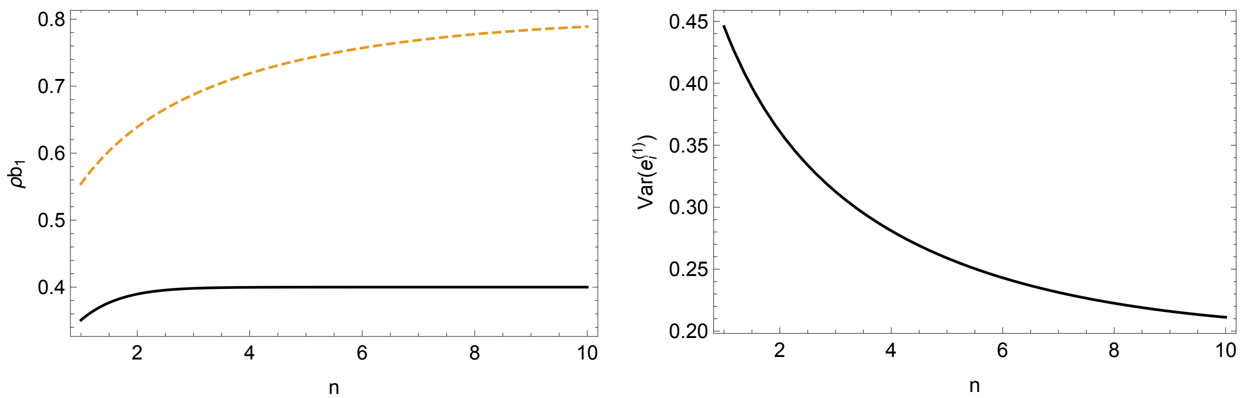

Figure 4: Left plot: ρb_1 (a measure of each firm's absolute sensitivity to non-local shocks), against n; parameters: $\tau_e = \tau_x = 1$; $\rho = \frac{1}{2}$ (solid line) and $\rho = 1$ (dashed line). Right plot: conditional variance of $e_i^{(1)}$ against n; parameters: $\tau_e = \tau_x = \rho = 1$.

a. the absolute sensitivity of stock prices and firm investment with respect to non-local shocks, ρb_1

b. investment efficiency, $E[Y_i - \frac{1}{2}K_i^2]$.

Proof: *See Appendix A.2.7.*

Corollary 4 formalizes these results and Figure 4 confirms the results for a specific set of parameters. It can be seen that ρb_1, which determines the firm's exposure to non-local shocks, increases in n. Moreover, this increase is higher if ρ is higher. The left panel, plots each manager's conditional variance of the non-local shock and shows that the firm manager becomes more informed about the non-local shock as he observes more prices. It follows from the expression for V_i in equation (4) that this decrease in the conditional variance leads to a more efficient investment decision.

4.3 Empirical Implications

The framework presented above yields several empirical implications. First, it implies that purely financial shocks, like e.g. liquidity shocks or market sentiment, can affect real decisions such as firm investment. If firm managers just rely on their private information these shocks do not play any role for real decisions and the stock market is just a "sideshow." However, with a feedback effect from the financial market to firm decisions, purely financial shocks affect managers' expectations and investment decisions. This exposure then gets implemented into stock prices. Importantly, if firm managers learn from their neighbors' prices, these shocks are transmitted through the entire network such that they not only affect local firms. Thus, even if a certain financial shock is fundamentally firm-specific, imperfect cross-learning implies that it affects many other firms in the network as well. Surprisingly, as shown in Corollary 2 and Figure 2, the impact can "skip" several firms, i.e. a certain shock can affect a number of firms, spare its direct neighbors, then affect other, more distant firms again, and so on.

In addition to the propagation of non-fundamental shocks, the framework also implies that fundamental shocks for a given firm affect real decisions for a completely unrelated firm. As a result, fundamentally unrelated firms have correlated investment decisions and generally appear more highly correlated than they are (based on their fundamentals). In particular, this endogenous comovement is stronger in times of higher uncertainty, when firms face a higher incentive to cross-learn. The model thus provides a learning-based explanation for the empirical findings, such as in Barrot and Sauvagnat (2016), that firm-level idiosyncratic shocks propagate in networks. In the cross-learning equilibrium with imperfect learning, firm-specific shocks effectively become systematic shocks that affect multiple firms in the economy.

Moreover, the model emphasizes that the fact that firm investment is affected by mispricing in a given firm's stock price, does *not* imply that the firm learns from this price. On the contrary, if two firms are fundamentally unrelated, mispricing in a given stock only spills over if the firm does *not* observe its price and cannot learn from it. Intuitively, the mispricing could be filtered out if this price was observed.[14]

5 Endogenous Price Acquisition

In this section I allow each firm to choose the number of observed prices, n_i. In particular, at $t = 0$ firm i chooses n_i to maximize the expected future firm value, V_i:

$$\max_{n_i} E\left[Y_i - \frac{1}{2}K_i^2 - C(n_i)\right]. \tag{9}$$

In this expression, $C(n_i)$ captures the price acquisition cost for each firm which is assumed to be strictly increasing in n. Moreover, I rule out corner solutions by assuming: $C(1) = 0$ and $C(N) = \infty$. Therefore, firms will always learn from at least one price, but never observe the entire vector of prices.

The next section focusses on the equilibrium outcome when each firm chooses n_i competitively. Subsequently, I contrast this outcome with the social optimum in which a benevolent social planner assigns n to all firms with the objective to maximize each firm's ex ante value. Both sections will focus on the firms' benefit to acquire price information without assuming a particular functional form for $C(\cdot)$. Section 5.3 numerically solves for the equilibrium values assuming specific functional forms for the price acquisition cost.

5.1 Equilibrium Price Acquisition

First, note that from equation (9) and the equilibrium expressions for Y_i and K_i, it follows that the expected firm value can be written as:

$$V_i = \frac{1}{2}\exp\left(2(1+\rho^2)\tau_e^{-1} - \rho^2\left(\tau_e + \tau_{z,i}\right)^{-1}\right) - C(n_i) \tag{10}$$

[14]Recent work by Dessaint et al. (2016) also emphasizes this point.

where $\tau_{z,i} \equiv \text{Var}^{-1}\left(e_i^1 | p_i^1, \ldots, p_i^{(n_i)}\right)$ denotes the precision of the price signal, firm i can compose using the stock prices of the next n_i firms. As shown before, a more precise price signal allows each manager to make a more informed investment decision. Importantly, when each firm manager decides on the optimal number of price signals, he performs the following cost-benefit analysis. On the one hand, an increase in n_i is associated with higher price acquisition cost $C(n_i)$, on the other hand it also leads to a more precise price signal and higher investment efficiency (captured by $\tau_{z,i}$).

Lemma 2 *In a symmetric equilibrium with $n_i = n$ for all $i \in \mathcal{N}$, a single firm j can remove all non-local noise from the composite price signal by observing $n_j = n + 1$ prices. The precision of this signal equals $\tau_{z,j} = 4\tau_x$.*
Proof: *See Appendix A.2.8.*

Lemma 2 formalizes the benefit for each individual firm to collect an additional price signal. In particular, assume all firms observe the prices of the next n firms, but firm j unilaterally observes $n_j = n + 1$ prices. Then, this firm is able to remove all non-local variation in its forward neighbor's stock price. As a result, it is able to recover the optimal signal $e_j^{(1)} + \frac{1}{2}x_j^{(1)}$ from the benchmark equilibrium without price acquisition cost (see Lemma 1). Of course, this signal is always more precise than the signal with only n observed prices such that all firms' marginal benefit of increasing n is always positive.[15]

Figure 5 plots the increase in τ_z for firm j if it chooses to observe an additional price signal while all other firms observe n prices. It can be seen that the increase in price informativeness for firm j decreases in n, the number of observed prices by its peers. Thus, the marginal benefit of collecting additional price information is decreasing. Furthermore,

[15]The lemma also shows why firms always want to observe the prices of following firms in the network. Only the price of firm $n + 1$ reduces the non-local noise in their existing price signal.

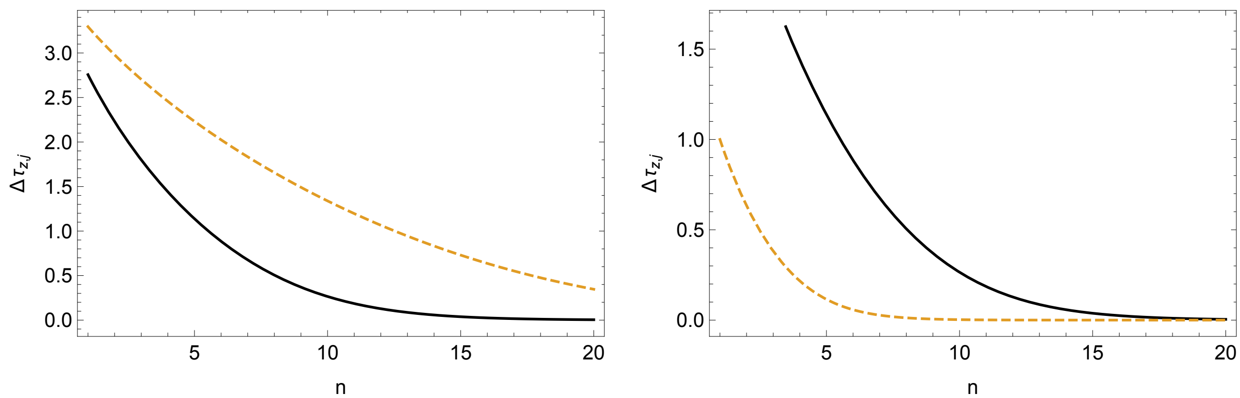

Figure 5: Both plots show the increase in τ_z for firm j if it observes $n+1$ prices, when all other firms observe n prices; $\rho = 1$ for both plots. Left plot: $\tau_x = 1$ and $\tau_e = 1$ (solid), $\tau_e = \frac{1}{2}$ (dashed). Right plot: $\tau_e = 1$ and $\tau_x = 1$ (solid), $\tau_x = \frac{1}{2}$ (dashed).

the left panel shows that it is more beneficial for an individual firm to increase n in times of high volatility (dashed line). Intuitively, higher values of σ_e imply low prior knowledge about the realization of $e_i^{(1)}$ and thus render price information more valuable. Similarly, the right plot shows that the marginal increase in price informativeness is higher when σ_x is lower, i.e. when the noisy supply shock is less volatile and the price signal is more informative in general.

Proposition 3 *Assume $C(n_i)$ is strictly increasing, $C(1) = 0$ and $C(N) = \infty$. There exists a unique, symmetric information acquisition equilibrium in which all firms choose to observe $n \in \mathcal{J}$ stock prices.*

Proof: *See Appendix A.2.9.*

Proposition 3 shows that the information acquisition equilibrium is unique. Intuitively, uniqueness follows from the decreasing marginal benefit of information acquisition (Figure 5) and the strictly increasing cost. Section 5.3 explicitly computes the equilibrium choices of n given a specific cost function $C(n_i)$ and discusses the implications for price efficiency, shock propagation, and welfare.

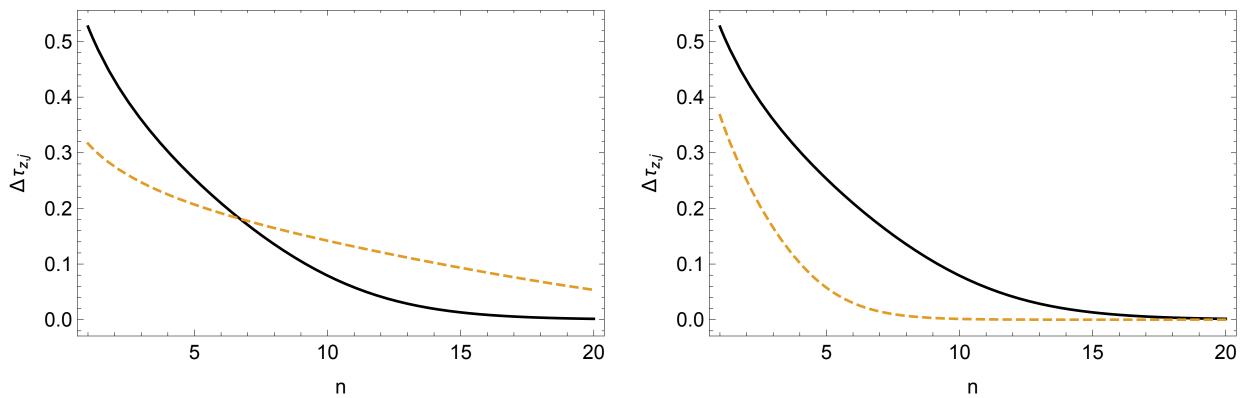

Figure 6: Both plots show the increase in τ_z for each firm if it observes $n+1$ prices, when all other firms also observe $n+1$ prices; $\rho = 1$ for both plots. Left plot: $\tau_x = 1$ and $\tau_e = 1$ (solid), $\tau_e = \frac{1}{2}$ (dashed). Right plot: $\tau_e = 1$ and $\tau_x = 1$ (solid), $\tau_x = \frac{1}{2}$ (dashed).

5.2 Socially Optimal Price Acquisition

In this section, I compare the information acquisition equilibrium to the socially optimal allocation. In particular, I consider a social planner who assigns n_i to all firms with the objective to maximize all firms' ex ante expected value V_i. The main difference to the equilibrium choice is that a social planner internalizes the impact of firm i's information acquisition decision on other firms. In particular, he internalizes that an increase in n_i poses a negative externality for the firm's backward neighbor. Intuitively, the price of firm i contains more non-local variation if it relies on more prices itself such that firm $i-1$ can extract less precise information from this signal.

Figure 6 plots the increase in price informativeness (τ_z) for each firm if all firms observe an additional price signal. There are two main differences to the equilibrium change in price informativeness depicted in Figure 5. First, it can be seen that the increase in price informativeness is an order of magnitude smaller if all firms simultaneously increase n. Second, the impact of σ_e can now be negative, i.e. for a given n the social benefit of acquiring additional price information can be lower in times of higher uncertainty (and small values of n).

Corollary 5 *The equilibrium number of observed prices is inefficiently high.*

Proof: *See Appendix A.2.10.*

Corollary 5 formalizes this informational externality. From each firm's individual perspective, increasing the number of observed firms promises a very informative price signal that is not contaminated with non-local noise. This reasoning does, however, not survive in equilibrium such that the private benefit of information acquisition does not equal the social benefit und the equilibrium price acquisition decision is inefficiently high.

5.3 Numerical Results

In this section, I provide numerical results regarding the equilibrium choice of observed prices and its implications for the main equilibrium. I also compare this solution to the socially optimal choice of observed prices and evaluate the loss in firm values resulting from the informational inefficiency discussed before.

For all of the numerical results in this section, I use a specific (intentionally simple) functional form for the price acquisition function:

$$C(n_i) = \begin{cases} 0 & \text{for } n_i = 1 \\ cn_i & \text{for } n_i = 2, \ldots, N-1 \\ \infty & \text{for } n_i = N. \end{cases} \tag{11}$$

This price acquisition cost is therefore strictly increasing in the number of observed prices and the constant $c > 0$ determines the marginal cost. Moreover, as before, I rule out corner solutions by imposing $C(1) = 0$ and $C(N) = \infty$.[16]

Tables 1 and 2 show the most important variables for different parameter specifications. First, the numerical exercises confirms the analytical results from before that the number

[16]Both conditions turn out to be redundant in the numerical solutions below.

of observed prices in equilibrium (n_i^{CE}) is always higher than the socially optimal number of prices (n_i^{SP}). Interestingly, the tables show that the percentage of infecting firms is strictly higher in the social optimum. Intuitively, precisely the fact that the social planner chooses a smaller number of observed prices implies that non-local shocks from a larger number of locations impact the stock price and investment decision of each firm. This ratio is lowest for the case $n_i^{CE} = 10$ in which shocks from only 19% of all firms in the network affect p_i and k_i. Note, however, that this number is still unusually high because firm i only shares a productivity shock with 2% (2 out of $N = 100$) of the firms.

The fact that firm managers collect more price signals in the competitive equilibrium implies that they can infer more information from these prices such that the precision of the composite price signal (τ_z) is always higher in equilibrium. Due to the increasing price acquisition cost, however, firm manager always refrain from collecting all prices such that the precision of their price signal is always below that in the benchmark equilibrium ($\tau_z^{Full} = 4\tau_x$). Even though τ_z is higher in equilibrium, the informational externality emphasized before leads to an efficiency loss for all firms. Therefore, each firm's ex ante value is always smaller in equilibrium. The associated percentage loss in firm value ranges from 1.3% to 7.4% depending on the model parameters.

Interestingly, this numerical exercise also shows that firm managers seem to have a higher incentive to collect price signals if they are more strongly entangled (higher ρ) with their direct neighbors. Consequently, n_i^{CE} is generally higher in Table 1 featuring the maximum value of $\rho = 1$ compared to Table 2 with $\rho = 0.8$. Similarly, the number of collected signals in equilibrium is highest if fundamentals are particularly volatile (low $\tau_e \equiv \sigma_e^{-2}$) and if the noisy supply shock is less volatile (high $\tau_x \equiv \sigma_x^{-2}$).

	$\{\tau_e = 1, \tau_x = 1\}$	$\{\tau_e = 1, \tau_x = \frac{5}{4}\}$	$\{\tau_e = \frac{5}{4}, \tau_x = 1\}$	$\{\tau_e = \frac{5}{4}, \tau_x = \frac{5}{4}\}$
n_i^{CE}	8	10	4	5
n_i^{SP}	4	4	2	2
n_i^{No}	0	0	0	0
n_i^{Full}	100	100	100	100
$\frac{\lvert\mathcal{J}^{inf,CE}\rvert}{\lvert\mathcal{N}\rvert}$	0.21	0.19	0.33	0.29
$\frac{\lvert\mathcal{J}^{inf,SP}\rvert}{\lvert\mathcal{N}\rvert}$	0.33	0.33	0.51	0.51
$\tau_z\left(n_i^{CE}\right)$	3.4945	4.3226	2.9169	3.5772
$\tau_z\left(n_i^{SP}\right)$	2.5594	2.7573	2.0677	2.2126
$\tau_z\left(n_i^{No}\right)$	0	0	0	0
$\tau_z\left(n_i^{Full}\right)$	4	5	4	5
$V_i\left(n_i^{CE}\right)$	17.8535	17.6232	7.6491	7.4711
$V_i\left(n_i^{SP}\right)$	18.6127	18.9200	8.0742	8.1894
% loss in V_i	4.2524	7.3585	5.5575	9.6144

Table 1: This table shows the number of observed prices n_i, the fraction of infecting locations $\frac{\lvert\mathcal{J}^{inf}\rvert}{\lvert\mathcal{N}\rvert}$, the informational content of the price signal τ_z, and the ex ante firm value V_i assuming a linear cost function $C(n_i) = cn_i$ and parameters: $c = \frac{1}{2}$, $\rho = 1$ and $N = 100$. CE: competitive equilibrium, SP: social optimum, No: no cross-learning ($n = 0$), $Full$: benchmark equilibrium ($n = N$).

	$\{\tau_e = 1, \tau_x = 1\}$	$\{\tau_e = 1, \tau_x = \frac{5}{4}\}$	$\{\tau_e = \frac{5}{4}, \tau_x = 1\}$	$\{\tau_e = \frac{5}{4}, \tau_x = \frac{5}{4}\}$
n_i^{CE}	3	3	2	2
n_i^{SP}	2	2	1	1
n_i^{No}	0	0	0	0
n_i^{Full}	100	100	100	100
$\frac{\lvert\mathcal{J}^{inf,CE}\rvert}{\lvert\mathcal{N}\rvert}$	0.41	0.41	0.51	0.51
$\frac{\lvert\mathcal{J}^{inf,SP}\rvert}{\lvert\mathcal{N}\rvert}$	0.51	0.51	1.00	1.00
$\tau_z\left(n_i^{CE}\right)$	3.0171	3.3964	2.6760	2.9700
$\tau_z\left(n_i^{SP}\right)$	2.3760	2.5973	1.8090	1.9525
$\tau_z\left(n_i^{No}\right)$	0	0	0	0
$\tau_z\left(n_i^{Full}\right)$	4	5	4	5
$V_i\left(n_i^{CE}\right)$	9.8309	9.9877	4.8582	4.9251
$V_i\left(n_i^{SP}\right)$	9.9932	10.1222	5.0937	5.1464
% loss in V_i	1.6509	1.3467	4.8475	4.4933

Table 2: This table shows the number of observed prices n_i, the fraction of infecting locations $\frac{\lvert\mathcal{J}^{inf}\rvert}{\lvert\mathcal{N}\rvert}$, the informational content of the price signal τ_z, and the ex ante firm value V_i assuming a linear cost function $C(n_i) = cn_i$ and parameters: $c = \frac{1}{2}$, $\rho = \frac{4}{5}$ and $N = 100$. CE: competitive equilibrium, SP: social optimum, No: no cross-learning ($n = 0$), $Full$: benchmark equilibrium ($n = N$).

6 Conclusion

This paper analyzes a feedback model with multiple firms that are pairwise connected through their common exposure to a productivity shock. Firms benefit from informative stock prices but have to decide how many prices to observe and analyze in equilibrium. This informational friction leads to the propagation of non-local financial and fundamental shocks throughout the network, particularly in times of high prior uncertainty, less noisy supply shocks and stronger correlation in firm fundamentals. Collecting additional price signals always raises investment efficiency for firms but due to an informational externality they over-invest in stock price information.

The framework's tractability offers several opportunities for future research on feedback effects with a finite number of interconnected firms. For example, it would be interesting to endogenize the shape of the underlying network by allowing firms to decide on their links to other firms. This analysis could shed more light on the relationship between a given network structure, the informational content of stock prices and welfare. More generally, the framework is well-suited for settings with strategic interaction between firms and their stock markets. For example, a firm could use its information disclosure decision to manipulate the informational content of its stock price to diminish the amount of information for its competitor who relies on this price signal.

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A Appendix

A.1 Notation and Preliminary Derivations

In this appendix, I sometimes work with the precision of a random variable instead of its variance. For a generic random variable x with variance σ_x^2, I define its precision by $\tau_x \equiv \sigma_x^{-2}$.

I also frequently use two standard results from Bayesian updating. Consider a random variable x that is Normally distributed with zero mean and precision τ_x. Further consider an unbiased signal s_x with precision τ_s, then the first two conditional moments can be written as:

$$\begin{aligned} E[x|s_x] &= \frac{\tau_s}{\tau_x + \tau_s} s_x \\ \text{Var}(x|s_x) &= (\tau_x + \tau_s)^{-1} . \end{aligned}$$

A.2 Proofs

A.2.1 Proof of Lemma 1

From the equation for the log stock price given in the text,

$$p_i = \pi_0 + 2e_i + 2\rho E\left[e_i^{(1)}|\vec{p}\right] + x_i,$$

it follows that the firm manager and insiders in location $i-1$ can transform this price into an unbiased signal about e_i because they observe all prices. This signal is given by:

$$z_i(\vec{p}) = \frac{p_i - \pi_0}{2} - \rho E\left[e_i^{(1)}|\vec{p}\right] = e_i + \frac{1}{2}x_i$$

and its precision equals $4\tau_x$.

A.2.2 Proof of Proposition 1

First note that optimal capital investment is given by equation (5), such that:

$$k_i = e_i + \rho E\left[e_i^{(1)}|\vec{p}\right] + \frac{\rho^2}{2}\mathrm{Var}\left(e_i^{(1)}|\vec{p}\right)$$

where I used the fact that manager i receives a perfect signal about e_i and observes all prices. From Lemma 1 it follows that:

$$E\left[e_i^{(1)}|\vec{p}\right] = \frac{4\tau_x}{\tau_e + 4\tau_x}\left(e_i + \frac{1}{2}x_i\right)$$

and

$$\mathrm{Var}\left(e_i^{(1)}|\vec{p}\right) = \left(\tau_e + 4\tau_x\right)^{-1}.$$

Therefore, optimal capital investment can be written as:

$$k_i = a_0 + e_i + a_1\rho\left(e_i^{(1)} + \frac{1}{2}x_i^{(1)}\right)$$

where $a_0 = \frac{\rho^2}{2}\left(\tau_e + 4\tau_x\right)^{-1}$ and $a_1 = \frac{4\tau_x}{\tau_e+4\tau_x}$.

Similarly, the log stock price for each firm is given by equation (6) in the text, which can be written as $p_i = 2k_i + x_i$, such that:

$$p_i = 2a_0 + 2e_i + 2a_1\rho\left(e_i^{(1)} + \frac{1}{2}x_i^{(1)}\right) + x_i.$$

A.2.3 Proof of Proposition 2

Given that managers and insiders only observe the prices for the next n firms, equation (5) in the text becomes:

$$k_i = e_i + \rho E\left[e_i^{(1)}|p_i^{(1)},\ldots,p_i^{(n)}\right] + \frac{\rho^2}{2}\mathrm{Var}\left(e_i^{(1)}|p_i^{(1)},\ldots,p_i^{(n)}\right)$$

and as before $p_i = 2k_i + x_i$. Due to the normality of all shocks, I can rewrite the conditional expectation as:

$$E\left[e_i^{(1)}|p_i^{(1)},\dots,p_i^{(n)}\right] = \gamma_0 + \sum_{j=1}^{n} \gamma_j p_i^{(j)}$$

for endogenous weights γ_j.

Then, it follows that from the perspective of an arbitrary firm $i-1$, the price of its forward neighbor can be transformed to:

$$\widetilde{p}_i = \frac{p_i}{2} - \frac{\rho^2}{2}\text{Var}\left(e_i^{(1)}|p_i^{(1)},\dots,p_i^{(n)}\right) - \rho\gamma_0 - \rho\sum_{j=1}^{n-1}\gamma_j p_i^{(j)} = e_i + \frac{1}{2}x_i + \gamma_n p_i^{(n)}$$

which represents an unbiased signal about e_i. Note that in the derivation of $\widetilde{p}_i$ I used that: (i) all conditional variances are constants, and (ii) firm $i-1$ observes $p_i,\dots,p_i^{(n-1)}$.

The precision of this signal is given by: $\tau_p^{-1} = \frac{1}{4}\tau_x^{-1} + \gamma_n^2\text{Var}\left(p_i^{(n)}\right)$. Plugging in the definition of p_i and using the law of total variance, I derive the following implicit equation for τ_p:

$$\tau_p^{-1} = \frac{1}{4}\tau_x^{-1} + \gamma_n^2\rho^2\left(\tau_e^{-1} + \frac{1}{4}\tau_x^{-1} + \rho^2\tau_e^{-1} - \rho^2\left(\tau_e+\tau_p\right)^{-1}\right)$$

In particular, the endogenous weight γ_n can be simplified by noting that:

$$E\left[e_i^{(1)}|p_i^{(1)},\dots,p_i^{(n)}\right] = \gamma_0 + \sum_{j=1}^{n}\gamma_j p_i^{(j)} = \frac{\tau_p}{\tau_e+\tau_p}\widetilde{p}_i^{(1)}$$

Plugging in the definition for $\widetilde{p}_i^{(1)}$ and matching coefficients on both sides yields: $\gamma_n = -\left(\frac{\tau_p}{\tau_e+\tau_p}\right)^n\rho^{n-1}$. Plugging this expression back into the equation for τ_p above gives:

$$\tau_p^{-1} = \frac{1}{4}\tau_x^{-1} + \left(\frac{\tau_p}{\tau_e+\tau_p}\right)^{2n}\rho^{2n}\left(\tau_e^{-1} + \frac{1}{4}\tau_x^{-1} + \rho^2\tau_e^{-1} - \rho^2\left(\tau_e+\tau_p\right)^{-1}\right)$$

It follows that equilibrium investment k_i can be written as:

$$k_i = b_0 + e_i + b_1\rho\left(e_i^{(1)} + \frac{1}{2}x_i^{(1)} + b_n\rho p_i^{(n+1)}\right)$$

with: $b_0 = \frac{\rho^2}{2}\left(\tau_e + \tau_p\right)^{-1}$, $b_1 = \frac{\tau_p}{\tau_e+\tau_p}$ and $b_n = -\left(\frac{\tau_p}{\tau_e+\tau_p}\right)^n \rho^{n-1}$.

The expression for p_i simply follows from $p_i = 2k_i + x_i$, as before.

A.2.4 Proof of Corollary 1

Note that Proposition 2 implies that $k_i = b_0 + e_i$ and $p_i = 2b_0 + 2e_i + x_i$ if $\rho = 0$. Therefore, k_i is only affected by e_i and p_i is only affected by e_i and x_i.

A.2.5 Proof of Corollary 2

This result follows directly from Proposition 2, after successively replacing $p_i^{(n+1)}$ in the expressions for k_i and p_i. The indicator function $D(j)$ determines the sign of these sensitivities and is defined as:

$$D(j) = \begin{cases} 1 & \text{if } j \in \{1, 1\times(n+1), 1\times(n+1)+1, 3\times(n+1), 3\times(n+1)+1, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

therefore the sign of these sensitivities alternates across the non-local pairs of firms that are part of the set of infecting firms, $\mathcal{J}^{inf}$.

A.2.6 Proof of Corollary 3

From Corollary 2 it follows that the magnitude of the sensitivity given by $|\rho| b_1$. Then, plugging in the equilibrium value for b_1 and simple differentiation gives:

$$\begin{aligned}
\frac{\partial b_1}{\partial \tau_e} &= \frac{-4\left(b_1^2\rho^2+1\right)\tau_x}{\left(4\left(b_1\rho^2+1\right)\tau_x+\tau_e\right)\left(b_1^{2n}\rho^{2n}\left(4\tau_x\left(2b_1(n+1)\rho^2+2n+1\right)+(2n+1)\tau_e\right)+\tau_e+4\tau_x\right)} \\
&< 0 \\
\frac{\partial b_1}{\partial \tau_x} &= \frac{b_1\tau_e\left(b_1^{2n}\rho^{2n}+1\right)}{\tau_x\left(b_1^{2n}\rho^{2n}\left(4\tau_x\left(2b_1(n+1)\rho^2+2n+1\right)+(2n+1)\tau_e\right)+\tau_e+4\tau_x\right)} > 0 \\
\frac{\partial b_1}{\partial \rho} &= -\rho^{-1}\frac{2b_1^{2n+1}\rho^{2n}\left(4\tau_x\left(b_1(n+1)\rho^2+n\right)+n\tau_e\right)}{b_1^{2n}\rho^{2n}\left(4\tau_x\left(2b_1(n+1)\rho^2+2n+1\right)+(2n+1)\tau_e\right)+\tau_e+4\tau_x}
\end{aligned}$$

where $\tau_e = \sigma_e^{-2}$ and $\tau_x = \sigma_x^{-2}$, such that $\frac{\partial y}{\partial \sigma_e} = -2\frac{\partial y}{\partial \tau_e}\frac{\tau_e}{\sigma_e}$ and $\frac{\partial y}{\partial \sigma_x} = -2\frac{\partial y}{\partial \tau_x}\frac{\tau_x}{\sigma_x}$. It follows that the magnitude always rises (falls) with σ_e (σ_x). Moreover, an increase (decrease) for ρ increases (decreases) ρb_1 if ρ is positive (negative).

A.2.7 Proof of Corollary 4

Starting with part a. Differentiating ρb_1 with respect to n yields:

$$\frac{\partial \rho b_1}{\partial n} = -\frac{b^{2m+1}\rho^{2m}(\log(b_1^2) + \log(\rho^2))\left(4(b_1\rho^2 + 1)\tau_x + \tau_e\right)}{b_1^{2n}\rho^{2n}\left(4\tau_x(2b_1(n+1)\rho^2 + 2n + 1) + (2n+1)\tau_e\right) + \tau_e + 4\tau_x}$$

where I used the expression for b_1 derived in Proposition 2. The fact that $b_1 \in (0, 1)$ and $\rho^2 \leq 1$ implies that $\frac{\partial \rho b_1}{\partial n} > 0$.

Next, part b. Note that from the definition of V_i and the equilibrium expressions for K_i, it follows that investment efficiency can be written as:

$$E_0[Y_i - \frac{1}{2}K_i^2] = \frac{1}{2}\exp\left(2(1+\rho^2)\tau_e^{-1} - \rho^2(\tau_e + \tau_z)^{-1}\right).$$

As a result, n only affects investment efficiency through its impact on price informativeness τ_z. Moreover, note that $\frac{\partial E_0[Y_i - \frac{1}{2}K_i^2]}{\partial \tau_z} > 0$, i.e. more informative prices lead to higher investment efficiency. Lastly, differentiating the implicit expression for τ_z in Proposition 2 with respect to n leads to $\frac{\partial \tau_z}{\partial n} > 0$. Taken together, an increase in the number of observed prices leads to higher investment efficiency.

A.2.8 Proof of Lemma 2

As shown in the Proof of Proposition 2, the price signal available to firm i can be written as:

$$\tilde{p}_i = e_i^{(1)} + \frac{1}{2}x_i^{(1)} + \gamma_n p_i^{(n+1)}.$$

If firm i observes an additional price signal, it is able to remove $p_i^{(n+1)}$ from this signal such that the new price signal equals:

$$\tilde{p}_i^* = e_i^{(1)} + \frac{1}{2} x_i^{(1)}$$

which equals the optimal price signal $z_i(\vec{p})$ in the benchmark equilibrium without price acquisition cost (see Proof of Lemma 1).

A.2.9 Proof of Proposition 3

Given that the information cost is strictly increasing in n_i, it is sufficient to show that the marginal benefit, the change in $V_i - C(n_i)$, for each firm is weakly decreasing in n_i. From the definition of V_i it follows that:

$$V_i(n_i) - C(n_i) = E[Y_i - \frac{1}{2} K_i^2].$$

Plugging in the expressions for Y_i and K_i, taking unconditional expectations and taking the first difference (i.e. $(V_i(n+1) - C(n+1)) - (V_i(n) - C(n))$ yields:

$$\Delta\left(V_i - C\right) = \frac{1}{2} \exp\left(2(1+\rho^2)\tau_e^{-1}\right)\left(\exp\left(-\rho^2(\tau_e + 4\tau_x)^{-1}\right) - \exp\left(-\rho^2(\tau_e + \tau_z(n))^{-1}\right)\right)$$

where I used the result from before that $\tau_z(n+1) = 4\tau_x$ if all other firms observe n prices.

Then, simple differentiation with respect to n yields:

$$\frac{\partial \Delta\left(V_i - C\right)}{\partial n} = \frac{\partial \Delta\left(V_i - C\right)}{\partial \tau_z}\frac{\partial \tau_z}{\partial n} = -\frac{\rho^2 e^{\frac{2\left(\rho^2+1\right)}{\tau_e} - \frac{\rho^2}{\tau_e + \tau_z}}}{2\left(\tau_e + \tau_z\right)^2}\frac{\partial \tau_z}{\partial n} \leq 0$$

which is weakly negative because $\frac{\partial \tau_z}{\partial n} > 0$ as shown before in Corollary 4.

A.2.10 Proof of Corollary 5

To show that the equilibrium number of observed prices is inefficiently high, I show that the social marginal benefit is always higher than the private marginal benefit. Thus, I compare $\Delta^S (V_i - C)$ with $\Delta^P (V_i - C)$. The key difference is that Δ^S increases n_i for all N firms, whereas Δ^P only increases n_i for one firm and thus does not internalize the negative impact of more observed prices on other firms.

As shown before, the informational content of firm i's price signal if it unilaterally observes one more price equals $\tau'_{zi} = 4\tau_x$ which is larger than τ_z if all other $N-1$ firms also increase n by one unit, as I will show below.

Mathematically,

$$\begin{aligned}
\Delta^P - \Delta^S &= \frac{1}{2}\exp\left(2(1+\rho^2)\tau_e^{-1}\right)\left(\exp\left(-\rho^2(\tau_e + 4\tau_x)^{-1}\right) - \exp\left(-\rho^2(\tau_e + \tau_z(n))^{-1}\right)\right) \\
&- \frac{1}{2}\exp\left(2(1+\rho^2)\tau_e^{-1}\right)\left(\exp\left(-\rho^2(\tau_e + \tau_z(n+1))^{-1}\right) - \exp\left(-\rho^2(\tau_e + \tau_z(n))^{-1}\right)\right) \\
&= \frac{1}{2}\exp\left(2(1+\rho^2)\tau_e^{-1}\right)\left(\exp\left(-\rho^2(\tau_e + 4\tau_x)^{-1}\right) - \exp\left(-\rho^2(\tau_e + \tau_z(n+1))^{-1}\right)\right)
\end{aligned}$$

Thus, to show that $\Delta^P - \Delta^S \geq 0$ is equivalent to showing $4\tau_x \geq \tau_z(n+1)$ or $\frac{1}{4}\tau_x^{-1} \leq \tau_z^{-1}(n+1)$.

Note that Proposition 2 shows that $\tau_z^{-1}(n+1)$ is given by:

$$\tau_z^{-1}(n+1) = \frac{1}{4}\tau_x^{-1} + \left(\frac{\rho\tau_z}{\tau_e + \tau_z}\right)^{2n+2}\left(\tau_e^{-1} + \frac{1}{4}\tau_x^{-1} + \rho^2\left(\tau_e^{-1} - (\tau_e + \tau_z)^{-1}\right)\right) \geq \frac{1}{4}\tau_x^{-1}$$

where the last inequality follows from the fact that $\tau_z > 0$ such that $\tau_e^{-1} > (\tau_e + \tau_z)^{-1}$.