Optimists, Pessimists, and the Equity Premium: The Role of Preferences and Market (In)Completeness

Nicole Branger* Patrick Konermann* Christian Schlag§

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We propose a long-run risk model with an optimistic and a pessimistic Epstein-Zin investor who disagree about the likelihood of jumps in expected consumption growth. We explicitly solve the model for the case of market incompleteness when expected growth risk cannot be insured. In our model the dependence of the equity premium on the relative wealth or consumption shares of the two investors is much less pronounced than in related models like Chen, Joslin, and Tran (2012). Furthermore, with the combination of recursive utility, investor heterogeneity, and substantial market incompleteness, our approach can explain empirical facts about the relation between varying degrees of disagreement and quantities like return volatilities and trading volume better than competing models. Furthermore, in terms of investor survival we find that for our parametrization the investor whose beliefs correspond to the *true* model tends to lose consumption share in the long run.

Keywords: Epstein-Zin utility, long-run risk, heterogeneous beliefs, market incompleteness, disagreement

JEL: D52, D53, E44, G11, G12

*Finance Center Muenster, University of Muenster, Universitaetsstr. 14-16, 48143 Muenster, Germany. E-mails: nicole.branger|patrick.konermann@wiwi.uni-muenster.de.

§Department of Finance, Goethe-University, 60323 Frankfurt am Main, Germany. E-mail: schlag@finance.uni-frankfurt.de.

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1 Introduction and Motivation

Ever since Mehra and Prescott (1985) showed that standard asset pricing models have a severe problem to generate a high enough equity premium, i.e., an expected excess return on the aggregate stock market close to the empirically observed value of around 6%, a vast literature has emerged and many new theoretical approaches to the explanation of the equity premium have been proposed. One prominent idea, originally put forward by Rietz (1988) and later taken up by Barro (2006, 2009), is that rare, but large negative jumps in the consumption process ('consumption disasters') make the stock market, i.e., the claim on aggregate consumption, very risky, so that a high premium is required to make the representative agent hold this asset in equilibrium.

As demonstrated in a recent paper by Chen, Joslin, and Tran (2012) (CJT hereafter) this mechanism can easily break down, however, once investors are heterogenous and thus trade with each other. A natural source of heterogeneity in such a model are the investors' beliefs about the probability of a disaster, and CJT show that already a small share of optimists (with low subjective disaster probability) in the economy is enough to provide insurance to pessimists (with high subjective disaster probability) and to reduce the equity premium substantially.

In our paper we also focus on the explanation of the equity premium in a model with heterogenous agents and disasters, but we take a closer look at the impact of both the preference specification and the structure of the asset market in the sense of whether all risks can be insured or the market is incomplete. With respect to preferences we replace constant relative risk aversion (CRRA) as used in CJT and Dieckmann (2011) (in the special case of log utility) by recursive preferences of the Epstein-Zin (EZ) type. In terms of the dynamics of the economy we propose, similar to Benzoni, Collin-Dufresne, and Goldstein (2011), long-run risk with downward jumps in the expected growth rate of aggregate consumption instead of consumption disasters. Analogously to CJT and Dieckmann (2011) the investors in our model are heterogenous with respect to their beliefs concerning the likelihood of a large negative jump-driven shock, but of course to the expected consumption growth rate instead of to aggregate consumption. Furthermore, we study both a complete and an incomplete market. On an incomplete market investors will not be able to insure the jump and diffusive risk generated by the stochastic variation in the state variable expected consumption growth.

In the analysis of our model we arrive at conclusions which are along several dimensions very different from those found in related papers. As an example, Figure 1 provides a graphical representation of the equity premium in the different models. The lines show the equity premium as a function of the pessimistic investor's consumption share, so that the left (right) boundary of the abscissa corresponds to an economy populated by optimists (pessimists) only.

Reading the curve for the CJT model (the gray dotted line), from right to left, we find exactly the effect described above, namely that the equity premium drops dramatically already when the optimists represent only a small share of the economy, whereas it is relatively insensitive to the distribution of consumption among the two groups as long as the share of optimists is already somewhat larger. The figure also shows the results for the model proposed by Dieckmann (2011), both for the case of a complete and an incomplete market (represented by the gray solid and dashed line, respectively). He finds that the equity premium is much less convex in the pessimist's consumption share than in the CJT model and that it is higher in an economy with a complete market than when the investors cannot share disaster risk in aggregate consumption.

Our setup also produces a much flatter curve relating the equity premium to the pessimist's consumption share than CJT. However, the relation between the complete and the incomplete market case is exactly reversed compared to Dieckmann (2011) in that we obtain a higher equity premium when the market is incomplete. This seems more intuitive to us, since in the case of incompleteness certain risks cannot be insured so that it should be more costly for investors to bear the overall risk of the equity market.[1] Also the level of the premium is higher in our model and closer to the usually assumed value of around 5 to 6%.

As important as it is as a core equilibrium quantity, the equity premium represents only one aspect of a heterogenous investor model. Due to the very fact that investors are heterogenous there is a motive for trading, so that one can investigate the volume of trading in the economy and its relation to the degree of disagreement between investors. As documented by Karpoff (1987) there is a positive association between return volatility and trading volume, and our model reproduces this result. Furthermore, as found recently by Carlin, Longstaff, and Matoba (2013), there is a positive relation between the amount of disagreement in the economy and expected returns, return volatility, and trading volume in the data, and this is also what we find in our model. Models with jumps in consumption and CRRA preferences like CJT and Dieckmann (2011) are not able to simultaneously

[1] Of course, in the boundary cases of a homogenous all-optimist or all-pessimist economy incompleteness no longer matters for the equity premium, and it is the same for the complete and the incomplete market.

generate all these results.

The key theoretical result in the CJT model is very striking, but at the same time it is not clear how strongly it depends on the assumptions concerning preferences and market completeness, and this is why we focus on exactly these two elements of the model.

To motivate our setup with jumps in the expected growth rate of consumption instead of in consumption itself, we take a look at the annual time series of US consumption growth rates from 1930 to 2008 taken from Beeler and Campbell (2012). The largest negative rate of consumption growth over this time span was -7.7% in 1932 during the Great Depression. This makes the jump sizes usually assumed in the disaster literature, e.g., -40% in CJT, look rather extreme, but one of course has to keep in mind that Barro (2006, 2009) looks at a cross-section of countries and not just the U.S. Applying standard filtering theory as presented in Liptser and Shiryaev (2001) to compute an estimate for the (unobservable) conditionally expected consumption growth rate (without jumps) shows that in this model the realized consumption growth of -7.7% in 1932 would be associated with a change in the conditionally expected growth rate of -3 percentage points, which seems very realistic.[2]

Introducing state variables like an expected growth rate is most powerful in models where the risks generated by these state variables are actually priced. In CRRA models risk generated by state variables is not priced, and these approaches also inherently suffer from implausible reactions of prices to changes in state variables. For example, an increase in expected consumption growth, which represents good news for the investor, would nevertheless lead to a decrease in prices. EZ preferences with standard parameter combinations avoid these problems and thus seem more appropriate to describe equilibrium prices and their dynamics.

In the CJT model the investors' ability to share disaster risk is crucial in undermining the role of disasters as an explanation of the equity premium. In other words, it matters a lot whether the market is complete, so that the investors can implement these risk-sharing trades, or not. CJT also briefly discuss (but do not explicitly compute the solution for) the case of an incomplete market. They conjecture that the outcome of the model would not change very much, if only one asset, e.g., the stock was traded. Given

[2]In a simple Gordon growth model, assuming an interest rate of 5%, a *permanent* change in the dividend growth rate from, e.g., 3% down to 0% would imply a drop in the price-dividend ratio of the stock market from 50 down to 20, so the effects would be even more dramatic than those of a negative 40% shock to dividends. Given that this value can be associated with the most negative shock to consumption over our sample, we will later on assume a jump size in the expected growth rate process of -3%.

the rather unequal contribution of jump and diffusive risk in their model it seems indeed plausible that, as long as consumption risk is mainly jump risk and the consumption claim can be traded, the equity premium will be very sensitive to the presence of only a small share of optimists willing to bear this large risk. However, when diffusion risk is also important not being able to trade both risk factors separately might matter much more.[3]

So, in summary, to provide a model which avoids the internal inconsistencies of CRRA, which features jumps in a state variable rather than in consumption itself, and which explicitly allows for market incompleteness, we suggest an economy with two EZ investors, who differ in their beliefs about the intensity of jumps in the expected growth rate of consumption. Incompleteness is introduced by making it impossible for the investors to share the risk generated by the stochastic variation in the expected growth rate of consumption.

We will now briefly describe the main findings of our analysis. In terms of the dependence of the equity premium on the share of optimists in the economy, we find a much flatter and a much more linear relationship than CJT. The expected excess return on the consumption claim in our model is around 3% in an all optimist economy and 3.9% in an all pessimist economy. Applying the usual leverage argument, i.e., considering dividends as levered by a factor of around 1.5 relative to consumption[4] this yields equity risk premia between 4.4% and 5.8%, i.e., values very close to the historical average observed in the data. This shows that disasters can contribute to the explanation of the equity premium, even if they are not modeled as extreme negative changes in consumption itself, but as jumps in state variables.

To assess the impact of market incompleteness we compare the results for complete and incomplete markets. Here we find that the equity risk premium on the incomplete market can be up to 5% higher. This not only represents a much smaller difference in absolute terms than the 30% reported by Dieckmann (2011) in a model with log utility, it also goes in the opposite direction, since in his model the equity premium is higher on the complete market. Second, in our model the return volatility on an incomplete market is higher, in contrast to the findings by Kübler and Schmedders (2012).

Overall we find that the effect of heterogeneity alone is limited, but market incompleteness turns out to be the key ingredient that is needed to reconcile model and

[3]Given the huge jump size assumed by CJT, incompleteness with respect to this source of risk would be a rather natural scenario, since, for example, index put options so far out of the money suffer from substantial illiquidity.

[4]According to Collin-Dufresne, Johannes, and Lochstoer (2013) this value is 'consistent with the average aggregate leverage ratio in the U.S. stock market'.

data. However, without heterogeneity incompleteness would not matter, since there would be zero trading volume in the first place. In that sense it is indeed the combination of heterogeneity and incompleteness which is driving the results.

From an asset pricing model with heterogenous investors one can also derive predictions concerning quantities like return volatility and trading volume and compare them to the data. In a recent empirical paper Carlin, Longstaff, and Matoba (2013) investigate the link between disagreement and asset prices in a model-free fashion, thereby providing very robust results. Their main findings suggest that higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume. Furthermore, in an earlier paper Karpoff (1987) finds that the correlation between return volatility and trading volume is positive.[5] Since models with heterogenous investors were developed, among other things, to explain trading between investors and its impact on prices, these stylized facts can serve as additional over-identifying restrictions, which such a model should satisfy.

From the set of models designed to deal with belief heterogeneity and market incompleteness in the context of the equity premium, ours (with an incomplete market) is the only one, which can qualitatively match all the above features from the data. In the CJT model and in the complete markets version of Dieckmann (2011) return volatility remains constant with varying disagreement, and the trading volume in the consumption claim is zero, irrespective of the amount of disagreement.[6] In the incomplete markets version of the model suggested by Dieckmann (2011) trading volume goes up with increasing disagreement, but neither the expected return on equity nor the return variance moves in the direction suggested by the data. Both of the models are furthermore unable to generate any correlation between return volatility and trading volume, whereas our model produces the empirically observed positive dependence between these two variables.

In summary, we conclude that in order to match not only the equity premium itself but also the link between disagreement, return volatility, and trading volume it seems necessary to include more general preferences than CRRA and to consider a material form of market incompleteness, resulting in a model in which the consumption share of optimists and pessimists is much less relevant for the equity premium than in the CJT case. In our model EZ preferences take care of the return volatility, incompleteness induces

[5] Wang (1994) suggests an explanation for this based on learning, for which Llorente, Michaely, Saar, and Wang (2002) provide empirical support.

[6] We vary the amount of disagreement via mean-preserving spreads, i.e., the average beliefs remain unchanged. We thus deviate from the original analysis in CJT and Dieckmann (2011), who only consider a variation in the pessimist's jump intensity, while leaving the optimist's unchanged.

the investors to trade the consumption claim with each other, and the combination of the two elements generates the correct sign for the correlation between trading volume and return volatility.

In a model with heterogeneous investors, long-run survival of both types of investors is usually an issue.[7] In a model with CRRA preferences, survival of both investors is a knife-edge case, and, with identical preferences, it is always the investor with the less biased beliefs who survives in the long run. Borovicka (2013) shows that, with EZ preferences, this is not necessarily true any longer, but that there can be many parameters combinations for which both investor groups or even only the investors with the worse bias survive in the long run. The latter is exactly what we find in our model. A Monte Carlo simulation shows that for our benchmark parametrization, it is the pessimistic investor who, by assumption, has correct beliefs, but vanishes in the long run. The pessimist is indeed right, but she is right only concerning very rare events, occurring on average once every 50 years. Therefore being right does not pay off for her.

The remainder of this paper is organized as follows. In Section 2, we introduce the model setup. Section 3 describes the model solution. In Section 4 we discuss the results of quantitative analysis of our model. Section 5 concludes. The technical details of the model solution as well as additional analyses are provided in the appendix.

2 Model Setup

We consider two investors with identical EZ preferences.[8] The individual value function of investor i ($i = 1, 2$) at time t is given as

$$J_{i,t} = E_{i,t}\left[\int_t^\infty f_i(C_{i,s}, J_{i,s})\, ds\right], \tag{1}$$

where $f_i(C_i, J_i)$ is her normalized aggregator function with

$$f_i\left(C_{i,t}, J_{i,t}\right) = \frac{\beta\, C_{i,t}^{1-\frac{1}{\psi}}}{\left(1-\frac{1}{\psi}\right)\left[(1-\gamma)\; J_{i,t}\right]^{\frac{1}{\theta}-1}} - \beta\, \theta\, J_{i,t}. \tag{2}$$

[7]See, e.g., Dumas, Kurshev, and Uppal (2009), Yan (2008), and Kogan, Ross, Wang, and Westerfield (2006, 2009).

[8]See e.g. Epstein and Zin (1989) for the discrete-time setup and Duffie and Epstein (1992) for the extension to continuous time.

β is the subjective discount factor, γ is the coefficient of relative risk aversion, ψ denotes the intertemporal elasticity of substitution (IES), and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. The well-known advantage of recursive utility is that it allows to disentangle the relative risk aversion and the IES, which in the CRRA case would be linked via $\gamma \equiv \psi^{-1}$, so that $\theta = 1$. In the following, we assume $\gamma > 1$ and $\psi > 1$, which implies $\gamma > \frac{1}{\psi}$ and $\theta < 0$, so that the investor has a preference for the early resolution of uncertainty.

Under the true probability measure $\mathbb{P}$ aggregate consumption C and the stochastic component of its expected growth rate X follow the system of stochastic differential equations

$$\begin{aligned}\frac{dC_t}{C_t} &= (\bar{\mu}_C + X_t)\; dt + \sigma_C' \, dW_t \\ dX_t &= -\kappa_X \, X_t \, dt + \sigma_X' \, dW_t + L_X \, dN_t\,(\lambda)\,,\end{aligned}$$

where $W = (W_c, W_x)'$ is a two-dimensional standard Brownian motion, and N represents a Poisson process with constant intensity λ and constant jump size $L_X < 0$. With the exception of the jump component this is the classical long-run risk setup from Bansal and Yaron (2004) written in continuous time. The volatility vectors are specified as $\sigma_C' = (\sigma_c, 0)$ and $\sigma_X' = (0, \sigma_x)$, so that consumption and the long-run growth rate are locally uncorrelated. The key feature of this model is that there are jumps representing disasters, which, however, do not occur in the consumption process itself, but in the state variable X.

The investors agree on all parameters of the model except the intensity of the Poisson process, i.e., roughly speaking they disagree about the likelihood of a disaster in the growth process over the next time interval. This implies that under the subjective probability measure $\mathbb{P}^i$ $(i = 1, 2)$ the stochastic growth rate evolves as

$$dX_t = -\kappa_X \, X_t \, dt + \sigma_X' \, dW_t + L_X \, dN_t\,(\lambda_i)\,.$$

Since the investors in our model do not learn about the unobservable intensity, they 'agree to disagree', i.e., they observe the same information flow, but interpret it differently. The 'agree to disagree' assumption is justified theoretically, e.g., in Acemoglu, Chernozhukov, and Yildiz (2007). They show that when investors are simultaneously uncertain about a random variable and the informativeness of an associated signal, even an infinite sequence of signals does not lead investors' heterogeneous prior beliefs about the random variable to converge.[9]

Since the issue of market completeness is central to our analysis, we have to fix the set of traded assets. When the market is complete, the investors can trade the claim on aggregate consumption, the money market account as well as two 'insurance products' linked to the Brownian motion and the jump component in X, respectively.[10] The consumption claim is in unit net supply, while the other three assets are in zero net supply. When we consider the incomplete market case, the insurance products will be no longer available.[11]

3 Equilibrium

All the equilibrium quantities in our model will be functions of the investors' consumption shares and the state variable X. Define w as investor 1's share in aggregate consumption, i.e., $w = \frac{C_1}{C}$.[12] Its dynamics can be written as a jump-diffusion process

$$dw = \mu_w(w, X)\, dt + \sigma_w(w, X)'\, dW + L_w(w, X)\, dN(\lambda_1), \tag{3}$$

where the coefficient functions $\mu_w(w, X)$, $\sigma_w(w, X)$, and $L_w(w, X)$ will be determined in equilibrium. The dynamics of investor 1's and investor 2's level of consumption then follow from Ito's lemma:

$$\begin{aligned} \frac{dC_1}{C_1} &= \left\{\bar{\mu}_C + X + \frac{1}{w}\,\mu_w + \frac{1}{w}\,\sigma_w'\,\sigma_C\right\} dt \\ &\quad + \left\{\sigma_C + \frac{1}{w}\,\sigma_w\right\}' dW + \left\{\frac{1}{w}\,L_w\right\} dN(\lambda_1) \\ &\equiv \mu_{C_1}\, dt + \sigma_{C_1}'\, dW + L_{C_1}\, dN(\lambda_1) \end{aligned} \tag{4}$$

[9]The reason is that investors have to update beliefs about two sources of uncertainty using one sequence of signals.

[10]The insurance products are described by their cash-flows. We assume that the first cash-flow has some (given) exposure to diffusion risk in X and no exposure to jumps, while the second cash-flow has no exposure to diffusion risk and some (given) exposure to jumps. For more details, see Appendix A.

[11]One could of course also analyze the case of intermediate incompleteness, where only one of the insurance products is available to the investors. The results we achieve in this setup typically lie between the two special cases we analyze in Section 4.

[12]In what follows we suppress the time index to simplify notation.

$$
\begin{aligned}
\frac{dC_2}{C_2} &= \left\{\bar{\mu}_C + X - \frac{1}{1-w}\,\mu_w - \frac{1}{1-w}\,\sigma_w'\,\sigma_C\right\}\,dt \\
&\quad + \left\{\sigma_C - \frac{1}{1-w}\,\sigma_w\right\}'\,dW + \left\{-\frac{1}{1-w}\,L_w\right\}\,dN\left(\lambda_2\right) \\
&\equiv \mu_{C_2}\,dt + \sigma_{C_2}'\,dW + L_{C_2}\,dN\left(\lambda_2\right).
\end{aligned} \tag{5}
$$

A key element of the solution will be the investors' individual log wealth-consumption ratios $v_i \equiv v_i(w, X)$. From Equation (1) we get

$$
\mathbb{E}_{i,t}\left[dJ_i + f_i\left(C_i, J_i\right)\right] \;=\; 0. \tag{6}
$$

Motivated by Campbell, Chacko, Rodriguez, and Viceira (2004) and Benzoni, Collin-Dufresne, and Goldstein (2011), we employ the following guess for the individual value function J_i:

$$
J_i = \frac{C_i^{1-\gamma}}{1-\gamma}\,\beta^\theta\,e^{\theta\,v_i} \tag{7}
$$

where, as shown in these papers, v_i is indeed investor i's log wealth-consumption ratio at time t. An application of Ito's Lemma yields

$$
\begin{aligned}
dv_i &= \left\{\frac{\partial v_i}{\partial w}\,\mu_w + \frac{1}{2}\,\frac{\partial^2 v_i}{\partial w^2}\,\sigma_w'\,\sigma_w - \frac{\partial v_i}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 v_i}{\partial X^2}\,\sigma_X'\,\sigma_X\right. \\
&\quad \left. + \frac{\partial^2 v_i}{\partial w\,\partial X}\,\sigma_w'\,\sigma_X\right\}\,dt + \left\{\frac{\partial v_i}{\partial w}\,\sigma_w + \frac{\partial v_i}{\partial X}\,\sigma_X\right\}'\,dW \\
&\quad + \left\{v_i\left(w + L_w, X + L_X\right) - v_i\left(w, X\right)\right\}\,dN\left(\lambda_i\right) \\
&\equiv \mu_{v_i}\,dt + \sigma_{v_i}'\,dW + L_{v_i}\,dN\left(\lambda_i\right).
\end{aligned} \tag{8}
$$

Plugging the guess in (7) into Equation (1) gives the following partial differential equation (PDE) for $v_i \equiv v_i\left(w, X\right)$ $(i = 1, 2)$:

$$
\begin{aligned}
0 &= e^{-v_i} - \beta + \left(1 - \frac{1}{\psi}\right)\,\left[\mu_{C_i} - \frac{1}{2}\,\gamma\,\sigma_{C_i}'\,\sigma_{C_i}\right] + \mu_{v_i} + \frac{1}{2}\,\theta\,\sigma_{v_i}'\,\sigma_{v_i} \\
&\quad + \left(1-\gamma\right)\,\sigma_{C_i}'\,\sigma_{v_i} + \frac{1}{\theta}\,\left[\left(1 + L_{C_i}\right)^{1-\gamma}\,e^{\theta\,L_{v_i}} - 1\right]\,\lambda_i.
\end{aligned} \tag{9}
$$

Following Duffie and Skiadas (1994), the pricing kernel ξ_i of investor i at time t is given as

$$
\xi_i = e^{-\beta\,\theta\,t - (1-\theta)\int_0^t e^{-v_{i,s}}\,ds}\,e^{(\theta-1)\,v_i}\,C_i^{-\gamma}\,\beta^\theta
$$

with dynamics

$$\begin{aligned}\frac{d\xi_i}{\xi_i} &= -\left\{\beta + \frac{1}{\psi}\,\mu_{C_i} - \frac{1}{2}\left(1+\frac{1}{\psi}\right)\gamma\,\sigma'_{C_i}\,\sigma_{C_i} - \frac{1}{2}\,(1-\theta)\,\sigma'_{v_i}\,\sigma_{v_i}\right. \\ &\qquad \left. -\,(1-\theta)\,\sigma'_{C_i}\,\sigma_{v_i} + \left(1-\frac{1}{\theta}\right)\left[(1+L_{C_i})^{1-\gamma}\,e^{\theta\,L_{v_i}}-1\right]\lambda_i\right\} dt \\ &\qquad -\left\{\gamma\,\sigma_{C_i} + (1-\theta)\,\sigma_{v_i}\right\}' dW + \left\{(1+L_{C_i})^{-\gamma}\,e^{(\theta-1)\,L_{v_i}}-1\right\} dN(\lambda_i)\,. \end{aligned} \tag{10}$$

From this we can obtain the investor-specific market prices of risk as the exposures of the pricing kernel to the different risk factors. The individual market prices of diffusion risks are given as

$$\eta_i^W = \gamma\,\sigma_{C_i} + (1-\theta)\,\sigma_{v_i}. \tag{11}$$

The first term represents the standard market price of (individual) consumption risk, which would also result in a CRRA economy, while the second term gives the extra market prices of risk for the diffusive volatility of the (log) wealth-consumption ratio and thus for the state variables w and X (assuming $\theta \neq 1$). Analogously the individual market prices of jump risk η_i^N are given by

$$\eta_i^N = (1+L_{C_i})^{-\gamma}\,e^{(\theta-1)\,L_{v_i}} - 1, \tag{12}$$

where the first term on the right-hand side is the product of the market price of consumption jump risk with CRRA utility and (assuming again $\theta \neq 1$) an adjustment for jump risk in the state variables.

Finally, the subjective risk-free rate r_i^f equals the negative drift of the pricing kernel, i.e.,

$$\begin{aligned} r_i^f &= \beta + \frac{1}{\psi}\,\mu_{C_i} - \frac{1}{2}\left(1+\frac{1}{\psi}\right)\gamma\,\sigma'_{C_i}\,\sigma_{C_i} - \frac{1}{2}\,(1-\theta)\,\sigma'_{v_i}\,\sigma_{v_i} - (1-\theta)\,\sigma'_{C_i}\,\sigma_{v_i} \\ &\qquad - \left[\eta_i^N - \left(1-\frac{1}{\theta}\right)\left[(1+L_{C_i})^{1-\gamma}\,e^{\theta\,L_{v_i}}-1\right]\right]\lambda_i. \end{aligned} \tag{13}$$

The terms have the usual interpretation of representing the impact of impatience, the individual consumption growth rate, precautionary savings due to uncertainty about individual consumption, about the evolution of the log wealth-consumption ratio and its covariance with individual consumption and, finally, precautionary savings due to jump risk (again composed of the CRRA result $\eta_i^N\,\lambda_i$ and an adjustment term due to EZ preferences).

The equilibrium is characterized by the fact that markets for the traded assets have to clear and that the investors agree on their prices. The exact procedures to compute the equilibria for the complete and the incomplete market economy are described in Appendix A.

4 Quantitative Analysis of the Model

4.1 Parameters

The parameters used in the quantitative analysis of the model are given in Table 1. They mostly represent standard values from the long-run risk literature.[13] If a disaster takes place the expected consumption growth drops by $L_X = -0.03$ which is about the same size as in Benzoni, Collin-Dufresne, and Goldstein (2011). With $\kappa_X = 0.1$, shocks have a half-life of about 6.9 years. The new element in the model is given by the agents' heterogenous beliefs with respect to the intensity of jumps in the stochastic component X of expected consumption growth. The pessimistic investor 1 assumes an intensity of $\lambda_1 = 0.020$, i.e., on average one X-jump every 50 years, while the optimist thinks there will be on average a jump only every 1,000 years, i.e., $\lambda_2 = 0.001$. In what follows we will assume that the pessimist's beliefs represent the true model.

Equity is considered a claim on levered consumption with a leverage factor of $\phi = 1.5$.[14] The two insurance products are characterized by $\mu_Z = -0.1$, $\sigma_Z = 0.001$, $\mu_I = -0.1$, and $L_I = 0.01$. All the model results are shown for the stochastic part of the expected growth rate of consumption at its long run mean of -0.006.[15]

[13] Cf. amongst others Benzoni, Collin-Dufresne, and Goldstein (2011) or Bansal, Kiku, and Yaron (2012).

[14] According to Collin-Dufresne, Johannes, and Lochstoer (2013) this value is 'consistent with the average aggregate leverage ratio in the U.S. stock market'.

[15] The long-run mean is computed as the value of X, where the expected change in X is equal to zero. This value is given by $\frac{\lambda L_X}{\kappa_X}$, which is equal to -0.006.

4.2 Results

4.2.1 Equity Risk Premium

Figure 2 presents the equity premium as well as its sources for a complete and an incomplete market. From left to right one finds the parts of the equity premium due to diffusive consumption risk, diffusive growth rate risk, and growth rate jump risk, and finally the total equity premium.[16]

On the complete market (upper row of graphs) the part of the premium due to consumption risk is the same we would also obtain in a CRRA economy, and it is the same for both investors. The investors also agree on the market price of risk for diffusive shocks to the stochastic growth rate X, but the part of the equity premium due to this factor is nevertheless not exactly constant across w. The reason for the very small variation in w is that the exposure of the return on individual wealth to W^X is higher for the pessimist.[17] Overall, this part of the equity premium is nevertheless sizable with a value of around 3.5%.

Since the two diffusive premia are basically independent of the consumption shares of the investors, the variation in the equity premium in w is almost exclusively caused by the jump part. The total equity premium increases in the pessimist's consumption share, which is intuitively clear, and ranges from 4.4% to 5.8%. As we had already seen in Figure 1 we thus find an overall much flatter and much less convex relationship between the equity premium and the share of optimists than CJT.

The incomplete market case is shown in the lower row of Figure 2. All the curves for aggregate wealth are pretty similar to the complete market case. However, the exposures of the investors' individual wealth to the risk factors are very different on an incomplete than on a complete market, as can be seen from comparing the lower rows of Figures 8 and 9. So it is the differences in the market prices of risk between the complete and the incomplete market (see Figure 10), which almost perfectly offset the differences in exposures. Overall, the total equity premium is now almost linear in w. Of course, the two boundary values for $w = 0$ and $w = 1$ coincide with the complete market case, so that

[16] Appendix C contains the results for all the basic equilibrium quantities, such as the wealth-consumption ratio, the dynamics of consumption shares, the risk-free rate, the market prices of risk, and the exposures of individual and aggregate wealth to the risk factors in the model. These results are not discussed here in detail, but delegated to Appendix C, since they only represent preliminary steps for the analysis of our model in this section.

[17] See Appendix C.4 and the third panel from the left in the lower row of Figure 8.

the range of the premium remains the same, and it is also still monotonically increasing in the pessimist's consumption share.

In our model the equity premium is up to 5% higher on the incomplete market. Our results here differ from the findings in Dieckmann (2011), who, in a setup with log utility and consumption jumps, reports an equity premium which can be up to 30% higher on a complete than on an incomplete market (see Figure 1). At the level of the equity premium alone one cannot say whether a model is more meaningful than the other, since in both models the derivations are fully consistent internally. This is why we look at other stylized facts, which can serve as over-identifying restrictions for a model.

4.2.2 Trading Volume and Return Volatility

Trading volume represents the change in asset holdings by the investors. So before we analyze trading volume in detail, we first take a look at the investors' portfolio holdings. The fractions of individual wealth invested in the different assets on a complete market are shown in the first row of Figure 3. As discussed above both agents invest 100% of their individual wealth into the consumption claim. To implement their desired exposures to the different sources of uncertainty they rely on the other three assets. The investors disagree on the amount of jump risk, and sharing this risk is the primary trading motive. The pessimist buys the jump insurance product I from the optimist, which gives him a positive cash-flow upon a jump. He offsets most of the resulting exposure to diffusive X-risk by selling the insurance product Z. Finally, he takes a long position in the money market account, which also reflects his overall lower position in risky assets.

To analyze trading volume in our model we basically follow (among others) Longstaff and Wang (2012) and Xiong and Yan (2010), who measure trading volume as the absolute volatility of an investor's asset holdings. However, since there are two sources of uncertainty (and not only one as in their framework), we adjust their measure slightly and define trading volume as the absolute volatility of the consumption share weighted average of both investors' holdings for asset j:[18]

$$TV_j = \left| \left(w \, \frac{\partial \pi_1^j}{\partial w} + (1-w) \, \frac{\partial \pi_2^j}{\partial w} \right) \sigma_w + \left(w \, \frac{\partial \pi_1^j}{\partial X} + (1-w) \, \frac{\partial \pi_2^j}{\partial X} \right) \sigma_X \right|, \tag{14}$$

where π_i^j is the fraction of investor i's wealth invested in asset j.

[18]To the best of our knowledge we are the first to deal with the problem of determining the trading volume in an economy with two diffusive shocks.

Looking at the complete market case first, the trading volume for each of the two insurance products shown in Figure 4 is inversely U-shaped in the pessimist's consumption share. Furthermore we had already seen that in this case both investors hold 100% of their respective individual wealth in the consumption claim, but they do not use this asset to trade with each other. Since the trading volume is zero, it has to be uncorrelated to the return volatility shown in the left graph in the upper row of Figure 5. This volatility is obtained by adding up the exposures of aggregate wealth to consumption risk and to shocks in the long-run growth rate and multiplying with the leverage factor ϕ. Since the consumption exposure is constant and the other one is (slightly) decreasing the return volatility is slightly decreasing as well.

When the insurance products are not available the pessimist reduces his jump exposure by selling the consumption claim to the optimist and invests the proceeds in the money market account (see the lower row in Figure 3). Consequently the trading volume in the consumption claim determined according to Equation (14) is positive and inversely U-shaped in w, as shown in the left graph in the lower row of Figure 5. The return volatility on the consumption claim is determined as before. It is above 8% and thus implies excess volatility, as on a complete market.[19] Like the trading volume, it is inversely U-shaped, as can be seen in the right graph in the upper row of Figure 5.

In the next step, we look at the relation between the trading volume in the consumption claim and its return volatility. As Karpoff (1987) points out, the positive correlation between these two quantities is one of the most robust patterns related to trading activity in the data. In the complete market, our model can not match this pattern. The constant positions in the consumption claim imply zero trading volume, so that return volatility has to be independent of the trading volume. On the incomplete market, however, the properties of trading volume and return volatility as functions of w imply a positive relationship between the two quantities as can be seen in the right graph in the lower row of Figure 5.

In the incomplete market case, our model is also in line with two further stylized facts if the pessimist has a smaller consumption share than the optimist ($w < 0.5$).[20] First, we find a positive relation between return volatility and expected stock returns which are both increasing in w. If stock prices drop, the volatility thus increases, which is the well-known the leverage effect first documented by Black (1976). Second, a larger trading

[19] The consumption volatility is around 3.5%. Note that excess volatility would still be present in an economy without leverage, i.e., $\phi = 1$.

[20] Figure 6 and Table 3 show that this condition is met in general.

volume is accompanied by larger absolute changes in prices and in return volatility. This result is consistent with many empirical studies, e.g. Gallant, Rossi, and Tauchen (1992).

Comparing our results for the complete and the incomplete market case, we find that market incompleteness induces a positive trading volume in the consumption claim. More importantly, return volatility on an incomplete market is higher than on a complete market, in contrast to the findings in Kübler and Schmedders (2012), who combine an OLG framework and two log utility investors with heterogeneous beliefs on the probability of exogenous i.i.d. shocks. Once again it becomes clear that combining log utility with incomplete markets leads to results that can be basically completely turned upside down when more sophisticated utility functions are introduced into the model.

Furthermore our results highlight the importance of market incompleteness to reproduce the empirically observed positive correlation between return volatility and trading volume. Of course, this incompleteness has to come together with investor heterogeneity, since otherwise there would be no trading.

As in the previous section we briefly compare our model to the two closest competitors, CJT and Dieckmann (2011). In CJT the return volatility is constant because investors do not share consumption risk but only jump risk due to market completeness. The latter fact also implies zero trading volume in the consumption claim. Therefore this model can reproduce neither the empirically observed positive correlation between return volatility and trading volume found by Karpoff (1987) nor the findings by Gallant, Rossi, and Tauchen (1992). Furthermore, since return volatility is not only constant but also coincides with consumption volatility, the model has an explanation neither for the leverage effect nor for excess volatility.

For Dieckmann (2011) we have to distinguish between completeness and incompleteness. Like in CJT the complete market version of his model generates a constant return volatility, again because investors share only jump risk, which in turn implies a zero trading volume in the consumption claim. Also in the incomplete market case the model has nothing to say about the relationship between volatility and trading volume, since also here return volatility is constant, so that it does not help that there is now a non-zero trading volume in the consumption claim. Since the expected return is constant due to log utility, the model also fails to reproduce the results of Gallant, Rossi, and Tauchen (1992), just like on the complete market. Furthermore the model is neither able to generate excess volatility nor the leverage effect, since the return volatility is constant for both market structures.

4.2.3 Varying Degree of Disagreement

An increase in disagreement means in our model that the difference between the subjective jump intensities becomes larger. The difference can increase when the beliefs of one or both investors become more extreme. In the following we will interpret an increase in disagreement as a mean preserving spread. In doing so we follow Carlin, Longstaff, and Matoba (2013) who investigate the link between disagreement and asset prices empirically in a model-free fashion and thus provide very robust results.[21] Their key findings relevant for our paper are that higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume.

To analyze the effect of varying disagreement we consider three scenarios, in all three of which the consumption share weighted average of the subjective intensities, however, remains constant at $\bar{\lambda} = 0.0200$ which coincides with the true model in our base case. In the first scenario the investors agree on the jump intensity, with $\lambda_1 = \lambda_2 = 0.0200$. Next we set $\lambda_1 = 0.0250$ and $\lambda_2 = 0.0150$, which represents minor disagreement, and finally take $\lambda_1 = 0.0300$ and $\lambda_2 = 0.0100$ as the case with the most pronounced disagreement between the investors. The other parameters of the model remain unchanged.

Table 2 presents the results of the analysis for the complete and the incomplete market case. On a complete market higher disagreement leads to a lower expected return (under the true belief) and a higher return volatility. The lower expected return is mainly caused by the smaller compensation for jump risk, which in turn is caused mainly by the increased risk sharing and thus the lower impact of the pessimist on the jump risk premium. When the degree of disagreement increases, the amount of diffusive growth rate risk shared between the investors increases, leading to a higher return volatility. However, the trading volume in the consumption claim still remains zero, since, no matter how much disagreement increases, both investors simply hold 100% of their wealth in the consumption claim. So, in summary, on a complete market only the reaction of the return volatility to increasing disagreement is in line with the empirical findings in Carlin, Longstaff, and Matoba (2013).

On the incomplete market higher disagreement leads to a higher expected return (under the true belief), a higher return volatility and to higher trading volume. The increase in the expected return is mainly due to the increased precautionary savings leading to a higher risk-free rate. When disagreement increases, the amount of shared

[21]Their disagreement index is based on the *standard deviation* of a normalized change in prepayment forecasts across dealers in the mortgage backed security market.

diffusion risk increases and this leads to a higher return volatility as well as to a higher trading volume. Overall, we find that our model with market incompleteness matches the facts from the data shown in Carlin, Longstaff, and Matoba (2013) very well. We conclude from this that market incompleteness together with heterogeneity (to generate a trading motive) are of first-order importance as the ingredients of an asset pricing model.

From their empirical findings described above Carlin, Longstaff, and Matoba (2013) draw the conclusion that there is a positive premium for disagreement. They do so, however, without explicitly showing that not only expected returns but also expected *excess* returns exhibit such a behavior, i.e., they implicitly assume that the impact on the interest rate would not over-compensate the effects on expected returns. In our general equilibrium model disagreement necessarily also has an effect on the risk-free rate, and, as can be seen from Table 2, higher disagreement indeed leads to a higher risk-free rate on both complete and incomplete markets. On the incomplete market this increase is actually larger than the increase in the expected return, thus leading to a lower overall risk premium. Without putting too much emphasis on this result, it may nevertheless be interpreted as an indication that the equilibrium effects on all relevant quantities, including interest rates, should be taken into account.

The papers closest to ours, CJT and Dieckmann (2011), represent varying degrees of disagreement via more extreme beliefs on the part of one investor while the other investor's beliefs do not change. However, for the purpose of confronting these models with the data from Carlin, Longstaff, and Matoba (2013) in the same way as our own model we vary disagreement by introducing a mean preserving spread.

Increasing disagreement leads to higher expected returns in the CJT model, which is compatible with the empirical findings. Due to market completeness, however, return volatility remains unchanged, and the trading volume is zero. In the Dieckmann (2011) model higher disagreement implies lower expected returns on a complete market. As in CJT return volatility remains unchanged and trading volume is zero. In the incomplete market case the model does a little better by matching the empirical results related to trading volume, but still fails in the other dimensions. In summary, our model with incomplete markets is the only one able to explain all three major empirical findings in Carlin, Longstaff, and Matoba (2013).

4.2.4 Investor Survival

Our economy is populated by investors with different and apodictic beliefs, i.e., they do not update their subjective estimate of the jump intensity given the realizations of consumption growth. This might cause a divergence from a heterogenous to a homogenous (one-)investor economy in the long run in the sense that only one of the two investors will have a non-negligible consumption share.

There is a rich literature dealing with natural selection in financial markets,[22] and it can be shown analytically in a model with CRRA investors and i.i.d. consumption growth, that the investor with the 'worse' model will lose all her consumption in the long run and disappear from the economy. 'Worse' here means that when otherwise identical investors disagree about one parameter in the model (mostly the expected growth rate of consumption), the one whose assumed value is further away from the true model will vanish in the long run. This is not necessarily true in models with EZ investors. As shown by Borovicka (2013), two investors with identical EZ preferences, who differ with respect to the expected growth rate of consumption, can both have non-zero expected consumption shares in the long run despite the fact that the beliefs of only one of them represent the true model.

In a sense our model represents a third case. This becomes clear from Table 3, which shows the results of a Monte Carlo simulation of the pessimist's consumption share over a period of 50, 100, 200, 500, and 1,000 years in our model and in the CJT model. The table shows that the consumption shares behave in the opposite way in the two models. While the pessimist's expected consumption share becomes smaller and smaller in our model, it increases over time in the model of CJT with CRRA preferences. This survival of the pessimist in our model is surprising, since her belief represents the true model, and so she should be the surviving agent in the long run. Figure 6 shows the density of the pessimist's consumption share on the complete and incomplete market. It confirms that it is indeed the pessimist who vanishes in the long run. The speed of extinction is larger in the incomplete than in the complete market. On the incomplete market, the pessimist earns the lower expected excess return on wealth (see Figure 2) and consumes more out of her wealth (see Figure 7). This also holds true when she is close to extinction, and by holding the less risky position and consuming more, she cannot escape extinction.[23] On

[22] Among others, see Dumas, Kurshev, and Uppal (2009), Yan (2008), and Kogan, Ross, Wang, and Westerfield (2006, 2009).

[23] See Borovicka (2013) for a detailed discussion of survival in case of EZ investors with heterogeneous beliefs.

the complete market, she still consumes more when she is small, but now earns a higher expected return on her wealth. However, the higher risk premium is not large enough to compensate the larger propensity to consume, and again, it is the pessimist who vanishes from the market in the long run. The pessimist is indeed right, but she is right only concerning very rare events, occurring on average once every 50 years. Therefore being right does not pay off for her.

5 Conclusion

Heterogenous beliefs are an important ingredient in state-of-the-art asset pricing models. While heterogeneity in most cases helps to explain the high equity risk premium that we observe empirically, Chen, Joslin, and Tran (2012) find that in disaster-risk models already a small share of optimists providing insurance to pessimists leads to a substantial decrease of the equilibrium expected excess return on equity. They rely on heterogenous beliefs about the intensity of large negative jumps in aggregate consumption, on CRRA preferences, and on market completeness.

In this paper we have suggested an alternative model featuring two heterogenous investors with recursive preferences who differ in their beliefs about the intensity of jumps not in the level of consumption itself, but in the long-run growth rate. Furthermore, we explicitly take market incompleteness into account when solving the model. To the best of our knowledge we are the first to combine all of these features in one model. In summary we find with respect to the quantities of interest that the effect of heterogeneity in isolation is limited, but the combination of incompleteness and heterogeneity is very powerful.

In terms of the dependence of the equity risk premium on the share of optimists, we find a rather flat and almost linear relationship . With market incompleteness the equity premium increases by up to 5% relative to the complete markets case, while it decreases by around 30% in the Dieckmann (2011) model with log utility, which seems to a certain degree counterintuitive. In our model the incomplete market also features the higher equity return volatility, in contrast to the findings by Kübler and Schmedders (2012), who also assume log utility.

We take the analysis one step further by considering not only moments of returns, but also trading volumes and the relationship between all these quantities and the degree of disagreement in the economy. We regard the empirical findings concerning these variables provided by Karpoff (1987) and Carlin, Longstaff, and Matoba (2013) as important over-identifying restrictions for a model like ours. These authors show that the correlation between return volatility and trading volume is positive and that higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume. It turns out that our model, in the incomplete markets version, is the only one from the set of competing approaches, which can qualitatively match these features of the data. In summary, we conclude that in order the match not only the equity premium itself but also the link between disagreement, return volatility, and trading volume is seems necessary to include more general preferences than CRRA and to consider a material form of market incompleteness.

Finally, in a model with two otherwise identical investors, who differ in their beliefs about a key parameter, survival becomes an important issue. In the usual diffusion-driven models under CRRA preferences it will always be the investor whose beliefs are closer to the true model (or even coincide with it) who survives. From the analysis in Borovicka (2013) we know that under EZ preferences both investors can survive in such a case. In our setup, the pessimist has beliefs, which coincide with the true model, but nevertheless she is the one who loses all her consumption share in the long run. The speed of extinction is faster on an incomplete than on a complete market.

The results of our analysis are also relevant for the interpretation of the work, e.g., by Backus, Chernov, and Martin (2011) and Julliard and Ghosh (2012) who determine implied disaster probabilities assuming CRRA preferences. The former authors explicitly say that changes in investor preferences or the introduction of heterogeneity could potentially also change their conclusions, and our results here show that this is indeed the case. In this sense our findings have much broader implications than just those related to the equity premium as a function of the general level of optimism in the economy.

References

Acemoglu, D., V. Chernozhukov, and M. Yildiz (2007): "Learning and Disagreement in an Uncertain World," Working Paper.

Backus, D., M. Chernov, and I. Martin (2011): "Disasters Implied by Equity Index Options," *Journal of Finance*, 66, 1969–2012.

Bansal, R., D. Kiku, and A. Yaron (2012): "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *Critical Finance Review*, 1, 183–221.

Bansal, R., and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59(4), 1481–1509.

Barro, R. (2006): "Rare Disasters And Asset Markets In The Twentieth Century," *Quarterly Journal of Economics*, 121(3), 823–866.

——— (2009): "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, 99(1), 243–264.

Beeler, J., and J. Y. Campbell (2012): "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1(1), 141–182.

Benzoni, L., P. Collin-Dufresne, and R. S. Goldstein (2011): "Explaining Asset Pricing Puzzles Associated with the 1987 Market Crash," *Journal of Financial Economics*, 101, 552–573.

Black, F. (1976): "Studies of Stock Price Volatility Changes," *Proceedings of the Business and Economics Section of the American Statistical Association*, pp. 177–181.

Borovicka, J. (2013): "Survival and long-run dynamics with heterogeneous beliefs under recursive preferences," Working Paper.

Campbell, J. Y., G. Chacko, J. Rodriguez, and L. M. Viceira (2004): "Strategic asset allocation in a continuous-time VAR model," *Journal of Economic Dynamics & Control*, 28, 2195–2214.

Carlin, B., F. Longstaff, and K. Matoba (2013): "Disagreement and Asset Prices," Working Paper.

Chen, H., S. Joslin, and N.-K. Tran (2012): "Rare Disasters and Risk Sharing," *Review of Financial Studies*, 25, 2189–2224.

Collin-Dufresne, P., M. Johannes, and L. Lochstoer (2013): "Parameter Learning in General Equilibrium: The Asset Pricing Implications," Working Paper.

Dieckmann, S. (2011): "Rare Event Risk and Heterogeneous Beliefs: The Case of Incomplete Markets," *Journal of Financial and Quantitative Analysis*, 46, 459–88.

Duffie, D., and L. G. Epstein (1992): "Stochastic Differential Utility," *Econometrica*, 60, 353–394.

Duffie, D., and C. Skiadas (1994): "Continuous-Time Security Pricing: A Utility Gradient Approach," *Journal of Mathematical Economics*, 23, 107–132.

Dumas, B., A. Kurshev, and R. Uppal (2009): "Equilibrium Portfolio Strategies in the Presence of Sentiment Risk and Excess Volatility," *Journal of Finance*, 64, 579–629.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework," *Econometrica*, 57, 937–969.

Gallant, A., P. Rossi, and G. Tauchen (1992): "Stock Prices and Volume," *Review of Financial Studies*, 5, 199–242.

Julliard, C., and A. Ghosh (2012): "Can Rare Events Explain the Equity Premium Puzzle?," *Review of Financial Studies*, 25(10), 3037–3076.

Karpoff, J. M. (1987): "The relation between price changes and trading volume: A survey," *Journal of Financial and Quantitative Analysis*, 22(1), 109–126.

Kogan, L., S. Ross, J. Wang, and M. Westerfield (2006): "The Price Impact and Survival of Irrational Traders," *Journal of Finance*, 61, 195–229.

——— (2009): "Market Selection," Working Paper.

Kübler, F., and K. Schmedders (2012): "Financial Innovation and Asset Price Volatility," *American Economic Review*, 103(3), 147–151.

Liptser, R., and A. Shiryaev (2001): *Statistics of Random Processes II. Applications*. Springer, Berlin.

Llorente, G., R. Michaely, G. Saar, and J. Wang (2002): "Dynamic Volume-Return Relation of Individual Stocks," *Review of Financial Studies*, 15(4), 1005–1047.

Longstaff, F. A., and J. Wang (2012): "Asset Pricing and the Credit Market," *Review of Financial Studies*, 25(11), 3169–3215.

Mehra, R., and E. Prescott (1985): "The equity premium: A puzzle," *Journal of Monetary Economics*, 15, 145–161.

Rietz, T. (1988): "The Equity Risk Premium. A Solution," *Journal of Monetary Economics*, 22(1), 117–131.

Wang, J. (1994): "A model of competitive stock trading volume," *Journal of Political Economy*, 102(1), 127–168.

Xiong, W., and H. Yan (2010): "Heterogeneous Expectations and Bond Markets," *Review of Financial Studies*, 23(4), 1433–1466.

Yan, H. (2008): "Natural Selection in Financial Markets: Does It Work?," *Management Science*, 54, 1935–1950.

A Solving for the Equilibrium

A.1 Complete Market

When the market is complete, the investors will in equilibrium agree on the risk-free rate, the market prices of diffusion risk and the risk-neutral jump intensity $\lambda_i^{\mathbb{Q}} \equiv \lambda_i^{\mathbb{P}} \left(1+\eta_i^N\right)$. We will use these restrictions to solve for the coefficients μ_w, σ_w, and L_w of the consumption share process (3).

First, σ_w is obtained by equating the investors' market prices of diffusion risk η_i^W, yielding

$$\sigma_w = \frac{w\,(1-w)\,(1-\theta)\,\left[\frac{\partial v_2}{\partial X} - \frac{\partial v_1}{\partial X}\right]}{\gamma + w\,(1-w)\,(1-\theta)\,\left[\frac{\partial v_1}{\partial w} - \frac{\partial v_2}{\partial w}\right]}\,\sigma_X. \tag{A.1}$$

The drift μ_w follows from the condition that the investors must agree on the risk-free rate, so that

$$\begin{aligned}
\mu_w \;=\; & -\sigma_w'\,\sigma_C + \psi\,w\,(1-w) \\
& \times \Bigg\{ \frac{1}{2}\left(1+\frac{1}{\psi}\right)\gamma\,\left[\sigma_{C_1}'\,\sigma_{C_1} - \sigma_{C_2}'\,\sigma_{C_2}\right] + \frac{1}{2}\,(1-\theta)\,\left[\sigma_{v_1}'\,\sigma_{v_1} - \sigma_{v_2}'\,\sigma_{v_2}\right] \\
& \quad + (1-\theta)\,\left[\sigma_{C_1}'\,\sigma_{v_1} - \sigma_{C_2}'\,\sigma_{v_2}\right] \\
& \quad + \left[\eta_1^N - \left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_1})^{1-\gamma}\,e^{\theta\,L_{v_1}} - 1\right]\right]\,\lambda_1 \\
& \quad - \left[\eta_2^N - \left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_2})^{1-\gamma}\,e^{\theta\,L_{v_2}} - 1\right]\right]\,\lambda_2 \Bigg\}.
\end{aligned} \tag{A.2}$$

Finally, the jump size L_w is found by using the condition that the investor-specific risk-neutral jump intensities $\lambda_i^{\mathbb{Q}}$ must be equal, implying

$$L_w = \frac{e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_1}-L_{v_2}\right)+\ln\frac{\lambda_1}{\lambda_2}\right]} - 1}{\frac{1}{w} + \frac{1}{1-w}\,e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_1}-L_{v_2}\right)+\ln\frac{\lambda_1}{\lambda_2}\right]}}. \tag{A.3}$$

The equilibrium solution is then found by simultaneously solving the two PDEs in (9) for v_1 and v_2 using the above equations for μ_w, σ_w and L_w.

Given these coefficients as well as the individual wealth-consumption ratios we can then compute other equilibrium quantities. For example, the total return (including consumption) on investor i's wealth follows the process

$$\begin{aligned}
\frac{dV_i}{V_i} + e^{-v_i}\,dt \;=\; & \left\{\left[\mu_{C_i} + \mu_{v_i} + \frac{1}{2}\,\sigma_{v_i}'\,\sigma_{v_i} + \sigma_{C_i}'\,\sigma_{v_i}\right] + e^{-v_i}\right\}\,dt + (\sigma_{C_i} + \sigma_{v_i})'\,dW \\
& + \left[(1+L_{C_i})\,e^{L_{v_i}} - 1\right]\,dN(\lambda_i) \\
\equiv\; & \left(\mu_{V_i} + e^{-v_i}\right)\,dt + \sigma_{V_i}'\,dW + L_{V_i}\,dN(\lambda_i),
\end{aligned} \tag{A.4}$$

where $V_i = C_i e^{v_i}$.

To find the agents' portfolio holdings one has to set the dynamics of individual wealth component by component equal to the dynamics of a portfolio containing the set of tradable assets, i.e., wealth changes and changes in the value of the portfolio have to have identical exposures to all the risk factors in the economy. In the complete market case the tradable assets are the claim on aggregate consumption, the money market account, and the insurance products linked to jump and diffusion risk in X, respectively.

We now specify the insurance products in more detail. To trade the diffusion risk in X the agents can use a claim labeled Z with cash flow dynamics

$$\frac{dZ}{Z} = \mu_Z \, dt + \sigma_Z' \, dW,$$

where μ_Z and $\sigma_Z' = (0, \sigma_z)$ are specified exogenously. Let ζ_i denote the log price-to-cash-flow ratio ζ_i of this asset from investor i's perspective. We write the dynamics of ζ_i as

$$d\zeta_i = \mu_{\zeta_i} \, dt + \sigma_{\zeta_i}' \, dW + L_{\zeta_i} \, dN\left(\lambda_i\right).$$

The coefficients as well as the PDE satisfied by ζ_i are presented in Appendix B. Since the investors agree on the price of the instrument, $\zeta_1 = \zeta_2 \equiv \zeta$.

In an analogous fashion, the payoff from the jump-linked instrument is denoted by I and evolves as

$$\frac{dI}{I} = \mu_I \, dt + L_I \, dN(\lambda_i),$$

where the coefficients μ_I and L_I are again given exogenously. The log price-to-cash-flow ratio ϖ_i follows the process

$$d\varpi_i = \mu_{\varpi_i} \, dt + \sigma_{\varpi_i}' \, dW + L_{\varpi_i} \, dN(\lambda_i).$$

As for Z, the coefficients and the PDE satisfied by ϖ_i are shown in Appendix B. Again, the investors must agree on the price of I, i.e., $\varpi_1 = \varpi_2 \equiv \varpi$.

Finally, the aggregate log wealth-consumption ratio $v \equiv \log\left(w \, e^{v_1} + (1-w) \, e^{v_2}\right)$ has dynamics

$$\begin{aligned} dv &= \left\{ \frac{\partial v}{\partial w} \mu_w + \frac{1}{2} \frac{\partial^2 v}{\partial w^2} \sigma_w' \, \sigma_w - \frac{\partial v}{\partial X} \kappa_X \, X + \frac{1}{2} \frac{\partial^2 v}{\partial X^2} \sigma_X' \, \sigma_X \right. \\ & \quad \left. + \frac{\partial^2 v}{\partial w \, \partial X} \sigma_w' \, \sigma_X \right\} dt + \left\{ \frac{\partial v}{\partial w} \sigma_w + \frac{\partial v}{\partial X} \sigma_X \right\}' dW \\ & \quad + \left\{ v\left(w + L_w, X + L_X\right) - v\left(w, X\right) \right\} \, dN\left(\lambda_i\right) \\ &\equiv \mu_v \, dt + \sigma_v' \, dW + L_v \, dN(\lambda_i). \end{aligned} \tag{A.5}$$

Investor i's total wealth V_i is equal to the value of his holdings (in units) $Q_{i,C}$, $Q_{i,M}$, $Q_{i,Z}$, and $Q_{i,I}$ in the consumption claim, the money market account, and the two insurance products with prices P^C, P^M, P^Z, and P^I, respectively. Let Π_i denote the value of this portfolio. With $\pi_{i,C}$, $\pi_{i,M}$ $\pi_{i,Z}$ and $\pi_{i,I}$ denoting the relative share of investor i's wealth invested in the four assets, the total return dR_i^{Π} on her portfolio can be represented as

$$\begin{aligned} dR_i^{\Pi} &= \pi_{i,C}\left(\frac{dP^C}{P^C}+e^{-v}dt\right)+\pi_{i,M}\,rdt+\pi_{i,Z}\left(\frac{dP^Z}{P^Z}+e^{-\zeta}dt\right)+\pi_{i,I}\left(\frac{dP^I}{P^I}+e^{-\varpi}dt\right)\\ &= \Bigg\{\pi_{i,C}\left(\bar{\mu}_C+X_t+\mu_v+\frac{1}{2}\,\sigma_v'\,\sigma_v+\sigma_C'\,\sigma_v+e^{-v}\right)+\pi_{i,M}\,r\\ &\quad+\pi_{i,Z}\left(\mu_Z+\mu_\zeta+\frac{1}{2}\,\sigma_\zeta'\,\sigma_\zeta+\sigma_Z'\,\sigma_\zeta+e^{-\zeta}\right)\\ &\quad+\pi_{i,I}\left(\mu_I+\mu_\varpi+\frac{1}{2}\,\sigma_\varpi'\,\sigma_\varpi+e^{-\varpi}\right)\Bigg\}\,dt\\ &\quad+\{\pi_{i,C}\,(\sigma_C+\sigma_v)+\pi_{i,Z}\,(\sigma_Z+\sigma_\zeta)+\pi_{i,I}\,\sigma_\varpi\}'\,dW\\ &\quad+\left\{\pi_{i,C}\,\left(e^{L_v}-1\right)+\pi_{i,Z}\,\left(e^{L_\zeta}-1\right)+\pi_{i,I}\,\left[(1+L_I)\,e^{L_\varpi}-1\right]\right\}\,dN(\lambda_i). \end{aligned} \tag{A.6}$$

The portfolio shares are determined by the condition that investor i's wealth and her financing portfolio have to react in the same way to the shocks in the model. With respect to the diffusions the condition is thus

$$\sigma_{V_i}\equiv\sigma_{C_i}+\sigma_{v_i}\stackrel{!}{=}\pi_{i,C}\,(\sigma_C+\sigma_v)+\pi_{i,Z}\,(\sigma_Z+\sigma_\zeta)+\pi_{i,I}\,\sigma_\varpi,$$

where the left-hand side is due to (A.4).

Look at $\pi_{i,C}$ first. From (4), (5), and (A.1) one can see that the first component in σ_{C_i} is equal to σ_c, since the first component of σ_w (a multiple of σ_X) is equal to zero. Equation (8) furthermore shows that σ_{v_i} is a multiple of σ_X, so that its first component is also equal to zero. Overall, the first component of the sum of vectors $\sigma_{C_i}+\sigma_{v_i}$ is thus equal to σ_c. The same is true for the volatility vectors of investor i's portfolio, as can be seen from the definitions of σ_C and σ_Z as well as Equations (B.1), (B.2), and (A.5). Taken together this implies $\pi_{i,C}=1$ $(i=1,2)$. So both agents invest all their wealth into the claim on aggregate consumption, implying that the positions in the other three assets add up to zero in value *for each agent individually.*

$\pi_{i,Z}$ and $\pi_{i,I}$ follow from equating the reactions of wealth and the portfolio to W_X shocks and jumps. This gives two conditions, where the first one refers to the second components of the vectors $\sigma_{C_i}+\sigma_{v_i}$ and $(\sigma_C+\sigma_v)+\pi_{i,Z}\,(\sigma_Z+\sigma_\zeta)+\pi_{i,I}\,\sigma_\varpi$, respectively. The second one is obtained by matching the terms in front of dN in the total return on wealth and on the financing portfolio, using $\pi_{i,C}=1$. This implies

$$(1+L_{C_i})\,e^{L_{v_i}}-1\equiv\left[e^{L_v}-1\right]+\pi_{i,Z}\,\left[e^{L_\zeta}-1\right]+\pi_{i,I}\,\left[(1+L_I)\,e^{L_\varpi}-1\right].$$

These two equations can be solved numerically for $\pi_{1,Z}$ and $\pi_{1,I}$. The portfolio weights for investor 2 are then found via the aggregate supply condition for the insurance products, which says that their total value in the economy has to equal zero, i.e., $\pi_{1,Z}\,V_1+\pi_{2,Z}\,V_2\equiv 0$ and $\pi_{1,I}\,V_1+\pi_{2,I}\,V_2\equiv 0$. Finally, investor i's position in the money market account is given as $\pi_{i,M} = -(\pi_{i,Z} + \pi_{i,I})$.

A.2 Incomplete Market

On the incomplete market the insurance products are no longer available to the investors, but they of course still have to agree on the prices of the claim on aggregate consumption and the money market account. Let ν_i denote investor i's subjective log price-dividend ratio of the claim on aggregate consumption. Its dynamics are given as follows:

$$
\begin{aligned}
d\nu_i &= \left\{ \frac{\partial \nu_i}{\partial w}\, \mu_w + \frac{1}{2}\, \frac{\partial^2 \nu_i}{\partial w^2}\, \sigma_w'\, \sigma_w - \frac{\partial \nu_i}{\partial X}\, \kappa_X\, X + \frac{1}{2}\, \frac{\partial^2 \nu_i}{\partial X^2}\, \sigma_X'\, \sigma_X \right. \\
&\quad \left. + \frac{\partial^2 \nu_i}{\partial w\, \partial X}\, \sigma_w'\, \sigma_X \right\} dt + \left\{ \frac{\partial \nu_i}{\partial w}\, \sigma_w + \frac{\partial \nu_i}{\partial X}\, \sigma_X \right\}' dW \\
&\quad + \left\{ \nu_i \left(w + L_w, X + L_X\right) - \nu_i \left(w, X\right) \right\} dN\left(\lambda_i\right). \qquad \text{(A.7)}
\end{aligned}
$$

Furthermore ν_i solves the following PDE

$$
\begin{aligned}
0 &= e^{-\nu_i} + \mu_{\xi_i} + \mu_C + \mu_{\nu_i} + \frac{1}{2}\, \sigma_{\nu_i}'\, \sigma_{\nu_i} + \sigma_{\xi_i}'\, \sigma_C + \sigma_{\xi_i}'\, \sigma_{\nu_i} + \sigma_C'\, \sigma_{\nu_i} \\
&\quad + \left[\left(1 + L_{C_i}\right)^{-\gamma}\, e^{(\theta - 1)\, L_{v_i}}\, e^{L_{\nu_i}} - 1 \right] \lambda_i, \qquad \text{(A.8)}
\end{aligned}
$$

which is obtained by first computing the differential $\frac{d(\xi_i\, C\, e^{\nu_i})}{\xi_i\, C\, e^{\nu_i}}$ and then using the fact that the sum of expected price change and cash flow must be equal to zero, i.e., that $E^{\mathbb{P}^i}\left[\frac{d(\xi_i\, C\, e^{\nu_i})}{\xi_i\, C\, e^{\nu_i}}\right] + e^{-\nu_i} = 0$. Since the investors agree on the price of the dividend claim, $\nu_i = \nu_2 \equiv \nu$.

Like before on a complete market the investor constructs his financing portfolio so that its return equals the return on his individual wealth. The return on wealth is the same as as on a complete market, while the return on the financing portfolio is now given as

$$
\begin{aligned}
dR_i^{\Pi} &= \pi_{i,C} \left(\frac{dP_i^C}{P_i^C} + e^{-\nu_i}\, dt \right) + \pi_{i,M}\, r\, dt \\
&= \left\{ \pi_{i,C} \left[\bar{\mu}_C + X_t + \mu_{\nu_i} + \frac{1}{2}\, \sigma_{\nu_i}'\, \sigma_{\nu_i} + \sigma_C'\, \sigma_{\nu_i} + e^{-\nu_i} \right] + \pi_{i,M}\, r \right\} dt \\
&\quad + \left\{ \pi_{i,C}\, \left(\sigma_C + \sigma_{\nu_i}\right) \right\}' dW + \left\{ \pi_{i,C}\, \left(e^{L_{\nu_i}} - 1\right) \right\} dN(\lambda_i).
\end{aligned}
$$

Since the investors' individual wealth and their financing portfolios have to have the same exposure to the two diffusions and the jump component, the following conditions have to hold for each investor:

$$
\begin{aligned}
\pi_{i,C}\, \left(\sigma_c + \sigma_{\nu_i,C}\right) &= \sigma_{C_i,C} + \sigma_{v_i,C} \qquad \text{(A.9)} \\
\pi_{i,C}\, \sigma_{\nu_i,X} &= \sigma_{C_i,X} + \sigma_{v_i,X} \qquad \text{(A.10)} \\
\pi_{i,C}\, \left(e^{L_{\nu_i}} - 1\right) &= \left(1 + L_{C_i}\right) e^{L_{v_i}} - 1, \qquad \text{(A.11)}
\end{aligned}
$$

where $\sigma_{\cdot,C}$ and $\sigma_{\cdot,X}$ refer to the first and second component of the respective volatility vector.

We want to solve for the following eight variables of interest: the two individual log wealth-consumption ratios v_1 and v_2, the log price-dividend ratio of the traded consumption claim ν, the drift μ_w, the two elements of the volatility vector σ_w, and the jump size L_w of the consumption share process, and the portfolio weight for the claim on aggregate consumption $\pi_{1,C}$. The portfolio weight for investor 2 is determined via the market clearing condition $\pi_{1,C}\, C_1\, e^{v_1} + \pi_{2,C}\, C_2\, e^{v_2} = C\, e^{v}$, and the weight of the money market account is given by $\pi_{i,M} \equiv 1 - \pi_{i,C}$.

There are eight equations we can use to find these quantities: the two PDEs for the individual log wealth-consumption ratios represented by equation (9) for $i = 1, 2$, the two PDEs for the individual log price-dividend ratios of the claim on aggregate consumption given in (A.8) for $i = 1, 2$, the equation obtained through the restriction that the individual risk-free rates given in (13) have to be equal, and the three equations for the portfolio weights (A.9) – (A.11).

B Pricing the Insurance Assets

Analogously to Equations (8) and (9) the dynamics of the log price-to-cash-flow ratio ζ_i of the insurance asset Z are given by

$$\begin{aligned} d\zeta_i &= \left\{ \frac{\partial \zeta_i}{\partial w}\, \mu_w + \frac{1}{2}\, \frac{\partial^2 \zeta_i}{\partial w^2}\, \sigma_w'\, \sigma_w - \frac{\partial \zeta_i}{\partial X}\, \kappa_X\, X + \frac{1}{2}\, \frac{\partial^2 \zeta_i}{\partial X^2}\, \sigma_X'\, \sigma_X \right. \\ &\qquad \left. + \frac{\partial^2 \zeta_i}{\partial w\, \partial X}\, \sigma_w'\, \sigma_X \right\} dt + \left\{ \frac{\partial \zeta_i}{\partial w}\, \sigma_w + \frac{\partial \zeta_i}{\partial X}\, \sigma_X \right\}' dW \\ &\quad + \left\{ \zeta_i \left(w + L_w, X + L_X\right) - \zeta_i\left(w, X\right) \right\}\, dN\left(\lambda_i\right) \\ &\equiv \mu_{\zeta_i}\, dt + \sigma_{\zeta_i}'\, dW + L_{\zeta_i}\, dN\left(\lambda_i\right), \end{aligned} \tag{B.1}$$

and ζ_i solves the PDE

$$\begin{aligned} 0 &= e^{-\zeta_i} + \mu_{\xi_i} + \mu_Z + \mu_{\zeta_i} + \frac{1}{2}\, \sigma_{\zeta_i}'\, \sigma_{\zeta_i} + \sigma_{\xi_i}'\, \sigma_Z + \sigma_{\xi_i}'\, \sigma_{\zeta_i} + \sigma_Z'\, \sigma_{\zeta_i} \\ &\quad + \left[\left(1 + L_{C_i}\right)^{-\gamma}\, e^{(\theta - 1)\, L_{v_i}}\, e^{L_{\zeta_i}} - 1 \right]\, \lambda_i. \end{aligned}$$

The insurance product I has a price-to-cash flow ratio denoted by ϖ_i with dynamics

$$\begin{aligned} d\varpi_i &= \left\{ \frac{\partial \varpi_i}{\partial w}\,\mu_w + \frac{1}{2}\,\frac{\partial^2 \varpi_i}{\partial w^2}\,\sigma_w'\,\sigma_w - \frac{\partial \varpi_i}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 \varpi_i}{\partial X^2}\,\sigma_X'\,\sigma_X \right. \\ &\qquad \left. + \frac{\partial^2 \varpi_i}{\partial w\,\partial X}\,\sigma_w'\,\sigma_X \right\} dt + \left\{ \frac{\partial \varpi_i}{\partial w}\,\sigma_w + \frac{\partial \varpi_i}{\partial X}\,\sigma_X \right\}' dW \\ &\quad + \left\{ \varpi_i\,(w + L_w, X + L_X) - \varpi_i\,(w, X) \right\}\, dN\,(\lambda_i) \\ &\equiv \mu_{\varpi_i}\,dt + \sigma_{\varpi_i}'\,dW + L_{\varpi_i}\,dN(\lambda_i). \end{aligned} \tag{B.2}$$

ϖ_i solves the PDE

$$\begin{aligned} 0 &= e^{-\varpi_i} + \mu_{\xi_i} + \mu_I + \mu_{\varpi_i} + \frac{1}{2}\,\sigma_{\varpi_i}'\,\sigma_{\varpi_i} + \sigma_{\xi_i}'\,\sigma_{\varpi_i} \\ &\quad + \left[(1 + L_I)\,(1 + L_{C_i})^{-\gamma}\,e^{(\theta-1)\,L_{v_i}}\,e^{L_{\varpi_i}} - 1 \right]\,\lambda_i. \end{aligned}$$

C Auxiliary Quantitative Results

C.1 Wealth-consumption ratios

Due to the recursive utility specification wealth-consumption ratios are key ingredients to asset pricing. The aggregate and individual wealth-consumption ratios on a complete market are shown in the upper row of Figure 7. Looking at the dependence on the pessimist's consumption share w first we see that when the optimist becomes small (i.e., when w tends to 1), he wants to avoid extinction and consumes less and saves more. The analogous logic (now for w going to 0) applies to the pessimist which is represented by the dotted line, although the pessimist is reacting in a much less extreme fashion than the optimist. The aggregate wealth-consumption ratio is downward sloping in w, since the optimist would save more in the respective single investor economy (left boundary) than the pessimist (right boundary). The right graph in the upper row confirms the intuition that a higher long-run growth rate implies more attractive investment opportunities which lead to less consumption and higher savings. The slope of all three curves in this graph is about the same, so the optimist and the pessimist react in pretty much the same relative fashion to changes in X, although the level is higher for the optimist.

In terms of the dependence on X the results for the incomplete market are very similar (right graph in the lower row). Concerning the dependence on w, however, incompleteness matters, at least for the pessimist. His individual wealth-consumption ratio is now increasing in w. Since the insurance products are not present anymore, saving becomes so unattractive for the pessimist that, even in the face of extinction, he still prefers to consume more than when he is large. Also the optimist is affected, but to a much lesser degree than the pessimist, and also for the market as a whole the results are unite similar to the case of completeness.

C.2 Consumption share dynamics

The upper row in Figure 8 shows (from left to right) the coefficients μ_w, $\sigma_{w,C}$, $\sigma_{w,X}$, and L_w of the consumption share process on a complete market. The curves for the optimist, the pessimist, and the aggregate market are all identical, so that there is only one line in the graphs. Note that the boundary values for $w = 0$ and $w = 1$ are equal to zero, since in a one-investor economy, the investor's consumption share is necessarily constant.

From the first graph, showing μ_w, it becomes obvious that in times without jumps the pessimist's consumption share decreases on average due to the compensation for risk sharing. Since jumps increase the pessimist's consumption share due to the payoffs from the associated insurance contract, the average compensation in times without jumps has to be negative.

As we can see from the graph for $\sigma_{w,C}$, consumption risk is not shared, since the investors have identical beliefs with respect to this source of risk. So the investors' consumption shares remain unchanged following a consumption shock. Also the reaction of w to a diffusive shock in X is not very pronounced, as we can see from the graph for $\sigma_{w,X}$. The small non-zero values for larger w are due to the fact that the optimist reacts stronger to an increase in the long-run growth rate than the pessimist.

The picture is quite different for jumps in X. For an equal consumption distribution the reaction to jumps can be up to approximately 10%. When a disaster strikes (among other things) the long-run growth rate in the economy drops and due to the less attractive investment opportunities, both investors save less and thus consume more. But the pessimist's reaction is much stronger than the optimist's, so that the term $L_{v_1} - L_{v_2}$ in Equation (A.3) is negative which leads to the shape of the graph. So on a complete market investors almost exclusively share jump risk.

The upper row of Figure 9 shows the results for the incomplete markets case. Compared to the complete markets case the situation has changed significantly. The reactions to the two types of diffusion risk become much more pronounced, whereas the reaction to jumps becomes much smaller. The reason is that on an incomplete market the only risky asset which the pessimist can use to reduce his jump exposure is the consumption claim. Reducing this exposure by reducing the amount of wealth invested in the consumption claim *automatically* implies a reduction in the diffusive exposure as well, so that in the end the investors mainly share diffusive risk. Of course, as indicated by the first graph in this row, the pessimist still accepts a decrease in his consumption share on average.

C.3 Risk-free rate and market prices of risk

On a complete market the investors have to agree on the risk-free rate, on the market prices for the diffusion risks W_c and W_x, and on the risk-neutral jump intensity which are all shown (in this order from left to right) in the upper row of Figure 10.

The first graph shows the risk-free rate from Equation (13). We can see that the precautionary savings due to jump risk overcompensate the impact of the individual consumption growth rate for the optimist and vice versa for the pessimist. Overall the risk-free rate decreases slightly in w and varies between 0.6% and 1.1%.

Next, the market price of risk for W_c is constant in w. Since X does not load on W_c and the investors do not share consumption risk, we end up with the usual CRRA result that the market price of risk is equal to $\gamma\sigma_C$ for both investors.

Also the market price of risk for W_x is basically a constant. It decreases very slowly in w, which is due to the fact that in the respective one-investor economies for $w = 0$ and $w = 1$ there is also only a small difference between the respective market prices of risk, since investors only disagree on the jump intensity.

While disagreement about the jump intensity has a negligible impact on the market prices of diffusion risk, it has a dramatic effect on the risk-neutral jump intensity $\lambda^{\mathbb{Q}}$. It ranges from below 1% in an all-optimist economy to almost 16% for $w = 1$. The investors' subjective market prices for *positive* jump exposure can be determined by comparing the risk-neutral with the subjective jump intensities, and here there are significant differences between the optimist and the pessimist. While the optimist has a negative market price of risk throughout, the sign switches for the pessimist, once he has reached a certain size in the economy.

The lower row in Figure 10 presents the result for the incomplete markets case. First, the risk-free rate on the incomplete market is basically indistinguishable from the one on the complete market. Next, the market price of consumption risk, represented by the first component of the vector shown in Equation (11), is mainly driven by what we called the market price of risk for relative size, which is negative for the pessimist and positive for the optimist. These terms are increasing in w, and so is the market price of consumption risk. Note that now, on an incomplete market, the individual market prices of risk no longer coincide. The story is basically the same for the market price of diffusive risk in X, only the numbers are different.

Finally, the pronounced differences between the optimist's and the pessimist's risk-neutral jump intensities are mostly determined by the different physical jump intensities assumed by the investors. In contrast to the complete markets case the market price of jump risk is now negative for both investors across the full range of w (and more negative for the pessimist). Both investors' risk-neutral jump intensities are much closer to being linear than on the complete market and are increasing much less in w than before.

C.4 Wealth exposures

To analyze the properties of the return on individual and aggregate wealth we go back to Figure 8 and look at the lower row of graphs, which show the drift and sensitivity of these returns with respect to diffusive consumption risk, diffusive growth rate risk, and jump risk (see Equation (A.4)).

The exposure of all the returns to consumption risk is constant and equal to σ_c, again since X does not load on W_c and the investors do not share consumption risk. They do, however, share the risk of diffusive shocks in the long-run growth rate, and this is why we see in the third graph that the exposure of the optimist's wealth is decreasing in w, and the opposite is true for the pessimist. In the aggregate the exposure to diffusive growth rate risk decreases slightly.

In terms of the jump exposure of individual and aggregate wealth we see that the sensitivity of the optimist's wealth to jumps in X is always negative, whereas the pessimist's is mostly positive, but also becomes negative when he is sufficiently large. Both exposures decrease in w, whereas in the aggregate the jump sensitivity is more or less constant.

The first column illustrates the average return on wealth in times without jumps. We have already seen that on a complete market, investors mainly share jump risk, so that the drift is mainly a compensation for jump risk.

The return on individual wealth and aggregate wealth on an incomplete market are presented in the lower row of Figure 9. The second and third graph show the exposures to consumption risk and to diffusive shocks in the long-run growth rate. The curves look very similar due to the similar way in which the investors share these two sources of risk by shifting exposures from the pessimist to the optimist. This is also true for jump risk. Concerning the drift of the wealth process the pessimist has to compensate the optimist for sharing mainly diffusion risks and a small amount of jump risk. Therefore, the optimist's wealth increases in times without jumps, whereas it decreases for the pessimist.

Whether the market is complete or not obviously only has a very small effect on the results for the aggregate wealth. It does, however, matter for the properties of the investors' individual wealth processes. On a complete market the investors mainly share mainly jump risk, which shows up in the wide range of the jump sizes of individual wealth as a function of w.

On an incomplete market the investors cannot adjust the exposures to the different sources of risk separately because they only have access to the money market account and the claim on aggregate consumption. So if the pessimist reduces his jump exposure this means both diffusion exposures will decrease automatically. Since jump risk is only a small fraction of the risk embedded in the aggregate consumption claim while it is mainly influenced by diffusion risk, we see large differences in the diffusion coefficients of the return on individual wealth.

D Numerical Implementation

We want to describe briefly the way we implemented the model using MATLAB and the corresponding Toolbox provided by the Numerical Algorithms Group. We rely on a numerical solution of the model on a two-dimensional grid over the pessimist's consumption share w and the long-run growth rate X. For the former we use 31 equidistant steps over the interval $[0, 1]$ and 29 steps over the interval $[-0.1560, 0.1440]$ for the latter.

D.1 Complete market

To obtain boundary conditions for the PDE in (9) we study the limiting cases if the pessimist's consumption share w goes to zero or one. In either case we have one very large and one very small investor. The large investor sets the prices, whereas the small one just takes the price as given.

Since w is very close to zero, $\frac{1}{1-w}\,\mu_w$, $\frac{1}{1-w}\,\sigma_w$ and L_w are zero as well, thus the PDE for the large investor 2 simplifies to

$$\begin{aligned} 0 &= e^{-v_2} - \beta + \left(1 - \frac{1}{\psi}\right)\left[\bar{\mu}_C + X_t - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right] - \frac{\partial v_2}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 v_2}{\partial X^2}\,\sigma_X'\,\sigma_X \\ &\quad + \frac{1}{2}\,\theta\left(\frac{\partial v_2}{\partial X}\right)^2 \sigma_X'\,\sigma_X + (1-\gamma)\,\frac{\partial v_2}{\partial X}\,\sigma_C'\,\sigma_X + \frac{1}{\theta}\left[e^{\theta\,[v_2(w,X+L_w)-v_2(w,X_t)]} - 1\right]\lambda_2. \end{aligned}$$

We use $v_2 = -\log\left[\beta - \left(1 - \frac{1}{\psi}\right)\left(\bar{\mu}_C - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right)\right]$, the solution for the wealth-consumption ratio in a one-investor economy without a state variable, as starting value for our numerical optimization.

For the small investor the fact that $1 - w$ is very close to one implies

$$\begin{aligned} \frac{1}{w}\,\mu_w &= \psi\left\{\frac{1}{2}\left(1 + \frac{1}{\psi}\right)\gamma\left[\sigma_{C_1}'\,\sigma_{C_1} - \sigma_{C_2}'\,\sigma_{C_2}\right] + \frac{1}{2}\,(1-\theta)\left[\sigma_{v_1}'\,\sigma_{v_1} - \sigma_{v_2}'\,\sigma_{v_2}\right]\right. \\ &\quad + (1-\theta)\left[\sigma_{C_1}'\,\sigma_{v_1} - \sigma_{C_2}\,\sigma_{v_2}\right] \\ &\quad + \left[e^{(\theta-1)\,[v_1(w,X+L_X)-v_1(w,X)]} - 1 - \left(1 - \frac{1}{\theta}\right)\left[e^{\theta\,[v_1(w,X+L_X)-v_1(w,X)]} - 1\right]\right]\lambda_1 \\ &\quad \left. - \left[e^{(\theta-1)\,[v_2(w,X+L_X)-v_2(w,X)]} - 1 - \left(1 - \frac{1}{\theta}\right)\left[e^{\theta\,[v_2(w,X+L_X)-v_2(w,X)]} - 1\right]\right]\lambda_2\right\} \\ \frac{1}{w}\,\sigma_w &= \frac{1}{\gamma}\,(1-\theta)\left(\frac{\partial v_2}{\partial X} - \frac{\partial v_1}{\partial X}\right)\sigma_X \end{aligned}$$

and $v_1\,(w, X + L_X) - v_1\,(w, X) = v_2\,(w, X + L_X) - v_2\,(w, X) - \frac{1}{\theta-1}\,\ln\left(\frac{\lambda_1}{\lambda_2}\right)$. Thus the PDE is given by

$$\begin{aligned} 0 &= e^{-v_1} - \beta + \left(1 - \frac{1}{\psi}\right)\left[\bar{\mu}_C + X_t + \frac{1}{w}\,\mu_w - \frac{1}{2}\,\gamma\left(\sigma_C'\,\sigma_C - \frac{1}{w^2}\,\sigma_w'\,\sigma_w\right)\right] - \frac{\partial v_1}{\partial X}\,\kappa_X\,X \\ &\quad + \frac{1}{2}\,\frac{\partial^2 v_1}{\partial X^2}\,\sigma_X'\,\sigma_X + \frac{1}{2}\,\theta\left(\frac{\partial v_1}{\partial X}\right)^2 \sigma_X'\,\sigma_X + (1-\gamma)\,\frac{\partial v_2}{\partial X}\left(\sigma_C + \frac{1}{w}\,\sigma_w\right)'\sigma_X \\ &\quad + \frac{1}{\theta}\left[e^{\theta\left[v_2(w,X+L_w)-v_2(w,X_t)-\frac{1}{\theta-1}\,\ln\left(\frac{\lambda_1}{\lambda_2}\right)\right]} - 1\right]\lambda_2. \end{aligned}$$

Again we rely on $v_1 = -\log\left[\beta - \left(1 - \frac{1}{\psi}\right)\left[\bar{\mu}_C - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right]\right]$ as starting value for our numerical optimization .

D.2 Incomplete market

On an incomplete market we also need starting values for the optimization problem described in Section A.2. In addition to the complete market solution we use $\nu = \frac{1}{2}\,v_1 + \frac{1}{2}\,v_2$ and $\pi_1^C = 1$ (for $w < 0.5$) resp. $\pi_2^C = 1$ (for $w \leq 0.5$).

Investors		
Relative risk aversion	γ	10
Intertemporal elasticity of substitution	ψ	1.5
Subjective discount rate	β	0.02
Aggregate consumption		
Expected growth rate of aggregate consumption	$\bar{\mu}_C$	0.02
Volatility of aggregate consumption	σ_C	0.0252
Stochastic growth rate		
Mean reversion speed	κ_X	0.1
Volatility	σ_x	0.0114
Jump size	L_X	-0.03
Jump intensity of the pessimistic investor 1	λ_1	0.020
Jump intensity of the optimistic investor 2	λ_2	0.001
Further parameters		
Leverage factor for dividends	ϕ	1.5
Drift of insurance product Z	μ_Z	-0.1
Volatility of insurance product Z	σ_Z	0.001
Drift of insurance product I	μ_I	-0.1
Jump size of insurance product I	L_I	0.01

Table 1: **Parameters**

	No disagreement	Medium disagreement	High disagreement
	$\lambda_1 = 0.0200$ $\lambda_2 = 0.0200$	$\lambda_1 = 0.0250$ $\lambda_2 = 0.0150$	$\lambda_1 = 0.0300$ $\lambda_2 = 0.0100$
		Complete market	
Expected Return	0.0642	0.0640	0.0634
Return Volatility	0.0801	0.0801	0.0803
Trading Volume	0.0000	0.0000	0.0000
Risk-free Rate	0.0063	0.0065	0.0071
Expected Excess Return	0.0579	0.0575	0.0563
		Incomplete market	
Expected Return	0.0642	0.0643	0.0647
Return Volatility	0.0801	0.0804	0.0813
Trading Volume	0.0000	0.0006	0.0026
Risk-free Rate	0.0063	0.0065	0.0072
Expected Excess Return	0.0579	0.0578	0.0575

Table 2: **Varying degrees of disagreement**

The table shows the effect of varying degrees of disagreement on the expected return, the return volatility, the trading volume of the consumption claim, the risk-free rate, and the expected excess return on a complete (upper panel) and incomplete market (lower panel). In all three scenarios the true model is represented by the average belief $\bar{\lambda} = 0.0200$. Trading volume is defined in Equation (14). The parameters are shown in Table 1.

T (years)	Our paper Complete market	 Incomplete market	CJT
50	0.349	0.250	0.588
100	0.260	0.111	0.661
200	0.161	0.019	0.777
500	0.056	0.000	0.934
1,000	0.013	0.000	0.992

Table 3: **Investor survival**

The table shows the pessimist's expected consumption share $E[w_T]$ for T years into the future under the respective true measure. The expectation is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (3) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are shown in Table 1.

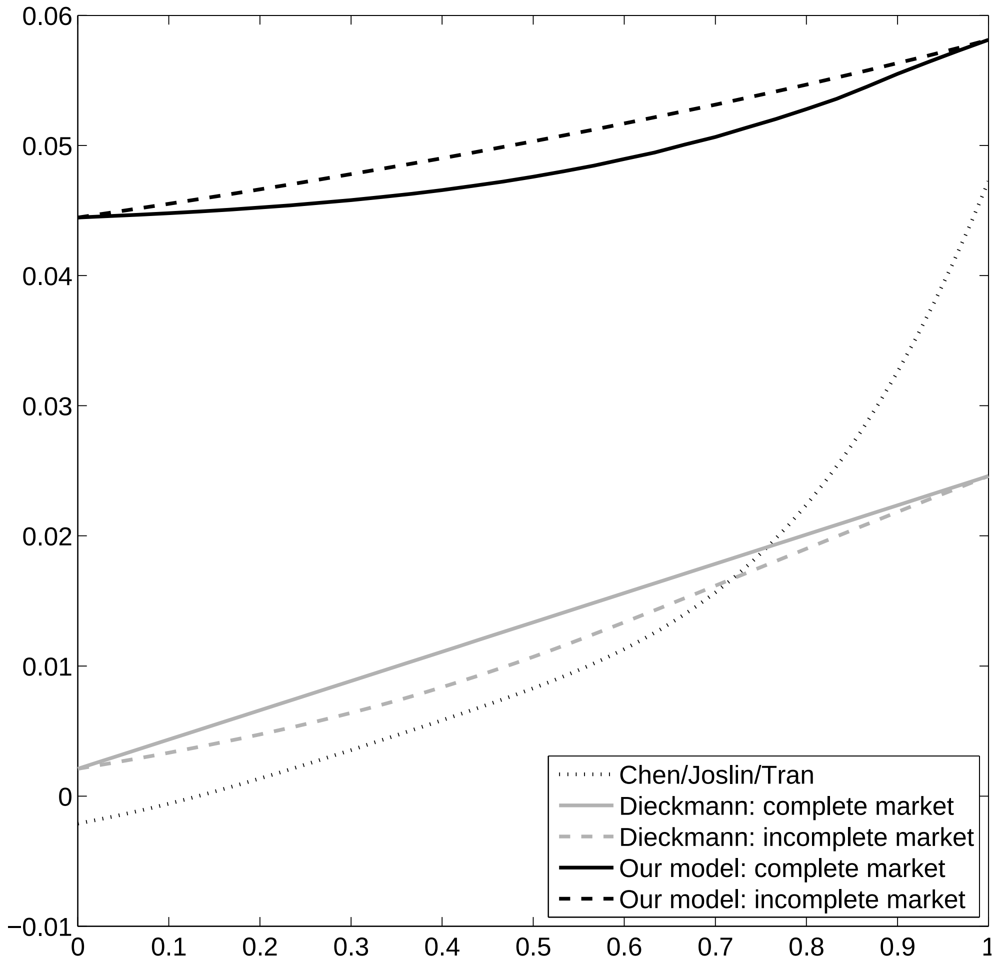

Figure 1:
Equity premium in Chen, Joslin, and Tran (2012), Dieckmann (2011), and our model

The graph shows the equity premium under the respective true probability measure for three different heterogenous investor models as a function of the pessimist's consumption share. The gray dotted line represents the model proposed by Chen, Joslin, and Tran (2012), the gray solid (dashed) line gives the equity premium in the model by Dieckmann (2011) when the market is complete (incomplete), and the black solid (dashed) line shows this quantity in our model for the complete (incomplete) market case.

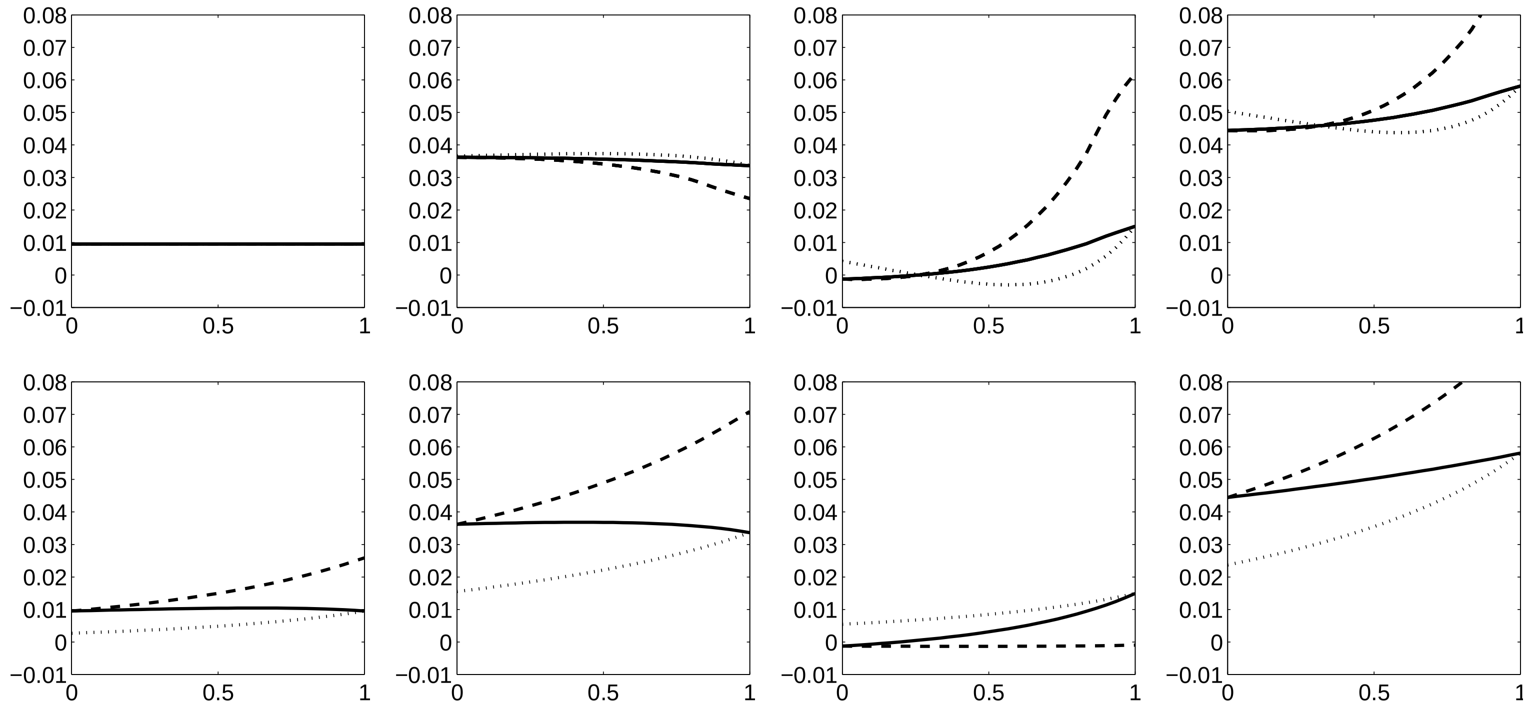

Figure 2:
Equity premium and its components

The figure shows (from left to right) the share of the equity premium due to diffusive consumption risk, diffusive growth rate risk, and jump risk, respectively, and the total equity premium as the sum of the three components. All quantities are shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The upper (lower) row of graphs shows the results for the complete (incomplete) market. The dotted (dashed) line depicts the expected excess return on the individual wealth of the pessimist (optimist) and the solid line the excess return on aggregate wealth. All quantities are determined under the true measure. The parameters are shown in Table 1.

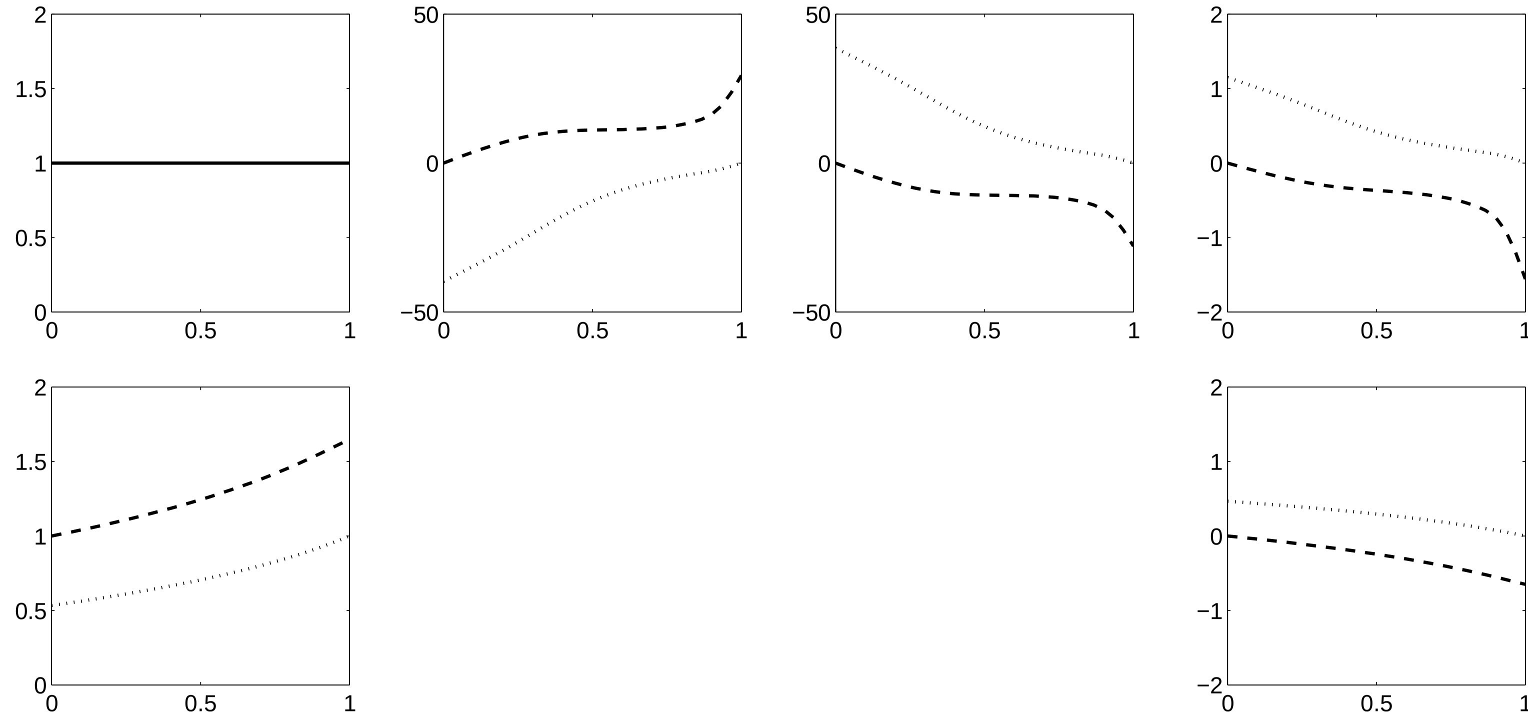

Figure 3:
Asset holdings

The figure shows the investors' asset holdings on the complete (upper row) and the incomplete market (lower row), respectively. In the upper row the graphs show from left to right show the fraction of wealth invested in the consumption claim, the diffusion insurance product Z, the jump insurance product I, and the money market account. In the lower row the left (right) graph is for the consumption claim (money market account). The pessimist's (optimist's) portfolio holdings are indicated by the dotted (dashed) line. All quantities are shown as functions of the pessimist's consumption share and for $X = -0.0060$. The parameters are shown in Table 1.

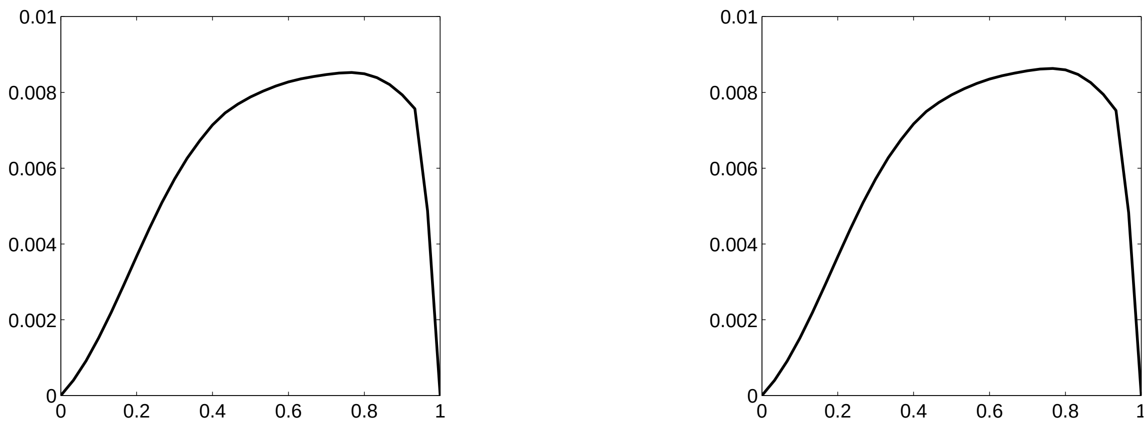

Figure 4:
Trading volume in the insurance assets

The figure shows the trading volume in the insurance assets Z (left) and I (right) as functions of the pessimist's consumption share and for $X = -0.0060$. The parameters are shown in Table 1.

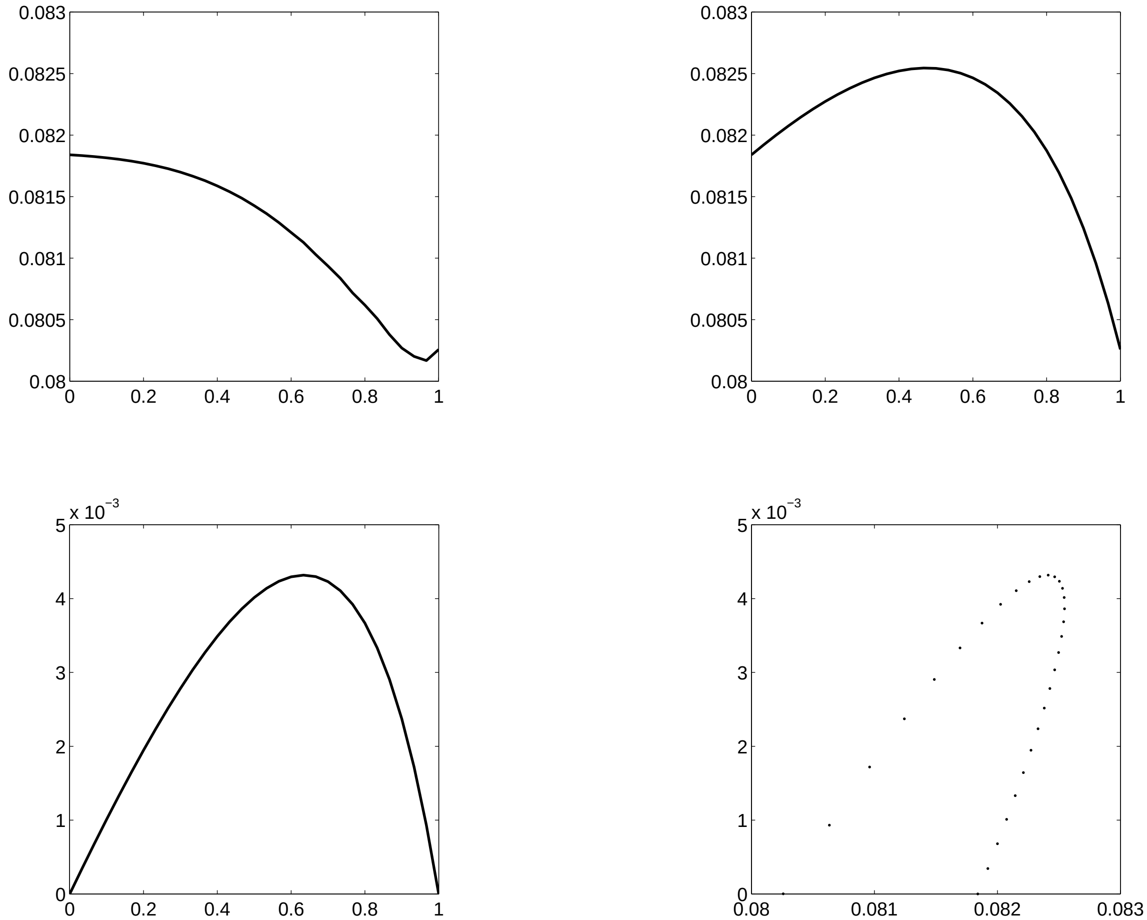

Figure 5:
Return volatility and trading volume

The figure shows in the upper row the return volatility of the consumption claim on a complete (left) and on an incomplete market (right). The graphs in the lower row depict the trading volume in the consumption claim (left) and the relationship between trading volume and the return volatility on an incomplete market. All quantities are shown as functions of the pessimist's consumption share and for $X = -0.0060$. The parameters are shown in Table 1.

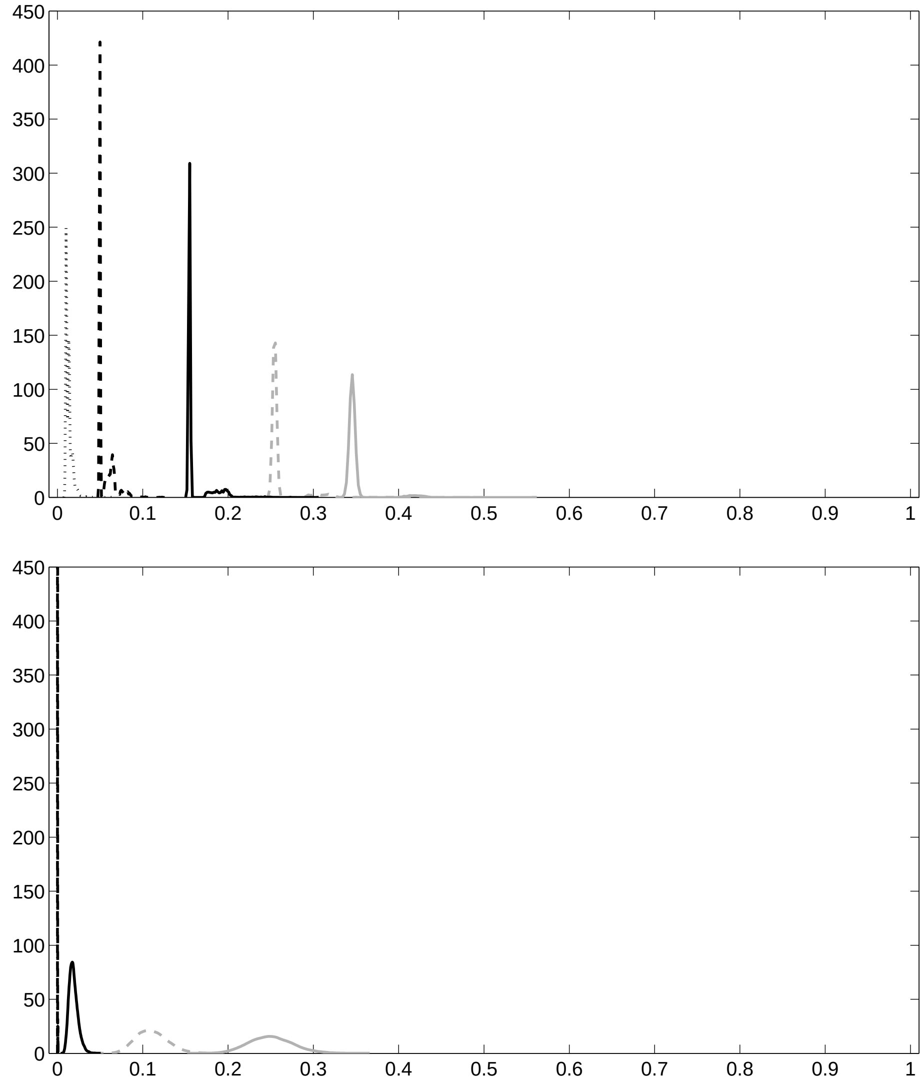

Figure 6:
Survival

The figure shows the kernel density estimates for the pessimist's consumption share determined by a Monte Carlo simulation under the true measure over 10,000 paths after 50 years (gray solid line), 100 years (gray dashed line), 200 years (black solid line), 500 years (black dashed line) and 1000 years (black dotted line). The upper panel shows the results on a complete market, the lower one those on an incomplete market.

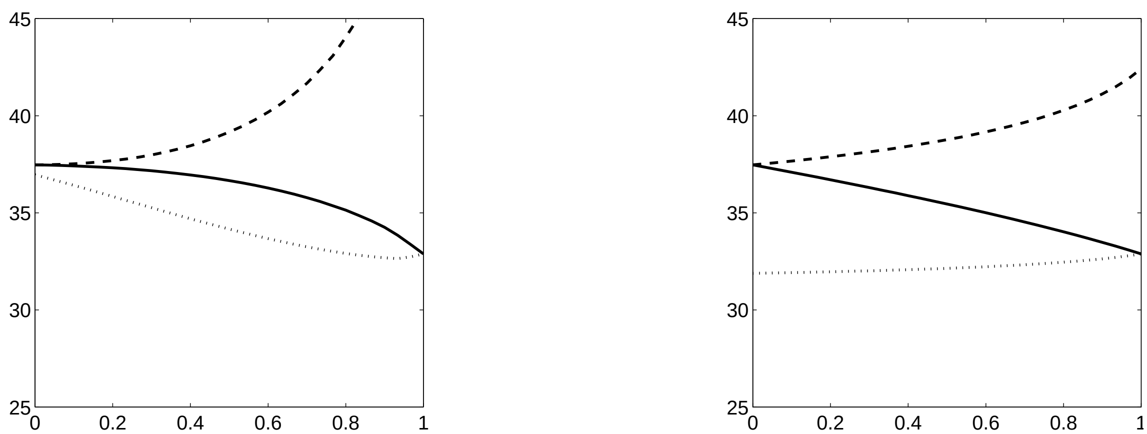

Figure 7:
Wealth-consumption ratios

The figure shows the aggregate and individual wealth-consumption ratios. The solid line represents the aggregate, the dotted (dashed) line shows the pessimist's (optimist's) individual wealth-consumption ratio. All quantities are shown as function of the pessimist's consumption share for $X = -0.0060$. The left (right) panel shows the results for the complete (incomplete) market. The parameters are shown in Table 1.

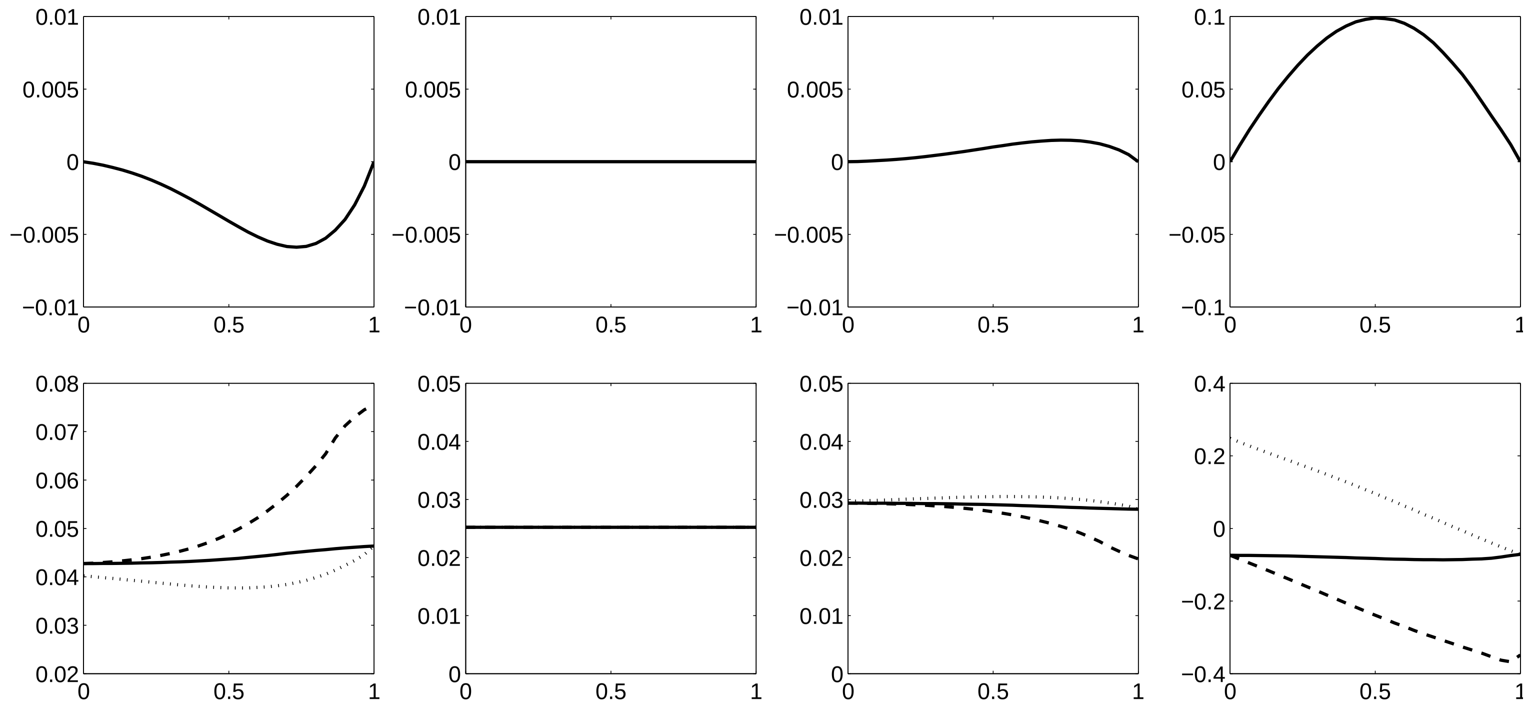

Figure 8:
Consumption share dynamics and wealth exposures (complete market)

The figure shows the coefficients in the dynamics of the pessimist's consumption share w (upper row) and the exposures of the return on aggregate and individual wealth on the risk factors in the economy (lower row) for the case of an complete market. In the lower row of graphs the solid line represents aggregate wealth, and the dotted (dashed) line shows the results for the pessimist (optimist). From left to right the graphs show the drift, and the coefficients for diffusive consumption shocks, diffusive expected growth rate shocks, and jumps in the expected growth rate, respectively. All quantities are shown as functions of the pessimist's consumption share w and for $X = -0.0060$. The parameters are shown in Table 1.

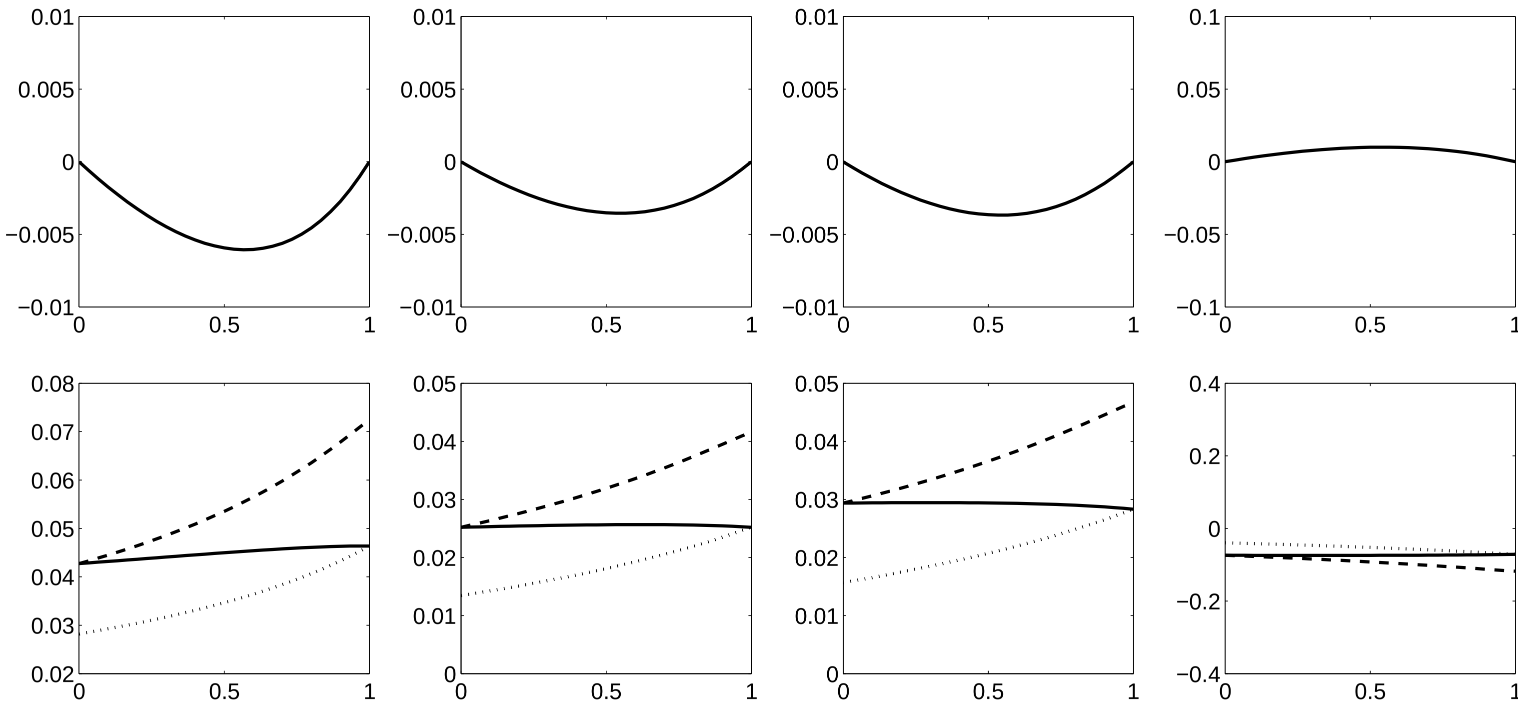

Figure 9:
Consumption share dynamics and wealth exposures (incomplete market)

The figure shows the coefficients in the dynamics of the pessimist's consumption share w (upper row) and the exposures of the return on aggregate and individual wealth on the risk factors in the economy (lower row) for the case of an incomplete market. In the lower row of graphs the solid line represents aggregate wealth, and the dotted (dashed) line shows the results for the pessimist (optimist). From left to right the graphs show the drift, and the coefficients for diffusive consumption shocks, diffusive expected growth rate shocks, and jumps in the expected growth rate, respectively. All quantities are shown as functions of the pessimist's consumption share w and for $X = -0.0060$. The parameters are shown in Table 1.

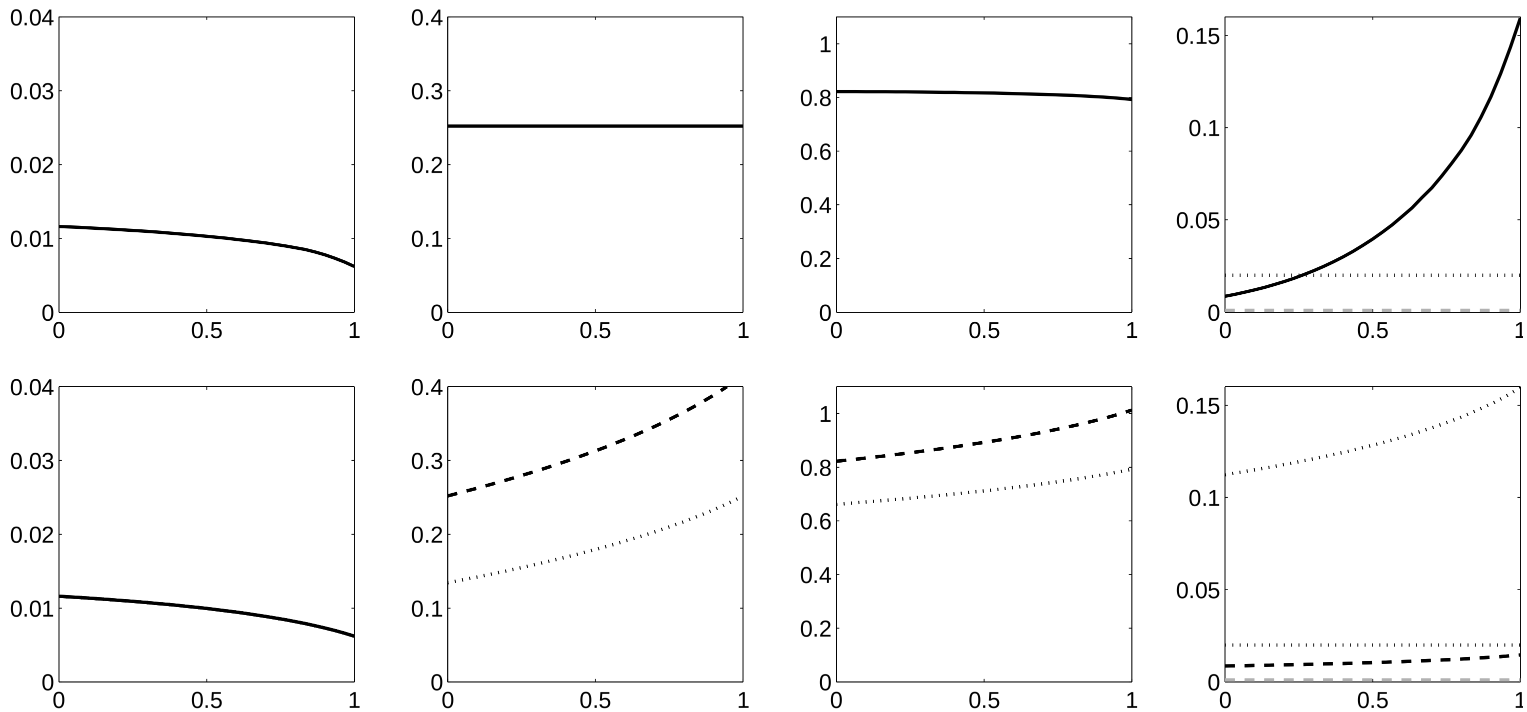

Figure 10:
Risk-free rate, market prices of risk, and risk-neutral jump intensities

The figure shows (from left to right) the risk-free rate, the market prices of risk for diffusive consumption and diffusive growth rate risk, and risk-neutral jump intensities on a complete (upper row) and an incomplete market (lower row). A solid black line indicates that both investors agree on the respective quantity. Otherwise, the dotted (dashed) black line represents the pessimist's (optimist's) view. The gray dotted (dashed) line shows the pessimist's (optimist's) subjective jump intensity. All quantities are shown as functions of the pessimist's consumption share and for $X = -0.0060$. The parameters are shown in Table 1.