

Sorting out low volatility stocks: Disentangling specific and systematic risk components

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***Abstract** It has been shown that portfolios of low volatility stocks have attractive risk-adjusted returns. This finding is known as the “low volatility anomaly”. As total stock volatility can be decomposed into idiosyncratic volatility (IV) and systematic risk (beta), a natural question is how the low volatility anomaly is related to each component. Is it the low beta or the low IV factor that drives the returns to low volatility stocks? An answer to this question has implications for plausible economic explanations of the low volatility effect, as the low beta and low IV effect have been explained by different economic mechanisms. However, their role in portfolios constructed from sorts on total volatility has not been examined empirically. We aim to fill this gap by assessing empirically how portfolios of low volatility stocks are related to the low beta and low IV factors. Our results suggest that low volatility sorts are driven mostly by the low IV factor but are less related to the low beta factor. We also find that the relation between volatility and beta is not only lower on average, but also more variable over time, than the relation between volatility and IV. An important implication of our results is that a convincing explanation of the low volatility anomaly needs to be consistent with the IV effect as the main driver of returns. Moreover, our results shed new light on some stylised facts in the low volatility literature. In fact, we provide an explanation for the finding that the low volatility effect is mostly driven by low returns of high volatility stocks rather than by high returns of low volatility stocks. While low volatility stocks have negligible exposure to the low beta factor, high volatility stocks have negative exposure to both the low beta and low IV factors, which explains their poor returns.*

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Introduction

Portfolios of low volatility stocks are widely used by equity investors to exploit the so-called “low volatility effect” (see Novy Marx 2014 for a discussion of product offerings). In fact, several authors have documented that sorting stocks based on volatility and holding portfolios which select the low volatility stocks leads to attractive performance (see in particular Haugen and Heins (1975) Haugen and Baker (1996, 2010), Blitz and Van Vliet (2007), and Baker, Bradley and Wurgler (2011)). Such low volatility strategies have become increasingly popular with investors over the recent years, as evidenced by numerous low volatility exchange-traded funds and indices. In addition, other risk measures have been proposed to sort stocks and document attractive performance of low risk versus high risk stocks. In particular, Frazzini and Pedersen (2013) sort stocks on their market beta as a measure of systematic risk and numerous papers sort stocks on their idiosyncratic volatility (IV) as measure of stock-specific risk (see Clarke et al (2010), Huang et al (2010), and the references therein). Li, Sullivan, and Garcia-Feijoo (2014a) analyse both portfolios sorted on low beta and low IV, and find that the low risk effect is not as pronounced as suggested by previous studies, in particular when considering implementation constraints. As total stock volatility can be naturally decomposed into an idiosyncratic (IV) and a systematic (beta) component, a natural question is how the low volatility anomaly is related to these two factors. Is it the low beta or the low IV factor that drives the returns to low volatility stocks? An answer to this question has implications for plausible economic explanations of the low volatility effect, as the low beta and low IV effect have been explained by different economic mechanisms.

Surprisingly, while various papers assess the role of the low beta and low IV factors in equity portfolio optimisation, such as minimum variance or equal risk contribution (see Scherer 2011 and De Leote et al 2013), their role in portfolios constructed from sorts on total volatility has not been examined. We aim to fill this gap by assessing empirically how portfolios of low volatility stocks are related to the low beta and low IV factors.

The literature documenting the performance of low volatility sorted portfolios in fact leaves unanswered the question of the relationship with low-beta and low idiosyncratic risk stocks. Instead, authors either acknowledge that the link between the different risk based factors is not clear, or assume that such a link is necessarily very strong (see section 1 below for a detailed discussion of related literature). Our paper provides an empirical assessment of these relationships, by breaking down the low volatility effect into systematic and idiosyncratic risk effects. Analysing the drivers of the low volatility effect is important for three reasons: First, understanding the role of the underlying factors is a prerequisite for assessing the relevance of various economic explanations of this effect. For instance, the low beta effect has been explained through leverage constraints, while such an explanation does not apply to the low idiosyncratic risk effect.

Second, our analysis helps to establish whether these two factors should be clearly separated or whether they are broadly similar in terms of empirical properties, as is often assumed.

Third, by breaking down the low volatility effect into different components, our analysis may provide a more detailed understanding of some of the stylised facts on return patterns of volatility-sorted portfolios.

Our results show that the low beta and low IV factors are indeed two separate factors with distinct empirical properties and a distinct role in the low volatility portfolios. When assessing portfolio composition of low volatility portfolios, we find strong evidence that low volatility sorts are mainly driven by the low IV stocks but are much more weakly related to low beta

characteristics of stocks. While the bottom quintile portfolio of stocks sorted by volatility contains almost only stocks that also belong to the bottom quintile sorted by IV, it contains stocks with a wide range of betas. Moreover, our results suggest that, the relation between volatility and beta is not only lower on average, but also more variable over time, than the relation between volatility and IV.

When assessing factor exposures of volatility-sorted portfolios, we find that the low beta factor alone only explains 18% of variability of the returns to a portfolio that is long the low volatility stocks and short the high volatility stocks. In contrast, the low IV factor alone explains 94% of this variability. Similarly, a returns attribution for this portfolio shows that 6.1% annual return comes from the low IV exposure while only 1.2% comes from the low beta exposure. A striking finding is that, for the low volatility quintile portfolio, exposure to the low beta factor is not statistically significant when it is included alongside the low IV factor.

Our analysis also provides novel evidence on an intriguing asymmetry between portfolios of low volatility and portfolios of high volatility stocks. In fact, while it is negligible in low volatility portfolios, the low beta factor plays a more pronounced role in high volatility stock portfolios. Portfolios of high volatility stocks have negative exposure not only to the low IV effect but also to the low beta effect. Given that they have negative exposure to both factors, it is not surprising that returns to high volatility portfolios are particularly poor. We thus shed new light on a stylised fact in the literature on the low volatility effect, namely that the effect is driven more by the poor returns of high volatility stocks than the high return of low volatility stocks.

Our conclusions are robust to changes in the setup including changing the stock weighting scheme, the data frequency used to estimate risk measures, and the factor model used to separate systematic and idiosyncratic risk. Disentangling volatility-based portfolio sorts thus shows great contrast of exposures to the different risk-based factors, namely the low idiosyncratic volatility and low beta factors. An important implication of our results is that a convincing explanation of the low volatility anomaly needs to be consistent with the IV effect as the main driver of returns.

The remainder of the paper is organized as follows. Section one shows how the low volatility literature points to the question of disentangling systematic and specific risk effects. Section two introduces the data and our methodology for assessing the importance of the two effects. Section three conducts an analysis of relationships between the different risk components in the cross section of stock characteristics while section four analyses returns of portfolios based on volatility sorts. A fifth section reports results of robustness checks and a final section provides concluding remarks.

1. Related Literature

The literature on volatility sorted portfolios focuses on documenting the anomalous (i.e. flat or negative) relation between volatility and expected returns, and the resulting attractive risk-adjusted returns of low volatility stocks. There is no attribution of this effect to separate components of stock-level risk. However, it is sometimes conjectured that low volatility stocks correspond to low beta stocks. For example, Blitz and van Vliet (2007)³, Roncalli

³ They write (emphasis added): “Ranking stocks on their historical volatility **bears a resemblance to ranking stocks on their historical CAPM beta**. Theoretically this follows from the fact that the beta of a stock is equal to its correlation with the market portfolio times its historical volatility and divided by the volatility of the market portfolio. Empirically we also

(2013)⁴, and Asness (2014)⁵ argue that the attractive performance of low volatility portfolios can be explained by their exposure to the low beta effect, i.e. the well-known empirical finding of a relatively flat relationship between beta and returns, which has been theoretically explained with borrowing restrictions by Black (1972). However, a recurring finding in the low volatility literature is that the effect of superior risk adjusted returns seems to be stronger when sorting stocks on volatility rather than beta, which points to potential differences between volatility sorted and beta sorted portfolios, suggesting that the two effects are not the same (see the results in Blitz and van Vliet (2007), Haugen and Baker (2009), Baker, Bradley and Wurgler (2011), Novy Marx (2014), as well as comments in Blitz, Falkenstein and van Vliet (2014)). These reported differences when sorting on different risk measures lead to the question of what really drives the return and risk of low volatility stock portfolios.

More recently, it has been recognized that the link between sorting stocks on volatility and sorting them on beta or IV is not clear. Li, Sullivan, and Garcia-Feijoo (2014a)⁶ were the first to point out that the Black (1972) explanation of the low risk effect strictly only applies to low beta stocks. Whether or not it also applies to low volatility stocks will depend on the importance of the low beta effect in portfolios of low volatility stocks. Likewise, Blitz, Falkenstein and van Vliet (2014) point out that the relationship between the low volatility and low idiosyncratic volatility effect is unclear. Our lack of understanding the link between low volatility sorted portfolios on the one hand, and the low beta and low IV effect on the other hand, points to an interesting research question. An empirical assessment of these relationships seems in order to advance our understanding of the low volatility effect.

Interestingly, several authors have developed the question of the contribution of the low beta and low IV effect to portfolio returns for a different class of portfolio strategies. In fact, Leote de Carvalho, Lu and Moulin (2012), as well as Scherer (2011) empirically analyse the exposure of optimised portfolios (such as minimum variance, maximum diversification and equal risk contribution portfolios) to the Fama and French (1993) factors, as well as a low beta and a low idiosyncratic volatility factor. They show that the optimised strategies have significant exposure to the low beta factor and somewhat lower exposure to the low idiosyncratic volatility factor (see in particular Exhibit 10 in Leote et al). Clarke et al. (2013) examine the dependence of asset weights in the same optimised portfolio strategies on the market beta and idiosyncratic volatility of stocks both analytically and empirically. They show that the three optimisation strategies lead to asset weights that depend negatively on market beta and idiosyncratic risk, but the dependence on market beta is much more pronounced than on idiosyncratic volatility.

observe that portfolios consisting of stocks with a low (high) volatility exhibit a low (high) beta as well. Since the earliest tests of the CAPM researchers have shown that the empirical relation between risk and return is too flat, e.g. Fama & MacBeth (1973). Similarly, others such as Black, Jensen & Scholes (1972) report that **low beta stocks** contain positive alpha.”

⁴ He writes (emphasis added): “the low volatility effect [...] states that stocks with high idiosyncratic volatility are less rewarded than stocks with low idiosyncratic volatility in a risk-return framework. This result can be interpreted as **a new formulation of the low beta effect** (Black et al. 1972) which states that low beta stocks produce positive alpha.”

⁵ He refers to risk-based factors as (emphasis added) “low beta (or low volatility, [...], or whatever **correlated versions** are floating around) factors”

⁶ They write (emphasis added): “**Though his focus was on market beta**, Black (1972) offers a theoretically consistent interpretation of why low risk stocks might do so well relative to high risk stocks. He shows that borrowing restrictions such as margin requirements might cause low-beta stocks to outperform.”

For portfolios based on low volatility sorts, several authors attempt to gain a better understanding of return drivers, without however attempting a separation of effects stemming from the low IV and low beta factors. For example, Chow et al (2014) analyse the factor exposures of several risk-based weighting methods including a low beta factor but excluding a low IV factor. Novy-Marx (2014) assesses factor exposures of low volatility portfolios to the size, value and profitability factors, without however considering exposures to the underlying risk-based factors (low-beta and low IV). Another related paper is Li, Sullivan and Garcia Feijoo (2014b) who disentangle two components of the returns to low volatility portfolios, namely the covariance of returns with a low risk factor, and the risk characteristics themselves⁷. Our analysis has a similar objective of disentangling returns sources but distinguishes between two different characteristics, low IV and low beta.

In contrast to the available evidence on the dependence of optimised portfolios on the idiosyncratic and systematic risk components, no such evidence is available for volatility-sorted portfolios. Our paper aims to examine the roles of idiosyncratic and systematic risk components in portfolios that are based on sorting stocks by volatility.

2. Data and methodology

Following Baker, Bradley and Wurgler (2011), we form quintile portfolios of stocks sorted by the 5-year trailing total volatility, where total volatility is estimated using monthly returns. To sort out the relative importance of the systematic risk component (low beta effect) and the specific risk component (low IV effect) on these portfolios of stocks sorted by volatility, we conduct two types of analysis.

First, we assess the relationship between different risk measures in the cross section of stock characteristics. This first type of analysis provides a direct assessment of the relationships between volatility and its components in the cross section and does not consider returns of portfolios based on the corresponding sorts. At each point in time, we compute the cross sectional volatility, IV and beta in our universe of stocks. We are thus able to assess the relationship between these variables in the cross section of stocks. In particular, we look at stock level risk measures to assess how strong the relation between stock volatility and its idiosyncratic and systematic components is across all stocks. Moreover, when forming portfolios of stocks sorted on risk measures, we assess the overlap in terms of inclusion of stocks in quintile portfolios formed on different risk measures. Intuitively, if the low volatility effect is driven for instance by the low beta effect, one would expect a high overlap in the stock weights of a portfolio selecting the low beta stocks, and the stock weights of a portfolio selecting the low volatility stocks.

Second, we analyse returns to portfolios of stocks sorted on volatility, by relating them to a set of factor returns. We introduce dedicated factors capturing the low beta and low IV effect, and analyse the exposure of low volatility portfolios to these factors. Moreover, we break down the returns and the volatility of low volatility portfolios into components that can be attributed to each of these factors. The remainder of this section provides a detailed description of our empirical setup, before we turn to the discussion of results for both types of analysis in section four and section five below.

⁷ They conclude that the evidence is more consistent with mispricing than with a common risk factor explanation of the low volatility effect.

2.1. Stock universe

Volatility and returns of the stocks used in our universe are computed using data from the CRSP database. For the period from January 1966 to December 2013, each month, we select all stocks traded on the NYSE, Amex and NASDAQ. We use the following rules to deal with illiquid stocks, avoid survivorship bias, and undue influence of extreme return observations. To avoid an undue influence of thinly traded stocks, we omit any monthly return equal to zero when computing risk measures. Moreover, stocks with less than 24 monthly returns recorded in the 5 years prior to portfolio formation are not included in the relevant cross section⁸. To avoid an undue influence of extreme return observations, stocks that, as of our portfolio formation date, are penny stocks (i.e. stocks with a price of less than USD 5) are excluded from our analysis. Moreover, return observations exceeding 200% are also excluded⁹. It should be noted that all these exclusions are based on data available prior to the portfolio formation date so as to not introduce any look-ahead-bias. To avoid survivorship bias, we include all stocks traded at the time of portfolio formation (at the end of the previous month). If a stock is delisted during the month following portfolio formation, the stock is included in our cross section, up to the return observation available on its last trading day.

2.2 Risk measures and Portfolio Formation

Stock level risk can be decomposed into a systematic and a stock specific component. The systematic risk component is the variance contributed from the various systematic risk factor exposures, while the stock specific component is the variance that is unique to the stock in the sense that it is unrelated to the variances of the systematic risk factors.

Using variance to proxy for risk, we can express the stock level risk in the following equation:

$$\sigma_i^2 = \underbrace{\mathbf{b}_i' \Sigma \mathbf{b}_i}_{\text{systematic}} + \underbrace{\sigma_{\varepsilon_i}^2}_{\text{stock specific}} \quad (1)$$

where σ_i^2 denotes the variance of the i^{th} stock in the cross section, \mathbf{b}_i denotes the $K \times 1$ vector of betas, i.e. loadings of the i^{th} stock return on the K systematic factors, Σ denotes the $K \times K$ variance covariance matrix of the returns to the K systematic factors, and $\sigma_{\varepsilon_i}^2$ denotes the variance of the residual returns of the i^{th} stock.

We run our analysis using different factor models to distinguish specific from systematic risk, notably a single factor model using the market factor (CAPM) and a multi factor model using the market, value, and size factors (Fama and French (1993)). In the single factor model, the risk for stock i is simply:

$$\sigma_i^2 = (\beta_m \sigma_m)^2 + \sigma_{\varepsilon_i}^2 \quad (2)$$

where β_m is the factor loading of i^{th} stock returns on the market factor, and σ_m is the variance of the market returns.

⁸ Omitting stocks with less than 24 monthly return observations is standard practice in empirical studies of the cross section of stock returns in general (see for example Fama and French (1993)) and when analyzing the relationship between stock volatility and returns in particular (see Bali and Cakici (2008))

⁹ This approach is similar to Fu (2009), where stock observations of stock returns greater than 300% are treated as outliers or possible data entry errors are discarded. In our case, after excluding the penny stocks, observations of stock returns exceeding 200% only consist of 40 observations out of over 3 million monthly observations.

For each month, the stock level risk (total volatility), systematic risk (beta) and stock specific risk (idiosyncratic volatility) are computed over the 5-year period prior to portfolio formation, using monthly total returns data (including reinvestment of dividends). Since we want to assess the overlap between sorts on total volatility and sorts on beta or IV, we actually categorise stocks into quintiles according to their ranking by each of the three risk measures, volatility, IV and beta. For example, in Jan 2012, Apple is ranked in the 4th highest beta quintile, but is ranked in the middle (3rd) quintile in the cross-section of stocks sorted on total volatility or IV. Using our sorts, we form our base portfolios sorted by stock level risk, these portfolios are cap weighted and revised monthly.

Our base portfolios of interest are these portfolios sorted by total volatility, notable the high and low quintiles, as well as the long/short portfolio that reflects the difference in return between low and high volatility stocks.

2.3 Factor models and attribution

We use different factor models drawing on various sets of factors to explain the returns to the volatility sorted portfolios. The full-fledged factor model follows the approach in Scherer (2011) and de Leote et al (2012) and augments the Carhart four factor model by a low beta factor and a low IV factor. To be specific, we run the following time series regression to estimate factor loadings.

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_{p,1}r_{mkt,t} + \beta_{p,2}r_{smb,t} + \beta_{p,3}r_{hml,t} + \beta_{p,4}r_{mom,t} + \beta_{p,5}r_{bab,t} + \beta_{p,6}r_{ivol,t} + \varepsilon_{p,t} \quad (3)$$

Where the factors are defined as follows:

- MKT: market factor, long a cap-weighted broad market portfolio and short the riskfree rate
- HML: value factor (Fama-French (1993)): long value stocks and short growth stocks
- SMB: size factor (Fama-French (1993)): long small market cap stocks and short big market cap stocks
- MOM: momentum factor (Jegadeesh-Titman (1993)): long stocks with high past returns and short stocks with low past returns, where returns are measured over the past year omitting the last month
- BAB: betting against beta factor (Frazzini and Pedersen 2014) long low beta stocks and short high beta stocks, adjusting the long and short weights so that factor beta is zero
- IVOL: Low idiosyncratic volatility factor (Ang et al. (2006), Clarke et al (2010)¹⁰): long low idiosyncratic volatility stocks and short high idiosyncratic volatility stocks

Our focus is on the coefficient estimates associated with the BAB and IVOL factors, as well as on goodness-of-fit of various specifications using different sets of factors. In particular, the multi factor regression framework provides a natural methodology for assessing the marginal impact of one factor in the presence of another factor (on this point see Fama and French (2014)). For example, if the BAB and IVOL factors are the main drivers of low vol portfolios,

¹⁰ It should be noted that Ang et al (2006) use a short term volatility measure based on daily return observations over the previous month. We use longer term estimate of idiosyncratic volatility following Clarke et al (2010) and Li, Sullivan, and Garcia-Feijoo (2014)

the goodness of fit of the multi factor model including MKT, IVOL and BAB should be better than that of other three factor models, including MKT, SMB and HML. Likewise, if the goodness of fit does not increase when adding the IVOL factor to say the market and BAB factor, we would conclude that IVOL does not provide additional explanation of portfolio returns already provided by BAB¹¹.

As much as possible, we prefer to use publically available research data for each factor. In particular, we use the monthly factor return series provided by Kenneth French¹² (Market, SMB, HML and Momentum factors) and Andrea Frazzini¹³ (Betting against Beta factor). For the low IV factor, we construct a return series that is long the low IV quintile and short the high IV quintile described in subsection 2.2. The market portfolio provided by Kenneth French's data library is the value weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. The risk-free rate is the 1 month Treasury bill return.

In addition to analyzing the coefficient estimates and goodness of fit of different factor models, we also report the contribution to portfolio return and portfolio volatility provided by the different factors in equation 3. The advantage of looking at risk and return contributions is that they allow for a more intuitive assessment of the comparative importance of different factors. In fact, the contributions to risk and return show the influence on the investment outcome of a given factor exposure. Contributions can be derived from the factor loadings of volatility sorted portfolios together with risk and return characteristics of the factor returns. In particular, we break down the returns for volatility sorted portfolios into two parts, the unexplained part, and the return attributable to each factor. We do this by multiplying the estimated factor exposure (i.e. betas) by the annual return premium associated with each factor.

Moreover, we break down the variance for volatility sorted portfolios into three parts, an unexplained part, variance attributable to each factor, and variance attributable to the interaction of factors. The following expression is used to break down the variance.

$$\begin{aligned}
 \text{Volatility}^2 &= \underbrace{\sum_F \beta^{F^2} \text{Var}(R_F)}_{\text{Variance contributed by factors}} + \underbrace{2 \left[\sum_{F_i \neq F_j} \beta^{F_i} \beta^{F_j} \text{Cov}(R_{F_i}, R_{F_j}) \right]}_{\text{Variance due to interaction of factors}} + \underbrace{\sigma_\varepsilon^2}_{\text{Unexplained variance}} \quad (4)
 \end{aligned}$$

¹¹ One needs to keep in mind that (even adjusted) R-squares tend to increase when one increases the number of factors, hence we focus on comparisons of models with an identical number of factors.

¹² See Kenneth French's data library at < http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html>

¹³ See Andrea Frazzini's data library at < http://www.econ.yale.edu/~af227/data_library.htm>

2.4 Robustness Checks

We conduct various robustness tests of our results. An overview of results is reported in section 5. Detailed results are provided in the appendix. In particular, we make the following changes to our setup.

- Using three years of weekly data rather than 5 years of monthly data to estimate the risk measures prior to portfolio formation.
- Equal-weighting stocks within the volatility sorted quintile portfolios instead of cap-weighting them
- Computing idiosyncratic risk relative to the Fama and French (1993) three factor model rather than relative to only the market factor.
- Using deciles rather than quintiles to sort stocks based on volatility into the high and low categories.
- Using a beta neutral IVOL factor by adjusting the weights to the long and short leg of the factor by their respective beta.

3. Sorting out stock characteristics

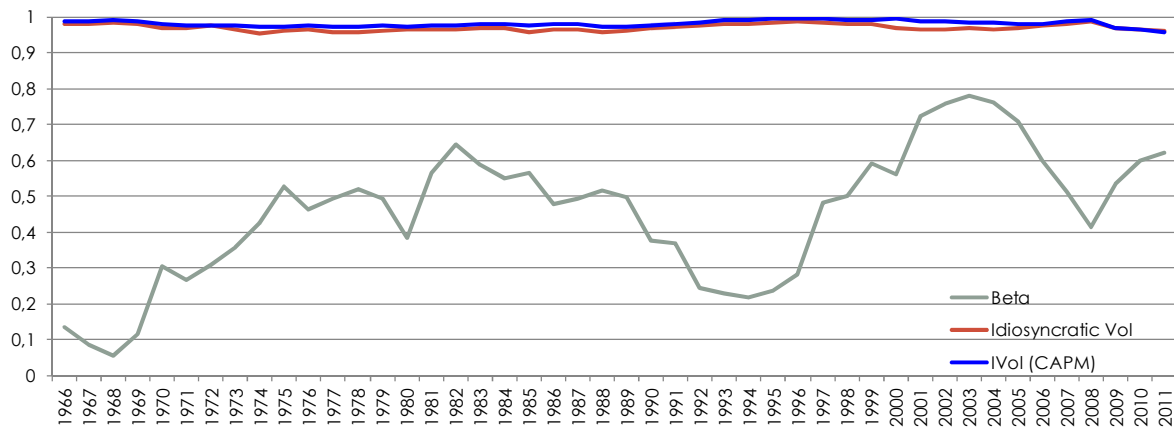
Before turning to an analysis of returns of portfolios of stocks sorted on volatility and their relation to risk-based factors, we assess directly whether sorts on volatility bear resemblance to sorts on its components: beta and IV. We conduct two types of analysis. First, we assess the correlation between volatility of stocks and different risk components of stocks. We report the Spearman rank correlation between volatility on the one hand, and beta or IV on the other hand. We also show the percentage of stock variance accounted for by systematic and specific risk components. Second, we look at cap-weighted quintile portfolios of stocks sorted on volatility. We assess the weight given in these portfolios to different categories of stocks, where we define the categories through sorts on either IV or beta.

3.1. Stock-level risk characteristics

Figure 1 displays the rank correlation between volatility on the one hand, and beta or IV on the other hand in the cross section of stock returns. The correlation is estimated at each monthly portfolio formation date, based on the prior five years of monthly returns data. In order to estimate idiosyncratic volatility, we use a single factor model, and alternatively we also use the Fama and French three factor model. It is clear from inspection of the results in Figure 1 that volatility and IV are strongly related. In fact, the correlation coefficient is close to one, irrespective of the date at which the analysis is conducted. The correlation between volatility and beta has very different properties. First, in spite of being positive, the correlation coefficient has an average value of about 0.5 over the time period considered. Second, the correlation between beta and volatility fluctuates widely over time, taking on values as low as about 0.1. Thus, the relation between volatility and beta is not only lower on average, but also more variable over time, than the relation between volatility and IV.

Figure 1: Cross-sectional correlation between volatility and IV / beta.

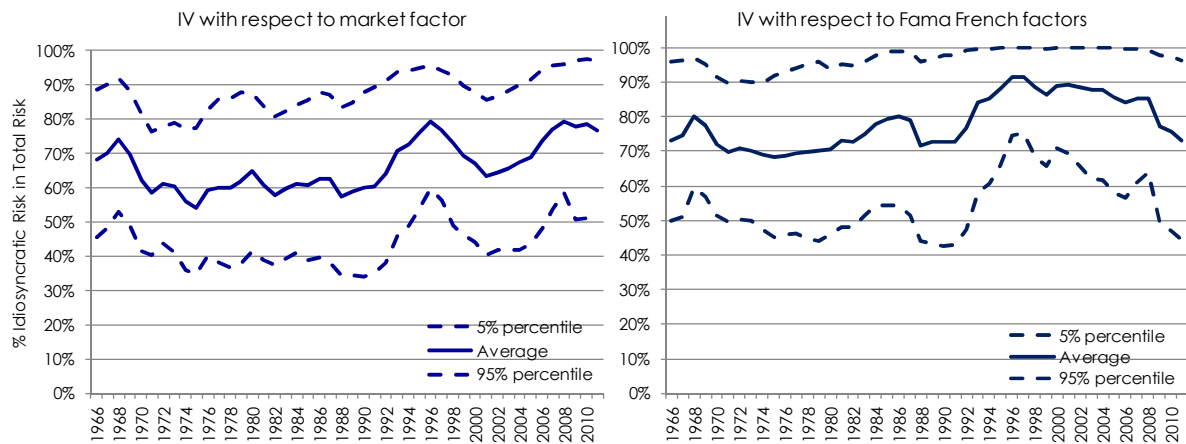
The graph displays the Spearman rank correlation coefficient calculate at the end of each month between the volatility and beta (respectively IV) of all stocks contained in the cross section for the respective month. All risk measures are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors.



In order to provide some perspective of the difference between beta and IV in terms of their correlation with volatility across stocks, we also display a direct assessment of the relative importance of each risk component, specific and systematic, in the overall volatility of stock returns. The left hand panel of Figure 2 displays the percentage of variance accounted for by the specific component, when considering only the market factor as a source of systematic risk, i.e. $1 - \beta_{M,i}^2 \sigma_M^2 / \sigma_i^2$. Inspection of Figure 2 shows that on average, the specific risk component (IV) accounts for about 70% of variance for a typical stock. While the percentage of volatility accounted for by the specific component across stocks is bound in the range from about 60% to about 80%. What is remarkable is that for 95% of stocks, the idiosyncratic risk component accounts for at least 50% of their volatility, as can be seen from the 5th percentile which takes on a value of about 50%. Moreover, the right hand panel of Figure 2 shows that the dominance of the idiosyncratic component is confirmed even when including additional systematic risk factors.

Figure 2: Idiosyncratic variance as percentage of total variance.

The graph displays the average, and selected percentiles of the cross sectional distribution of idiosyncratic volatility as a percentage of total stock volatility, calculated at the end of each month. All risk measures are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors.



3.2. Portfolio Composition

We assign stocks to five groups according to their idiosyncratic volatility (respectively their beta) prior to forming the total volatility quintile portfolios. We then show the composition of the low volatility quintile in terms of categories of stocks resulting from the IV (respectively beta) sort. If there is a strong relationship between volatility sorts and IV (respectively beta), we would expect to see considerable weight given to low IV stocks (respectively low beta) in the low volatility portfolio, and very little weight given to high IV stocks (respectively high beta stocks). Figure 3 reports the results for the low volatility portfolio.

Figure 3: Composition of low volatility portfolio by IV (respectively beta) categories

The graph displays the weight of IV and beta category in the low volatility quintile portfolio. All risk measures are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

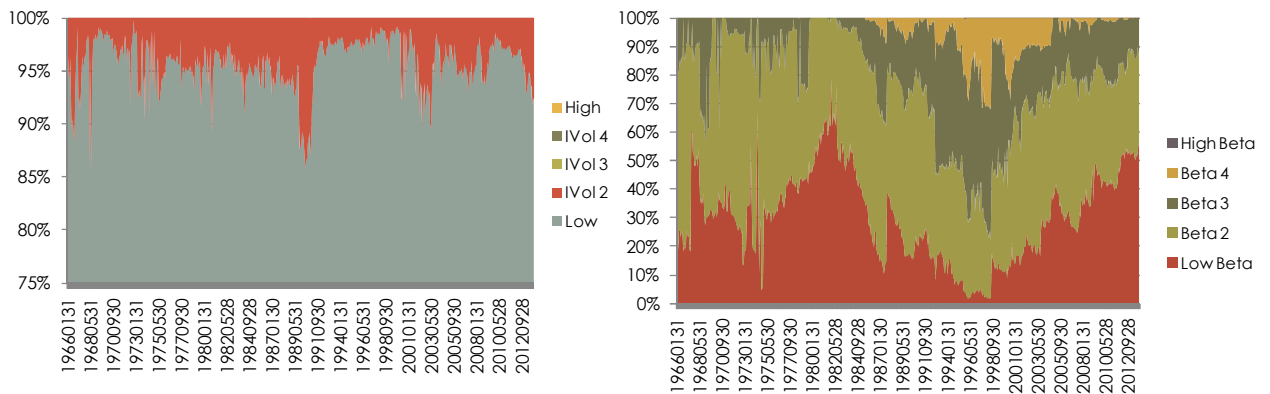
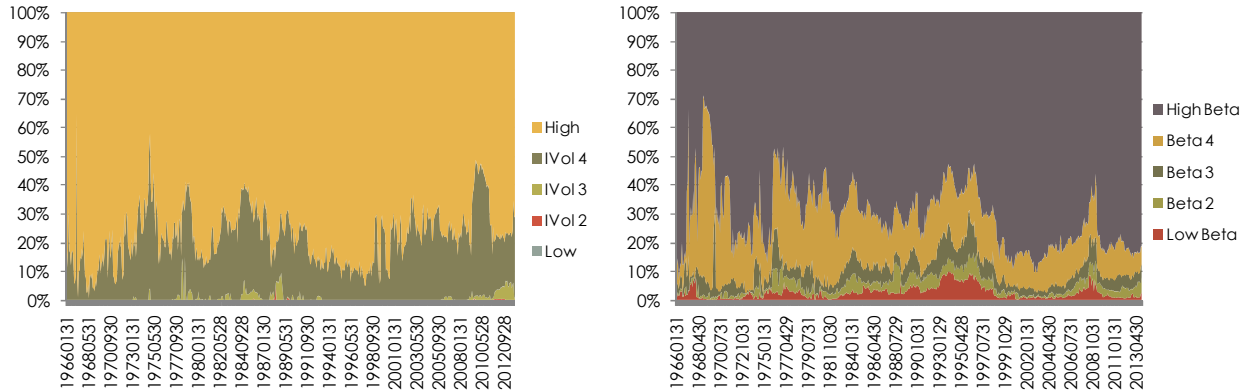


Figure 3 shows that, while the stocks in the low volatility portfolio belong mainly to the low IV category, they come from a wide range of beta categories, including stocks from four of the five quintiles created by sorting on beta. Consistent with the findings reported in Figure 1, we again find evidence that the relation between volatility and beta is not only weaker, but also more variable over time, than the relation between volatility and IV. In particular, the weight of low beta stocks in the low volatility portfolio fluctuates from near zero to about 60 percent. In contrast, the weight of the low IV stocks in the low volatility portfolio is relatively stable with levels from about 90% to close to 100%.

Figure 4 shows results for the other side of the volatility sort, notably the composition of the cap-weighted high volatility portfolio. As we did for the low volatility portfolio in Figure 3, we now indicate the composition of the high volatility portfolio in terms of categories of stocks resulting from the IV (respectively beta) sort. Here too, the overlap of the high volatility portfolio with the high IV categories is more pronounced than the overlap with the high beta category. The high vol portfolio puts substantial weight on the medium and low beta stocks. However, the overlap of the high volatility portfolio with high beta sorts (shown in Figure 4) is more pronounced than the overlap of the low volatility portfolio with low beta sorts (shown in Figure 3). Despite this nuance, the results in Figure 3 and Figure 4 suggest that when creating cap-weighted portfolios of low volatility and high volatility stocks, there is a remarkable overlap with IV sorted portfolios but little overlap with beta sorted portfolios, thus confirming the results in sub section 3.1 of a strong link between volatility and IV and a weaker albeit positive link between volatility and beta.

Figure 4: Composition of high volatility portfolio by IV (respectively beta) categories.

The graph displays the weight of IV and beta category in the high volatility quintile portfolio. All risk measures are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.



To provide additional perspective on the relation between volatility sorts versus sorts on the components of volatility, we report the weighted average beta, and weighted average IV for the high and low volatility portfolios in table 0. Note that the weighted average beta is equal to the portfolio beta, while the weighted average IV is not equal to portfolio IV since part of the stock-level idiosyncratic volatility will be diversified away when combining low or high volatility stocks in a portfolio. Our objects of interest however are stock-level characteristics, and therefore we report weighted averages rather than risk measures at the portfolio level. Note that we weight stock-level risk measures by the market cap weight of the respective stock in the relevant portfolio to come up with the weighted average. We report both the time series average, as well as the standard deviation over time, for the monthly observations of weighted average beta and IV. Note that, for ease of comparison of the results across IV and beta, we have normalised stock-level IV measure by dividing it by the weighted average across all stocks. Thus, a value of 1 indicates that the stock-level risk is identical to the weighted average across all stocks. In the case of beta, this property results directly from the definition of beta and no normalisation is needed.

Table 0: Stock-level characteristics in Volatility Sorted Portfolios: Beta and IV

The table displays the time series average, as well as the time series standard deviation of weighted average risk measures calculated each month for volatility sorted portfolios. For comparison, the table also indicates this information for portfolios that have been built by sorting on the relevant risk measure itself. All stock-level risk measures are normalised to one by dividing the risk measure by the weighted average risk measure across all stocks. Monthly average weighted risk measures are calculated from stock-level risk measures estimated with returns prior to the portfolio formation date. The time series average and time series standard deviation of weighted averages are calculated from the time series of all monthly weighted averages over the period January 1966 to December 2013.

Panel A:	Weighted Average Beta across stocks in...			
	Low Vol Quintile Portfolio	Low Beta Quintile Portfolio	High Vol Quintile Portfolio	High Beta Quintile Portfolio
Time Series Mean	0.68	0.38	2.04	2.10
Time Series Std Dev	0.12	0.17	0.35	0.28

Panel B:	Weighted Average Normalised IV across stocks in...			
	Low Vol Quintile Portfolio	Low IV Quintile Portfolio	High Vol Quintile Portfolio	High IV Quintile Portfolio
Time Series Mean	0.72	0.71	2.46	2.57
Time Series Std Dev	0.04	0.04	0.22	0.20

Table 0 compares the weighted average stock-level risk measures of volatility-sorted portfolios to those of portfolios that have been sorted directly on the relevant risk measure. This provides a different assessment of the overlap between volatility sorts, and sorts on IV or beta. The results in Table 0 confirm our conclusions from looking at the weight allocated to different categories of stocks. We see in Table 0 that stocks in the low volatility portfolio have an average IV (0.72) that is very close to stocks in the low IV quintile (0.71). In contrast, stocks in the low vol portfolio have an average beta (0.68) that is much higher than for stocks in the low beta portfolio (0.38). Moreover, the time variation in the average stock-level characteristics is noteworthy. While the Low Vol portfolio's average beta varies over time with a standard deviation of 0.12, its weighted average IV only varies with a standard deviation that is three times smaller at 0.04. Thus, it becomes clear that the beta of stocks in low volatility portfolios is not only quite different from that displayed by low-beta stocks, but it also varies greatly over time, suggesting that portfolios of low volatility stocks do not reliably consist of low beta stocks.

Overall, the results discussed in this section suggest that the IV component is dominant in explaining low volatility sorts, simply because stock level volatility is mainly driven by the firm specific risk component as opposed to systematic risk which plays a lesser role. Moreover, when looking at the time series of risk characteristics of stocks, we find that the relationship between low volatility stocks and low beta is not only weak, but also highly unstable.

After having assessed the link between volatility and its systematic and specific components in the cross section of individual securities, we now turn to an analysis of portfolio returns of volatility sorts.

4. Sorting out portfolio returns

While assessing the relationship between different risk measures, and the overlap of weights in volatility sorted portfolios reported in section 3 provides direct evidence on how different risk components are related to volatility, this section assesses which influence the different risk components have on the risk and return properties of volatility sorted portfolios. We augment the standard single factor and four factor models with risk-based factors (a low beta factor and a low IV factor), and assess factor exposures of portfolios based on volatility sorts. We also attribute the returns and the risk of volatility sorted portfolio to these factor exposures to provide intuitive insights into the relative importance of the different risk-based factors. Below, we first introduce the risk-based factors that we employ, before turning to a discussion of factor exposures and attribution results.

4.1. The factors

We introduce two risk-based factors which correspond to return differences obtained when sorting stocks based on beta, and respectively on idiosyncratic volatility. Similar to the analysis based on characteristics in the cross section of stock returns, our aim is to assess the risk components driving volatility sorts. In order to capture the two components underlying low volatility sorts, we use a low beta factor and a low IV factor, which have been detailed in subsection 3.3. In particular, we use a low beta factor introduced by Frazzini and Pedersen (2014), which is constructed from a long position in low beta stocks and a short position in high beta stocks. In addition, following Scherer (2011) and De Leote et al (2012) we further

augment our factor model with an IV factor. We construct this factor from the sorts discussed above, i.e. we create quintiles portfolios of stocks sorted on IV relative to the market factor, as well as relative to the Fama-French 3 factor model.

Before analyzing how volatility sorted portfolios are exposed to these factors, it is insightful to assess the properties of the factors themselves. In particular, for the analysis to be meaningful, we would like to understand whether these two factors really capture different cross sectional dimensions of returns. The summary statistics in table 1 provide an overview of the properties of the two idiosyncratic volatility factors (relative to the single factor model, denoted “CAPM IVOL”, and relative to the three factor model denoted “FF IVOL”), as well as the BAB factor of Frazzini and Pedersen (2014) and the market factor (excess returns over 1 month Treasury Bills of the CRSP value weighted index).

Table 1: Risk-based long/short factor portfolios. Descriptive statistics.

*The table displays descriptive statistics for monthly returns over the period January 1966 to December 2013. Average returns, standard deviations and alphas are annualized. The Market factor is from Kenneth French’s website. The BAB factor is from Andrea Frazzini’s website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-statistics marked with * correspond to estimates of the mean that are significantly different from zero at the 5% level. Skewness and kurtosis are estimated using the Kim and White (2004) robust skewness and kurtosis methodology.*

Panel I: Summary statistics of factor returns				
	Mkt	BAB	CAPM IVOL	FF IVOL
Mean	5.71%	10.38%	5.80%	6.93%
Std Dev	15.86%	11.55%	24.27%	23.50%
t-statistic	2.49*	6.22*	1.66	1.85
Skewness	-0.04	0.02	-0.06	-0.09
Kurtosis	-0.06	0.26	0.15	0.27
Panel II: Average returns left unexplained by exposure to other risk factors				
CAPM Alpha	-	10.70%	9.90%	10.94%
t-statistic	-	4.56*	4.13*	3.49*
3-Factor Alpha	-	8.07%	8.89%	8.29%
t-statistic	-	3.97*	5.09*	4.20*
Panel III: Correlations between factor returns				
	Mkt	BAB	CAPM IVOL	FF IVOL
Mkt	1	-0.08	-0.54	-0.55
BAB		1	0.36	0.38
CAPM IVOL			1	0.97
FF IVOL				1

The correlation coefficients for pairs of factor returns in Panel III of Table 1 show that the systematic risk factor (BAB) and the idiosyncratic risk factor (IVOL) have positive but low correlation at around 0.4. Moreover, the results in Panel I of Table 1 confirm that there is a pronounced difference for these two factors, as risk and return characteristics differ widely between the BAB factor (with an annual mean of about 10% and volatility of about 12%) and the IVOL factors (with annualised mean of about 6% and annualised volatility of about 24%).

Moreover, standard factor models fails to explain the returns to the BAB and IVOL factors, as evidence by the results in Panel II. In fact, the BAB and IVOL factor returns show significant positive alphas when adjusting their returns for exposure to common, risk factors.

It also appears that the IVOL factor calculated relative to a Fama-French systematic return component is similar to the factor using only the market factor to define systematic risk. The correlation is 0.97. In the following discussion, we focus on this CAPM version of IVOL, and provide a robustness check in section 5 and in the appendix where we use an IVOL factor that is defined relative to the Fama and French three factor systematic return component.

4.2. Factor exposures

We assess the factor exposures of volatility sorted portfolios in basic factor models, notably the single factor model and the Carhart (1997) four factor model. We then test several augmented versions of these two basic factor models, where we augment them with only the IVOL factor, with only the BAB factor, or with both factors. A question of interest is how the models augmented with a single risk-based factor fare compared to the models augmented by both risk-based factors. Answering this question will lead to a better understanding of the drivers of the low volatility effect. For example, if both factors are equally important in explaining volatility sorted portfolio returns, one would expect significant exposures to both factors, even when they are included together in the factor model. Moreover, one would expect a clear increase in the explanatory power of the model over models which include only one of the factors. To the contrary, if one of the factors dominates the other in explaining volatility sorted portfolio returns, adding the second (dominated) risk-based factor to an existing model with only the first (dominant) risk-based factor is expected to yield no considerable improvements in explanatory power. In the extreme, the volatility sorted portfolios may not even load significantly on the second factor in the presence of the first. The elegance of multivariate regressions is that the factor loadings provide an estimate of the marginal impact of a factor in the presence of the other factors.

We report results for various specifications of factor models in explaining the returns to the low volatility portfolio, high volatility portfolio, and the low-minus-high (LMV) volatility portfolio, in Table 2, Table 3, and Table 4. Conducting the analysis of the high and low quintiles separately before turning to the LMH (low minus high) portfolio is of interest, as it has been shown that the low risk effect is typically most pronounced in the high quintile as opposed to the low quintile, i.e. the return properties of the low volatility quintile is not very different from the middle quintiles, while the high risk quintile does display differences and drives the low risk effect (see in particular Li, Sullivan and Garcia-Feijoo (2014a) and Bali and Cakici (2008)).

Table 2: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio.

*The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (adjusted R-square) of regressions of excess returns of the (cap-weighted) low volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.*

	Market + Risk-based factors				Carhart + Risk-based factors			
	CAPM	+BAB	+IVOL	All	Carhart	+BAB	+IVOL	All
Alpha	0.01	-0.01	-0.02	-0.02	0.00	-0.01	-0.02	-0.02
<i>t-statistic</i>	0.92	-1.38	-2.92*	-3.35*	0.06	-0.91	-2.96*	-3.09*
BAB		0.20		0.04		0.13		0.02

<i>t</i> -statistic		3.46*		1.57		4.71*		1.30
IVOL			0.25	0.24			0.23	0.22
<i>t</i> -statistic			23.04*	25.30*			16.39*	14.01*
Mkt	0.70	0.71	0.91	0.90	0.81	0.80	0.91	0.91
<i>t</i> -statistic	21.01*	32.56*	80.64*	80.49*	44.24*	50.65*	71.86*	72.07*
SMB					-0.30	-0.30	-0.02	-0.03
<i>t</i> -statistic					-10.27*	-11.27*	-0.63	-0.85
HML					0.22	0.15	0.07	0.06
<i>t</i> -statistic					4.96*	4.05*	2.21	2.10
MOM					0.02	-0.01	0.01	0.01
<i>t</i> -statistic					0.45	-0.35	0.41	0.24
R ²	0.76	0.79	0.91	0.91	0.85	0.86	0.91	0.91

When assessing the results in Table 2 it is useful to distinguish between the augmented versions of the single factor model and the augmented version of the Carhart model.

The left hand side of Table 2 shows that the market factor alone accounts for about 76% of the variability of returns of the low volatility quintile portfolio. This is not surprising as the market factor tends to be dominant in any long only stock portfolio, as long as it is reasonably well-diversified. Augmenting the single factor model by the BAB factor leads to a statistically significant factor loading for BAB, but only to a marginal increase in the goodness-of-fit to about 79%. These results suggest that, while it is true that low volatility portfolios load on a low systematic risk factor, this loading is weak in explaining the variability of returns. In contrast, when adding the low IV factor to the market factor, not only is the factor loading to IVOL highly significant, but also do we observe a substantial increase in the percentage of variability of returns that is explained by the two factors (market factor and IVOL factor) with an R-squared of about 91%. Interestingly, when adding the BAB factor in addition to the IVOL factor, the R-squared remains at 91%. These results suggest that the low systematic risk factor does not add to the explanation of low volatility returns over and beyond what is explained by the low idiosyncratic risk factor. Moreover, it should be noted that in the presence of the IVOL factor, the factor loading for the BAB factor is not statistically significant at conventional levels with a t-statistic of about 1.6.

The right hand side of Table 2 is based on the Carhart four factor model as the base model. Compared to the results with augmented versions of the single factor model, we obtain broadly similar results concerning the importance of adding the different risk-based factors. The Carhart model allows capturing return components of the low volatility portfolio that are related to value, momentum and size factors. In fact, the low volatility portfolio displays a significantly positive value exposure and significantly negative size exposure. Overall, the four Carhart factors allow capturing 85% of variability of returns of the low vol portfolio, compared to 76% that were captured by the market factor alone. This increase in goodness-of-fit is unsurprising as the higher number of factors should lead to a higher R-squared. However, when assessing the impact of augmenting the model with a BAB factor, an IVOL factor, or both risk-based factors, the conclusions obtained from the analysis using the single factor model carry over to the analysis with the Carhart model. In particular, while exposure to the BAB factor is significant in the absence of the IVOL factor, it becomes statistically indistinguishable from zero when the BAB factor is included alongside the IVOL factor (t-statistic of 1.3). Moreover, the goodness of fit of the model including BAB and IVOL factors is indistinguishable from the one including only the IVOL factor, with both models leading to an R-squared of 91%.

Overall, it thus appears that the low IV factor dominates the low beta factor in explaining returns of the cap-weighted low volatility portfolio.

Table 3 shows our analysis of factor exposures for the high volatility quintile portfolio.

Table 3: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio.

The table displays regression coefficients and associated *t*-statistics, as well as the goodness of fit (*R*-squared) of regressions of excess returns of the (cap-weighted) high volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. *t*-stats marked with * correspond to coefficient estimates that are significant at the 5% level. *T*-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	Market + Risk-based factors				Carhart + Risk-based factors			
	CAPM	+BAB	+IVOL	All	Carhart	+BAB	+IVOL	All
Alpha	-0.10	-0.04	-0.02	-0.01	-0.07	-0.05	-0.01	-0.01
<i>t</i> -statistic	-3.60*	-1.38	-2.45	-1.41	-3.99*	-2.68*	-1.10	-0.79
BAB		-0.56		-0.09		-0.41		-0.07
<i>t</i> -statistic		-5.17*		-3.13*		-5.49*		-2.39
IVOL			-0.72	-0.70			-0.72	-0.70
<i>t</i> -statistic			-26.04*	-25.33*			-23.86*	-22.49*
Mkt	1.63	1.60	1.03	1.04	1.33	1.36	1.01	1.02
<i>t</i> -statistic	21.69*	31.82*	37.45*	39.18*	26.96*	29.71*	37.92*	38.29*
SMB					0.87	0.87	-0.02	0.00
<i>t</i> -statistic					14.39*	14.89*	-0.39	0.02
HML					-0.52	-0.30	-0.04	-0.02
<i>t</i> -statistic					-6.39*	-3.36*	-0.94	-0.38
MOM					-0.11	-0.03	-0.09	-0.08
<i>t</i> -statistic					-1.89	-0.51	-3.09*	-2.52*
R ²	0.73	0.78	0.96	0.97	0.86	0.88	0.97	0.97

A notable difference in the factor exposures of the high volatility portfolio is that the loadings on the BAB factor remain significant (with *t*-statistics of around -5) even in the presence of the IVOL factor. This is true in both the augmented versions of the single factor model and of the four factor model. While the results in Table 2 suggest that low volatility portfolios are really mainly about gaining positive exposure to the IVOL factor, Table 3 shows that the high volatility portfolio is about gaining negative exposure to both the IVOL and BAB factor. In this sense, the exposure across both risk components is stronger for the high volatility portfolio than for the low volatility portfolio.

Interestingly, it has been pointed out that the empirical evidence of what is termed the “low volatility” effect, i.e. the positive return difference between portfolios of low volatility stocks over high volatility stocks, is mainly driven by the returns to high volatility stocks. In fact, an empirical regularity is that high volatility stocks tend to have low returns compared to the average across all stocks, while low volatility stocks have returns that are not much different from the average across all stocks (see for example Novy-Marx 2014). Our finding that high volatility stocks are negatively exposed to both the IVOL and the BAB factor, while low volatility stocks are only exposed to the IVOL factor but not to the BAB factor, may provide an explanation of this empirical regularity. It should also be noted that this result concerning factor exposures is consistent with the result we obtained when analyzing the overlap of portfolio weights in section 3 (Figures 3 versus Figure 4).

However, despite the high volatility portfolio maintaining significant loadings to the BAB factor in the presence of the IVOL factor, the additional explanation provided by the BAB factor is small. In fact, the R-squared increases from 96% to 97% when adding the BAB factor to the market factor and IVOL, and remains almost unchanged at 97% when adding the BAB factor to the Carhart factors and the IVOL factor. Overall, while the systematic risk effect is more pronounced in the high volatility portfolio than it is in the low volatility portfolio, the results still suggest that the idiosyncratic component is the main driver of returns.

To assess overall differences in the cross section of stocks resulting from sorts on volatility, we now turn to the low minus high volatility portfolio. Table 4 presents the results of our factor regressions.

Table 4: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio.

The table displays regression coefficients and associated *t*-statistics, as well as the goodness of fit (*R*-squared) of regressions of excess returns of the return difference between the (cap-weighted) low and high volatility quintile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. *t*-stats marked with * correspond to coefficient estimates that are significant at the 5% level. *T*-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	Market + Risk-based factors				Carhart + Risk-based factors			
	CAPM	+BAB	+IVOL	All	Carhart	+BAB	+IVOL	All
Alpha	0.11	0.02	0.00	-0.01	0.07	0.04	-0.01	-0.01
<i>t</i> -statistic	3.04*	0.73	0.35	-0.73	3.23*	1.85	-1.05	-1.40
BAB		0.76		0.13		0.55		0.09
<i>t</i> -statistic		4.68*		4.62*		5.42*		3.18*
IVOL			0.97	0.94			0.95	0.93
<i>t</i> -statistic			41.63*	40.35*			32.90*	30.89*
Mkt	-0.93	-0.88	-0.12	-0.14	-0.52	-0.56	-0.10	-0.12
<i>t</i> -statistic	-8.71*	-13.09*	-4.10*	-5.03*	-8.28*	-10.08*	-3.80*	-4.38*
SMB					-1.16	-1.17	0.00	-0.03
<i>t</i> -statistic					-14.58*	-15.77*	0.09	-0.47
HML					0.75	0.45	0.11	0.08
<i>t</i> -statistic					6.69*	4.03*	3.47*	2.47
MOM					0.12	0.02	0.10	0.08
<i>t</i> -statistic					1.58	0.24	3.82*	3.07*
R ²	0.34	0.46	0.95	0.95	0.68	0.73	0.95	0.95

Table 4 confirms that the BAB factor is dominated by the IVOL factor when it comes to capturing the time series variation in returns to volatility sorted portfolios. In particular, we can see that the percentage of return variance of the LMH portfolio captured by the market factor and the IVOL factor is 95% and adding the BAB factor does not increase this percentage. Again, similar results hold for the augmented versions of the Carhart model. Moreover, even though the LMH portfolio loads significantly on the BAB factor in the presence of the IVOL factor (*t*-statistics of 4.62 for the augmented single factor model and 3.18 for the augmented Carhart model), the BAB factor loadings (close to 0.1) are much lower in magnitude than the IVOL factor loadings (close to 0.9).

However, factor loadings alone are difficult to interpret from a practical perspective. Therefore, in the next section, we present an analysis of return and risk attribution of the volatility sorted portfolios with respect to the risk factors.

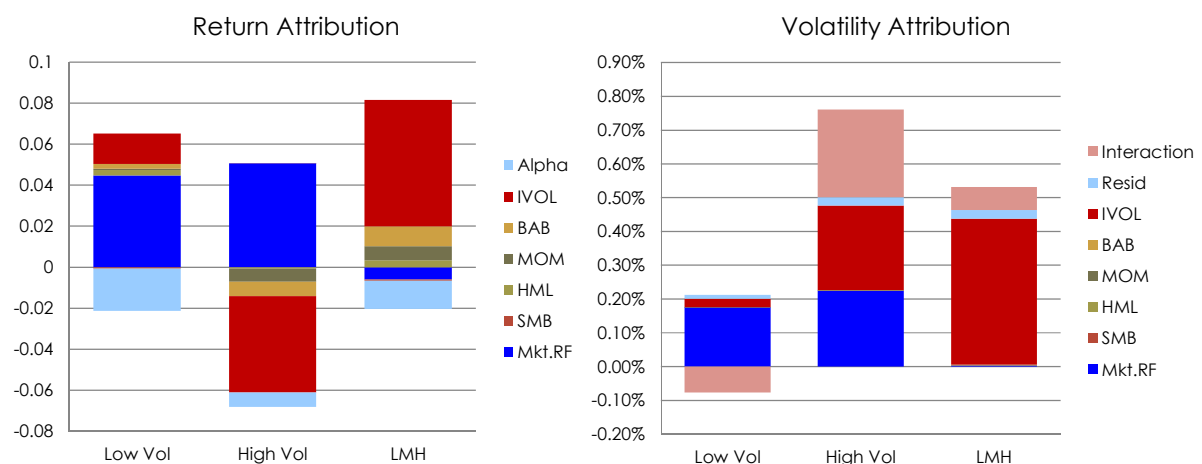
4.3. Performance and risk attribution

Based on the factor exposures of a portfolio it is possible to break down the contribution of different factors to the overall portfolio outcome in terms of return and risk. The analysis of factor exposures has a key advantage and a key drawback. The advantage is that return contributions and risk contribution are easily interpretable and provide an idea of the *economic significance* of the different effects. The drawback is that these contributions do not allow gaining insights into the *statistical significance* of the effects, as one loses the information on standard errors around point estimates that is provided in regression outputs. We thus see this analysis as a useful complement to the analysis of factor exposures provided in Tables 2, 3 and 4.

Figure 5 shows the contributions of different risk factors when we consider the factor exposures we have estimated using the full fledged six factor model including the market, value, size, momentum, BAB and IVOL factors. The left hand side of figure 5 shows the return attribution, which is based on the estimated factor exposures in conjunction with the estimated factor return (average annualized premium) for the respective factor. The right hand side of Figure 5 displays the contribution of each factor to portfolio volatility, based on estimated factor exposures and the estimated covariance across factors (see subsection 2.3 for a more detailed description).

Figure 5: Return and Risk Attribution for Portfolios based on volatility sorts.

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. Return contributions are expressed in terms of contribution to annualised portfolio returns, and are computed by multiplying factor exposures of the respective portfolio with the annualised returns of the corresponding factor. Volatility contributions are expressed in terms of contribution to monthly variance. See section 2.3 for a detailed description of factor models and attribution methods. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, from Kenneth French's website for the Carhart factors and from Andrea Frazzini's website for the BAB factor. We compute the IVOL factor as in tables 2, 3 and 4. Portfolios are formed once a month for the period January 1966 to December 2013.



The left hand side of Figure 5 shows the contributions of the different factors to annualised portfolio returns of the low volatility portfolio, the high volatility portfolio, and the return difference between the two, i.e. the low minus high volatility portfolio. The IVOL factor

contributes about 1.5% annual returns to the low volatility portfolio, while it contributes -4.7% to the annual return of the high volatility portfolio. In contrast, the BAB exposure only contributes 0.2% annual returns to the low volatility portfolio and -0.7% annual return to the high volatility portfolio. For the LMH portfolio, the low IV factor exposure contributes about 6 percent annual returns while the low beta factor contributes about 1 percent.

The right hand side of Figure 5 shows the contribution of the different factors to monthly variance of the three portfolios derived from volatility-sorts. Visual inspection of the plots representing the volatility attribution show that the low IV factor accounts for about one tenth of the volatility of the low volatility portfolio, for about one third of the volatility of the high volatility portfolio, and for about four fifths of the volatility of the LMH portfolio while the low beta factor has a negligible contribution to volatility for all three portfolios.

Overall, the conclusions derived from the analysis of factor exposures thus carry over to return and volatility attribution: BAB and IVOL contribute to the return and risk of portfolios that are based on sorting stocks by their volatility, but the low idiosyncratic risk factor makes a much larger contribution than the low systematic risk factor.

5. Robustness Checks

We conduct five different robustness checks of the main results presented above by varying the setup in different ways. For the sake of brevity, we provide a brief overview of the results of these robustness checks in this section, while providing detailed results in the appendix to this paper. The changes in setup have been described in more detail in subsection 2.4 above. Our general conclusion across the different parts of the analysis above is that volatility sorted portfolios are exposed both to an idiosyncratic volatility effect and a beta effect, though the idiosyncratic volatility effect clearly dominates both the portfolio composition and the risk factor exposures of portfolios based on volatility sorts.

In order to provide some perspective on the robustness of this conclusion in a brief overview, Table 5 shows the ratio between the IV effect and the low beta effect observed in different tests. For example, we report the ratio between the percentage weight of low IV stocks in low volatility portfolios, and the percentage weight of low beta stocks in the low volatility portfolio. While these percentages are time varying, for brevity we consider the average percentage weight observed over all monthly points of analysis from 1966 to 2013. If the low IV effect is stronger than the low beta effect in explaining the composition of low volatility portfolios, we would observe a larger percentage weight of low IV stocks than of low beta stocks, and the ratio would be greater than 1. Similarly, if the low beta effect dominated in the low volatility portfolio, we would expect the ratio would be less than 1.

Table 5 reports several key results such as the described ratio of weights, in order to assess the relative importance of IV effect compared to the beta effect. For all considered measures, if the resulting ratio exceeds one, our key conclusion of the dominance of the IV effect holds across different setups.

Table 5: Overview of Robustness Checks.

The table displays an extract of results from tables in the appendix. A key results ratio of one indicates that the low IV and low beta effect are of equal importance. A ratio greater than (less than) one indicates that the IV effect (beta effect) dominates.

Key result ratio	Change in setup				
	Weekly data	Equal weighted portfolios	Fama French IV	Deciles instead of quintiles	Beta adjusted IVOL
Avg. weight of low IV stocks in low vol portfolio / Avg. weight of low beta stocks in low vol portfolio	5.82	1.61	5.53	23.40	-
Avg. weight of high IV stocks in high vol portfolio / Avg. weight of high beta stocks in high vol portfolio	1.08	1.59	1.12	1.36	-
R-squared for regression of LMH returns on Mkt+IVOL / R-squared for regression of LMH returns on Mkt+BAB	1.99	1.67	2.03	1.89	1.92
R-squared for regression of LMH returns on Carhart+IVOL / R-squared for regression of LMH returns on Carhart+BAB	1.26	1.06	1.27	1.19	1.21
Returns to LMH attributable to IVOL factor / Returns to LMH attributable to BAB factor /	2.30	1.23	4.64	4.66	2.48
Volatility of LMH attributable to IVOL factor / Volatility of LMH attributable to BAB factor /	240	15	217	219	62

The results displayed in Table 5 show that the low IV effect is more pronounced than the low beta effect across the different setups we used to test the robustness of results in our base case. Detailed results can be obtained in the appendix.

6. Conclusions

Researchers studying portfolios based on volatility sorts have recently recognized that, despite the substantial amount of research on these strategies, and despite their wide use in practice, the precise link between such portfolios and the low beta effect on the one hand, and the low idiosyncratic risk effect on the other hand, is still unclear. In fact, there has not been any detailed empirical evidence on such links.

We provide such an assessment, establishing first that the low beta and low IV factors are two separate factors with distinct risk/return properties. When assessing the portfolio composition and the factor exposures of low volatility portfolios, we find that they are mainly driven by exposure to low IV effects rather than exposure to low beta effects. This result is consistent with the dominance of the idiosyncratic component in individual stock return volatilities, leading portfolio sorts based on volatility to make stock selection decisions mainly based on idiosyncratic risk.

We hope that our assessment of the return and risk drivers behind this commonly used stock ranking method provides useful information for a better understanding of such strategies. Moreover, the dominance of idiosyncratic volatility in portfolios based on volatility sorts implies that any explanation of the low volatility effect needs to be consistent with low IV as the main driver of returns.

We also observe an interesting asymmetry in exposures between low volatility and high volatility portfolios. In fact, while we find significant exposure to the low IV factor but negligible exposure to the low beta factor for low volatility stocks, we find significant exposures to both the low beta and low IV factor for high volatility stocks. This asymmetry may provide an explanation for a stylised fact observed in the literature on low volatility

portfolios, namely that the low volatility effect stems mainly from the poor returns of high volatility stocks rather than high returns of low volatility stocks.

There are several possible extensions of our analysis in future research. For example, it may be of interest to assess the time variation or state dependency of low IV and low beta exposures of low volatility portfolios. Indeed, though this is not a focus of our analysis, our results in section 3 point to considerable time series variability of the relationship between volatility and beta or idiosyncratic risk. Another interesting question left for further research is that of the identification of the underlying economic risk factors or state variables for BAB and IVOL factors which may shed light on the economic mechanisms at work behind these risk-based factors.

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Appendix

Below, we report detailed results from the robustness checks. These results were used to extract the summary provided in Table 5 in Section 5 above. Each of the subsections below provides the results for a given change in setup.

A.1. Portfolios calibrated using weekly data

Instead of using five years of monthly data to compute total volatility, IV and beta for sorting portfolios, we now use a calibration period of three year and use returns data with weekly frequency. This is similar to the analysis in Blitz and van Vliet (2007) for example. Overall, the results presented below suggest that the change in calibration period and frequency does not change our conclusion on dominance of the IV component within volatility sorted portfolios.

Figure A.1: Composition of low volatility portfolio by IV (respectively beta) categories (weekly data)

The graph displays the weight of each category in the low volatility quintile portfolio. All risk measures are estimated from weekly return observations over the prior three years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French’s website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

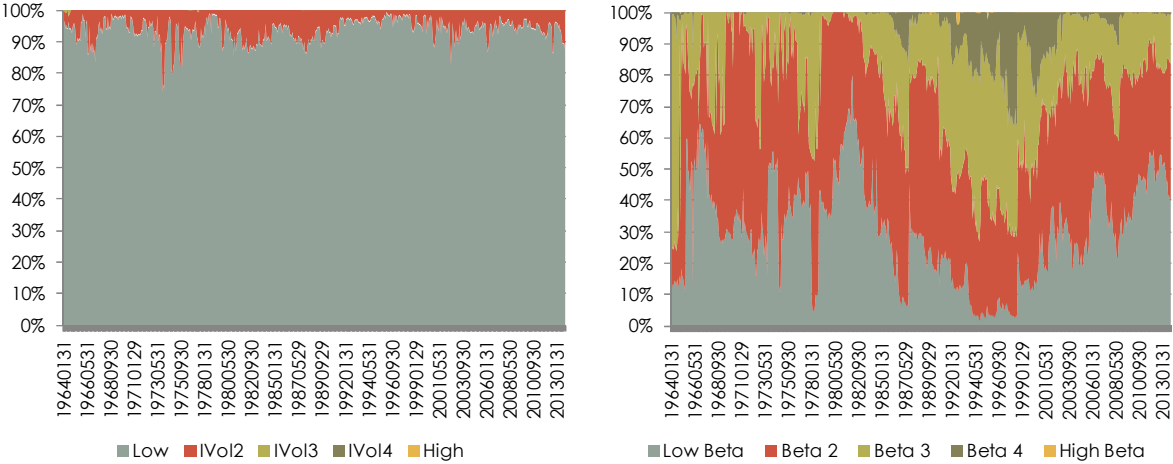


Figure A.2: Composition of high volatility portfolio by IV (respectively beta) categories (weekly data).

The graph displays the weight of each category in the high volatility quintile portfolio. All risk measures are estimated from weekly return observations over the prior three years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

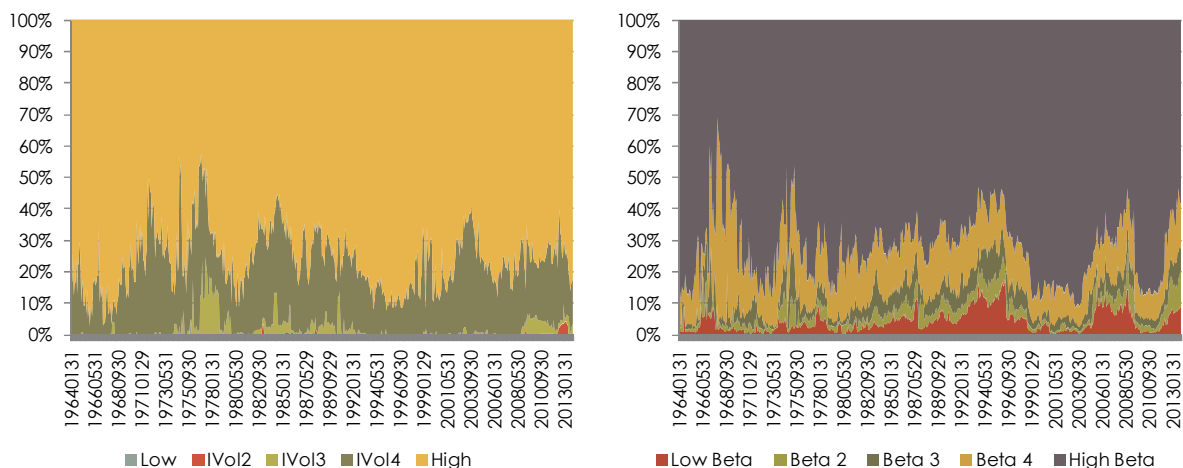


Table A.1: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio (weekly data).

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) low volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01
t-statistic	0.96	-1.33	-1.54	-2.41	0.18	-0.87	-1.80	-2.21
BAB		0.20		0.05		0.14		0.05
t-statistic		3.59*		1.84		5.16*		2.20
CAPM IVOL			0.23	0.22			0.21	0.20
t-statistic			17.88*	19.66*			13.19*	11.37*
Mkt	0.71	0.72	0.91	0.91	0.81	0.80	0.91	0.91
t-statistic	20.33*	29.52*	52.34*	53.02*	37.86*	45.39*	53.46*	53.52*
SMB					-0.29	-0.29	-0.03	-0.04
t-statistic					-10.01*	-11.45*	-0.90	-1.30
HML					0.19	0.12	0.04	0.03
t-statistic					3.86*	3.04*	1.32	0.82
MOM					0.03	0.00	0.03	0.02
t-statistic					0.82	0.09	0.99	0.77
R2	0.78	0.81	0.91	0.91	0.86	0.87	0.91	0.91

Table A.2: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio (weekly data).

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) high volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	-0.07	-0.01	-0.01	0.00	-0.05	-0.02	0.00	0.00
t-statistic	-2.44	-0.34	-1.35	-0.28	-2.59*	-1.30	-0.13	0.22
BAB		-0.62		-0.11		-0.45		-0.09
t-statistic		-5.70*		-3.79*		-6.26*		-3.25*
CAPM IVOL			-0.77	-0.75			-0.77	-0.74
t-statistic			-36.03*	-35.67*			-35.82*	-33.30*
Mkt	1.69	1.65	1.01	1.03	1.36	1.39	1.00	1.01
t-statistic	20.63*	33.93*	41.59*	44.22*	25.88*	32.24*	41.18*	42.49*
SMB					0.92	0.91	-0.02	0.01
t-statistic					13.39*	13.70*	-0.43	0.24
HML					-0.62	-0.39	-0.08	-0.05
t-statistic					-6.98*	-4.68*	-1.87	-1.21
MOM					-0.08	0.00	-0.07	-0.06
t-statistic					-1.16	0.04	-2.33	-1.83
R2	0.72	0.77	0.97	0.97	0.86	0.89	0.97	0.97

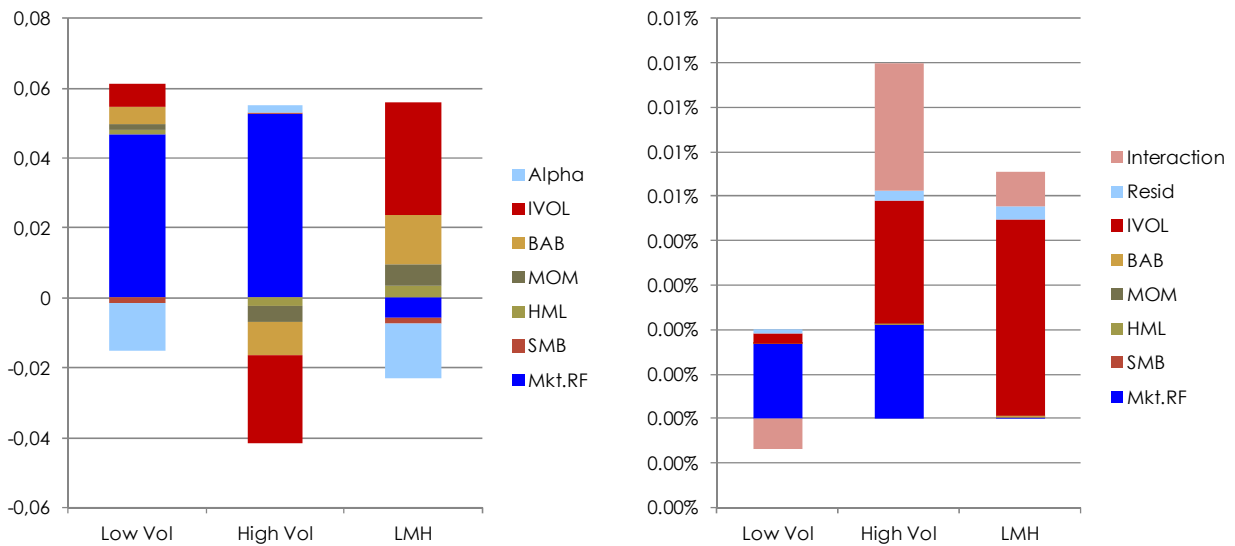
Table A.3: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio. (weekly data)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the return difference between the (cap-weighted) low and high volatility quintile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.08	0.00	0.00	-0.01	0.05	0.02	-0.01	-0.02
t-statistic	2.19	-0.05	0.32	-1.03	2.13	0.75	-1.13	-1.59
BAB		0.82		0.16		0.59		0.13
t-statistic		5.14*		6.21*		6.50*		4.93*
CAPM IVOL			1.00	0.96			0.98	0.94
t-statistic			51.95*	54.22*			36.13*	34.19*
Mkt	-0.98	-0.93	-0.10	-0.12	-0.55	-0.59	-0.08	-0.11
t-statistic	-8.64*	-13.43*	-2.95*	-4.00*	-7.93*	-10.75*	-2.74*	-3.65*
SMB					-1.20	-1.20	-0.01	-0.05
t-statistic					-13.85*	-14.97*	-0.10	-0.87
HML					0.81	0.50	0.12	0.08
t-statistic					6.61*	4.91*	2.98*	1.92
MOM					0.11	0.00	0.10	0.07
t-statistic					1.18	0.00	3.22*	2.47
R2	0.35	0.47	0.94	0.95	0.70	0.75	0.95	0.95

Figure A.3: Return and Risk Attribution for Portfolios based on volatility sorts. (weekly data)

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. See section 2.3 for a detailed description of factor models and attribution methods. All risk measures and risk premiums are estimated from the weekly return observations over the prior three years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Portfolios are formed once a month for the period January 1966 to December 2013.



A.2. Equal-weighting stocks in volatility sorted portfolios

Instead of weighting stocks by their market cap within the quintile portfolio sorted by volatility, we hold stocks with equal weights and run the same analysis as in the base case otherwise. The results are reported below.

Figure A.4: Composition of low volatility portfolio by IV (respectively beta) categories (equal weighted portfolios)

The graph displays the weight of each category in the low volatility quintile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

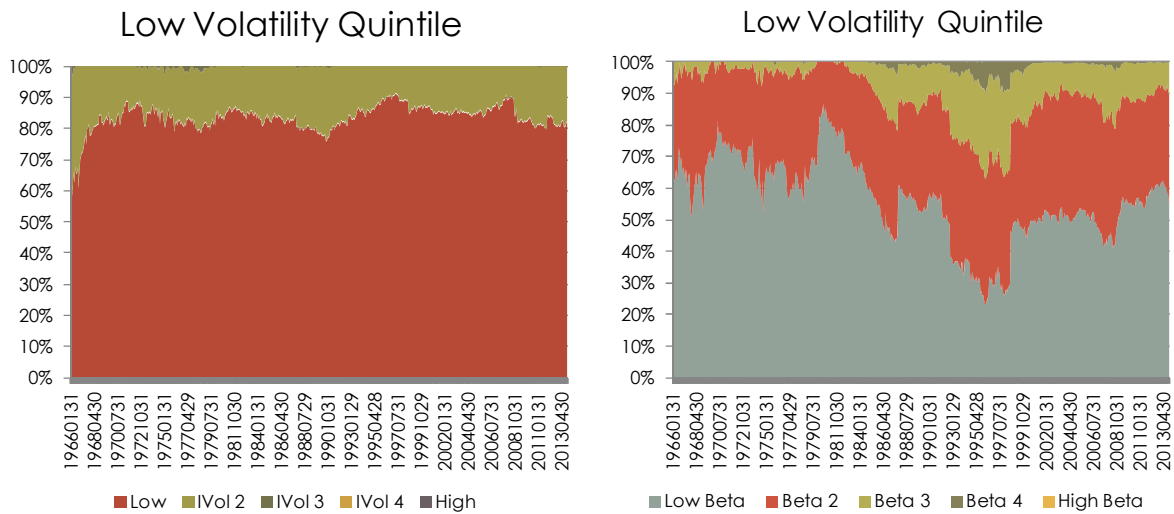


Figure A.5: Composition of high volatility portfolio by IV (respectively beta) categories (equal weighted portfolios).

The graph displays the weight of each category in the high volatility quintile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

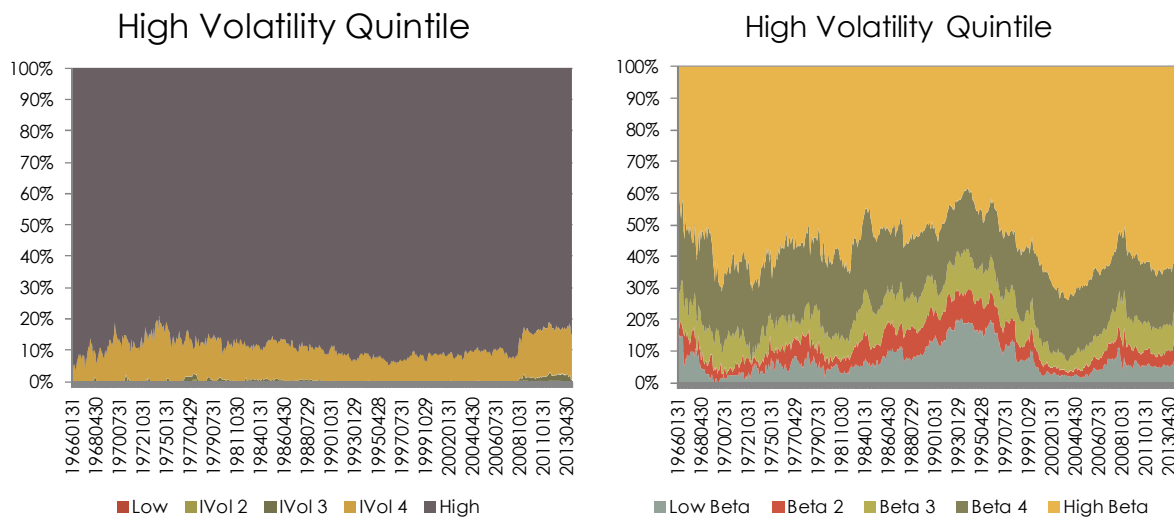


Table A.4: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio (equal weighted portfolios).

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (equal-weighted) low volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.04	0.00	0.02	0.00	0.01	0.00	0.00	-0.01
t-statistic	2.68*	0.09	1.91	-0.12	1.37	-0.14	-0.19	-0.97
BAB		0.32		0.28		0.24		0.18
t-statistic		6.84*		7.67*		9.96*		7.52*
CAPM IVOL			0.13	0.07			0.17	0.12
t-statistic			8.58*	4.12*			8.92*	5.90*
Mkt	0.58	0.60	0.69	0.65	0.64	0.62	0.71	0.68
t-statistic	18.38*	26.00*	23.52*	25.16*	27.62*	34.09*	34.87*	36.86*
SMB					0.09	0.09	0.30	0.24
t-statistic					2.90*	3.23*	8.14*	6.56*
HML					0.43	0.30	0.32	0.25
t-statistic					11.11*	8.83*	8.17*	7.26*
MOM					-0.03	-0.08	-0.04	-0.07
t-statistic					-1.16	-3.59*	-1.68	-3.47*
R2	0.63	0.74	0.69	0.75	0.77	0.82	0.81	0.83

Table A.5: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio (equal weighted portfolios)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (equal-weighted) high volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	-0.01	0.03	0.06	0.06	0.00	0.01	0.03	0.03
t-statistic	-0.37	1.07	4.16*	3.03*	-0.17	0.88	2.76*	2.98*
BAB		-0.38		0.09		-0.26		-0.08
t-statistic		-3.41*		1.28		-4.29*		-1.67
CAPM IVOL			-0.68	-0.70			-0.39	-0.37
t-statistic			-27.67*	-23.95*			-15.03*	-12.71*
Mkt	1.57	1.55	1.00	0.99	1.20	1.22	1.04	1.05
t-statistic	28.05*	31.29*	23.45*	19.63*	41.96*	40.02*	45.49*	40.05*
SMB					1.40	1.40	0.92	0.95
t-statistic					30.20*	32.00*	17.05*	17.07*
HML					-0.20	-0.07	0.05	0.08
t-statistic					-2.82*	-0.95	1.31	1.87
MOM					-0.23	-0.18	-0.22	-0.20
t-statistic					-4.14*	-4.23*	-5.89*	-5.81*
R2	0.67	0.69	0.88	0.88	0.92	0.92	0.95	0.95

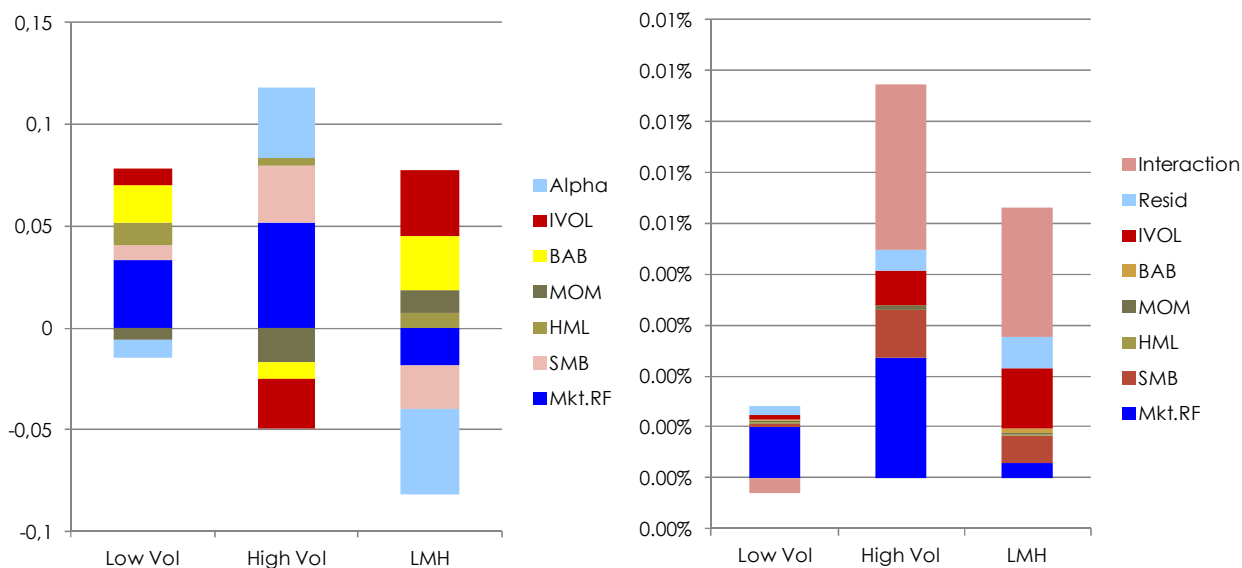
Table A.6: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio (equal weighted portfolios)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the return difference between the (equal-weighted) low and high volatility quintile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.05	-0.03	-0.04	-0.06	0.02	-0.01	-0.03	-0.04
t-statistic	1.35	-0.97	-2.55*	-4.08*	0.79	-0.80	-2.39	-3.55*
BAB		0.71		0.19		0.50		0.26
t-statistic		4.94*		3.36*		6.39*		5.00*
CAPM IVOL			0.82	0.78			0.56	0.49
t-statistic			34.41*	25.75*			16.59*	13.24*
Mkt	-0.99	-0.95	-0.31	-0.34	-0.57	-0.60	-0.32	-0.37
t-statistic	-11.66*	-18.89*	-8.64*	-7.69*	-12.06*	-15.74*	-10.67*	-11.20*
SMB					-1.31	-1.31	-0.63	-0.71
t-statistic					-19.48*	-22.33*	-9.01*	-10.79*
HML					0.64	0.37	0.27	0.17
t-statistic					6.33*	4.04*	4.69*	3.02*
MOM					0.19	0.10	0.18	0.13
t-statistic					2.48	1.75	4.01*	3.24*
R2	0.39	0.49	0.83	0.83	0.78	0.82	0.87	0.88

Figure A.6: Return and Risk Attribution for Portfolios based on volatility sorts (equal weighted portfolios)

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. See section 2.3 for a detailed description of factor models and attribution methods. All risk measures and risk premiums are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Portfolios are formed once a month for the period January 1966 to December 2013.



A.3. Idiosyncratic Volatility Estimated by Fama-French Model rather than CAPM

Instead of computing idiosyncratic volatility with respect to the market factor, we compute idiosyncratic volatility as the volatility of residual returns in a Fama and French three factor model. This is to maintain comparability with the literature on idiosyncratic volatility, see in particular Bali and Cakici (2008). We calculate IV with respect to the multifactor model both when analysing the cross sectional relationship between risk measures and when creating the IVOL factor in the returns-based analysis. As in the base case our calibration period is five years of monthly data, thus our IV remains a long term calibration rather than the short term IV computed over a one month window. See the discussion in the main part of the paper.

Figure A.7: Composition of low volatility portfolio by IV categories (IV with respect to multi-factor model)

The graph displays the weight of each category in the low volatility quintile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

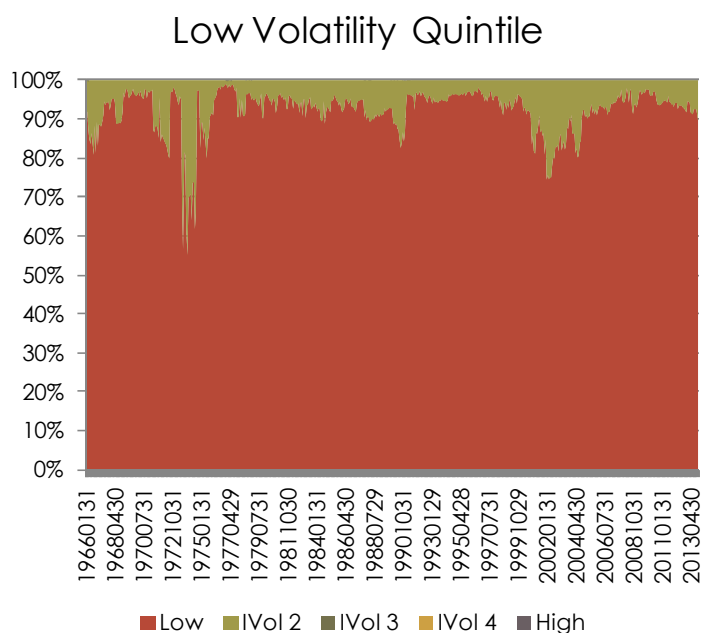


Figure A.8: Composition of high volatility portfolio by IV categories (IV with respect to multi-factor model)

The graph displays the weight of each category in the high volatility quintile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

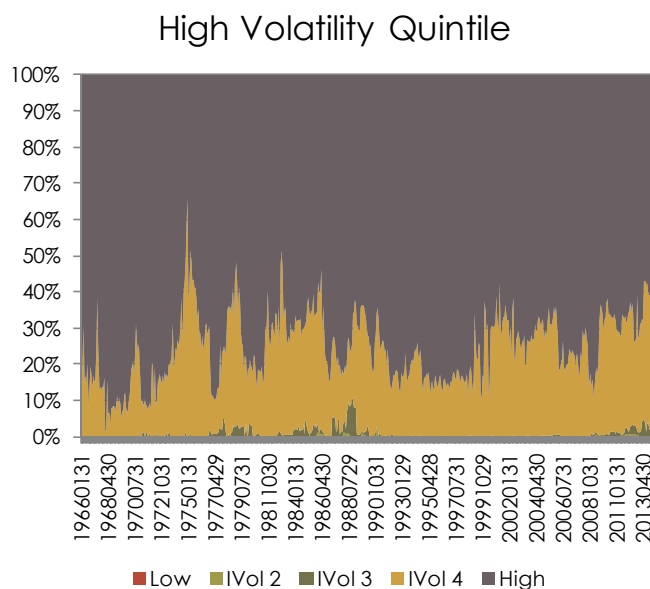


Table A.7: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio (IV with respect to multi-factor model)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) low volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.01	-0.01	-0.02	-0.02	0.00	-0.01	-0.02	-0.02
t-statistic	0.92	-1.38	-3.11*	-3.60*	0.06	-0.91	-2.61*	-2.95*
BAB		0.20		0.03		0.13		0.04
t-statistic		3.46*		1.31		4.71*		1.92
CAPM IVOL			0.25	0.24			0.22	0.21
t-statistic			16.11*	17.66*			13.01*	11.24*
Mkt	0.70	0.71	0.90	0.90	0.81	0.80	0.90	0.89
t-statistic	21.01*	32.56*	82.49*	85.60*	44.24*	50.65*	77.73*	83.88*
SMB					-0.30	-0.30	-0.05	-0.07
t-statistic					-10.27*	-11.27*	-2.00	-2.25
HML					0.22	0.15	0.03	0.02
t-statistic					4.96*	4.05*	0.99	0.70
MOM					0.02	-0.01	0.01	0.00
t-statistic					0.45	-0.35	0.25	-0.03
R2	0.76	0.79	0.90	0.90	0.85	0.86	0.90	0.90

Table A.8: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio (IV with respect to multi-factor model)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) high volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	-0.10	-0.04	-0.02	-0.01	-0.07	-0.05	-0.01	-0.01
t-statistic	-3.60*	-1.38	-1.65	-1.02	-3.99*	-2.68*	-1.44	-1.06
BAB		-0.56		-0.07		-0.41		-0.09
t-statistic		-5.17*		-2.21		-5.49*		-2.94*
CAPM IVOL			-0.73	-0.72			-0.75	-0.73
t-statistic			-36.66*	-34.85*			-30.67*	-29.24*
Mkt	1.63	1.60	1.03	1.04	1.33	1.36	1.02	1.03
t-statistic	21.69*	31.82*	34.76*	36.48*	26.96*	29.71*	38.28*	39.47*
SMB					0.87	0.87	0.04	0.07
t-statistic					14.39*	14.89*	0.77	1.35
HML					-0.52	-0.30	0.12	0.14
t-statistic					-6.39*	-3.36*	2.71*	3.33*
MOM					-0.11	-0.03	-0.08	-0.06
t-statistic					-1.89	-0.51	-2.51*	-1.80
R2	0.73	0.78	0.96	0.96	0.86	0.88	0.96	0.96

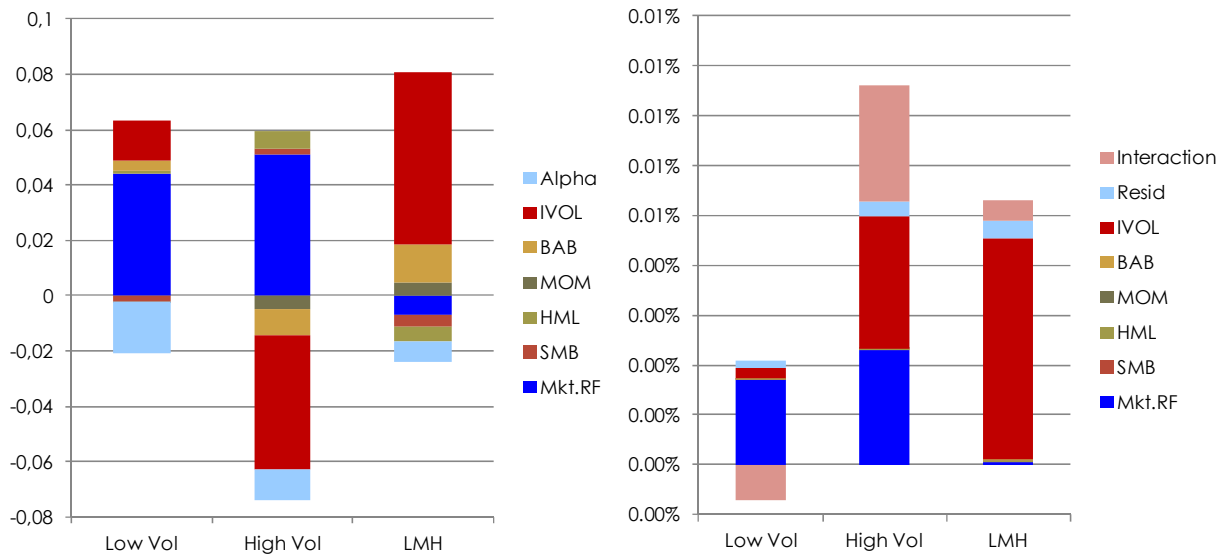
Table A.9: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio (IV with respect to multi-factor model)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the return difference between the (cap-weighted) low and high volatility quintile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.11	0.02	0.00	-0.01	0.07	0.04	0.00	-0.01
t-statistic	3.04*	0.73	-0.20	-0.85	3.23*	1.85	-0.24	-0.70
BAB		0.76		0.10		0.55		0.13
t-statistic		4.68*		3.04*		5.42*		3.63*
CAPM IVOL			0.98	0.96			0.98	0.94
t-statistic			63.66*	57.70*			41.35*	38.77*
Mkt	-0.93	-0.88	-0.12	-0.14	-0.52	-0.56	-0.12	-0.14
t-statistic	-8.71*	-13.09*	-4.32*	-4.86*	-8.28*	-10.08*	-4.11*	-5.26*
SMB					-1.16	-1.17	-0.09	-0.13
t-statistic					-14.58*	-15.77*	-2.02	-2.88*
HML					0.75	0.45	-0.08	-0.12
t-statistic					6.69*	4.03*	-2.29	-3.41*
MOM					0.12	0.02	0.08	0.06
t-statistic					1.58	0.24	2.81*	1.90
R2	0.34	0.46	0.93	0.93	0.68	0.73	0.93	0.93

Figure A.9: Return and Risk Attribution for Portfolios based on volatility sorts (IV with respect to multi-factor model)

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. See section 2.3 for a detailed description of factor models and attribution methods. All risk measures and risk premiums are estimated from the 24-60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Portfolios are formed once a month for the period January 1966 to December 2013.



A.4. Decile Portfolios instead of Quintile Portfolios

Below, we show the results when sorting stocks into deciles based on their volatility, rather than into quintiles in the base case. It should be noted that the IVOL factor is unchanged, i.e. based on the returns of the bottom quintile versus those of the top quintile portfolio. In this sense, the analysis is directly comparable to the base case. Only the convention for defining the “high” and “low” volatility sorted portfolios is changed from using the extreme quintiles to using the extreme deciles.

Figure A.10: Composition of low volatility portfolio by IV and beta categories (decile portfolios used for volatility sorts)

The graph displays the weight of each category in the low volatility decile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French’s website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

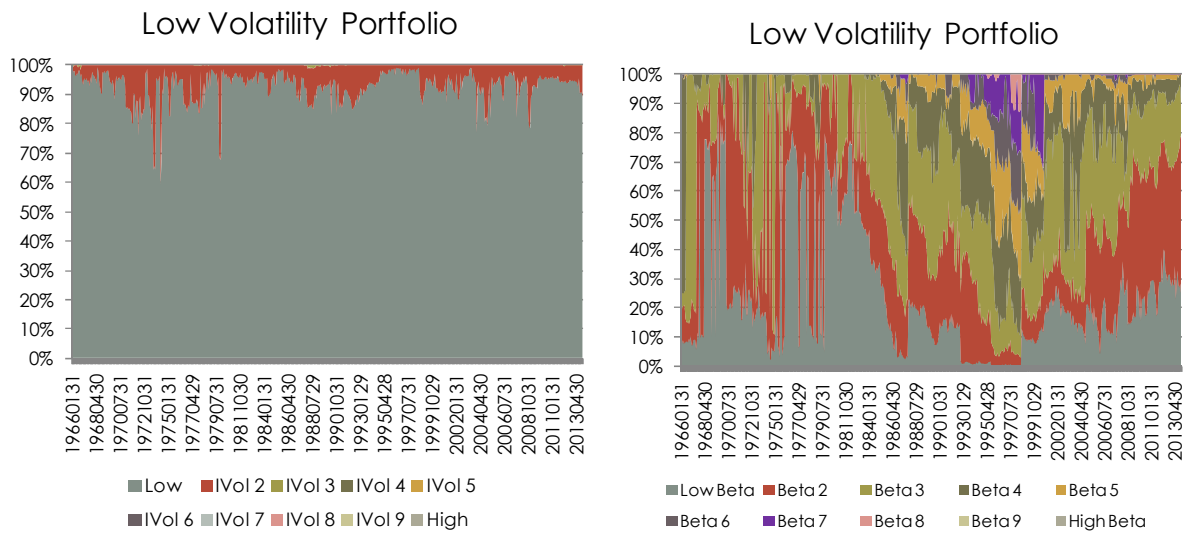


Figure A.11: Composition of high volatility portfolio by IV and beta categories (decile portfolios used for volatility sorts)

The graph displays the weight of each category in the high volatility decile portfolio. All risk measures are estimated from monthly return observations over the prior five years. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. Portfolios are formed once a month for the period January 1966 to December 2013.

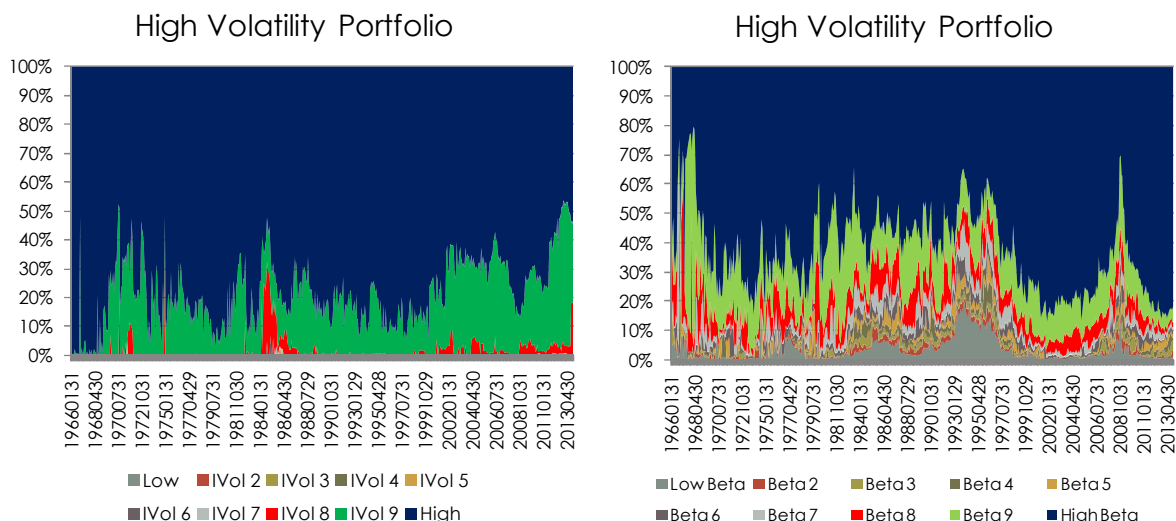


Table A.10: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio (decile portfolios used for volatility sorts)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) low volatility decile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.02	-0.01	-0.01	-0.01	0.01	0.00	-0.01	-0.01
t-statistic	1.66	-0.53	-0.68	-1.37	0.62	-0.22	-1.21	-1.42
BAB		0.23		0.08		0.14		0.05
t-statistic		4.37*		2.32		4.19*		1.60
CAPM IVOL			0.24	0.23			0.20	0.19
t-statistic			17.14*	15.02*			8.97*	7.42*
Mkt	0.60	0.61	0.80	0.79	0.72	0.71	0.81	0.80
t-statistic	15.50*	21.65*	42.60*	39.57*	28.51*	29.68*	39.53*	37.11*
SMB					-0.27	-0.27	-0.02	-0.04
t-statistic					-8.44*	-8.25*	-0.53	-0.93
HML					0.30	0.23	0.17	0.15
t-statistic					6.05*	5.02*	3.66*	3.52*
MOM					0.01	-0.02	0.01	0.00
t-statistic					0.30	-0.42	0.21	-0.08
R2	0.58	0.62	0.74	0.74	0.70	0.71	0.75	0.75

Table A.11: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio (decile portfolios used for volatility sorts)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) high volatility decile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	-0.11	-0.05	-0.02	-0.02	-0.09	-0.06	-0.03	-0.02
t-statistic	-3.42*	-1.51	-1.47	-1.14	-3.84*	-2.78*	-1.43	-1.26
BAB		-0.57		-0.04		-0.41		-0.07
t-statistic		-4.66*		-0.77		-4.98*		-1.51
CAPM IVOL			-0.80	-0.79			-0.72	-0.70
t-statistic			-22.57*	-24.19*			-13.23*	-12.59*
Mkt	1.74	1.71	1.07	1.08	1.39	1.42	1.07	1.09
t-statistic	20.69*	27.55*	32.37*	31.46*	25.61*	29.49*	31.43*	30.78*
SMB					1.12	1.13	0.24	0.27
t-statistic					14.49*	14.82*	2.35	2.52*
HML					-0.49	-0.26	-0.01	0.02
t-statistic					-4.64*	-2.66*	-0.13	0.27
MOM					-0.14	-0.06	-0.12	-0.11
t-statistic					-2.08	-1.11	-2.85*	-2.67*
R2	0.68	0.72	0.91	0.91	0.83	0.85	0.92	0.92

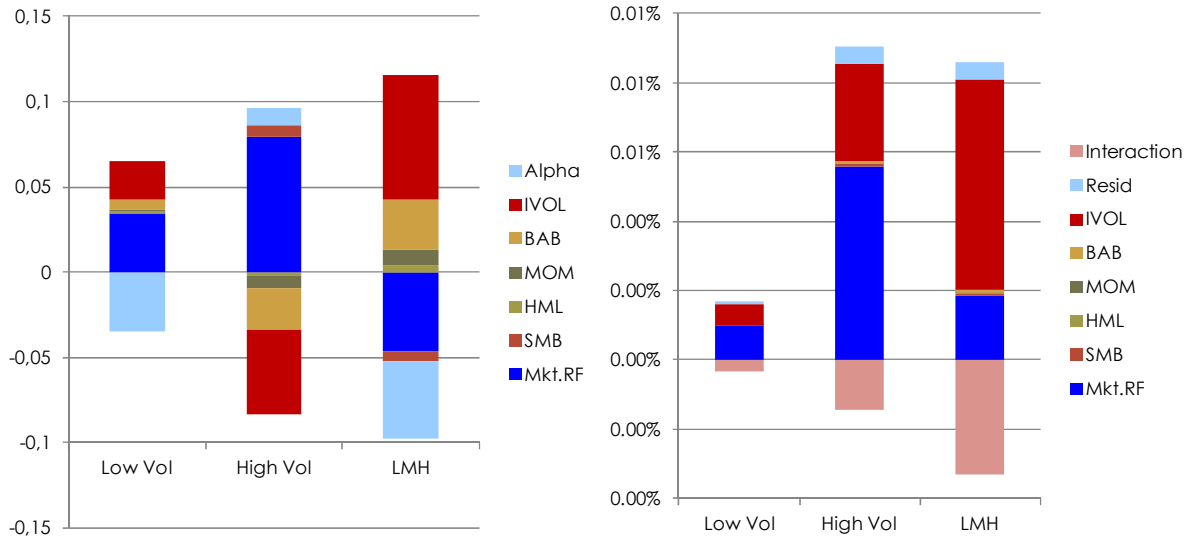
Table A.12: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio (decile portfolios used for volatility sorts)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the return difference between the (cap-weighted) low and high volatility decile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.13	0.04	0.02	0.01	0.10	0.06	0.01	0.01
t-statistic	3.21*	1.14	0.94	0.40	3.45*	2.32	0.75	0.50
BAB		0.80		0.12		0.56		0.13
t-statistic		4.97*		2.26		5.51*		2.26
CAPM IVOL			1.04	1.02			0.92	0.89
t-statistic			27.26*	26.59*			14.24*	12.94*
Mkt	-1.14	-1.09	-0.27	-0.29	-0.67	-0.71	-0.27	-0.29
t-statistic	-10.11*	-13.89*	-7.22*	-7.57*	-10.02*	-11.95*	-6.42*	-6.93*
SMB					-1.39	-1.40	-0.26	-0.30
t-statistic					-15.13*	-15.39*	-2.30	-2.58*
HML					0.79	0.49	0.18	0.13
t-statistic					6.20*	4.18*	2.33	1.85
MOM					0.15	0.04	0.13	0.11
t-statistic					1.75	0.60	2.62*	2.12
R2	0.36	0.45	0.86	0.86	0.69	0.73	0.87	0.87

Figure A.12: Return and Risk Attribution for Portfolios based on volatility sorts (decile portfolios used for volatility sorts)

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. See section 2.3 for a detailed description of factor models and attribution methods. All risk measures and risk premiums are estimated from the 60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Portfolios are formed once a month for the period January 1966 to December 2013.



A.5. Beta-neutral IVOL factor

Below, we show the results where we change the construction of the IVOL factor so as to make it beta neutral. We obtain beta neutrality (i.e. a beta of zero) by adjusting the investment into the low and high IV portfolios so as to have a beta of one both on the long and short side. This approach is similar to the construction of the BAB factor in Frazzini and Pedersen (2014). We run this analysis in order to see whether our results are driven by the fact that one of the two factors is beta neutral by construction (BAB) whereas the other is not (IVOL). The results below suggest that there is no palpable difference in the results when making the IVOL factor beta neutral compared to the base case from the main part of the paper.

Table A.13: Factor loadings and goodness of fit for factor models for returns of low volatility portfolio (beta-neutral IVOL factor)

The table displays regression coefficients and associated *t*-statistics, as well as the goodness of fit (*R*-squared) of regressions of excess returns of the (cap-weighted) low volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio versus the high quintile portfolio. *t*-stats marked with * correspond to coefficient estimates that are significant at the 5% level. *T*-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.01	-0.01	-0.03	-0.03	0.00	-0.01	-0.03	-0.03
t-statistic	0.92	-1.38	-4.94	-5.91	0.06	-0.91	-5.16	-5.53
BAB		0.20		0.07		0.13		0.05
t-statistic		3.46		3.47		4.71		3.31
CAPM IVOL			0.37	0.35			0.36	0.35
t-statistic			18.11	21.82			16.47	15.59
Mkt	0.70	0.71	0.67	0.67	0.81	0.80	0.68	0.68
t-statistic	21.01	32.56	33.19	41.90	44.24	50.65	41.51	43.79
SMB					-0.30	-0.30	0.01	0.00
t-statistic					-10.27	-11.27	0.53	0.06
HML					0.22	0.15	0.06	0.04
t-statistic					4.96	4.05	2.52	1.70
MOM					0.02	-0.01	0.03	0.02
t-statistic					0.45	-0.35	1.39	0.97
R2	0.76	0.79	0.93	0.93	0.85	0.86	0.93	0.93

Table A.14: Factor loadings and goodness of fit for factor models for returns of high volatility portfolio (beta-neutral IVOL factor)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the (cap-weighted) high volatility quintile portfolio on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	0.07	0.07	0.11	-0.10	-0.04	-0.01	0.01	-0.07
t-statistic	1.23	1.63	2.17	-3.60	-1.38	-0.45	1.19	-3.99
BAB	-0.73		-0.51		-0.56		-0.25	
t-statistic	-2.73		-2.10		-5.17		-4.00	
CAPM IVOL		-0.73	-0.61			-0.92	-0.86	
t-statistic		-4.87	-4.67			-24.52	-20.53	
Mkt				1.63	1.60	1.71	1.69	1.33
t-statistic				21.69	31.82	32.25	40.65	26.96
SMB								0.87
t-statistic								14.39
HML								-0.52
t-statistic								-6.39
MOM								-0.11
t-statistic								-1.89
R2	0.08	0.12	0.16	0.73	0.78	0.92	0.93	0.86

Table A.15: Factor loadings and goodness of fit for factor models for returns of low minus high volatility portfolio (beta-neutral IVOL factor)

The table displays regression coefficients and associated t-statistics, as well as the goodness of fit (R-squared) of regressions of excess returns of the return difference between the (cap-weighted) low and high volatility quintile portfolios on a set of factor returns. For monthly returns over the period January 1966 to December 2013. The Market factor is from Kenneth French's website. The BAB factor is from Andrea Frazzini's website. The IV factor is constructed with stock level data from CRSP, using the returns of the low quintile portfolio vs. the high quintile portfolio. t-stats marked with * correspond to coefficient estimates that are significant at the 5% level. T-stats reported are adjusted for heteroscedasticity and serial correlation using Newey-West method.

	CAPM + Risk Components				FF/Carhart + Risk Components			
	CAPM	+BAB	+IVOL	All	FF/ Carhart	+BAB	+IVOL	All
Alpha	3.04	0.73	-1.57	-3.70	3.23	1.85	-2.80	-3.87
t-statistic		0.76		0.31		0.55		0.29
BAB		4.68		4.49		5.42		4.58
t-statistic			1.28	1.21			1.17	1.10
CAPM IVOL			30.68	27.27			19.48	17.07
t-statistic	-0.93	-0.88	-1.04	-1.02	-0.52	-0.56	-0.94	-0.93
Mkt	-8.71	-13.09	-15.45	-20.11	-8.28	-10.08	-18.56	-22.18
t-statistic					-1.16	-1.17	-0.15	-0.22
SMB					-14.58	-15.77	-2.17	-2.95
t-statistic					0.75	0.45	0.22	0.09
HML					6.69	4.03	3.40	1.63
t-statistic					0.12	0.02	0.16	0.10
MOM					1.58	0.24	3.17	2.72
t-statistic	0.34	0.46	0.88	0.89	0.68	0.73	0.89	0.90
R2	3.04	0.73	-1.57	-3.70	3.23	1.85	-2.80	-3.87

Figure A.13: Return and Risk Attribution for Portfolios based on volatility sorts (beta-neutral IVOL factor)

The graph displays the contribution of common risk factors to returns and volatility of different volatility-sorted portfolios. See section 2.3 for a detailed description of factor models and attribution methods. All risk measures and risk premiums are estimated from the 60 prior monthly return observations. Data is from CRSP for all stocks listed on the NYSE/Amex/Nasdaq, and from Kenneth French's website for the Fama and French factors. The factor model corresponds to the Carhart model augmented with BAB and IVOL factors, as on the rightmost column of tables 2, 3, and 4. Portfolios are formed once a month for the period January 1966 to December 2013.

