# Returns on Cyclical and Defensive Stocks in Times of Scarce Information about the Business Cycle

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## Abstract

We study the implications of the quality of information about the business cycle for the pricing of defensive and cyclical stocks in a general equilibrium framework. We rely on a two-tree Lucas-style endowment economy in which the business cycle is modeled as an unobservable mean reverting process that influences one of the two assets. The representative Epstein-Zin investor learns about the latent business cycle variable from past dividends and from an additional signal. We find that expected excess returns on both cyclical and defensive stocks increase when information quality deteriorates, i.e. when the signal becomes more noisy. Variance reacts in an asymmetric way: a lower information quality induces a higher return variance of cyclical stocks and a lower return variance of defensive stocks. These findings are in line with stylized facts from the data when we use the dispersion in analysts' GDP forecasts as a measure for information quality.


**Keywords:** Asset Pricing, Lucas Orchard, Learning, Information Quality

**JEL:** G12

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# 1 Introduction

One objective of asset pricing is to explain the cross-section of stock returns by differences between the stocks with respect to, e.g., market beta, size, book-to-market, duration, or growth options.[1] In this paper, we focus on differences with respect to the exposure of the stock's dividend to the business cycle. We compare firms whose dividends are tightly linked to the business cycle, such as producers of machines or steel, with firms whose businesses' success hardly depends on economic circumstances, such as producers of food or consumer goods. Information about the business cycle is usually less than perfect, and the quality of information varies over time. In long run risks models business cycle risk (or growth rate risk) is an important driver of returns, and we expect information quality to have a significant impact too. What we are interested in is how the prices of cyclical and defensive stocks depend on the level of uncertainty about how the business cycle evolves. Do returns on cyclical stocks contain an additional positive risk premium for noisy information? Do returns on defensive stocks contain an uncertainty premium at all? How do the volatilities of cyclical and defensive stocks change?

This paper provides an empirical and theoretical analysis of the impact of business cycle risk and information quality on the cross-section of returns on cyclical and defensive stocks. In the empirical part, we analyze the relation between stock returns and the overall uncertainty about the business cycle. Uncertainty and the quality of information are measured by the dispersion of GDP forecasts in the survey of professional forecasters.[2] We document two major stylized facts: First, a decrease in information quality induces larger expected excess returns on both cyclical and defensive assets. Second, the return variance of cyclical assets increases during times of high economic uncertainty, while that of defensive assets decreases. In the theoretical part, we study a general equilibrium asset pricing model with two assets (a cyclical and a defensive asset), business-cycle risk, and different levels of information quality. We find that this model can explain these stylized empirical facts.

We rely on a long-run risk model with two trees. The representative investor has recursive preferences as proposed by Epstein and Zin (1989). The future dynamics of consumption depend on a business cycle variable which determines expected growth rates. Aggregate consumption is generated by the output of two "Lucas trees", i.e.

[1]See e.g. Fama and French (1992), Berk, Green, and Naik (1999), Lettau and Wachter (2007), and Nagel (2012).

[2]Other studies that use this data include Keane and Runkle (1990), Ang, Bekaert, and Wei (2007), Bansal and Shaliastovich (2009) and Anderson, Ghysels, and Juergens (2009).

claims to two different payment streams. The two trees stand for two segments of the economy, which are formed by sorting stocks with respect to their exposure to the business cycle.[3] The dividend growth rate is constant for the defensive asset (which we call asset $N$ for *non-business cycle related* in the following) and proportional to the (in general unobservable) business cycle for the cyclical asset (which we call asset $B$ for *business cycle related* in the following). The expected growth rate of cyclical assets, i.e. our business cycle variable, follows an Ornstein-Uhlenbeck process with high persistence. The investor cannot observe the current value of the business cycle, but in general has to infer it from past dividends and some noisy signal as additional source of information. The noise of the signal describes the overall quality of information in the economy, as in Veronesi (2000). The higher this noise, the worse the information available to the investor, and the higher the overall uncertainty in the economy.

Learning about the business cycle from dividend innovations and a signal has two major effects. First, when the investor has to learn about the business cycle from signal and dividend innovations, the perceived business cycle is smoother than the true business cycle. The larger the volatility of the signal, the lower the volatility of the perceived business cycle. A low information quality, i.e. a high volatility of the signal, thus *reduces* the perceived amount of long-run risk in the economy. Second, learning changes the characteristics of the risk factors. With perfect information about the business cycle, the investor is able to distinguish pure dividend fluctuations from far reaching business cycle shocks. In uncertain times, she misinterprets fluctuations in the dividends of the cyclical asset as shocks induced by changes in the business cycle. Even if the true business cycle and dividend innovations are uncorrelated, learning thus induces an endogenous correlation between the perceived business cycle and dividend innovations which is the larger the lower the quality of information.

In our model, poorer information quality implies higher risk premia for cyclical *and* defensive stocks. The defensive asset provides a hedge against business cycle risk, which lowers its expected excess return. Poorer information quality reduces the amount of perceived business cycle risk. This reduces the hedging pressure and leads to a higher expected excess return on the defensive asset. The cyclical asset has a positive exposure to business cycle risk. The lower amount of business cycle risk thus lowers its expected excess return. At the same time the investor interprets

[3]For ease of exposition, we will speak of two *assets* in the following, with the implicit understanding that these two assets represent the two segments of the economy.

parts of the dividend news as business cycle news. Since the market price of risk for business cycle risk is high, this increases the premium paid on business cycle risk. This latter effect on the expected return dominates, so that a poorer information quality also induces a higher risk premium on cyclical assets.

The impact of information quality on the return variances depends on the asset. While the return variance of the cyclical asset increases during uncertain times, the return variance of the defensive asset decreases. First, poorer information quality leads to a less volatile perceived business cycle which lowers the return volatility of both assets. For the cyclical asset, positive $B$-dividend innovations are double good news, since they do not only imply a higher level of current dividends, but also suggest better economic conditions due to a higher expected level of future dividends. This induces a larger reaction of the cyclical asset to $B$-dividend innovations which increases the volatility of its return. For the defensive asset, positive B-dividend innovations first lead to a higher price, since they decrease the relative size of defensive assets and since small assets are more valuable.[4] With learning, they also imply good news about the growth rate, which are actually bad news for the defensive asset, whose price decreases. Learning thus dampens the reaction to B-dividend innovations and thereby also the volatility.

Our paper is related to the literature on asset pricing in economies with two or more Lucas trees. Cochrane, Longstaff, and Santa-Clara (2008) study a two tree model with i.i.d. dividends and a representative investor with log preferences. Martin (2013) extends their setup to the case of general CRRA preferences and furthermore to an economy with $n$ trees. In an economy similar to ours, Branger, Schlag, and Wu (2011) look at the impact of learning about a stochastic growth rate under CRRA preferences. Branger, Dumitrescu, Ivanova, and Schlag (2012) analyze the impact of Epstein-Zin preferences in a two tree economy, but assume that the state variables are perfectly observable.

Our paper is also related to the literature which studies the impact of learning and information quality on asset prices. Pastor and Veronesi (2009) review the impact of learning in financial markets. Veronesi (2000) looks at an economy with $n$ assets whose dividend growth rates are stochastic and modeled by a hidden Markov process. The CRRA investor cannot observe the current growth rates, but relies on a signal to infer the current states. He finds decreasing equity premia if information quality gets worse. Brevik and d'Addona (2010) consider an investor with recursive preferences in this economy and find contrary results. The discussion in their pa-

[4]See Cochrane, Longstaff, and Santa-Clara (2008).

per indicates that recursive preferences are necessary to get economically plausible premia on information uncertainty. Ai (2010) looks at an Epstein-Zin-investor in a production economy with multiple assets. He also finds positive premia for information uncertainty. Both papers focus on explaining market equity premia and interest rates. In contrast to that, our focus is on studying the impact of information quality on cyclical and defensive assets. Johannes, Lochstoer, and Mou (2011) study asset pricing implications of learning not only about the state variable, but also about parameters and model specifications. In a single-tree general equilibrium model they find that innovations in beliefs are strongly related to realized aggregate equity returns. Croce, Lettau, and Ludvigson (2012) calculate prices of dividend claims in a single tree economy in cases of full and limited information. The mechanism behind their results is mainly the same as in our paper: With limited information the investor cannot distinguish between short and long-run risk. However, since in all of the papers named above, endowment from different sectors of the economy does not add to aggregate consumption, it is impossible to take spillover effects into account.

The remainder of the paper is organized as follows. Section 2 empirically investigates the impact of information quality on the return characteristics of cyclical and defensive assets. Section 3 contains the model setup and describes the model solution. In Section 4, we present and explain the results of our numerical example. Section 5 concludes.

# 2 Return predictability

In this section, we analyze the relation between the quality of information about the business cycle, and the expectations and variances of returns on assets that are positively or negatively correlated with the business cycle. The first subsection reviews the empirical literature on the uncertainty-return tradeoff in general. In the second subsection, we analyze the link between our proxy for information quality and the first two return moments.

## 2.1 Literature

A large strand of the literature deals with the impact of dispersion in analysts' earnings forecast on stock returns. Examples are Diether, Malloy, and Scherbina (2002), Johnson (2004), and Avramov, Chordia, Jostova, and Philipov (2009). They show that stocks with higher analysts' forecasts dispersion earn lower future returns. Another strand investigates the connection between the *market wide* level of uncertainty and conditional return expectations on which we focus in the following.

Anderson, Ghysels, and Juergens (2009) provide empirical evidence for an uncertainty-return tradeoff besides the well-known risk-return tradeoff. They find a considerable positive premium on uncertainty. Their measure for uncertainty is based on the quarterly survey conducted by the Federal Reserve Bank of Philadelphia (Fed), in which professional analysts forecast up to 32 economic variables. They construct a time series of expected market returns from this survey of professional forecasters (SPF) and approximate uncertainty by the variation in return expectations among forecasters.

Baltussen, van Bekkum, and van der Grient (2012) use the volatility of option-implied volatilities of index options to measure uncertainty. They do not find a positive premium on this kind of uncertainty. Drechsler (2012) investigates the predictive power of the variance risk premium. He shows that it is high especially for short return horizons. Many other studies, including Bollerslev, Tauchen, and Zhou (2009), Bollerslev, Gibson, and Zhou (2011), and Drechsler and Yaron (2011) confirm this finding. He also shows that the variance risk premium is tightly linked to uncertainty, measured by the dispersion in analysts' forecasts of GDP from the SPF. Ulrich (2012) investigates the connection between this uncertainty measure and the risk-free rate and the dividend yield. The latter two studies come closest to ours since they look at the information content of uncertainty in macro variables.

There is thus a considerable evidence for a tight link between uncertainty about a variable, i.e. information quality about the variable's future realizations, and asset returns in general, irrespective of the assets' relationship to that variable. In our paper, we look at the impact of uncertainty about the business cycle (approximated by the dispersion in analysts' GDP forecasts) on assets that are exposed to the business-cycle (cyclical assets) and assets that are not (defensive assets).

## 2.2 Predictive power of forecasting dispersion

As a measure of information quality, we use the dispersion in GDP forecasts from the survey of professional forecasters (SPF).[5] If information quality is close to perfect, there is no reason for dispersion in analysts' forecasts. Hence, we interpret a

[5]Since 1968:Q4, analysts are asked to predict the GDP (GNP from 1968:Q4 to 1991:Q4) of the current and the following four quarters.

low dispersion as an indicator for high information quality. Anderson, Ghysels, and Juergens (2009) justify this intuition by a model in which disagreement is directly related to uncertainty. For our empirical analysis, we use data from 1968:Q4 until 2007:Q4, i.e. we explicitly exclude the financial crisis.

Analysts are asked to predict the GDP (GNP up to 1991:Q4) of the current and the following four quarters. We denote the dispersion in time $i$ forecast of time $j$ GDP by $FD_i^j$. $FD_{t+1}^{t+1}$, e.g., is the dispersion of the forecast of the GDP in quarter $[t, t+1]$, which is based on a questionnaire, sent in the middle of this quarter. $FD_t^{t+1}$ is the dispersion of the forecast based on the questionnaire sent in the middle of the quarter $[t-1, t]$. We first show that $FD_t^{t+1}$ has predictive power for $FD_{t+1}^{t+1}$. The regression

$$FD_{t+1}^{t+1} = \alpha + \beta\, FD_t^{t+1} + \varepsilon_{t+1},$$

gives a coefficient $\beta = 0.4833$ (standard error 0.0258). Since the questionnaires are sent in the middle of a quarter, the dispersion in forecasts of actual periods is systematically lower than forecasts for later periods, since a part of the information is already known at that time. Hence, $\beta < 1$ could have been expected. The $R^2$ is 76.66% which indicates a strong persistence.

We include the dispersion $FD_t^{t+1}$ in time $t$ forecasts of GDP in $t+1$ in the predictive regressions for future returns.

$$r_{t+j} = \alpha + \beta_{FD} FD_t^{t+1} + \beta_{PD} \log(P/D)_t + \beta_{YS} YS_t + \beta_f r_t^f + \varepsilon_{t+1}.$$

Here, $r_{t+j}$ denotes the log excess return on the S&P 500 from the end of quarter $t$ to the end of quarter $t+j$. $\log(P/D)_t$, $YS_t$, and $r_t^f$ denote the time $t$ log price-dividend ratio of the S&P 500, the spread between Aaa and Baa rated bond-yields, and the return on a 3-month treasury bill, which are common control variables in the literature.[6] We also report results if the variable $FD$ is omitted to control for the additional predictive power of this variable.

Table 1 gives the results of the predictive regressions. For all three tested return horizons of one, four and eight quarters, the coefficient of $FD$ is positive, which indicates that high uncertainty about next period's GDP predicts high returns. While $\beta_{FD}$ is not significant for a return horizon of one quarter, it is significant at the 5%- and 1%-level (Newey-West standard errors) for returns horizon of one

[6]See for example Chen, Roll, and Ross (1986), Cremers (2002), Lewellen (2004), and Ang and Bekaert (2007).

and two years. For these horizons, uncertainty about future GDP adds a substantial amount of predictive power. For a return horizon of two years, the $R^2$ more than triples from 7.85% to 24.80%.

We also investigate the impact which uncertainty has on the returns on cyclical and defensive portfolios. Out of 30 industry portfolios[7], we choose 5 that are typically known as cyclical industries and 5 typical defensive industries. Boudoukh, Richardson, and Whitelaw (1994) name *Food and Beverages*, *Tobacco* and *Utilities* as examples of defensive industries. Levhari and Levy (1977) identify 10 defensive and 10 cyclical stocks. The defensive ones are stocks of producers of food, beverages, tobacco, healthcare products, and metal cans (included in French's industry portfolio *Business Supplies*). Makarov and Papanikolaou (2006) state that "Food, Beer, Consumer Goods, and Health (...) are less sensitive to the business cycles". We choose French's portfolios 1 (Food), 2 (Beverage), 6 (Consumer Goods), 8 (Healthcare), and 24 (Business Supplies). As examples of typical cyclical industries Boudoukh, Richardson, and Whitelaw (1994) name *Machines*, *Transportation Equipment*, and *Primary Metals* (corresponding to French's portfolio *Steel*). Levhari and Levy (1977) select firms from the sectors *Steel*, *Mining*, *Oil*, and *Business Equipment*. Hence, we select French's portfolios 12 (Steel), 13 (Machines), 17 (Mining), 19 (Oil), and 23 (Business Equipment).

For the excess returns on each of these ten portfolios, we perform the same regression as for the excess return on the S&P 500 with a return horizon of one year. Results are reported in Table 2. In line with intuition, the returns on the selected defensive portfolios have a negative correlation with quarterly log GDP growth, while those of the selected cyclical portfolios are positively correlated with GDP growth. We find positive coefficients of $FD$ throughout all investigated portfolios, 8 out of 10 are significant. All portfolios, regardless of the sign of their dependence on GDP innovations, pay an additional premium if uncertainty about GDP is high. The substantial increases in the $R^2$'s of almost all portfolios indicate that uncertainty about this macro variable is an important factor in predicting returns.

We also investigate the predictability of return variances of these portfolios. For each asset $i$, we perform the regression

$$RV_{t+1}(r_i) = \alpha + \beta_{FD} FD_t^{t+1} + \beta_m RV_{t+1}(r_m) + \varepsilon_{t+1},$$

where $RV_{t+1}(r_i)$ denotes the realized return variance of portfolio $i$ from $t$ to $t+1$,

[7]The portfolios are taken from Kenneth French's homepage `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

estimated from daily returns. We control for the variance of the return $r_m$ on the market portfolio (S&P 500) to account for possible heteroscedasticity that is not explained by the level of uncertainty about the GDP. Hence, we test whether the dispersion in analysts' GDP forecasts explains *excess variance* of portfolios over the market variance. Stated differently, we investigate if returns on cyclical and defensive stocks become more or less volatile during times of scarce information about the business cycle compared to a benchmark volatility, which is the volatility of the aggregate stock market.[8]

Table 3 reports the results. We find that the coefficients of $FD$ are positive for all cyclical portfolios while they are negative for the defensive portfolios. Although most of the coefficients are significant, including $FD$ into the regression adds only little explanatory power. The results show that cyclical assets tend to have more volatile returns in periods of high uncertainty, i.e. low information quality. In contrast, returns on defensive assets fluctuate less if information quality is low. This opposite behavior of the second moments of returns on cyclical and defensive assets stands in stark contrast to the behavior of the expected returns.

# 3 The Model

We propose a general equilibrium asset pricing model to explain the impact of information quality on the first two moments of returns on cyclical and defensive assets. In Section 3.2, we introduce a variable $x$, that represents the business cycle and two dividend dynamics. One of both depends on the current realization $x_t$ of the business cycle which makes the asset exposed to long run risk which is priced because the investor has recursive preferences (Section 3.1). In Section 3.3, we account for uncertainty by letting $x$ be latent. The level of information quality about $x$ depends on an additional signal, which provides more or less noisy information.

[8]Another interesting approach would be to use the VIX and thus information from the option market. This would allow a prediction of return variances, not only excess variances. However, the VIX is not available for periods before 1990.

## 3.1 The Investor

We consider a representative investor with Epstein-Zin (EZ) preferences.[9] Her value function $J$ is

$$J_t = E_t \left[ \int_t^\infty f(C_s, J_s)\, ds \right], \tag{1}$$

where $f(C,J)$ is the normalized aggregator function that aggregates current consumption and the continuation value

$$f(C,J) = \delta \left(1 - \frac{1}{\psi}\right)^{-1} C^{1-\frac{1}{\psi}} \left[(1-\gamma) J\right]^{1-\frac{1}{\theta}} - \delta\theta J.$$

$\delta$ is the subjective time preference rate, $\gamma$ is the coefficient of relative risk aversion, and $\psi$ denotes the intertemporal elasticity of substitution (IES). The parameter $\theta$ is defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Recursive utility allows to disentangle relative risk aversion and IES. In the following, we assume $\gamma > 1$ and $\psi > 1$. This implies $\theta < 0$, so that the investor has a preference for early resolution of uncertainty.

## 3.2 The Economy

We consider a continuous-time pure exchange economy. The business cycle variable $x$ follows an Ornstein-Uhlenbeck process

$$dx_t = \kappa_x (\overline{x} - x_t)\, dt + \sigma_x\, dW_{x,t}$$

with long-run mean $\bar{x}$ and mean-reversion speed $\kappa_x$. There is one perishable consumption good which serves as the numeraire. This consumption good is produced by two Lucas trees, which stand for two different sectors of the economy.[10] We denote the dividends of these trees by $B$ (business cycle related) and $N$ (non-business cycle related) in the following. They follow the diffusion processes

$$\begin{aligned} \frac{dB_t}{B_t} &= x_t\, dt + \sigma_B\, dW_{B,t} \\ \frac{dN_t}{N_t} &= \mu_N\, dt + \sigma_N\, dW_{N,t}. \end{aligned}$$

[9] See e.g. Epstein and Zin (1989) for the discrete-time setup and Duffie and Epstein (1992) for the extension to continuous time.

[10] This setup is similar to Cochrane, Longstaff, and Santa-Clara (2008), Branger, Schlag, and Wu (2011) and Martin (2013).

The dividend volatilities $\sigma_N$ and $\sigma_B$ are constant. The expected growth rate $\mu_N$ of asset $N$ is constant, too, while the expected growth rate of asset $B$ coincides with the business cycle variable. The Wiener processes $W_N$, $W_B$, and $W_x$ are independent.[11] We assume that $N_t$, $B_t$ and $x_t$ are adapted to a sigma algebra $\mathcal{F}_t$, which contains all information available at time $t$.

The model is a long-run risk model (see e.g. Bansal and Yaron (2004)) with the business cycle variable as the long-run risk factor. A shock in the *short-run risk factors* $W_N$ or $W_B$ has an immediate impact on the level of dividends, but does not change their future dynamics. In contrast, a shock in the *long-run risk factor* $W_x$, i.e. in the business cycle, has no immediate impact on consumption, but influences the future dynamics of the dividend processes.

In equilibrium, aggregate consumption is given by $C_t = N_t + B_t$ with dynamics

$$\frac{dC_t}{C_t} = [p_t\mu_N + (1-p_t)x_t]\,dt + p_t\sigma_N\,dW_{N,t} + (1-p_t)\sigma_B\,dW_{B,t},$$

where $p_t = \frac{N_t}{N_t+B_t}$ is the consumption share of asset $N$. Its dynamics are given by

$$dp_t = p_t(1-p_t)\left\{\left[\mu_N - x_t + (1-p_t)\sigma_B^2 - p_t\sigma_N^2\right]\,dt + \sigma_N\,dW_{N,t} - \sigma_B\,dW_{B,t}\right\}.$$

Changes in the state variables $x$ and $p$ change the level and volatility of the expected growth rate of aggregate consumption $\mu_{C,t}$ as well as its local volatility $\sigma_{C,t}$:

$$\begin{aligned}\mu_{C,t} &= p_t\mu_N + (1-p_t)x_t\\ \sigma_{C,t} &= \sqrt{p_t^2\sigma_N^2 + (1-p_t)^2\sigma_B^2}.\end{aligned}$$

## 3.3 Learning

In general, the investor cannot observe the business cycle $x$. She can learn about it from observing the dividend realizations $B$ of the cyclical asset. Since we are interested in the impact of information quality, we furthermore assume that there is an additional signal $s$ with dynamics

$$\frac{ds_t}{s_t} = x_t\,dt + \sigma_{s,t}\,dW_{s,t}.$$

The drift of the signal is equal to $x$, so that it contains information about the business cycle. For ease of exposition, we assume that the standard Brownian motions $W_N$, $W_B$, $W_x$, and $W_s$ are mutually independent.

[11] The model can easily be generalized to the case of correlated processes. However, this makes the notation and the interpretation more involved without adding to the main points of the paper.

The lower the conditional volatility $\sigma_{s,t}$ of the signal, the higher its quality. In the limiting case $\sigma_{s,t} = 0$, the investor can perfectly observe the business cycle variable $x$, and the filtered dynamics coincide with the true dynamics of the model. If $\sigma_{s,t} \to \infty$, the signal becomes useless, and the investor has to rely only on observations of the dividends of asset $B$ to learn about the business cycle.

In line with our findings in Section 2, we assume that the noise of the signal $(\sigma_{s,t})_t$ is a highly persistent process. Since we want to highlight this feature, we assume that the investor considers $\sigma_{s,t}$ to be constant, i.e. maximally persistent. We follow Cogley and Sargent (2008) and Johannes, Lochstoer, and Mou (2011) and apply the "anticipated utility approach" by Kreps (1998).

The investor's estimate of the business cycle variable is $\widehat{x}_t = E[x_t \mid \widehat{\mathcal{F}}_t]$, where $\widehat{\mathcal{F}}_t$ denotes the information set of the investor at time $t$. It comprises past realizations of the dividends and the signal, but not (in contrast to the larger set $\mathcal{F}_t$) the business cycle variable itself. We assume that the investor knows the structure of the model and all its parameters. The only unknown she has to infer is thus the business cycle variable $x$. Furthermore, we assume that the investor starts with a prior belief about $x$ which is normally distributed with mean $x_0$ and variance $\sigma_{x,0}^2$ which is equal to the steady-state variance.

The processes under the investor's filtration $\widehat{\mathcal{F}}_t$ are:[12]

$$\frac{dN_t}{N_t} = \mu_N \, dt + \sigma_N \, dW_{N,t} \tag{2}$$

$$\frac{dB_t}{B_t} = \widehat{x}_t \, dt + \sigma_B \, d\widehat{W}_{B,t} \tag{3}$$

$$\frac{ds_t}{s_t} = \widehat{x}_t \, dt + \sigma_{s,t} \, d\widehat{W}_{s,t} \tag{4}$$

$$d\widehat{x}_t = \kappa_x \left(\overline{x} - \widehat{x}_t\right) \, dt + \sigma_{xB,t} \, d\widehat{W}_{B,t} + \sigma_{xs,t} \, d\widehat{W}_{s,t}. \tag{5}$$

The sensitivities $\sigma_{xB,t}$ and $\sigma_{xs,t}$ of the filtered variable $\widehat{x}$ will be given below. $\widehat{W}_B$ and $\widehat{W}_s$ are standard Brownian motions under the filtration $\widehat{\mathcal{F}}_t$. Intuitively, the investor uses the current estimate of the business cycle $\widehat{x}$ to infer the innovations $d\widehat{W}_{B,t}$ in the $B$ dividend and $d\widehat{W}_{s,t}$ in the signal from Equations (3) and (4). She then relies on these innovations to update her estimate of $\widehat{x}$ beyond the deterministic mean-reversion component. Positive innovations in the $B$ dividend can be due to positive innovations $dW_B$ in $B$, but may also indicate that the business cycle variable is larger than its current estimate. The investor thus revises her estimate $\widehat{x}$ upwards,

[12] A detailed derivation of the filter equation can be found in Appendix B.

where the strength of her reaction to $d\widehat{W}_{B,t}$ is given by $\sigma_{xB,t}$. The same holds true for positive innovations in the signal.

The sensitivities are given by

$$\sigma_{xB,t} = \frac{\sqrt{\kappa_x^2 + \sigma_x^2 \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_{s,t}^2}\right)} - \kappa_x}{\sigma_B \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_{s,t}^2}\right)} \tag{6}$$

$$\sigma_{xs,t} = \frac{\sqrt{\kappa_x^2 + \sigma_x^2 \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_{s,t}^2}\right)} - \kappa_x}{\sigma_{s,t} \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_{s,t}^2}\right)}. \tag{7}$$

They depend on the uncertainty parameters of the model, and, in particular, on the information quality $\sigma_{s,t}$. Figure 1 depicts the dependence of the squared sensitivities $\sigma_{x,B}^2$ and $\sigma_{x,s}^2$ on $\sigma_{s,t}$. The larger $\sigma_{s,t}$, the less informative the signal is about the business cycle. Consequently, the reaction $\sigma_{xs,t}$ of the investor to news in the signal $s$ decreases in $\sigma_{s,t}$. At the same time, the investor relies more on dividend innovations to learn about the business cycle and her reaction $\sigma_{xB,t}$ increases in $\sigma_{s,t}$.

For $\sigma_{s,t} \to \infty$, it holds that $\sigma_{xs,t} \to 0$, while $\sigma_{xB,t}$ converges to the finite positive value

$$\lim_{\sigma_{s,t}\to\infty} \sigma_{xB,t} = \sigma_B \left( \sqrt{\kappa_x^2 + \frac{\sigma_x^2}{\sigma_B^2}} - \kappa_x \right).$$

For $\sigma_{s,t} = 0$, the investor can perfectly observe the business cycle. It holds that $\widehat{x} = x$, and we can replace the filtered dynamics by the true dynamics. For the sake of brevity we include this special case in our setup (2) - (5) and therefore replace Equation (5) by

$$d\widehat{x}_t = \kappa_x \left(\overline{x} - \widehat{x}_t\right) dt + \sigma_{xB,t}\, d\widehat{W}_{B,t} + \sigma_{xs,t}\, d\widehat{W}_{s,t} + \sigma_{xx,t}\, dW_{x,t}.$$

For $\sigma_{s,t} > 0$, we set $\sigma_{xx,t} = 0$ (the innovation $dW_{x,t}$ is not observable then) and define $\sigma_{xB,t}$ and $\sigma_{xs,t}$ via Equations (6) and (7). For $\sigma_{s,t} = 0$, we set $\sigma_{xx,t} = \sigma_x$, while $\sigma_{xB,t} = \sigma_{xs,t} = 0$ (there is no need to learn about the observable business cycle from dividend innovations, and with zero noise, there is no way to infer $d\widehat{W}_{s,t}$ from the signal). We define $\Sigma_{x,t} := (0, \sigma_{xB,t}, \sigma_{xx,t}, \sigma_{xs,t})'$. $||\Sigma_{x,t}||^2$ is the local variance of the inferred business cycle.

When it comes to asset pricing, two implications of learning will turn out to be important. First, the local variance $||\Sigma_{x,t}||^2$ of the business cycle variable (which is also depicted in Figure 1) decreases in $\sigma_{s,t}$. The higher the volatility of the signal, the less additional information the investor gets about the business cycle beyond $B$

dividends, and the smoother her estimate $\widehat{x}_t$. A higher volatility of the signal thus reduces the amount of long-run business cycle risk in the economy.

Second, innovations in the $B$ dividend are uncorrelated with innovations in the true business cycle $x_t$, but they are positively correlated with innovations in the estimated business cycle $\widehat{x}_t$. This correlation is increasing in $\sigma_{s,t}$, i.e. it is the larger the less precise the signal and the more the investor has to rely on $B$ innovations to learn about $x$. This changes the character of innovations in the dividend. With perfect information, these innovations are pure short-run risk, with no impact on the future dynamics of the $B$ dividend. With learning, they partly become long-run risk, since they do not only drive the current level of the dividend, but also its estimated future dynamics.

## 3.4 Model solution

In equilibrium, the representative investor has to consume the aggregate consumption $C_t = N_t + B_t$. Her wealth is given by the price of the claim to aggregate consumption. The log wealth-consumption ratio $v$ is a function of the state variables $p_t$ and $\widehat{x}_t$. From (1), we get

$$E_t \left[ dJ_t + f(C_t, J_t)\ dt \right] \;=\; 0. \tag{8}$$

Motivated by Campbell, Chacko, Rodriguez, and Viceira (2004) and Benzoni, Collin-Dufresne, and Goldstein (2011), we employ the guess

$$J_t \;=\; \frac{C_t^{1-\gamma}}{1-\gamma}\,\delta^\theta\, e^{\theta\, v(p_t,\widehat{x}_t)},$$

where $v\colon\, ]0,1[\times\mathbb{R} \longrightarrow \mathbb{R}$ is a continuous function which grows more slowly than the logarithm if $p$ approaches 0 or 1 (this includes, e.g., all polynomials in $p$).[13] As shown in the papers cited above, the quantity $v(p_t, \widehat{x}_t)$ is the log wealth-consumption ratio. Plugging this guess into Equation (8) results in a partial differential equation (PDE) for $v_t := v(p_t, \widehat{x}_t)$, which is given in Equation (18) in Appendix C. There, we also discuss the numerical solution and the boundary conditions for the PDE.

Following Duffie and Skiadas (1994), the pricing kernel in our economy is:

$$\xi_t \;=\; C_t^{-\gamma}\,\delta^\theta\, e^{-\delta\theta t-(1-\theta)\left(\int\limits_0^t e^{-v_s}ds+v_t\right)} \tag{9}$$

[13]This implies that the first derivative can at most have a pole of order 1 at $p = 0$ or $p = 1$, and that the second derivative can at most have a pole of order 2. This assures that the terms $p(1-p)\frac{\partial v_t}{\partial p_t}$ and $p^2(1-p)^2\frac{\partial^2 v_t}{\partial \widehat{\mu}_t^2}$ go to zero, so that the PDE (18) collapses to the corresponding PDE in the one-tree economy.

The risk-free interest rate $r_t^f$ is the negative drift of the pricing kernel. Applying Itô's lemma to (9) leads to

$$\begin{aligned} r_t &= \delta + \frac{1}{\psi}\,\widehat{\mu}_{C,t} - \frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\sigma_C^2 \\ &\quad -\frac{1}{2}(1-\theta)\Big[2p_t(1-p_t)\left(p_t\sigma_N^2-(1-p_t)\sigma_B^2\right)\upsilon_p + p_t^2(1-p_t)^2(\sigma_N^2+\sigma_B^2)(\upsilon_p)^2 \\ &\qquad + 2(1-p_t)\left(1-p_t\upsilon_p\right)\upsilon_{\widehat{x}}\sigma_B\sigma_{xB,t} + (\upsilon_{\widehat{x}})^2||\Sigma_{x,t}||^2\Big], \end{aligned} \tag{10}$$

where $\upsilon_p = \frac{\partial \upsilon_t}{\partial p_t}$ and $\upsilon_{\widehat{x}} = \frac{\partial \upsilon_t}{\partial \widehat{x}_t}$ denote partial derivatives, and $\widehat{\mu}_{C,t} = p_t\mu_N + (1-p_t)\widehat{x}_t$ is the perceived growth rate of aggregate consumption. If the investor's preferences display constant relative risk aversion (i.e. $\gamma = \frac{1}{\psi}$), we end up with the usual CRRA risk free rate, which is given by the first line of (10). We will look at the risk free rate more thoroughly in Section 4.5.

The market prices of risk follow from the exposures of the pricing kernel to the risk factors. They are given in Equation (13) and discussed in detail in Section 4.4.

Given the pricing kernel $\xi$, we can price any asset. In particular, we are interested in the prices of asset $N$ and asset $B$. For a generic asset with dividend stream $D$ given by

$$\frac{dD_t}{D_t} = \mu_{D,t}\,dt + \sigma_{D,t}\,dW_t,$$

the price at time $t$ is

$$P_t = E_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\,D_s\,ds\right].$$

If we denote the log price-dividend ratio by $w\left(p_t,\widehat{x}_t\right)$, the pricing equation becomes

$$\xi_t\,D_t\,e^{w(p_t,\widehat{x}_t)} = E_t\left[\int_t^\infty \xi_s\,D_s\,ds\right]. \tag{11}$$

Applying the formula of Feynman-Kac to Equation (11) leads to the PDE (19) in Appendix C. It can be solved similarly to Equation (18).

# 4 Quantitative Analysis of the Model

In this chapter we investigate whether the model proposed in Section 3 is able to explain the two stylized facts discussed in Section 2. First, expected excess returns should be larger on cyclical as well as on defensive assets if information quality is low. Second, the return variance of cyclical assets should increase during uncertain times, while that of defensive assets should decrease.

To match return moments, many asset pricing studies define dividends as levered consumption. We do not aim to match first and second moments of returns on a large set of portfolios. We rather investigate the returns on the two generic assets defined in Section 3 and focus on explaining the mechanisms that link return moments to the quality of information.

## 4.1 Parameters

The preference and model parameters for our numerical example are given in Table 4.[14] Since our focus is on the impact of information quality about the business cycle, we assume that the two trees are – apart from one having a deterministic and the other one having a stochastic growth rate – as similar as possible. Both dividend processes grow on average by 1.8% each year. The short-run volatility of the dividends is 3.52%, which results in a consumption volatility of 2.5% if both trees account for half of the economy. A one standard deviation shock in $x_t$ changes the expected growth rate of the $B$ dividends by 1.5% and that of aggregate consumption by up to 1.5%, where the impact increases in the relative size of the cyclical asset $B$. The mean-reversion speed is set to 0.3, so that business cycle shocks have a half-life of slightly more than 2 years.

In the following, we compare three cases which differ in the amount of information the investor has access to. In the *perfect signal* case ($\sigma_{s,t} = 0$) the business cycle is perfectly observable from the signal, and the filtered dynamics coincide with the true dynamics. In the *noisy signal* case, the signal is less than perfect (we set $\sigma_{s,t} = 0.015$), and information quality is lower than before. The investor now has to learn about the business cycle $x$ from both the dividend realizations of asset $B$ and the signal. Finally, we consider a case in which the signal has infinite noise ($\sigma_{s,t} = \infty$). The investor then relies only on the dividend of asset $B$ to infer $x$, and we call this the *no signal* case.

[14]These parameters are based on those used in Bansal, Kiku, and Yaron (2012), who consider a discrete time model with stochastic growth, stochastic volatility and jumps in the state variable. We focus on a stochastic growth rate only, which also implies that the risk premia in our model are somewhat lower than in the data.

## 4.2 Valuation Ratios

The wealth-consumption ratio and the price-dividend ratios of the two assets are depicted in Figure 2. The upper row gives the ratios as a function of the consumption share $p$ of asset $N$, while $\widehat{x}$ is set equal to its long-run mean $\overline{x}$. The lower row gives the ratios as a function of the business cycle variable $\widehat{x}$ when each asset accounts for half of the economy. The dependence of the valuation ratios on the consumption share displays the typical pattern known from the literature on two tree models.[15] The price-dividend ratios of the two assets are the larger the smaller their respective consumption share. The less they contribute to aggregate consumption risk, the more they become a safe haven and the more valuable they are.[16] The wealth-consumption ratio is a concave function of $p_t$ and largest for intermediate values of $p_t$. The reason is that aggregate consumption volatility is smallest for $p_t$ around 0.5.

The price-dividend ratio of the cyclical asset is lower than the one of the defensive asset with the same size. Its business cycle dependence lowers its attractiveness while asset $N$ offers a hedge against business cycle risk and is therefore more valuable. In line with that, the wealth-consumption ratio for high consumption shares of the cyclical asset $B$ ($p \to 0$) is lower than for high consumption shares of the defensive asset $N$ ($p \to 1$).

The lower row of Figure 2 shows the dependence of the valuation ratios on the estimated business cycle $\widehat{x}$ if both assets account for half of the economy. In booms (high $x$), the price-dividend ratio of the cyclical asset $B$ and the wealth-consumption ratio are high, while the price-dividend ratio of the defensive asset $N$ decreases. A large $\widehat{x}$ implies that expected future aggregate consumption is large. The larger drift of consumption leads to a higher risk-free rate (see also Equation (10)), which in turn leads to lower prices (sdf effect). For the cyclical asset and aggregate wealth, a higher $\widehat{x}_t$ also implies higher expected future cash flows. This increases the valuation ratios (cash flow effect). For asset $N$, there is only the sdf effect, and consequently, its price decreases in $\widehat{x}_t$. Economically, good news for the cyclical asset are bad news for the defensive asset in relative terms. For the cyclical asset and aggregate wealth, there is both an sdf and a cash flow effect. With an IES $\psi$ above one, the cash flow effect dominates, and prices increase in $\widehat{x}_t$.

Finally, the quality of information about the business cycle basically does not

[15]For a more detailed discussion, see Cochrane, Longstaff, and Santa-Clara (2008), Branger, Dumitrescu, Ivanova, and Schlag (2012) and Martin (2013).

[16]The price of course still goes to zero when its dividend goes to zero.

matter. The dependence of the valuation ratios on the consumption share $p$ and the estimated drift $\widehat{x}$ barely depends on the level of signal noise. Nevertheless, we will see that the dynamics of prices and the characteristics of returns depend strongly on information quality.

## 4.3 Return Exposures

To understand equity premia and return volatilities of the different assets, we look at their exposures to the different sources of risk in the economy, which are shown in Figure 3.

The right column of Figure 3 gives the exposure of the return on the cyclical claim to $B$ dividends. Its price is $B_t e^{\omega_t^B}$, where $\omega^B$ is the log price-dividend ratio. The exposure of the return is thus

$$\sigma_t^B = \begin{pmatrix} \sigma_{N,t}^B \\ \sigma_{B,t}^B \\ \sigma_{x,t}^B \\ \sigma_{s,t}^B \end{pmatrix} = \begin{pmatrix} 0 \\ \sigma_B \\ 0 \\ 0 \end{pmatrix} + \omega_p^B p_t(1-p_t) \begin{pmatrix} \sigma_N \\ -\sigma_B \\ 0 \\ 0 \end{pmatrix} + \omega_{\widehat{x}}^B \begin{pmatrix} 0 \\ \sigma_{xB,t} \\ \sigma_{xx,t} \\ \sigma_{xs,t} \end{pmatrix},$$

where $\omega_p^B$ and $\omega_{\widehat{x}}^B$ denote the partial derivatives of the log price-dividend ratio with respect to the consumption share $p$ and the estimated business cycle variable $\widehat{x}$.

The first term captures the *cash flow effect*, i.e. positive shocks to the $B$ dividend increase the price of asset $B$. The second term describes the *size effect*. Smaller assets are less exposed to aggregate consumption risk and thus more valuable. The size effect lowers the positive exposure to $B$-shocks and, analogously, induces a positive exposure to shocks in the other asset, i.e. in $N$.[17] The impact of learning and information quality shows up in the last term, driven by what we call the *outlook effect* in the following. As this effect causes a large part of the results, we now explain it in detail.

As argued in Section 4.2, the price of asset $B$ is increasing in the business cycle variable $\widehat{x}$, so that $\omega_{\widehat{x}}^B$ is positive. If the business cycle is observable (shown by the solid line in the graphs), the outlook effect induces a positive exposure to shocks in $x$. If the investor learns about $x$, the exposure to the (non-observable) shocks in $x$ of course drops to zero. It is replaced by a positive exposure to shocks in $\hat{x}$, i.e. shocks

[17] The exposure to $N$- and $B$-risk are both positive, since the cash flow effect dominates the size effect. An intertemporal elasticity of substitution above one is crucial for that, c.f. Branger, Dumitrescu, Ivanova, and Schlag (2012).

in the $B$ dividend and in the signal. Learning thus increases the already positive exposure to $B$ innovations and adds a positive exposure to $s$-innovations (shown by the dotted line). The lower the information quality, the more the investor relies on $B$ dividends to estimate $\widehat{x}_t$, which further increases the exposure to $B$. Moreover she learns less from the signal, which lowers the exposure to $s$-innovations. In the limit the signal becomes worthless, the exposure to $s$ drops to zero, and the exposure to $B$ innovations is largest (shown by the dashed line in the graphs). Learning therefore amplifies the exposure to short run dividend shocks in $B$, which the investor partly misinterprets as long run business cycle shocks. Besides this change in the structure of the exposures, information quality also has a second effect. The lower the information quality, the lower the absolute volatility of the estimated business cycle, as also discussed at the end of Section 3.3. Therefore, the additional exposure to $B$ innovations due to learning from $B$ is smaller than the exposure to $x$-innovations which it replaces.

The exposure of the return on the defensive asset $N$ is shown in the middle column of Figure 3 and given by

$$\sigma_t^N = \begin{pmatrix} \sigma_N \\ 0 \\ 0 \\ 0 \end{pmatrix} + \omega_p^N p_t(1-p_t) \begin{pmatrix} \sigma_N \\ -\sigma_B \\ 0 \\ 0 \end{pmatrix} + \omega_{\widehat{x}}^N \begin{pmatrix} 0 \\ \sigma_{xB,t} \\ \sigma_{xx,t} \\ \sigma_{xs,t} \end{pmatrix}. \tag{12}$$

The structure and the mechanisms are similar to those of the cyclical asset. The main difference is that the price of $N$ decreases in $\widehat{x}$ (c.f. Section 4.2). Therefore, in the *perfect signal* case, the defensive asset $N$ has a negative exposure to business cycle innovations. If $x$ is no longer observable, the negative exposure shifts to signal- and $B$ innovations. Positive dividend news for the cyclical asset $B$ are then both good and bad news. First, they decrease the consumption share of the defensive asset, which increases its price. Second, they are interpreted as good business cycle news, which are bad news for the defensive asset in relative terms, so that its price decreases. The worse the information quality and the more the investor learns from $b$ dividend realizations, the more the second effect lowers the initially positive exposure to $B$ innovations.

Finally, we turn to the exposure of the return on wealth, which is shown in the left column of Figure 3. The cash flow effect with respect to dividend innovations is increasing in the consumption share of the respective asset. The size effect is rather small, since the wealth-consumption ratio has a much lower dependence on the consumption share $p$ than the individual price-dividend ratios. Finally, the outlook effect has the same qualitative impact on the exposures as for the cyclical asset $B$. The absolute size of its impact is increasing in the consumption share $1-p_t$ of asset $B$.

## 4.4 Risk premia

The risk premium of asset $i$ ($i \in \{N, B, W\}$) is given by

$$RP_t^i = \lambda_t' \cdot \sigma_t^i = \lambda_{N,t}\sigma_{N,t}^i + \lambda_{B,t}\sigma_{B,t}^i + \lambda_{x,t}\sigma_{x,t}^i + \lambda_{s,t}\sigma_{s,t}^i,$$

where $\lambda_t$ denotes the market prices of risk and $\sigma_{j,t}^i$ is the exposure of the return on asset $i$ to risk factor $j$ ($j \in \{N, B, x, s\}$).

The market prices of risk are shown in Figure 4 and given by

$$\lambda_t = \gamma \begin{pmatrix} p_t\sigma_N \\ (1-p_t)\sigma_B \\ 0 \\ 0 \end{pmatrix} + (1-\theta)\upsilon_p p_t(1-p_t) \begin{pmatrix} \sigma_N \\ -\sigma_B \\ 0 \\ 0 \end{pmatrix} + (1-\theta)\upsilon_{\widehat{x}} \begin{pmatrix} 0 \\ \sigma_{xB,t} \\ \sigma_{xx,t} \\ \sigma_{xs,t} \end{pmatrix} \quad (13)$$

The first term captures the risk premium for short-run consumption risk. This premium is proportional to the relative risk aversion $\gamma$ and to the contribution of each asset to aggregate consumption risk. The second term is a risk premium for the risk factor ‘relative size’, which drives the variance of aggregate consumption. This premium vanishes for time-additive CRRA preferences.

The third term gives the market prices of long-run business cycle risk which also vanishes in case of CRRA utility. The level of information quality determines the risk factors driving the business cycle and thus the risk factors which earn a premium. If $x_t$ is observable (i.e. $\sigma_{s,t} = 0$), it is only driven by shocks in the Brownian motion $W_{x,t}$, and the business cycle risk premium is paid exclusively on this factor. As information quality deteriorates (i.e. $\sigma_{s,t} > 0$), shocks in the state variable are no longer observable, but the investor updates her estimate $\widehat{x}_t$ based on the innovations in the $B$ dividend and in the signal. The market price of business cycle risk is then paid for $B$- and $s$-shocks. The worse the information quality, the higher the contribution of business-cycle risk to the market price of risk of $B$, and the smaller the market price of risk of signal innovations. The crucial point to keep in mind for the analysis of expected excess returns is that $B$ innovations do not only command a risk premium on short-run consumption risk, but also on long-run business cycle risk if information quality is low.

Figure 5 shows the risk premia on both assets and on wealth. The risk premia on assets $N$ and $B$ are increasing in their respective consumption shares. The defensive asset $N$ has a positive exposure to innovations in its own dividend, on which it earns a positive premium. Since the market price of risk for innovations in $N$ is increasing in $N$'s consumption share $p_t$, i.e. in its contribution to aggregate consumption risk, the premium earned by asset $N$ on its exposure to $N$ innovations is increasing in $p_t$, too.A similar result holds for the risk premium on the cyclical asset $B$. The premium on business cycle risk is positive for the cyclical asset $B$, while it is negative for the defensive asset $N$ which hedges business cycle risk. Consequently, the risk premium of the cyclical asset $B$ is larger than the risk premium earned by an equally large defensive asset $N$. For aggregate wealth, the premium on business cycle risk is increasing in the consumption share of asset $B$. Furthermore, the volatility of aggregate consumption and thus also its risk premium is a convex function of $p_t$.

If the investor has to learn about the business cycle, risk premia on both assets and on wealth increase. As in our empirical study, risk premia on both kinds of assets are the larger, the lower the quality of information about the business cycle.

To understand the rationale, we look at the two effects of learning. First, the volatility of the inferred business cycle decreases, which lowers the overall market price of long-run risk. Second, since the business cycle has to be estimated from signal and dividend observations, the premium on long-run business cycle risk is no longer paid via a premium for $x$ innovations, but via a premium for signal and for $B$ innovations. When information quality deteriorates, the market price of risk for $B$ innovations increases, since they now also carry (part of) the compensation for long-run business cycle risk, and the market price of risk for signal innovations decreases. At the same time, the exposures to the different risk factors change, too, as described in Section 4.3.

The risk premium on asset $B$ can be rewritten as

$$RP_t^B = \alpha_{N,t}^B \sigma_N^2 + \alpha_{B,t}^B \sigma_B^2 + \alpha_{Bx,t}^B \sigma_B \sigma_{xB,t} + \alpha_{x,t}^B ||\Sigma_{x,t}||^2, \tag{14}$$

where the coefficients $\alpha_{j,t}^B$, $j \in \{N, B, Bx, x\}$ are given in Equation (20) in Appendix D. The risk premium depends on the size of dividend shocks ($\sigma_B$ and $\sigma_N$), business cycle shocks ($\Sigma_x$), and on the covariance between dividend and business cycle innovations ($\sigma_B \sigma_{xB}$). Learning lowers the overall volatility of the business cycle and thus also the risk premium, as can be seen from the last term in Equation (14). The shift in the structure of exposures and of the market prices of risk causes the "co-variance" term, which adds a significant risk premium. The investor can no longer separate long-run business cycle risk from short-run dividend risk. She thus claims an additional long-run risk premium on the short-run part of her exposure to $B$, and she claims an additional short-run risk premium on the long-run part of her exposure to $B$.

Intuitively, if information quality is low, short-run shocks in the dividend of the cyclical asset $B$ earn an additional premium because they are interpreted as long-run business cycle shocks *and* the investor reacts even more sensitive to them. Both effects taken together overcompensate the decrease in the pure long-run risk premium due to a less volatile perceived business cycle.

The same mechanisms yield an increase of the risk premium on wealth. However, the effect is less pronounced than for the business cycle related asset, since the exposure to $B$ innovations is smaller.

To explain the risk premium of asset $N$, we again consider the two effects explained above

$$RP_t^N = \alpha_{N,t}^N \sigma_N^2 + \alpha_{B,t}^N \sigma_B^2 + \alpha_{Bx,t}^N \sigma_B \sigma_{xB,t} + \alpha_{x,t}^N ||\Sigma_{x,t}||^2, \tag{15}$$

where the coefficients can again be found in Appendix D. The exposure of asset $N$ to $x$-innovations is negative, so that long-run risk has a negative contribution to $RP^N$. Therefore, the lower volatility of the perceived $\widehat{x}_t$ causes an increase in the risk premium of $N$. In contrast to asset $B$, the shift in the structure of exposures and market prices of risk now has a negligible impact. While the market price of $B$-risk increases, the positive exposure to $B$-risk decreases.

Intuitively, the defensive asset $N$ provides a hedge against fluctuations in the business cycle (i.e. innovations in $x$), which lowers its risk premium. If information quality deteriorates, perceived business cycle risk decreases and asset $N$ has to pay a higher premium.

In Section 2, we found that risk premia on cyclical *and* defensive stocks increase in times of high macro-economic uncertainty. Our model matches these findings and, furthermore, shows that the explanations are quite different. Cyclical assets pay a higher premium because it is more difficult to distinguish short run from long run innovations, while defensive assets pay a higher premium because overall long run risk decreases which makes them less valuable for hedging.

## 4.5 Risk-free Rate

The risk-free rate is shown in Figure 6 and given by Equation (10). Consumption volatility is a U-shaped function of the consumption share $p_t$, which - via its impact on precautionary savings - implies that the risk-free rate is an inversely U-shaped function of $p_t$. The figure shows that lowering information quality decreases the risk-free rate.

First, learning from a noisy signal (instead of observing the business cycle) reduces the variance of $\widehat{x}$, which lowers the precautionary savings motive and increases the risk-free rate. Second, learning also introduces a positive correlation between the $B$ dividend and $\widehat{x}$ as explained in Section 4.4. This induces a positive correlation between aggregate consumption and the overall state of the economy, so that bad news about consumption are double bad news, which magnifies the precautionary savings motive. In Equation (10) the last two terms capture these counterbalancing effects of learning. Again, the increase in the correlation dominates the reduction in variance. Thus, a lower information quality results in a lower risk-free rate.

## 4.6 Return Volatilities

From the exposures, we can calculate the local volatilities which are depicted in Figure 7. In line with our empirical findings in Section 2, the return volatility of the cyclical asset increases if the quality of information is low, while that of the defensive asset decreases.

The return volatility of the cyclical asset $B$ is shown in the right graph of Figure 7 and given by

$$(\sigma_t^B)' \cdot \sigma_t^B = \beta_{B,t}^B \sigma_B^2 + \beta_{N,t}^B \sigma_N^2 + \beta_{Bx,t}^B \sigma_B \sigma_{xB,t} + \beta_{x,t}^B ||\Sigma_{x,t}||^2, \qquad (16)$$

where the coefficients can be found in Appendix D. The first two terms capture the cash flow effect, which would result in a return volatility equal to the dividend volatility $\sigma_B$, and the size effect. The latter lowers the exposure to innovations in the own dividend and induces a positive exposure to innovations in the other dividend, which in turn lowers the return volatility.

The last two terms capture the impact of the business cycle and of learning. Fluctuations in $x$ represent an additional risk factor the return is exposed to. This last term in Equation (16) increases the return volatility. When the business cycle is no longer observable, but has to be inferred by the investor, the volatility $||\Sigma_{x,t}||$ of

$\widehat{x}$ decreases. At the same time, B-dividend innovations become positively correlated with innovations in the estimated business cycle, i.e. the local covariance $\sigma_B \sigma_{xB,t}$ between the exposures to short-run risk and long-run risk increases. The increase in return volatility due to this positive covariance exceeds the decrease due to the less volatile business cycle. The volatility of the return on the business cycle related asset $B$ is thus higher in case of a lower quality of the signal.

The return volatility of aggregate wealth, which is shown in the left graph in Figure 7, can be explained along the same lines. Similar to the volatility of aggregate consumption, it is lower for intermediate consumption shares than for extreme consumption shares. It also increases if the information quality deteriorates. The increase is the larger, the larger the consumption share of the business cycle related asset, i.e. the larger the impact of the business cycle $x_t$ on the economy.

The return volatility of the defensive asset is shown in the middle graph of Figure 7 and given by

$$(\sigma_t^N)' \cdot \sigma_t^N = \beta_{B,t}^N \sigma_B^2 + \beta_{N,t}^N \sigma_N^2 + \beta_{Bx,t}^N \sigma_B \sigma_{xB,t} + \beta_{x,t}^N ||\Sigma_{x,t}||^2, \tag{17}$$

where the coefficients can be found in Appendix D. In uncertain times, positive shocks in $B$ dividends make the investor revise her estimate $\widehat{x}_t$ upwards, which leads to a decrease in the price of asset $N$ (outlook effect). Thus, the positive exposure due to the size effect is reduced, and the overall exposure to $B$ innovations becomes smaller. In the end, price reactions to signal- and $B$-shocks are therefore less pronounced which leads to a lower return volatility of asset $N$.

# 5 Conclusion

In this paper, we have analyzed the impact of learning and information quality on the prices of cyclical and defensive assets. Empirically, two major stylized facts can be found. First, expected excess returns are larger on cyclical as well as on defensive assets if information quality is low. Second, the return variance of cyclical assets increases during uncertain times, while that of defensive assets decreases. We explain these facts in a general equilibrium model with two assets. The dividends of the cyclical asset in our economy are tightly linked to the business cycle, which we model as an exogenous mean-reverting process. It turns out that the quality of information about the business cycle has significant implications for the joint behavior of the prices of the two assets and for their return characteristics.

For the cyclical asset in our model, both the volatility and the expected excess return increase. Somehow surprisingly, the additional excess return is not caused by a higher premium for long-run risk (the overall amount of long-run risk even decreases due to the lack of information about the business cycle). It can rather be attributed to a higher premium for dividend fluctuations of the cyclical asset, which usually only carry a (rather small) premium for short-run risk. With learning from these dividends about the business cycle, the investor struggles in discriminating between short- and long-run shocks. Hence, dividend innovations also have long-run risk character, which increases their market price of risk significantly.

For the defensive asset in our model, the volatility decreases, while its expected excess return increases. This increase is driven by the lower volatility of the perceived business cycle, which leads to a lower premium for long-run risk. Since the defensive asset provides a hedge against business cycle risk, the lower market price of risk lowers the negative contribution of long-run risk to the risk premium and thus increases the expected excess return. The less volatile perceived business cycle, in combination with the smaller reaction to innovations in dividends of the cyclical asset, also leads to a lower return volatility of the defensive asset in uncertain times.

Our model offers a basis for further extensions. As in Bansal and Shaliastovich (2011), one may introduce jumps in the growth rate of one asset (i.e. the business cycle in our terms), or in the signal as in Shaliastovich (2009).[18] A further natural extension would be to allow the investor to recognize that the volatility of the signal evolves stochastically over time. It would also be interesting to study the term structure of the equity risk premia in our model, as well as the model's implications for bonds and, in particular, for uncertainty-sensitive assets like options.

[18] In contrast to our model, both papers consider a single tree economy, i.e. only look at cyclical assets.

# A Data

**Forecasting dispersion:** We use the dispersion in nominal GDP forecasts as reported in *Dispersion_1.xls* from the homepage of the Philadelphia Fed (`http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files/NGDP`). The dispersion measure is constructed by taking the difference between the 75th and 25th percentile. For a detailed description of the survey data, see the URL above.

**Stock returns:** All stock returns are taken from Kenneth French's homepage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). These include the CRSP value weighted stock return index, which we use as proxy for the return on the stock market and 30 portfolios sorted on industry at the end of June each year. We use quarterly log excess returns. For the calculation of the variance measures, we use daily log returns. For a detailed description of the return data, see the URL above.

**Price-dividend ratio:** The price-dividend ratio (based on the S&P Composite index) is taken from Robert Shiller's homepage (`http://www.econ.yale.edu/~shiller/data.htm`).

**Yield spread:** We use the difference between Moody's seasoned Aaa and Baa rated industrial bond rates from the H.15 release of the Federal Reserve Board of Governors (`http://www.federalreserve.gov/releases/h15/data.htm`).

**Risk-free rate:** We use the 3-month secondary market Treasury bill rate from the H.15 release of the Federal Reserve Board of Governors (`http://www.federalreserve.gov/releases/h15/data.htm`) as risk-free rate.

# B Learning

We employ the Kalman-Bucy scheme in its so-called *conditionally Gaussian* version to calculate the investor's state belief $\widehat{x}_t$. The true model is given by

$$\begin{aligned} d\begin{pmatrix}\log N_t\\ \log B_t\\ \log s_t\end{pmatrix} &= [\begin{pmatrix}\mu_N+\frac{1}{2}\sigma_N^2\\ \frac{1}{2}\sigma_B^2\\ \frac{1}{2}\sigma_{s,t}^2\end{pmatrix}+\begin{pmatrix}0\\1\\1\end{pmatrix}x_t]dt+\begin{pmatrix}\sigma_N&0&0\\0&\sigma_B&0\\0&0&\sigma_{s,t}\end{pmatrix}\begin{pmatrix}dW_{N,t}\\ dW_{B,t}\\ dW_{s,t}\end{pmatrix}, \\ dx_t &= [\kappa_x(\overline{x}-x_t)]dt+\sigma_x\, dW_x \end{aligned}$$

Since the investor's prior belief $\widehat{x}_0$ is assumed to be Gaussian, $\widehat{x}_t$ will also be Gaussian, $\mathcal{N}(m_t,\alpha_t)$. According to Theorem 12.1 of Liptser and Shiryaev (2001) $m_t$ and $\alpha_t$ satisfy the equations

$$\begin{aligned} dm_t &= \kappa_x(\overline{x}-m_t)dt \\ &\quad +\alpha_t\begin{pmatrix}0\\1\\1\end{pmatrix}^t\begin{pmatrix}\sigma_N^{-2}&0&0\\0&\sigma_B^{-2}&0\\0&0&\sigma_{s,t}^{-2}\end{pmatrix}\left\{d\begin{pmatrix}\log N_t\\ \log B_t\\ \log s_t\end{pmatrix}-\left[\begin{pmatrix}\mu_N+\frac{1}{2}\sigma_N^2\\ \frac{1}{2}\sigma_B^2\\ \frac{1}{2}\sigma_{s,t}^2\end{pmatrix}+\begin{pmatrix}0\\1\\1\end{pmatrix}m_t\right]dt\right\} \\ \dot{\alpha}_t &= -2\kappa_x\alpha_t+\sigma_x^2-\left(\alpha_t\begin{pmatrix}0\\1\\1\end{pmatrix}^t\begin{pmatrix}\sigma_N^{-2}&0&0\\0&\sigma_B^{-2}&0\\0&0&\sigma_{s,t}^{-2}\end{pmatrix}\right)^2 \end{aligned}$$

Assuming that we are in the steady state, we can consider $\dot{\alpha}_t = 0$ and solve for $\alpha_t$ which gives

$$\alpha_t = \frac{\sqrt{\kappa_x^2 + \sigma_x^2(\sigma_B^{-2} + \sigma_{s,t}^{-2})} - \kappa_x}{\sigma_B^{-2} + \sigma_{s,t}^{-2}}.$$

Substituting into the above equation gives

$$\begin{aligned} dm_t &= \kappa_x(\overline{x} - m_t)dt \\ &+\alpha_t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}^t \begin{pmatrix} \sigma_N^{-1} & 0 & 0 \\ 0 & \sigma_B^{-1} & 0 \\ 0 & 0 & \sigma_{s,t}^{-1} \end{pmatrix} \left\{ \begin{pmatrix} 0 \\ \sigma_B^{-1}(x_t - m_t) \\ \sigma_{s,t}^{-1}(x_t - m_t) \end{pmatrix} + \begin{pmatrix} dW_{N,t} \\ dW_{B,t} \\ dW_{s,t} \end{pmatrix} \right\} \\ &= \kappa_x(\overline{x} - m_t)dt + \sigma_{xB,t} d\widehat{W}_{B,t} + \sigma_{xs,t} d\widehat{W}_{s,t}, \end{aligned}$$

where

$$\begin{aligned} \sigma_{xB,t} &= \frac{\alpha_t}{\sigma_B} = \frac{\sqrt{\kappa_x^2 + \sigma_x^2(\sigma_B^{-2} + \sigma_{s,t}^{-2})} - \kappa_x}{\sigma_B\ (\sigma_B^{-2} + \sigma_{s,t}^{-2})}, \\ d\widehat{W}_{B,t} &= dW_{B,t} - \frac{m_t - x_t}{\sigma_B} dt, \\ \sigma_{xs,t} &= \frac{\alpha_t}{\sigma_{s,t}} = \frac{\sqrt{\kappa_x^2 + \sigma_x^2(\sigma_B^{-2} + \sigma_{s,t}^{-2})} - \kappa_x}{\sigma_{s,t}\ (\sigma_B^{-2} + \sigma_{s,t}^{-2})}, \\ d\widehat{W}_{s,t} &= dW_{s,t} - \frac{m_t - x_t}{\sigma_{s,t}} dt. \end{aligned}$$

Obviously $\widehat{W}_B$ and $\widehat{W}_s$ are standard Brownian motions under the investor's subjective measure.

# C Model solution

Plugging the guess for the value function $J_t = \frac{C_t^{1-\gamma}}{1-\gamma}\,\delta^\theta\, e^{\theta\, \upsilon(p_t, \widehat{x}_t)}$ in Equation (8) and using Itô's lemma yields the partial differential equation for the log wealth-consumption ratio $\upsilon_t$

$$\begin{aligned} 0 = \;& e^{-\upsilon_t} - \delta + \left(1 - \frac{1}{\psi}\right)\left[\widehat{\mu}_{C,t} - \frac{1}{2}\gamma\sigma_C^2\right] \\ +\;& \left[\kappa_x\left(\overline{x} - \widehat{x}_t\right) + (1-\gamma)(1-p_t)\sigma_B\sigma_{xB,t}\right]\frac{\partial \upsilon_t}{\partial \widehat{x}_t} \\ +\;& p_t(1-p_t)\Big[\mu_N - \widehat{x}_t - \gamma\left(p_t\sigma_N^2 - (1-p_t)\sigma_B^2\right)\Big]\frac{\partial \upsilon_t}{\partial p_t} \\ +\;& \frac{1}{2}\,||\Sigma_{x,t}||^2\left(\frac{\partial^2 \upsilon_t}{\partial \widehat{x}_t^2} + \theta\left(\frac{\partial \upsilon_t}{\partial \widehat{x}_t}\right)^2\right) \\ +\;& \frac{1}{2}\,p_t^2(1-p_t)^2(\sigma_N^2 + \sigma_B^2)\left(\frac{\partial^2 \upsilon_t}{\partial {p_t}^2} + \theta\left(\frac{\partial \upsilon_t}{\partial p_t}\right)^2\right) \\ -\;& p_t(1-p_t)\sigma_B\sigma_{xB,t}\left(\frac{\partial^2 \upsilon}{\partial \widehat{x}_t \partial p_t} + \theta\frac{\partial \upsilon_t}{\partial \widehat{x}_t}\frac{\partial \upsilon_t}{\partial p_t}\right), \end{aligned} \tag{18}$$

where $\widehat{\mu}_{C,t} = p_t\mu_N + (1-p_t)\widehat{x}_t$ denotes the perceived growth rate of aggregate consumption. This PDE has to be solved numerically, where we rely on the method of finite differences. To obtain a unique solution, we need adequate boundary conditions. The condition explained in footnote 13 assures that we are back in the single tree economies in which only asset $B$ or asset $N$ exist, if $p_t$ goes to the limiting values 0 and 1. Since the dividends of the defensive asset $N$ are not exposed to shocks in $\widehat{x}_t$, the single tree economy with asset $N$ is not exposed to $\widehat{x}_t$ as well. Plugging $\upsilon_{\widehat{x}_t} = 0$ and $p_t = 1$ into (18) leads to

$$\lim_{p_t \to 1} e^{\upsilon_t} = \left\{ \delta - \left(1 - \frac{1}{\psi}\right)\left(\mu_N - 0.5\gamma\sigma_N^2\right) \right\}^{-1}$$

The single tree economy in which only asset $B$ exists results in a standard affine asset pricing model with one state variable. Imposing an affine guess for the log price dividend ratio of asset $B$ and utilizing the Campbell-Shiller linearization for its return yields an approximate analytical solution. Eraker and Shaliastovich (2008) provide details on this solution technique.

The price dividend ratio of a generic asset with dividend stream $D$ solves the PDE

$$0 = -r_t + \mu_{D,t} + \mu_{w,t} + \frac{1}{2}||\sigma_{w,t}||^2 - \lambda_t'\left(\sigma_{D,t} + \sigma_{w,t}\right) + \sigma_{D,t}'\sigma_{w,t} + e^{-w_t}, \tag{19}$$

where $\mu_{w,t}$ and $\sigma_{w,t}$ denote the drift and the volatility of the log price dividend ratio, which follow via Itô's lemma. It can be solved similarly to Equation (18).

# D Return moments

The expected equity premium on asset $B$ is given by the scalar product of the vector of market prices of risk and the vector of return exposures of asset $B$ to the different risk factors:

$$RP_t^B = \lambda_t' \cdot \sigma_t^B = \alpha_{N,t}^B\sigma_N^2 + \alpha_{B,t}^B\sigma_B^2 + \alpha_{Bx,t}^B\sigma_B\sigma_{xB,t} + \alpha_{x,t}^B||\Sigma_{x,t}||^2,$$

where

$$\begin{aligned}
\alpha_{N,t}^B &= \left[\gamma p_t + (1-\theta)\upsilon_p p_t(1-p_t)\right] p_t(1-p_t)\omega_p^B, \\
\alpha_{B,t}^B &= \left[\gamma(1-p_t) - (1-\theta)\upsilon_p p_t(1-p_t)\right]\left[1 - p_t(1-p_t)\omega_p^B\right], \\
\alpha_{Bx,t}^B &= \left[\gamma(1-p_t) - (1-\theta)\upsilon_p p_t(1-p_t)\right]\omega_{\widehat{x}}^B + (1-\theta)\upsilon_{\widehat{x}}\left[1 - p_t(1-p_t)\omega_p^B\right], \\
\alpha_{x,t}^B &= (1-\theta)\upsilon_{\widehat{x}}\omega_{\widehat{x}}^B.
\end{aligned} \tag{20}$$

Proceeding similarly with the return exposures of asset $N$ yields

$$RP_t^N = \lambda_t' \cdot \sigma_t^N = \alpha_{N,t}^N\sigma_N^2 + \alpha_{B,t}^N\sigma_B^2 + \alpha_{Bx,t}^N\sigma_B\sigma_{xB,t} + \alpha_{x,t}^N||\Sigma_{x,t}||^2,$$

with

$$\begin{aligned}
\alpha_{N,t}^N &= \left[\gamma p_t + (1-\theta)\upsilon_p p_t(1-p_t)\right]\left[1 + p_t(1-p_t)\omega_p^N\right], \\
\alpha_{B,t}^N &= \left[\gamma(1-p_t) - (1-\theta)\upsilon_p p_t(1-p_t)\right]\left[-p_t(1-p_t)\omega_p^N\right], \\
\alpha_{Bx,t}^N &= \left[\gamma(1-p_t) - (1-\theta)\upsilon_p p_t(1-p_t)\right]\omega_{\widehat{x}}^N + (1-\theta)\upsilon_{\widehat{x}}\left[-p_t(1-p_t)\omega_p^N\right], \\
\alpha_{x,t}^N &= (1-\theta)\upsilon_{\widehat{x}}\omega_{\widehat{x}}^N.
\end{aligned} \tag{21}$$

The local return variance of asset $B$ is given by the sum of squared exposures of the return on asset $B$ to the different risk factors:

$$(\sigma_t^B)' \cdot \sigma_t^B = \beta_{N,t}^B \sigma_N^2 + \beta_{B,t}^B \sigma_B^2 + \beta_{Bx,t}^B \sigma_B \sigma_{xB,t} + \beta_{x,t}^B ||\Sigma_{x,t}||^2,$$

with

$$\begin{aligned}
\beta_{N,t}^B &= \left[p_t(1-p_t)\omega_p^B\right]^2 \\
\beta_{B,t}^B &= \left[1 - p_t(1-p_t)\omega_p^B\right]^2 \\
\beta_{Bx,t}^B &= 2\left[1 - p_t(1-p_t)\omega_p^B\right]\omega_{\widehat{x}}^B \\
\beta_{x,t}^B &= (\omega_{\widehat{x}}^B)^2
\end{aligned} \tag{22}$$

Proceeding similarly with the vector of return exposures of asset $N$ yields

$$(\sigma_t^N)' \cdot \sigma_t^N = \beta_{N,t}^N \sigma_N^2 + \beta_{B,t}^N \sigma_B^2 + \beta_{Bx,t}^N \sigma_B \sigma_{xB,t} + \beta_{x,t}^N ||\Sigma_{x,t}||^2,$$

with

$$\begin{aligned}
\beta_{N,t}^N &= \left[1 + p_t(1-p_t)\omega_p^N\right]^2 \\
\beta_{B,t}^N &= \left[p_t(1-p_t)\omega_p^N\right]^2 \\
\beta_{Bx,t}^N &= -2p_t(1-p_t)\omega_p^N \omega_{\widehat{x}}^N \\
\beta_{x,t}^N &= (\omega_{\widehat{x}}^N)^2
\end{aligned} \tag{23}$$

## References


Ai, H. (2010): "Information Quality and Long-run Risk: Asset Pricing Implications," *Journal of Finance*, 65(4), 1333–1367.

Anderson, E., E. Ghysels, and J. Juergens (2009): "The Impact of Risk and Uncertainty on Expected Returns," *Journal of Financial Economics*, 94, 233–263.

Ang, A., and G. Bekaert (2007): "Stock Return Predictability: Is it There?," *Review of Financial Studies*, 20, 651–707.

Ang, A., G. Bekaert, and M. Wei (2007): "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?," *Journal of Monetary Economics*, 54, 1163–1212.

Avramov, D., T. Chordia, G. Jostova, and A. Philipov (2009): "Dispersion in analysts' earnings forecasts and credit rating," *Journal of Financial Economics*, 91, 83–101.

Baltussen, G., S. van Bekkum, and B. van der Grient (2012): "Unknown Unknowns: Vol-of-Vol and the Cross Section of Stock Returns," Working Paper.

Bansal, A., and I. Shaliastovich (2009): "Confidence Risk and Asset Prices," Working Paper.

——— (2011): "Learning and Asset-price Jumps," *Review of Financial Studies*, 24, 2738–2780.

Bansal, R., D. Kiku, and A. Yaron (2012): "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *Critical Finance Review*, 1(4), 183–221.

Bansal, R., and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59(4), 1481–1509.

Benzoni, L., P. Collin-Dufresne, and R. S. Goldstein (2011): "Explaining Asset Pricing Puzzles Associated with the 1987 Market Crash," *Journal of Financial Economics*, 101, 552–573.

Berk, J., R. Green, and V. Naik (1999): "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54, 1553–1607.

Bollerslev, T., M. Gibson, and H. Zhou (2011): "Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities," *Journal of Econometrics*, 160(5), 102–118.

Bollerslev, T., G. Tauchen, and H. Zhou (2009): "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22(5), 4463–4492.

Boudoukh, J., M. Richardson, and R. Whitelaw (1994): "Industry Returns and the Fisher Effect," *Journal of Finance*, 49(5), 1595–1615.

Branger, N., I. Dumitrescu, V. Ivanova, and C. Schlag (2012): "Two Trees the EZ Way," Working Paper.

Branger, N., C. Schlag, and L. Wu (2011): "Pricing Two Heterogenous Trees," *Journal of Financial and Quantitative Analysis*, 46(5), 1437–1462.

Brevik, F., and S. d'Addona (2010): "Information Quality and Stock Returns Revisited," *Journal of Financial and Quantitative Analysis*, 45(6), 1419–1446.

Campbell, J. Y., G. Chacko, J. Rodriguez, and L. M. Viceira (2004): "Strategic asset allocation in a continuous-time VAR model," *Journal of Economic Dynamics & Control*, 28, 2195–2214.

Chen, N., R. Roll, and S. Ross (1986): "Economic Forces and the Stock Market," *Journal of Business*, 59, 383–403.

Cochrane, J., F. A. Longstaff, and P. Santa-Clara (2008): "Two Trees," *Review of Financial Studies*, 21, 347–385.

Cogley, T., and T. Sargent (2008): "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making," *International Economic Review*, 49(1), 185–221.

Cremers, M. (2002): "Stock Return Predictability: A Bayesian Model Selection Perspective," *Review of Financial Studies*, 15, 1223–1249.

Croce, M. M., M. Lettau, and S. Ludvigson (2012): "Investor Information, Long-Run Risk, and the Term Structure of Equity," Working Paper.

Diether, K., C. Malloy, and A. Scherbina (2002): "Differences of Opinion and the Cross-Section of Stock Returns," *Journal of Finance*, 57(3), 2113–2141.

Drechsler, I. (2012): "Uncertainty, Time-Varying Fear, and Asset Prices," *Journal of Finance*, forthcoming.

Drechsler, I., and A. Yaron (2011): "What's Vol Got to Do With it," *Review of Financial Studies*, 24(1), 1–45.

Duffie, D., and L. G. Epstein (1992): "Stochastic Differential Utility," *Econometrica*, 60, 353–394.

Duffie, D., and C. Skiadas (1994): "Continuous-Time Security Pricing: A Utility Gradient Approach," *Journal of Mathematical Economics*, 23, 107–132.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework," *Econometrica*, 57, 937–969.

Eraker, B., and I. Shaliastovich (2008): "An Equilibrium Guide to Designing Affine Pricing Models," *Mathematical Finance*, 18, 519–543.

Fama, E., and K. French (1992): "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47(2), 427–465.

Johannes, M., L. Lochstoer, and Y. Mou (2011): "Learning about Consumption Dynamics," Working Paper.

Johnson, T. (2004): "Forecast Dispersion and the Cross Section of Expected Returns," *Journal of Finance*, 59(1), 1957–1978.

Keane, M., and D. Runkle (1990): "Testing the Rationality of Price Forecasts: New Evidence From Panel Data," *American Economic Review*, 80, 714–735.

Kreps, D. (1998): "Anticipated Utility and Dynamic Choice," in *Frontiers of Research in Economic Theory*, ed. by D. Jacobs, E. Kalai, M. Kamien, and N. Schwartz, pp. 242–274. Cambridge University Press.

Lettau, M., and J. Wachter (2007): "Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium," *Journal of Finance*, 62, 55–92.

Levhari, D., and H. Levy (1977): "The Capital Asset Pricing Model and the Investment Horizon," *Review of Economics and Statistics*, 59(1), 92–104.

Lewellen, J. (2004): "Predicting returns with financial ratios," *Journal of Financial Economics*, 74, 209–235.

Liptser, R., and A. Shiryaev (2001): *Statistics of Random Processes II. Applications*. Springer, Berlin.

Makarov, I., and D. Papanikolaou (2006): "Sources of Systematic Risk," Working Paper.

Martin, I. (2013): "The Lucas Orchard," *Econometrica*, 81(1), 55–111.

Nagel, S. (2012): "Empirical Cross-Sectional Asset Pricing," Working Paper.

Pastor, L., and P. Veronesi (2009): "Learning in Financial Markets," *Annual Review of Financial Economics*, 1, 361–381.

Shaliastovich, I. (2009): "Learning, Confidence and Option Prices," Working Paper.

Ulrich, M. (2012): "The Real Risk-Free Rate and the Price of Equity: What Role does Macroeconomic Uncertainty Play?," Working Paper.

Veronesi, P. (2000): "How Does Information Quality Affect Stock Returns?," *Journal of Finance*, 55(2), 807–837.

| Dependent | $FD_t^{t+1}$ | $\log(P/D)_t$ | $YS_t$ | $r_t^f$ | $R^2(\%)$ | $\bar{R}^2(\%)$ |
|---|---|---|---|---|---|---|
| $r_{t+1}$ | | -0.0346 | 3.6904 | -3.6151 *** | 5.7153 | 3.8544 |
| | | [0.0231] | [2.5662] | [1.3106] | | |
| $r_{t+1}$ | 0.0545 | -0.0536 ** | 2.6770 | -3.6645 *** | 6.5403 | 4.0645 |
| | [0.0412] | [0.0262] | [2.4373] | [1.3272] | | |
| $r_{t+4}$ | | -0.1229 * | 5.3529 | -8.4905 ** | 8.6305 | 6.7909 |
| | | [0.0716] | [5.8818] | [3.9160] | | |
| $r_{t+4}$ | 0.3350 ** | -0.2345 *** | -0.8039 | -8.7070 ** | 16.2310 | 13.9669 |
| | [0.1383] | [0.0830] | [4.7815] | [3.5719] | | |
| $r_{t+8}$ | | -0.2046 | -3.7823 | -7.6088 | 7.8513 | 5.9447 |
| | | [0.1265] | [7.3509] | [6.6672] | | |
| $r_{t+8}$ | 0.6636 *** | -0.4135 *** | -15.8401** | -7.7728 | 24.8013 | 22.7125 |
| | [0.2040] | [0.1291] | [6.2094] | [5.3967] | | |

Table 1: **Predictability of excess returns on S&P 500**

This table presents results of return predictability regressions. The sample is quarterly from 1968:Q4 to 2007:Q4. The dependent variable $r_{t+j}$ ($j \in \{1,4,8\}$) is the log excess return on the S&P 500 Index over the following one, four, or eight quarters, as indicated. The four and eight months return series are overlapping. The independent variables are the dispersion in analysts' forecasts for the next period's GDP, the log price dividend ratio of the S&P 500 in the middle month of the quarter, the spread between Aaa and Baa rated bond-yields and the return on a 3-month treasury bill. We report regression coefficients, Newey-West (HAC) standard errors in brackets, $R^2$, and adjusted $R^2$. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

| Portf | $\rho(\Delta$gdp,r) | $FD_t^{t+1}$ | $\log(P/D)_t$ | $YS_t$ | $r_t^f$ | $R^2(\%)$ | $\bar{R}^2(\%)$ |
|---|---|---|---|---|---|---|---|
| Panel 1: Returns on defensive assets | | | | | | | |
| **1** | -0.1318 | 0.3263 ** | -0.1393 * | 2.0211 | -0.1519 | 13.1099 | 10.7615 |
| Food | | [0.1254] | [0.0817] | [5.7702] | [3.3913] | (5.9594) | (4.0659) |
| **2** | -0.1334 | 0.4607 *** | -0.2221 *** | -6.5374 | 1.8494 | 15.2273 | 12.9362 |
| Beer | | [0.1701] | [0.0579] | [7.0005] | [4.0619] | (5.1665) | (3.2571) |
| **6** | -0.0718 | 0.3838 ** | -0.1561 ** | -0.1714 | -4.0097 | 11.1030 | 8.7004 |
| Hshld | | [0.1721] | [0.0785] | [6.1872] | [3.7085] | (2.4659) | (0.5021) |
| **8** | -0.0707 | 0.2069 | -0.1173 | -9.4243 | 2.2872 | 4.9889 | 2.4211 |
| Hlth | | [0.1515] | [0.0812] | [6.2006] | [3.7701] | (2.4301) | (0.4656) |
| **24** | -0.0178 | 0.3180 *** | -0.1483 ** | 7.1834 | -6.2256 * | 18.3119 | 16.1041 |
| Paper | | [0.1018] | [0.0584] | [4.7420] | [3.5524] | (10.5717) | (8.7711) |
| Panel 2: Returns on cyclical assets | | | | | | | |
| **12** | 0.0793 | 0.4224 ** | -0.2825 * | 1.5365 | -18.8432 *** | 19.2776 | 17.0959 |
| Steel | | [0.2060] | [0.1595] | [7.1890] | [6.0397] | (13.5468) | (11.8061) |
| **13** | 0.0208 | 0.2781 * | -0.2188 *** | 2.5680 | -16.5853 *** | 19.8665 | 17.7007 |
| FabPr | | [0.1568] | [0.0795] | [5.0649] | [4.1972] | (16.0709) | (14.3811) |
| **17** | 0.0454 | 0.4623 *** | -0.2697 ** | -5.2416 | -15.2412 ** | 16.4939 | 14.2370 |
| Mines | | [0.1727] | [0.1218] | [8.7931] | [6.8156] | (9.2470) | (7.4197) |
| **19** | 0.0387 | 0.3670 ** | -0.2221 *** | -6.4006 | -10.9071 ** | 15.8117 | 13.5363 |
| Oil | | [0.1487] | [0.0844] | [7.4321] | [4.8121] | (8.1129) | (6.2628) |
| **23** | 0.0462 | 0.2575 | -0.2737 | 3.6031 | -14.9192 * | 8.8595 | 6.3962 |
| BusEq | | [0.2413] | [0.2462] | [8.0328] | [8.2728] | (7.3868) | (5.5221) |

Table 2: **Predictability of portfolio returns**

This table presents results of return predictability regressions. The sample is quarterly from 1968:Q4 to 2007:Q4. The dependent variables are annual log excess returns on 10 selected industry sorted portfolios. The return series are overlapping. The second column reports the correlation between $r_i$ and quarterly GDP growth. The independent variables are as in Table 1. We report regression coefficients and Newey-West (HAC) standard errors in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. We also report $R^2$ and adjusted $R^2$ in the last two columns and $R^2$ and adjusted $R^2$ of a regression on the controls $\log(P/D)$, $YS$, and $r^f$ in parentheses below.

| Portf | $\rho(\Delta$gdp,r) | $FD_t^{t+1}$ | $RV_{t+1}(r_m)$ | $R^2(\%)$ | $\bar{R}^2(\%)$ |
|---|---|---|---|---|---|
| Panel 1: Returns on defensive assets | | | | | |
| **1** | -0.1318 | -0.0034 * | 0.9614 *** | 83.7362 | 83.5236 |
| Food | | [0.0019] | [0.0858] | (83.1408) | (83.0292) |
| **2** | -0.1334 | -0.0030 | 0.8771 *** | 62.6011 | 62.1122 |
| Beer | | [0.0034] | [0.1137] | (62.5458) | (62.2978) |
| **6** | -0.0718 | -0.0083 *** | 1.2887 *** | 73.6230 | 73.2782 |
| Hshld | | [0.0023] | [0.0845] | (72.0339) | (71.8487) |
| **8** | -0.0707 | -0.0039 * | 1.2738 *** | 83.9815 | 83.7721 |
| Hlth | | [0.0021] | [0.0648] | (83.7945) | (83.6872) |
| **24** | -0.0178 | -0.0026 * | 1.2903 *** | 89.0203 | 88.8768 |
| Paper | | [0.0019] | [0.1425] | (88.7916) | (88.7173) |
| Panel 2: Returns on cyclical assets | | | | | |
| **12** | 0.0793 | 0.0177 *** | 1.9270 *** | 76.8483 | 76.5456 |
| Steel | | [0.0051] | [0.1812] | (73.7739) | (73.6002) |
| **13** | 0.0208 | 0.0069 *** | 1.4700 *** | 92.1241 | 92.0211 |
| FabPr | | [0.0017] | [0.0701] | (91.0072) | (90.9476) |
| **17** | 0.0454 | 0.0131 * | 1.0983 *** | 51.4566 | 50.8221 |
| Mines | | [0.0072] | [0.1601] | (48.6539) | (48.3139) |
| **19** | 0.0387 | 0.0033 | 0.9847 *** | 69.7045 | 69.3085 |
| Oil | | [0.0033] | [0.0595] | (69.2801) | (69.0767) |
| **23** | 0.0462 | 0.0264 ** | 1.8145 *** | 51.1433 | 50.5047 |
| BusEq | | [0.0110] | [0.4448] | (45.7805) | (45.4214) |

Table 3: **Predictability of portfolio variances**

This table presents results of return variance predictability regressions. The sample is quarterly from 1968:Q4 to 2007:Q4. The dependent variable is the return variance $RV_{t+1}(r_i)$ as described in Section 2.2, where $r_i$ denotes log returns on 10 selected industry sorted portfolios. The second column reports the correlation between $r_i$ and quarterly GDP growth. The independent variables are the dispersion in analysts' forecasts for the next period's GDP and the variance $RV_{t+1}(r_m)$ of the S&P 500. We report regression coefficients and Newey-West (HAC) standard errors in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. We also report $R^2$ and adjusted $R^2$ in the last two columns and $R^2$ and adjusted $R^2$ of a regression on the control variable $RV_{t+1}(r_m)$ in parentheses below.

| | | |
|---|---|---|
| *Parameters for investor's preference* | | |
| Relative risk aversion | $\gamma$ | 10 |
| Intertemporal elasticity of substitution | $\psi$ | 1.5 |
| Subjective discount rate | $\delta$ | 0.02 |
| *Parameters for dividend and signal processes* | | |
| Expected dividend growth rate of the $N$-tree | $\mu_N$ | 0.018 |
| Long run mean of the business cycle variable | $\overline{x}$ | 0.018 |
| Mean reversion speed in the process of $x_t$ | $\kappa_x$ | 0.3 |
| Volatility of dividend growth rate of the $N$-tree | $\sigma_N$ | 0.0352 |
| Volatility of dividend growth rate of the $B$-tree | $\sigma_B$ | 0.0352 |
| *Perfect signal case: Business cycle variable is stochastic but perfectly observable* | | |
| Volatility of the mean reversion process of $x_t$ | $\sigma_x$ | 0.015 |
| Volatility of the signal process | $\sigma_{s,t}$ | 0 |
| *Noisy signal case: Business cycle variable is stochastic and unobservable, signal with finite noise* | | |
| Volatility of the mean reversion process of $x_t$ | $\sigma_x$ | 0.015 |
| Volatility of the signal process | $\sigma_{s,t}$ | 0.015 |
| *No signal case: Business cycle variable is stochastic and unobservable, signal with infinite noise* | | |
| Volatility of the mean reversion process of $x_t$ | $\sigma_x$ | 0.015 |
| Volatility of the signal process | $\sigma_{s,t}$ | $\infty$ |

Table 4: **Parameters**

The table gives the parameters used in the quantitative analysis of our model. They are based on the parameters used in Bansal, Kiku, and Yaron (2012).

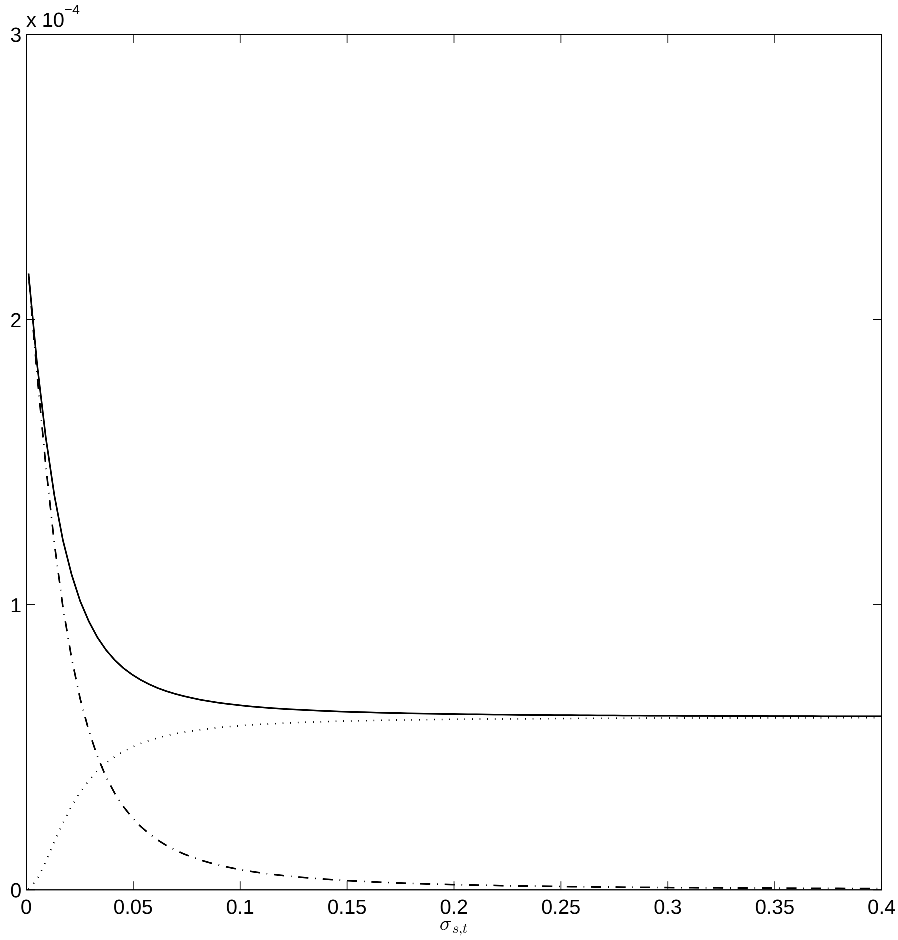


Figure 1:
**Risk factors of the perceived drift**

The figure shows the squared sensitivities of the estimated business cycle variable $\widehat{x}$ to perceived $B$-innovations (dotted line), to signal-innovations (dashed-dotted line), and the local variance of the estimated business cycle $||\Sigma_{x,t}||^2 = \sigma^2_{xB,t} + \sigma^2_{xs,t}$ (solid black line) as a function of the signal volatility $\sigma_{s,t}$. We use the parametrization given in Table 4, except for the conditional volatility of the signal $\sigma_{s,t}$.

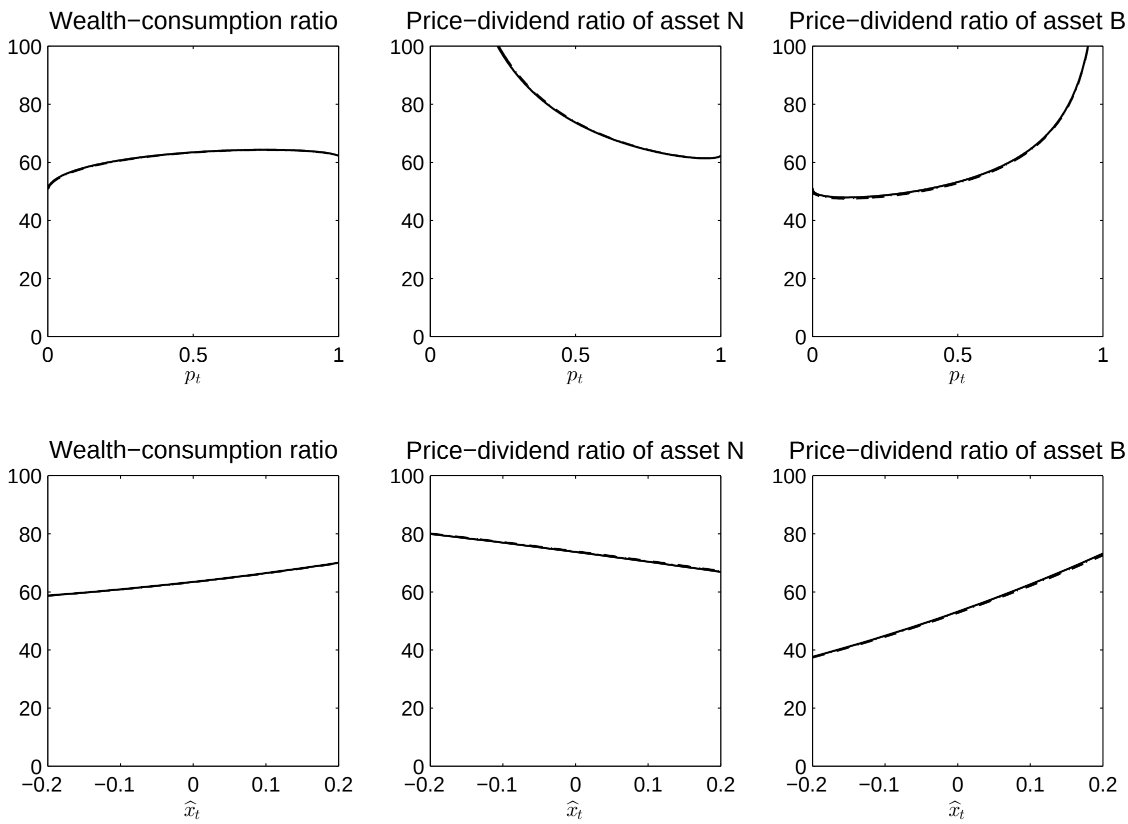


Figure 2:
**Valuation ratios**

The figure shows the wealth-consumption ratio and the price-dividend ratios of the two assets as a function of the consumption share $p$ of asset $N$ for $x_t = 0.018$ (in the upper row) and as a function of the estimated business cycle variable $\widehat{x}_t$ for a consumption share $p_t = 0.5$ (in the lower row). The lines for the perfect signal case ($\sigma_{s,t} = 0$), the noisy signal case ($\sigma_{s,t} = 0.015$) and the no signal case ($\sigma_{s,t} = \infty$) coincide. We use the parametrization of Table 4.

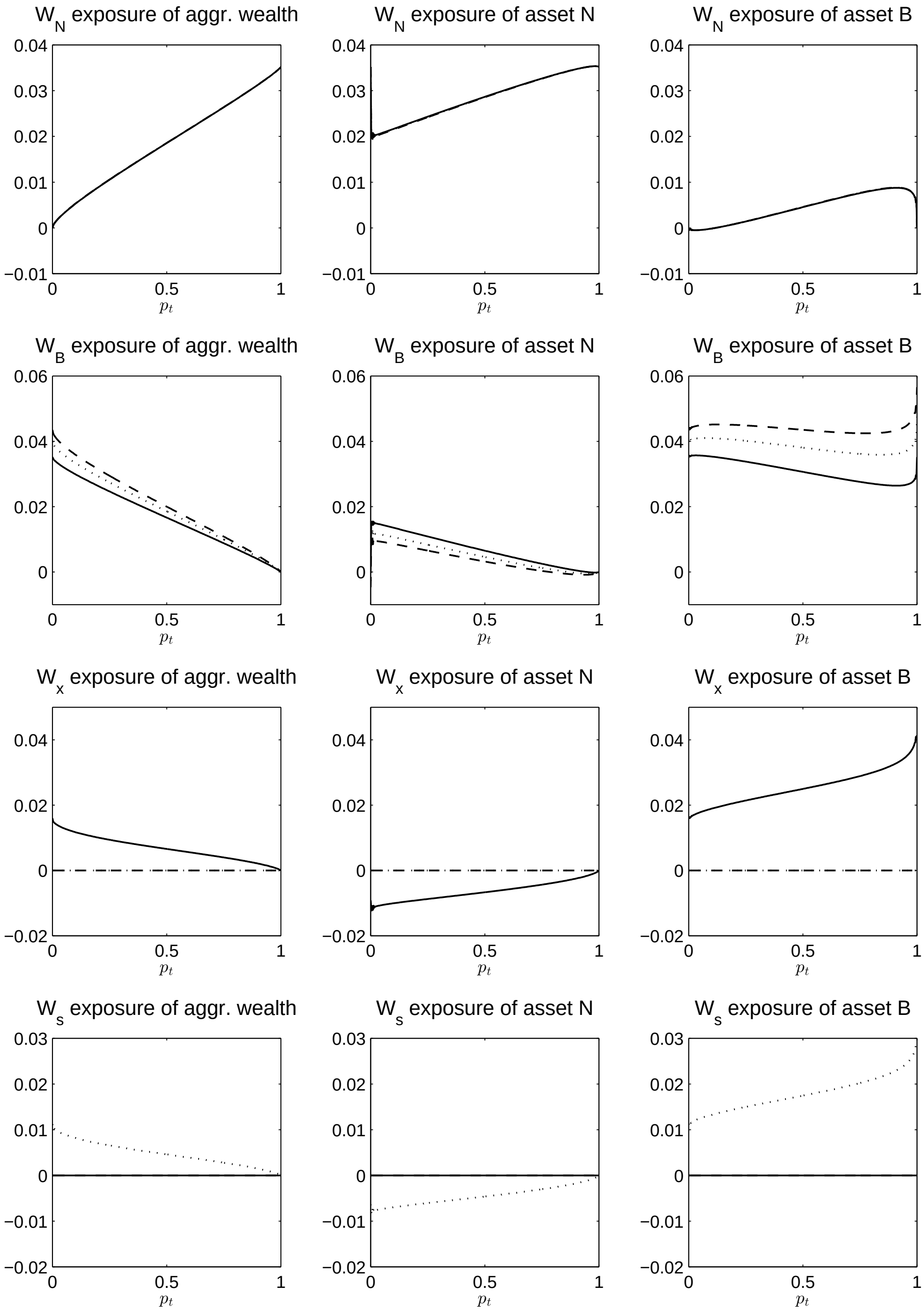


Figure 3:
**Return Exposures**

The figure shows the price-sensitivities of the return on aggregate wealth (left column), the defensive asset $N$ (middle column) and the cyclical asset $B$ (right column) to shocks in the dividend of $N$ (first row), in the dividend of $B$ (second row), in the business cycle variable $x$ (third row) and in the signal (fourth row) as a function of the consumption share $p_t$ of asset $N$. The solid line represents the perfect signal case ($\sigma_{s,t} = 0$), the dotted line the noisy signal case ($\sigma_{s,t} = 0.015$) and the dashed line the no signal case ($\sigma_{s,t} = \infty$). We use the parametrization of Table 4.

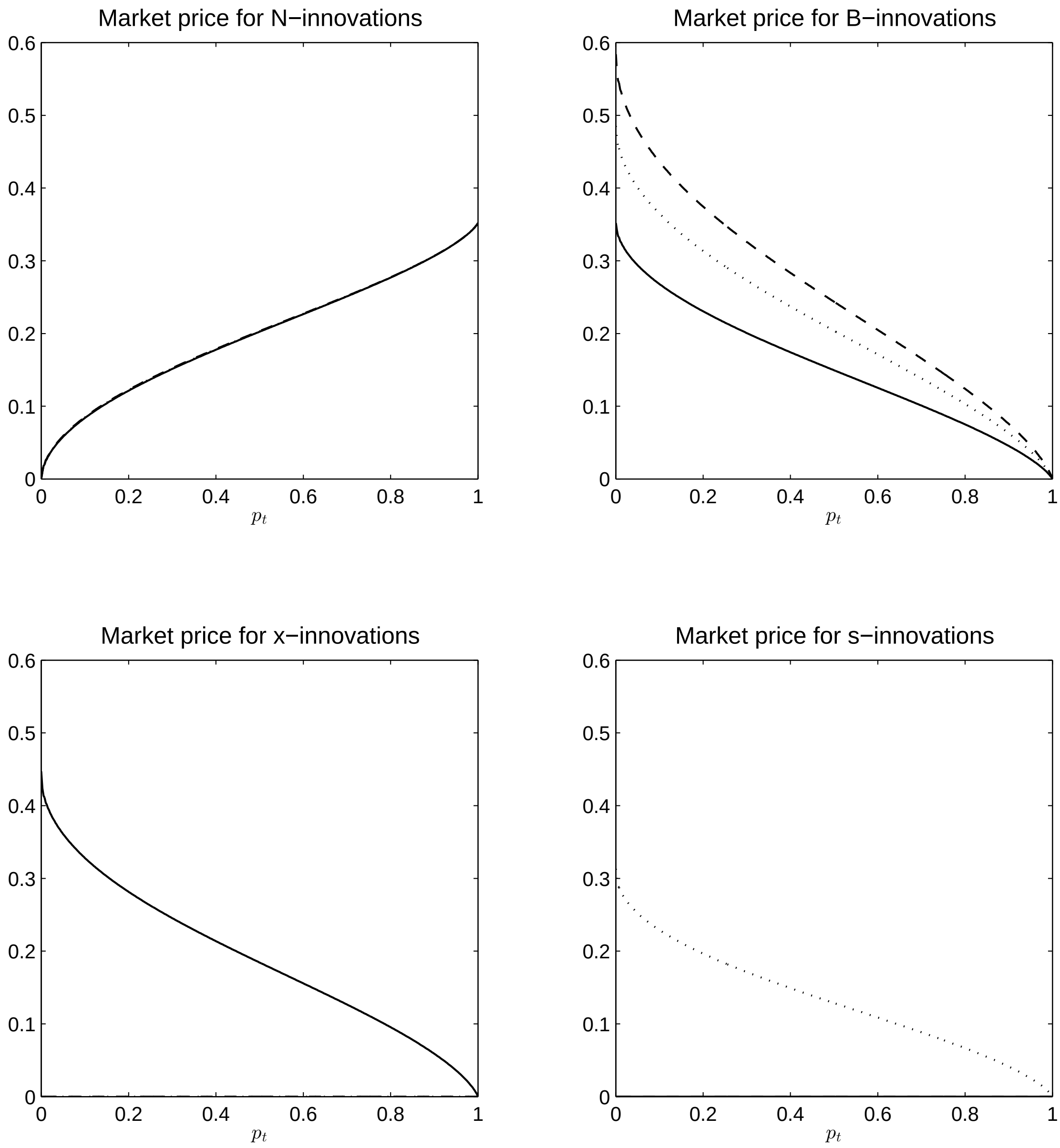


Figure 4:
**Market Prices of Risk**

The figure shows the market prices of risk for shocks in the dividend of $N$ (upper row, left column), in the dividend of $B$ (upper row, right column), in the business cycle variable (lower row, left column) and in the signal (lower row, right column) as a function of the consumption share $p_t$ of asset $N$. The solid line represents the perfect signal case ($\sigma_{s,t} = 0$), the dotted line the noisy signal case ($\sigma_{s,t} = 0.015$) and the dashed line the no signal case ($\sigma_{s,t} = \infty$). We use the parametrization of Table 4.

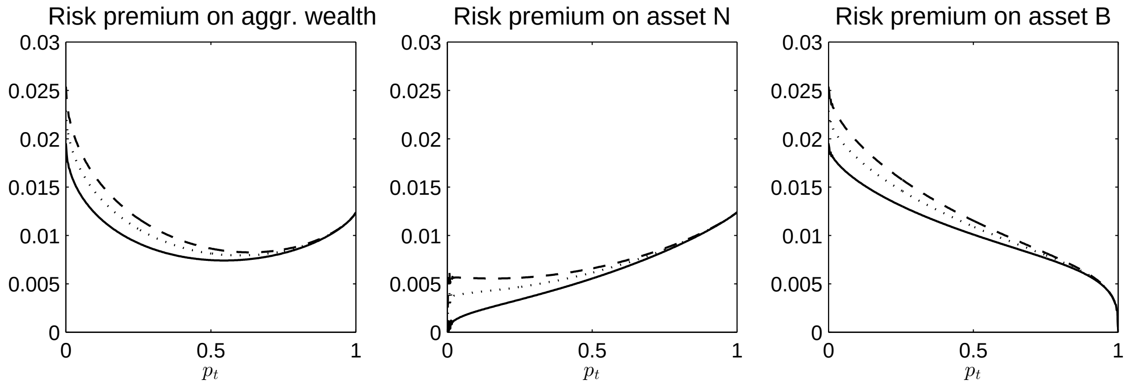


Figure 5:
**Risk Premia**

The figure shows the risk premia on aggregate wealth and on the two assets as a function of the consumption share $p_t$ of asset $N$. The solid line represents the perfect signal case ($\sigma_{s,t} = 0$), the dotted line the noisy signal case ($\sigma_{s,t} = 0.015$) and the dashed line the no signal case ($\sigma_{s,t} = \infty$). We use the parametrization of Table 4.

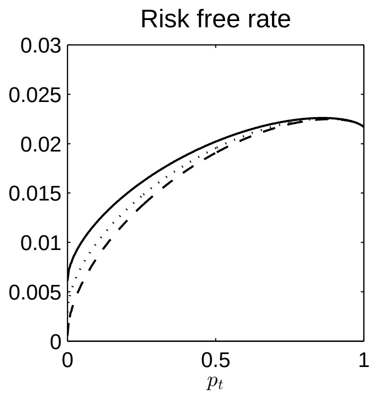


Figure 6:
**Risk-free rate**

The figure shows the risk-free rate as a function of the consumption share $p_t$ of asset $N$. The solid line represents the perfect signal case ($\sigma_{s,t} = 0$), the dotted line the noisy signal case ($\sigma_{s,t} = 0.015$) and the dashed line the no signal case ($\sigma_{s,t} = \infty$). We use the parametrization of Table 4.

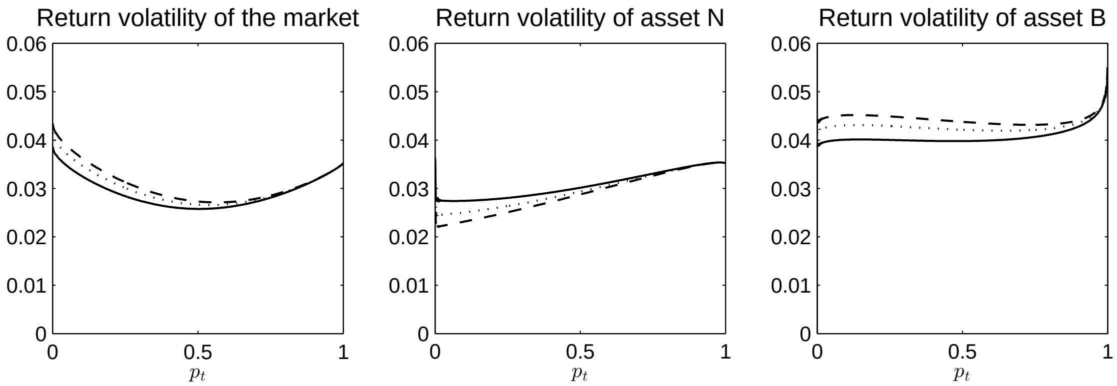


Figure 7:
**Return volatilities**

The figure shows the return volatilities of aggregate wealth and of the two assets as a function of the consumption share $p_t$ of asset $N$. The solid line represents the perfect signal case ($\sigma_{s,t} = 0$), the dotted line the noisy signal case ($\sigma_{s,t} = 0.015$) and the dashed line the no signal case ($\sigma_{s,t} = \infty$). We use the parametrization of Table 4.