# Heuristic Portfolio Trading Rules with Capital Gain Taxes*

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## Abstract

### Heuristic Portfolio Trading Rules with Capital Gain Taxes

This paper studies the out-of-sample performance of portfolio trading strategies when an investor faces capital gain taxation and proportional transaction costs. Under no capital gain taxation and no transaction costs, we show that, consistent with past literature such as DeMiguel, Garlappi, and Uppal (2009b), a simple 1/N trading strategy is not dominated out-of-sample by a variety of optimizing trading strategies. A notable exception of a strategy that does outperform 1/N in our analysis is the parametric portfolios of Brandt, Santa-Clara, and Valkanov (2009). With dividend and realization-based capital gain taxes, the welfare costs of the taxes are large with the cost being as large as 30% of wealth in some cases. Overlaying simple tax trading heuristics on these trading strategies improves out-of-sample performance. In particular, the 1/N trading strategy's welfare gains improve when a variety of tax trading heuristics are also imposed. For medium to large transaction costs, no trading strategy can outperform a 1/N trading strategy augmented with a tax heuristic, not even the most tax- and transaction cost efficient buy-and-hold strategy. Our results thus show that optimal trading strategies trade risk and return considerations off against tax considerations and neither solely focus on any of the two.



**Keywords:** portfolio choice, capital gain taxation, limited use of capital losses, heuristic trading rules

**JEL Classification: G11, H20**

# 1 Introduction

While capital gain taxation is an important friction faced by individual investors, it is notoriously difficult to model. In particular, most capital gain taxation schemes are realization-based, implying taxes are only assessed when a trading position is closed. Furthermore, computing capital gain taxes typically involves tracking the past purchase prices of securities to correctly establish a position's tax basis. This combination makes solving for optimal portfolios especially problematic as the complexity of the problem faced is similar to solving a state-dependent transaction cost problem. In particular, solving portfolio choice problems with a large number of assets is especially vexing given that a large number of state variables beyond just asset holdings must be used to describe the problem.[1]

Progress has been made on how capital gain taxation influences optimal portfolio choice by studying less complex problems with simplifying assumptions. For example, the seminal work of Constantinides (1983) explores a setting where investors can effectively undo the effect of capital gain taxation by engaging in "shorting-the-box" trades when capital gains and losses are treated symmetrically. Later work has focused on settings more consistent with current tax codes where it is more difficult to circumvent capital gain taxes.

Given the complexity of incorporating a capital gain tax that cannot be circumvented, this later work relies on numerically solving portfolio choice problems by restricting the number of assets considered as well as simplifying the evolution of each asset's tax basis by typically using an average purchase price basis rule. Dammon, Spatt, and Zhang (2001b) numerically study a stock and bond portfolio choice problem where the investor must potentially realize capital gains to rebalance for risk versus return motives. Building from this paper, other works study even richer environments such as incorporating two risky stocks (Gallmeyer, Kaniel, and Tompaidis (2006), Garlappi, Naik, and Slive (2001), and Dammon, Spatt, and Zhang (2001a)), the exact purchase price tax basis rule (DeMiguel and Uppal (2005)), separate taxable and tax-deferred accounts for the same investor (Dammon, Spatt, and Zhang (2004), Garlappi and Huang (2006), and Huang (2008)), the limited use of realized capital losses (Ehling, Gallmeyer, Srivastava, Tompaidis, and

[1]Appendix C of Gallmeyer, Kaniel, and Tompaidis (2006) describes some of the computational issues related to just a two stock capital gain tax portfolio choice problem. In particular, they solve a lifetime portfolio choice problem by using parallel computing techniques. See also Lo and Haugh (2001b) for a broader discussion of the computational problems faced when computing optimal portfolios.

Yang (2011), and Marekwica (2010)), and wash-sale constraints (Jensen and Marekwica (2011)). This work however is not easily applicable to portfolios with a large number of stocks due to the well-known curse of dimensionality. This is the focus of our work where we attempt to develop heuristic trading rules that take capital gain taxation into account for a large number of assets.

In parallel to the work on portfolio choice with capital gain taxation, recent work such as DeMiguel, Garlappi, and Uppal (2009b) stresses the importance of out-of-sample performance of portfolio choice rules in the presence of estimation risk. Such an analysis should be especially important in the context of capital gain taxation as out-of-sample performance is driven by how efficiently taxes are paid in addition to risk versus return concerns. To study estimation risk, we build tax-optimized heuristic trading rules by modifying existing no-tax portfolio choice strategies and analyzing their performance out of sample across a variety of data sets. In particular, our work has two goals. First, we want to understand how capital gain taxation and proportional trading costs influence the performance of portfolio strategies empirically that have been shown to perform well in a frictionless setting. Second, we want to understand how tax-optimized heuristic trading rules that modify existing strategies in the literature might perform better.

Overall, we do find a role for tax-optimized heuristic trading rules to improve the performance of trading strategies out of sample. Reconfirming the results of DeMiguel, Garlappi, and Uppal (2009b), we show that, under no capital gain taxation and no transaction costs, a simple 1/N trading strategy is not dominated out-of-sample by a variety of optimizing trading strategies with the notable exception being the parametric portfolios of Brandt, Santa-Clara, and Valkanov (2009). Once capital gain taxes are imposed, the welfare costs are large with the cost of capital gain taxation being as large as 30% of wealth in some cases.

Overlaying simple tax trading heuristics on these trading strategies improve out-of-sample performance. In particular, the 1/N trading strategy's welfare gains improve with a variety of tax trading heuristics imposed. For medium to large transaction costs, no trading strategy can outperform a 1/N trading strategy augmented with a tax heuristic, not even the most tax- and transaction cost efficient buy-and-hold strategy. Our results thus show that optimal trading strategies trade risk and return considerations off against tax considerations and neither solely focus on any of the two.

The paper proceeds as follows. Section 2 introduces the portfolio strategies considered and explores their performance in a no tax setting. Section 3 imposes a capital gain tax and asks how the performance of the trading strategies out of sample are impacted. The performance of our tax trading heuristics is explored in Section 4. Section 5 concludes. Additional details on the construction of our out-of-sample portfolios are provided in the Appendix.

## 2 Trading Strategy Efficiency under No Taxation

Before turning to how capital gain taxation influences portfolio choice, we first establish a benchmark set of portfolio choice strategies that have been previously studied in an out-of-sample context with no market imperfections. We also describe the data sets used to test out-of-sample performance for an individual investor maximizing utility over terminal wealth with a $T$ month investment horizon.

The investor's preferences are assumed to be CRRA expected utility with a relative risk aversion coefficient $\gamma$. To maximize utility, the investor chooses an allocation in $N$ risky assets each month where the vector of portfolio weights is denoted $\mathbf{w}_t$. Individual portfolio weights are referred to as $w_{1,t}, \ldots, w_{N,t}$.

### 2.1 Portfolio Choice Strategies

Our candidate portfolio choice strategies are chosen from past work such as DeMiguel, Garlappi, and Uppal (2009b), and DeMiguel, Garlappi, Nogales, and Uppal (2009a) who study the out-of-sample properties of a variety of portfolio choice models. Given our objective is to study portfolios that could potentially be implemented by individual investors, we do not explicitly focus on strategies with short selling except in one case discussed below. Additionally, we want to mitigate the amount of short selling that an investor can do to rule out negative terminal wealth paths. Given we compute certainty equivalents for a CRRA investor, he is never willing to accept a wealth profile with the possibility of a negative terminal wealth. Below we outline the portfolio choice strategies we consider which are summarized in Table 1.

**1/N Strategy**

DeMiguel, Garlappi, and Uppal (2009b) evaluate 14 different portfolio choice models across 7 data sets and show that none of those portfolio choice models performs consistently better in terms of Sharpe ratios, certainty equivalent returns, or turnover as compared to 1/N. They argue that potential diversification gains from more advanced portfolio choice strategies are outweighed by estimation error. The naïve diversification strategy of holding an equal share of wealth in all assets at each and every point in time defines our "1/N" strategy. From a tax perspective, a 1/N investor

| Portfolio Strategy | Abbreviation |
|---|---|
| Equally-weighted portfolio | 1/N |
| Value-weighted market portfolio | VW |
| Buy and hold portfolio initially invested in the 1/N portfolio | BH |
| Minimum-variance portfolio with short sales constrained as in Jagannathan and Ma (2003) | MV |
| 1-norm-constrained minimum-variance portfolio as in DeMiguel, Garlappi, Nogales, and Uppal (2009a) with a short-selling budget of 5% | NC |
| Parametric portfolio as in Brandt, Santa-Clara, and Valkanov (2009) using the factors Size, Book-to-Market, and Momentum with the value-weighted market portfolio as initial portfolio weights | PPVW |
| Parametric portfolio as in Brandt, Santa-Clara, and Valkanov (2009) using the factors Size, Book-to-Market, and Momentum with the equally-weighted market portfolio as initial portfolio weights | PPNC |

Table 1: **Portfolio Strategies Considered.**

tends to incur tax as he always sells winners and buys losers to adjust portfolio weights back to 1/N.

### Value-Weighted Market Portfolio Strategy

The only portfolio all investors could hold at the same time that would lead to market clearing is the value-weighted market portfolio. For the value-weighted market portfolio defined as our "VW" strategy, the share of the investor's wealth held in an asset is equal to the share of the total market value of this type of asset to the total market value of all assets considered. Any dividends paid are reinvested in the securities to preserve the value-weighting. We refer to this benchmark portfolio choice strategy as VW throughout.

### Buy-and-Hold Strategy

From a capital gain tax perspective, a buy-and-hold strategy is an obvious strategy to employ to help minimize capital gain tax trading costs. It is also particularly easy to implement given no rebalancing is necessary. Previous works such as Lo and Haugh (2001a) and Rogers (2001) show that buy-and-hold strategies perform well relative to fully-optimized dynamic portfolio choice strategies. Additionally, the results in the appendix to DeMiguel, Garlappi, and Uppal (2009b) (DeMiguel, Garlappi, and Uppal (2006)) suggest that buy and hold might result in a slightly higher certainty equivalent return than the 1/N portfolio strategy with rebalancing. Our buy-and-hold strategy, denoted "BH," initially invests in the 1/N portfolio and does not rebalance portfolio weights during the entire investment horizon except to reinvest dividends in the assets they stem from. Whereas the other benchmark models we consider focus on the risk versus return tradeoff and ignore tax effects, BH can be thought of as the other extreme, in only focusing on tax-optimization but ignoring dynamic risk versus return considerations.

### Minimum Variance Strategies

Moving beyond the 1/N, VW, and BH strategies requires the estimation of return statistics. One of the simplest portfolio strategies to implement both in computation and the estimation of return statistics is a minimum variance portfolio. From the work of Markowitz (1952), we know optimal portfolio weights are the solution to a simply quadratic programming problem that can be solved for a large number of assets. The only quantity that needs to be estimated is the return variance-covariance matrix. Solving for other points on the mean-variance frontier is not considered here as estimation errors in the mean returns have a substantial impact on estimated portfolio weights (Best and Grauer (1991), Green and Hollifield (1992), and Chopra and Ziemba (1993)).

In our strategy denoted "MV", portfolio weights at time $t$ are found by solving

$$\arg\min_{\mathbf{w}_t} \mathbf{w}_t^\top \Sigma_t \mathbf{w}_t \tag{1}$$

s.t.

$$\mathbf{1}^\top \mathbf{w}_t = 1, \quad w_{i,t} \geq 0, \tag{2}$$

where $\Sigma_t$ denotes the variance-covariance matrix of returns and $\mathbf{1}$ denotes a vector of ones. Portfolio weights are short-sale constrained for two reasons. First, Jagannathan and Ma (2003) show that constraints can help improving out-of-sample performance. Second, short sale constraints prevent the investor's total final wealth from becoming negative in some states of the world. Again, such a wealth profile would never be held by a CRRA investor.

Recent work by DeMiguel, Garlappi, Nogales, and Uppal (2009a) extends the work of Jagannathan and Ma (2003) by imposing a norm constraint on portfolio weights. They demonstrate that such an approach can out-of-sample outperform a 1/N strategy. Portfolios in this class can be viewed as shrinking the portfolio weight vector instead of shrinking estimators for the moments of assets to reduce estimation risk.

DeMiguel, Garlappi, Nogales, and Uppal (2009a) propose a relatively wide range of ways for constraining portfolio norms. In our strategy "NC," we constrain the portfolio weight vector by a 1-norm not to exceed some exogenously given $\delta \geq 1$. The investor's portfolio at time $t$ is then found by solving

$$\arg\min_{\mathbf{w}_t} \mathbf{w}_t^\top \Sigma_t \mathbf{w}_t \tag{3}$$

s.t.

$$\mathbf{1}^\top \mathbf{w}_t = 1, \quad ||\mathbf{w}_t||_1 = \sum_{i=1}^{N} |w_{i,t}| \leq \delta. \tag{4}$$

When $\delta = 1$, the optimal solution collapses to Jagannathan and Ma (2003), a short sale constrained minimum variance portfolio.

Focusing on constraining the portfolio weight vector by a 1-norm is motivated by two reasons. First, $\delta$ can be economically motivated. The quantity $\frac{\delta-1}{2}$ is the maximum amount of short-selling allowed in the investor's portfolio. Second, constraining the 1-norm of the portfolio weight vector allows us to choose the investor's maximum short selling budget small enough to ensure that total final wealth of our CRRA investor does not become negative. We set $\delta = 1.1$, indicating that the investor's portfolio may not contain short-positions exceeding 5% of his wealth invested. Due to space constraints, we only present minimum variance results for the NC strategy. MV strategy results, which are similar, are available from the authors.

### Parametric Portfolio Strategies

Recent work by Brandt, Santa-Clara, and Valkanov (2009) suggests modelling portfolio weights as a function of asset-specific characteristics such as size, book-to-market, and momentum. Estimating portfolio weights directly from asset-specific characteristics helps alleviate estimation errors as asset return moments are not directly estimated, improving out-of-sample performance.

The Brandt, Santa-Clara, and Valkanov (2009) parametric portfolios are constructed as follows. Assume that the portfolio weight of asset $i$ at time $t$, $w_{i,t}$, can be expressed as a function of that asset's characteristics at time $t$, $x_{i,t}$, implying $w_{i,t} = f(x_{i,t}; \theta)$ where the function $f(\cdot)$ is common across all assets and time. We again constrain the investor's portfolio weights to be non-negative. That is, after computing unconstrained portfolio weights $w_{i,t}$, we adjust them as follows

$$w_{i,t}^{+} = \frac{\max[w_{i,t}, 0]}{\sum_{j=1}^{N} \max[w_{j,t}, 0]} = \frac{\max[f(x_{i,t}, \theta), 0]}{\sum_{j=1}^{N} \max[f(x_{j,t}, \theta), 0]}. \tag{5}$$

Maximizing utility from the gross portfolio return $R_{p,t}$ from time $t$ to time $t+1$ is then equivalent to maximizing $\max_{\theta} \mathbb{E}\left[U\left(\sum_{i=1}^{N} w_{i,t}^{+} R_{i,t}\right)\right]$. For applying the parametric portfolio approach to data, the assets' characteristics to consider and the functional relation $f$ between them and the portfolio weights have to be determined. We follow Brandt, Santa-Clara, and Valkanov (2009) and use the log of the assets' market equity, the log of one plus book equity divided by market equity, and the lagged one-year return from time $t-13$ to time $t-1$ as the assets' characteristics. For the functional relation between these characteristics and the unconstrained portfolio weights we assume a relationship of the form

$$w_{i,t} = \overline{w}_{i,t} + \frac{1}{N}\theta^{\top}\hat{x}_{i,t}, \tag{6}$$

where $\overline{w}_{i,t}$ denotes some exogenously pre-specified initial portfolio weight and $\hat{x}_{i,t}$ are the characteristics of asset $i$ standardized cross-sectionally across all assets at time $t$ to have zero mean and unit standard deviation. That is, our portfolio weights, constrained to be non-negative, are given by

$$w_{i,t}^{+} = \frac{\max\left[\overline{x}_{i,t} + \frac{1}{N}\theta^{\top}\hat{x}_{i,t}, 0\right]}{\sum_{j=1}^{N} \max\left[\overline{x}_{j,t} + \frac{1}{N}\theta^{\top}\hat{x}_{j,t}, 0\right]}. \tag{7}$$

For estimating the vector $\theta$ of coefficients describing the linear relationship between $\hat{x}_{i,t}$ and $w_{i,t}^{+}$ based on our estimation window of length $M$, the investor solves the optimization problem

$$\max_{\theta} \frac{1}{M} \sum_{m=1}^{M} U\left(\sum_{i=1}^{N} \frac{\max\left[\overline{x}_{i,m} + \frac{1}{N}\theta^{\top}\hat{x}_{i,m}, 0\right]}{\sum_{j=1}^{N} \max\left[\overline{x}_{j,m} + \frac{1}{N}\theta^{\top}\hat{x}_{j,m}, 0\right]} R_{i,m}\right). \tag{8}$$

We consider two different choices for the initial portfolio weights $\overline{w}_{i,t}$. First, following Brandt, Santa-Clara, and Valkanov (2009), we use the value-weighted market portfolio as the initial portfolio, denoted as the "PPVW" strategy. Second, we choose the 1/N portfolio as our initial portfolio, denoted as the "PPN" strategy. Due to space considerations, we only present results for the PPN strategy. The PPVW strategy results are similar to the PPN strategy results and are available from the authors.

### 2.2 Data

To evaluate a set of benchmark no-tax and tax-optimized portfolio strategies out-of-sample requires return data for a set of assets. We use monthly data from five different data sets. These data sets consist of 6 and 25 Fama-French portfolios sorted by size and book-to-market (denoted FF06 and FF25), 10 and 48 industry portfolios representing the US stock market (denoted IN10 and IN48), and 50 randomly chosen stocks from the CRSP/Compustat merged database (denoted CRSP). All the data sets used, including the time period used to build out-of-sample returns, are summarized in Table 2.

| Data set | Abbreviation | Obs. in sample | Source |
|---|---|---|---|
| 6 Fama-French portfolios sorted by size and book-to-market | FF06 | 01/1938-12/2008 | K. French's website |
| 25 Fama-French portfolios sorted by size and book-to-market | FF25 | 01/1943-12/2008 | K. French's website |
| 10 Industry portfolios representing the US stock market | IN10 | 01/1938-12/2008 | Own construction |
| 30 Industry portfolios representing the US stock market | IN30 | 01/1938-12/2008 | Own construction |
| 50 randomly chosen stocks from the CRSP database | CRSP | 01/1963-12/2008 | CRSP/Compustat merged database |

Table 2: **Data Sets Used**

Data for the Fama-French portfolios is from Kenneth French's website.[2] For the industry portfolios, the website only provides data sets containing total returns. To allow for a different tax treatment of capital gains and dividend payments, we follow the construction outlined on Kenneth

[2] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

French's website and also construct ex-dividend time series. The details for the construction of our industry portfolios can be found in Appendix A. Data for the CRSP data set is from the CRSP/Compustat merged database. To avoid a potential survivorship bias in the CRSP data, we allow for stocks to be delisted from our randomly chosen 50 stocks. We replace each delisted stock by randomly choosing a new stock at the time of delisting.

The parametric portfolio choice strategies require, in addition to return data, data on book-to-market ratios and market equity. For the Fama-French portfolios, value weighted book-to-market data is already available in the data sets from Kenneth French's website. Following standard timing conventions, we use this data with a lag of at least six months. For each industry portfolio considered, we compute the market value as the product of the number of firms in that portfolio and the average value weighted firm size. For the Fama-French portfolios where the average value weighted firm size is not available, we use the average firm size, which should be a close proxy.

For the CRSP data set, we follow Brandt, Santa-Clara, and Valkanov (2009) in constructing these factors. A detailed description of the selection process of the 50 randomly selected stocks from the CRSP database and the computation of the factors book-to-market and size for those assets is given in Appendix A. For all data sets, we construct the time series for the momentum factor as the lagged compounded one-year return from time $t-13$ to $t-1$. We use the log of 1 plus the book-to-market ratio, the log of firm size, and the momentum factor as the assets' characteristics in the parametric portfolio approach.

To assess out-of-sample performance, our computations rely on a "rolling-sample" approach where the length of our data set is assumed to be $J$ monthly observations. We use $M$ months of data to estimate parameters needed to form portfolios. Beginning at time $t = M+1$ to $t = M+T$, we estimate portfolio weights at each of those points in time for the past $M$ months of data and compute the pre-tax portfolio return for each period. Using these portfolio returns, we then compute the investor's final wealth at the end of the investment horizon. We then increase the beginning and the end of the investment horizon successively by one period until we reach the end of our data set. By doing so, we generate a series of $J - M - T + 1$ out-of-sample observations for the investor's terminal wealth. Based on these observations, we then evaluate the desirability of several investment strategies by comparing expected utility levels across strategies.

## 2.3 Investor Characteristics and Transaction Costs

We first analyze the performance of our portfolio choice strategies in a setting when investors are not subject to taxation. Unless otherwise stated, we consider an investor with a relative risk aversion of $\gamma = 5$ and an investment horizon of $T = 120$ months. The investor uses $M = 120$ months of historical data to estimate parameters for determining present portfolio weights.

We also incorporate proportional transaction costs into our analysis to capture how the cost of trading impacts the performance of each strategy. Three different proportional transaction costs are considered in our analysis: 0%, 0.5% (50 basis points), and 1.5% (150 basis points). The zero transaction cost case is meant to isolate the impact of each strategy by itself. The proportional transaction costs of 0.5% and 1.5% are comparable to those estimated in Lesmond, Ogden, and Trzcinka (1999) where they estimate proportional "round-trip transaction costs from 1963 to 1990 that are 1.2% and 10.3% for large and small decile firms, respectively." Our proportional transaction costs are also in line with those used by Lynch and Tan (2010) where they calibrate a portfolio choice problem with proportional transaction costs ranging from 0.2% to 1.455%.

## 2.4 Results under No Taxation

Under no transaction costs, Table 3 summarizes certainty equivalent gains relative to the 1/N strategy (Panel A), Sharpe ratios (Panel B), standard deviations (Panel C), and trading volume (Panel D) for the trading strategies considered. Before discussing the results, it is useful to describe how the summary statistics in the table are computed.

We define the certainty equivalent gain relative to the 1/N strategy as the difference between the investor's certainty equivalent under each benchmark portfolio choice strategy and the 1/N benchmark portfolio strategy divided by the certainty equivalent under the 1/N benchmark portfolio strategy. For example, the PPN trading strategy under the FF06 dataset has a certainty equivalent increase of 21.53% implying that a 1/N investor's wealth would have to increase by that percentage to be just as well off.

The other three panels are computed as follows. For trading strategy $i$, let $w^i_{j,t,obs}$ denote the portfolio weight of asset $j$ at the beginning of the $t$-th month of the investment horizon in the out-of-sample observation *obs* after trading has occurred. The weight $w^i_{j,t-,obs}$ is the cor-

Panel A: Gain Relative to 1/N

| Data | VW | BH | NC | PPN |
|---|---|---|---|---|
| FF06 | **-20.32** | 1.96 | **-23.53** | 21.53 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| FF25 | **-20.30** | 2.56 | **-18.45** | 18.82 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| IN10 | **-4.68** | **-0.88** | **-11.99** | 5.41 |
| | (1.00) | (1.00) | (1.00) | (0.00) |
| IN30 | **-10.45** | 0.76 | **-17.53** | 58.65 |
| | (1.00) | (0.01) | (1.00) | (0.00) |
| CRSP | **-28.86** | **-10.13** | **-20.60** | 73.78 |
| | (1.00) | (1.00) | (1.00) | (0.00) |

Panel B: Sharpe Ratios

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | 0.260 | 0.249 | 0.264 | 0.265 | 0.277 |
| FF25 | 0.253 | 0.249 | 0.258 | 0.282 | 0.276 |
| IN10 | 0.277 | 0.260 | 0.273 | 0.276 | 0.264 |
| IN30 | 0.261 | 0.260 | 0.261 | 0.282 | 0.303 |
| CRSP | 0.279 | 0.251 | 0.262 | 0.266 | 0.309 |

Panel C: Standard deviations

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | 4.65 | 4.20 | 4.66 | 3.95 | 4.64 |
| FF25 | 4.76 | 4.19 | 4.74 | 3.89 | 4.67 |
| IN10 | 4.00 | 4.23 | 4.08 | 3.35 | 4.32 |
| IN30 | 4.36 | 4.23 | 4.42 | 3.36 | 4.87 |
| CRSP | 4.76 | 4.58 | 4.75 | 3.96 | 5.56 |

Panel D: Trading Volume

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | 1.44 | 1.22 | 0.31 | 6.02 | 20.93 |
| FF25 | 1.67 | 1.82 | 0.29 | 10.72 | 17.20 |
| IN10 | 2.09 | 1.92 | 0.31 | 5.93 | 25.91 |
| IN30 | 2.62 | 2.38 | 0.30 | 8.22 | 27.63 |
| CRSP | 5.59 | 4.04 | 0.62 | 13.05 | 37.97 |

Table 3: **Optimal Portfolio Performance under No Taxes and No Transaction Costs.** The values in parenthesis in Panel A are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

responding portfolio weight before trading at time $t$. The return $r_{j,t,obs}$ defined as the return of asset $j$ in the $t$-th month of the investment horizon in the out-of-sample observation $obs$. With the expected average monthly portfolio return of our out-of-sample observations given by $\mu^i = \frac{1}{Y \cdot T} \sum_{obs=1}^{Y} \sum_{t=1}^{T} \sum_{j=1}^{N} w^i_{j,t,obs} \cdot r_{j,t,obs}$, the standard deviations $\sigma^i$, Sharpe ratios $SR^i$ and average trading volume over all out-of-sample observations for trading strategy $i$ are defined as

$$\sigma^i = \sqrt{\frac{1}{Y \cdot T - 1} \sum_{obs=1}^{Y} \sum_{t=1}^{T} \left( \sum_{j=1}^{N} w^i_{j,t,obs} \cdot r_{j,t,obs} - \mu^i \right)^2}, \tag{9}$$

$$SR^i = \frac{\mu^i}{\sigma^i}, \tag{10}$$

$$\text{Trading volume}^i = \frac{1}{Y \cdot (T-1)} \sum_{obs=1}^{Y} \sum_{t=2}^{T} \sum_{j=1}^{N} |w^i_{j,t,obs} - w^i_{j,t-,obs}|. \tag{11}$$

To not bias the reported trading volumes, initial trades to establish the portfolio as well as final trades to liquidate the portfolio are not included.

Turning to our results, Panel A of Table 3 reports certainty equivalent gains of our portfolio choice strategies relative to the 1/N portfolio choice strategy. Values in parenthesis are p-values from 1,000 Efron (1979)-bootstraps under the null hypothesis that the certainty equivalent gain is non-positive. We use the 1/N strategy as our benchmark to compare with the other strategies given DeMiguel, Garlappi, and Uppal (2009b) report that it is difficult to find portfolio choice strategies that systematically outperform it. Confirming the findings of DeMiguel, Garlappi, and Uppal (2009b), our results show that the value-weighted (VW), buy-and-hold (BH), and the mean-variance norm-constrained (NC) strategies do not outperform 1/N systematically out of sample as most of the certainty equivalents are not rejected as being non-positive. However, our results suggest that the parametric portfolio strategy with 1/N as initial portfolio weights (PPN) does systematically outperform the 1/N portfolio choice strategy.[3] All reported utility gains are strongly positive relative to 1/N.

Panels B through D of Table 3 summarize the characteristics of these portfolios. The out-of-sample Sharpe ratios in Panel B show that parametric portfolio PPN and the norm-constrained minimum-variance portfolio NC tend to have the highest Sharpe ratios across the datasets, while the value-weighted portfolio VW has the lowest. The monthly portfolio return standard deviations in Panel C demonstrate that not surprisingly the norm-constrained minimum-variance portfolio NC has the lowest volatility. The parametric portfolio PPN has one of the highest volatilities consistently.

[3]The same is also true for the PPVW strategy. These results are available from the authors.

Panel D reports the average monthly trading volume under each portfolio choice strategy. Not surprisingly, the NC and PPN portfolios have substantially higher trading volumes compared to the other strategies. While in a CAPM-based world, the trading volume from holding the value-weighted market portfolio should be zero, this is not the case for our VW strategy. For the Fama-French and the industry portfolios, volume for the VW strategy can be generated due to changes in the composition of the portfolios over time. Additionally trading volume of all portfolios, including the buy-and-hold portfolio BH, are impacted by dividend payments, seasoned equity offerings, mergers, acquisitions, and assets exiting the data set.

Table 3 presented results when there are no costs to rebalance portfolios. From the large trading volume for the NC and PPN strategies in Panel D, transaction costs could significantly impact the performance of these portfolios. Tables 4 and 5 present certainty equivalent gains relative to the 1/N portfolio strategy and Sharpe ratios when a proportional transaction cost of 50 basis points (Panel A) and 150 basis points (Panel B) is imposed. Imposing a transaction cost now impacts the performance of each trading strategy under 1/N. At a 50 basis points transaction cost, the PPN strategy still largely outperforms the 1/N strategy except for the IN10 dataset. However, at a transaction cost of 150 basis points, the performance gain of the PPN strategy relative to 1/N largely disappears as seen in Table 4. The 150 basis point transaction cost case also highlights the viability of the buy-and-hold strategy BH relative to 1/N. With the increase in trading costs, the BH strategy now dominates the 1/N strategy for all datasets except the CRSP dataset. The Sharpe ratios in Table 5 tell a similar story. With an increase in transaction costs, the PPN Sharpe ratio drops. BH's Sharpe ratio remains relatively stable as the strategy requires much less rebalancing to implement.

# 3 Trading Strategy Efficiency under Capital Gain Taxation

In the previous section, we explored how a variety of portfolio choice strategies performed out-of-sample when the investor faces no capital gain taxation. Here, we document how each of these trading strategies behaves once a capital gain tax is imposed.

Panel A: Gain Relative to 1/N for 50 bp Transaction Cost

| Data | VW | BH | NC | PPN |
|---|---|---|---|---|
| FF06 | **-20.19** | 2.79 | **-25.14** | 7.53 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| FF25 | **-20.35** | 3.51 | **-22.24** | 6.29 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| IN10 | **-4.39** | 0.20 | **-14.45** | **-8.42** |
| | (1.00) | (0.19) | (1.00) | (1.00) |
| IN30 | **-10.09** | 2.25 | **-20.70** | 35.97 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| CRSP | **-27.75** | **-7.30** | **-24.06** | 42.73 |
| | (1.00) | (1.00) | (1.00) | (0.00) |

Panel A: Gain Relative to 1/N for 150 bp Transaction Cost

| Data | VW | BH | NC | PPN |
|---|---|---|---|---|
| FF06 | **-19.93** | 4.47 | **-28.31** | **-16.34** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| FF25 | **-20.45** | 5.45 | **-29.31** | **-15.60** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| IN10 | **-3.79** | 2.42 | **-19.23** | **-31.31** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| IN30 | **-9.37** | 5.30 | **-26.83** | **-1.12** |
| | (1.00) | (0.00) | (1.00) | (0.59) |
| CRSP | **-25.48** | **-1.38** | **-30.57** | **-4.03** |
| | (1.00) | (0.91) | (1.00) | (0.96) |

Table 4: **Optimal Portfolio Performance under No Taxes with Transaction Costs.** The values in parenthesis are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

### 3.1 Capital Gain and Dividend Taxation

To isolate the role capital gain taxation plays on the performance of each trading strategy, we impose a capital gain tax on the investor. We focus on the realization-based feature of capital gain taxation; namely, that capital gain taxes are only paid when a position in a particular security is reduced. Dividends are assumed to be taxed once they are paid out. We also abstract away from the rate of capital gain taxation being a function of the holding period. In particular, we assume that there is a single capital gain tax rate $\tau_g$ which is assessed portfolio-wide across all realized gains. Realized capital losses are either used to offset against current realized gains or carried

Panel A: 50 Basis Point Transaction Cost

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | 0.257 | 0.247 | 0.263 | 0.256 | 0.254 |
| FF25 | 0.251 | 0.246 | 0.257 | 0.267 | 0.257 |
| IN10 | 0.273 | 0.256 | 0.272 | 0.266 | 0.233 |
| IN30 | 0.257 | 0.256 | 0.260 | 0.268 | 0.274 |
| CRSP | 0.272 | 0.246 | 0.260 | 0.249 | 0.275 |

Panel B: 150 Basis Point Transaction Cost

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | 0.252 | 0.242 | 0.260 | 0.238 | 0.207 |
| FF25 | 0.245 | 0.240 | 0.255 | 0.237 | 0.218 |
| IN10 | 0.266 | 0.250 | 0.269 | 0.246 | 0.171 |
| IN30 | 0.249 | 0.248 | 0.257 | 0.241 | 0.216 |
| CRSP | 0.259 | 0.235 | 0.257 | 0.214 | 0.205 |

Table 5: **Sharpe Ratios under No Taxes with Transaction Costs.**

forward as described below.[4]

A variety of methods exist in world tax codes to determine the tax basis, or the price used to subtract from the current price when computing realized capital gains or losses. We consider three methods — a weighted-average purchase price rule, an exact identification rule, and an accrual-based method.[5] For space considerations, we only report results for the exact identification rule.[6] The weighted-average purchase price rule constructs the tax basis for a particular asset based on past purchase prices weighted by the number of shares purchased. It is commonly used in work that studies optimal portfolio choice under capital gain taxation numerically as it greatly reduces the dimensionality of the state space needed to describe the optimization problem. See for example Dammon, Spatt, and Zhang (2001b). The exact identification rule computes the tax basis by always reducing the position in a particular stock using the shares that lead to the smallest realized capital gain. DeMiguel and Uppal (2005) study the welfare benefits of the exact identification rule versus the weighted-average purchase price rule. The accrual-based method simply assumes that each trading period all capital gains and losses are realized. Hence, it removes the tax-timing option commonly studied in the capital gain tax portfolio choice literature

[4]Many capital gain tax codes impose different tax rates based on the holding period of a particular asset. For an analysis of long-term and short-term capital gain taxes in the U.S., see for example Dammon and Spatt (1996).

[5]The U.S. tax code allows for a choice between the weighted-average purchase price rule and the exact identification rule of the shares to be sold. The Canadian tax code uses the weighted-average purchase price rule.

[6]Results for the other two methods are available from the authors.

and provides a convenient benchmark by providing an upper-bound on the cost of a capital gain tax.

Each month, the portfolio under each portfolio choice strategy is rebalanced to its optimal no tax allocation. We assume that we tax loss sell any asset with an embedded capital loss. We do not impose any wash sale restrictions on the portfolio choice decision, so positions in securities with realized capital losses can be immediately re-established. Consistent with most tax codes, we assume the limited use of capital losses implying that realized capital losses can only be used to offset realized gains now or in the future implying that unused capital losses must be tracked over the portfolio's life. This more realistic feature of the tax code has been studied in a no-arbitrage context by Gallmeyer and Srivastava (2011) and an optimal portfolio choice context by Ehling, Gallmeyer, Srivastava, Tompaidis, and Yang (2011) and Marekwica (2010).

Throughout, dividends are taxed at a rate of $\tau_d = 35\%$. Realized capital gains are taxed at a rate of $\tau_g = 20\%$. While this rate is higher than the current U.S. rate of $\tau_g = 15\%$, it is more consistent with historical U.S. capital gain tax rates as well as tax rates in several European countries. For a comprehensive summary of U.S. capital gain tax rates through time, see Figure 1 in Sialm (2009).

### 3.2 Results under Capital Gain Taxation

The out-of-sample performance of each portfolio choice strategy is summarized in Tables 6 and 7 where taxation is now introduced. Each table presents results for a 0 basis point (Panel A), a 50 basis point (Panel B), and a 150 basis point transaction cost (Panel C). Table 6 computes the certainty equivalent wealth loss for a particular strategy relative to the no-tax performance of that strategy. Hence, this table summarizes how costly it is for an investor to face taxation under each of the benchmark portfolio choice strategies. Table 7 allows for a comparison across different benchmark portfolio choice strategies in the presence of the capital gain tax. The table computes the certainty equivalent wealth loss of each benchmark portfolio strategy relative to the 1/N strategy when both pay capital gain tax. In both tables, a negative quantity denotes a wealth loss. The values in parenthesis in Table 7 are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

From Table 6, the cost of taxation is large across all data sets and all trading strategies with certainty equivalent wealth losses ranging from $-14.99\%$ to $-40.27\%$. Consistent with its large trading volume, the PPN strategy generates the largest cost of taxation across all datasets and transaction costs.

We now ask, from an after-tax perspective, how each trading strategy performs relative to the 1/N strategy. From Panel A of Table 7, the results, under no transaction cost, are similar to the no tax case. Across all datasets, the PPN strategy still outperforms 1/N even from an after-tax perspective. As the transaction cost increases, the PPN strategy's dominance over 1/N diminishes. At a 50 basis point transaction cost, the PPN strategy only dominates the 1/N strategy for two datasets. At a 150 basis point transaction cost, the 1/N strategy dominates the PPN strategy. The buy-and-hold strategy BH, given its low trading volume from Panel D of Table 3, dominates the 1/N strategy especially in the high transaction cost case of Panel C. That is, our results for the BH strategy suggest that for reasonable levels of transaction costs it is important to reduce trading costs and defer capital gains tax payments. Whereas the BH strategy solely focuses on this motive and ignores risk versus return considerations, the other portfolio strategies solely focus on risk versus return considerations but ignore tax effects. We now turn to augmenting the latter portfolio strategies with heuristic tax trading overlays, or modifications, to mitigate tax trading costs.

# 4 Benefits of Heuristic Tax Trading Rules

The previous section, especially Table 6, highlighted that taxation can have a large impact on an investor who attempts to implement a portfolio strategy by naïvely ignoring the effect of capital gain taxation. Given solving optimal portfolio choice problems with capital gain taxation becomes quickly intractable for a large number of assets and trading periods, we now construct a set of heuristic tax trading strategies that are meant to augment the no tax optimal portfolio choice. We ask how much performance improves out-of-sample with these modifications to the frictionless portfolio choice strategies.

Panel A: 0 Basis Point Transaction Cost

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | -16.19 | -13.64 | -16.55 | -15.47 | -22.84 |
| FF25 | -15.97 | -13.31 | -16.50 | -15.89 | -21.09 |
| IN10 | -15.30 | -14.29 | -14.99 | -17.04 | -18.79 |
| IN30 | -15.52 | -14.30 | -15.54 | -16.92 | -23.36 |
| CRSP | -19.15 | -13.96 | -17.94 | -19.30 | -29.80 |

Panel B: 50 Basis Point Transaction Cost

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | -16.40 | -13.78 | -16.54 | -15.55 | -22.91 |
| FF25 | -16.25 | -13.58 | -16.48 | -15.98 | -22.09 |
| IN10 | -15.49 | -14.36 | -14.96 | -16.71 | -18.94 |
| IN30 | -16.07 | -14.44 | -15.53 | -16.58 | -28.68 |
| CRSP | -20.30 | -14.12 | -18.27 | -19.86 | -40.27 |

Panel C: 150 Basis Point Transaction Cost

| Data | 1/N | VW | BH | NC | PPN |
|---|---|---|---|---|---|
| FF06 | -16.46 | -13.75 | -16.52 | -15.66 | -23.35 |
| FF25 | -16.26 | -13.47 | -16.45 | -16.53 | -21.37 |
| IN10 | -15.59 | -14.41 | -14.92 | -17.29 | -18.54 |
| IN30 | -16.12 | -14.49 | -15.51 | -17.13 | -24.34 |
| CRSP | -20.75 | -14.21 | -18.19 | -20.81 | -33.41 |

Table 6: **Certainty Equivalent Wealth Loss under Taxation.** This table reports the percentage loss in initial wealth an investor would accept to trade under a no-tax regime under each dataset and trading strategy.

| Panel A: 0 Basis Point Transaction Cost | | | | |
|---|---|---|---|---|
| Data | VW | BH | NC | PPN |
| FF06 | **-17.89** | 1.53 | **-22.87** | 11.89 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| FF25 | **-17.77** | 1.92 | **-18.38** | 11.58 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| IN10 | **-3.54** | **-0.52** | **-13.80** | 1.07 |
| | (1.00) | (0.99) | (1.00) | (0.09) |
| IN30 | **-9.15** | 0.73 | **-18.90** | 43.92 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| CRSP | **-24.29** | **-8.79** | **-20.75** | 50.90 |
| | (1.00) | (1.00) | (1.00) | (0.00) |

| Panel B: 50 Basis Point Transaction Cost | | | | |
|---|---|---|---|---|
| Data | VW | BH | NC | PPN |
| FF06 | **-17.68** | 2.62 | **-24.37** | **-0.85** |
| | (1.00) | (0.00) | (1.00) | (0.78) |
| FF25 | **-17.81** | 3.22 | **-21.99** | **-1.13** |
| | (1.00) | (0.00) | (1.00) | (0.92) |
| IN10 | **-3.12** | 0.82 | **-15.69** | **-12.16** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| IN30 | **-8.35** | 2.90 | **-21.18** | 15.55 |
| | (1.00) | (0.00) | (1.00) | (0.00) |
| CRSP | **-22.15** | **-4.95** | **-23.64** | 6.96 |
| | (1.00) | (1.00) | (1.00) | (0.00) |

| Panel C: 150 Basis Point Transaction Cost | | | | |
|---|---|---|---|---|
| FF06 | **-17.33** | 4.40 | **-27.62** | **-23.23** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| FF25 | **-17.80** | 5.21 | **-29.53** | **-20.75** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| IN10 | **-2.45** | 3.23 | **-20.86** | **-33.71** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| IN30 | **-7.61** | 6.05 | **-27.71** | **-10.81** |
| | (1.00) | (0.00) | (1.00) | (1.00) |
| CRSP | **-19.33** | 1.81 | **-30.63** | **-19.36** |
| | (1.00) | (0.02) | (1.00) | (1.00) |

Table 7: **Certainty Equivalent Wealth Gain/Loss Relative to the 1/N Strategy.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular portfolio choice strategy earns compared to the 1/N strategy under each data set and trading strategy. The values in parenthesis are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

### 4.1 Heuristic Tax Trading Strategies Considered

Our choice of heuristic tax trading strategies is driven by the intuition of the capital gain tax portfolio choice literature such as Constantinides (1983), Dammon, Spatt, and Zhang (2001b), and Gallmeyer, Kaniel, and Tompaidis (2006). In this literature, an investor always trades off rebalancing a portfolio for risk and return incentives against the tax costs of rebalancing. One complication is how to handle the realization of capital losses. With no transaction costs, Gallmeyer and Srivastava (2011) show that it is weakly optimal for an investor to always realize all capital losses even if they cannot be immediately used to offset realized capital gains as these losses can be carried forward.

With a proportional transaction cost, this result breaks down given the desire to also minimize transaction costs. Under the 0 basis point transaction cost case, we assume capital losses are realized immediately. With transaction costs, we only realize losses when the present tax savings dominates the transaction cost incurred when realizing the loss. The exact procedure used for realizing losses with transaction costs is outlined in Appendix B.

Our heuristic modifications to the portfolio choice strategies differ in how they choose to rebalance securities with embedded capital gains. The heuristic strategies we consider are as follows:

1. **Never Realize Gains Strategy (NRG).** Under the NRG strategy, the investor never realizes any capital gains before the end of the investment horizon. Hence, this strategy allows for a strong capital gain lock-in effect in that no capital gain taxes are ever paid on the portfolio until possibly the end of the investment horizon. Securities that appreciate in value are not rebalanced to their no-tax optimal portfolio weights. The investor does realize all capital losses in the portfolio each period in the 0 basis point transaction cost case. Once the losses are realized, these securities can be rebalanced. The method used to rebalance is assumed to be the same for all the heuristic strategies. It is explained below after describing all the heuristic strategies. Dividends paid in the NRG strategy, as well the other heuristic strategies, are used to rebalance portfolio weights as described below. In contrast, the BH strategy always reinvests dividends back into the security that paid them.

2. **Only Realize Gains when Endowed with Losses (ORL)**. In contrast to the NRG

strategy, the ORL strategy allows for rebalancing positions with embedded capital gains, but only if there are existing capital losses, either current or unused from the past in the form of a tax loss carry forward. When the capital loss available is not large enough to cover all the capital gains necessary for rebalancing, we assume that the investor first realizes the gains on those positions with the highest deviation from the optimal portfolio weight.[7]

3. **Portfolio Weight Percent Deviation of $X\%$ (PX).** Under the PX strategy, each portfolio weight can deviate away from its benchmark portfolio weight by a maximum of $X$ percent. For example, if the optimal portfolio weight in asset $i$ is $\hat{w}_i$, the portfolio weight of asset $i$, $w_i$, must always be contained in the set $w_i \in [(1-X)\hat{w}_i, (1+X)\hat{w}_i]$. Results are presented when $X = 10\%$.

4. **Portfolio Weight Percentage Point Deviation of $X$ (PPX).** Under the PPX strategy, each portfolio weight can deviate away from its benchmark portfolio weight by a maximum of $X/N$ percentage points where $N$ is the total number of assets in the portfolio. For example, if the optimal portfolio weight in asset $i$ is $\hat{w}_i > 0$, the portfolio weight of asset $i$, $w_i$, must always be contained in the set $w_i \in [\max\{0, \hat{w}_i - \frac{X}{N}\}, \min\{\hat{w}_i + \frac{X}{N}, 1\}]$. The PPX and the PX strategies behave similarly when portfolio weights are close to 1/N. When portfolio weights deviate from 1/N, small portfolio weights are allowed to move more before rebalancing under the PPX strategy than the PX strategy. Results are presented when $X = 25$, 50, 100, and 150.

5. **Portfolio Weight Deviation as a Multiple $X$ of Unrealized Gains (GX).** Under the GX strategy, the investor allows for a deviation from his benchmark portfolio weight which is a multiple $X$ of the investor's average level of unrealized capital gains per dollar held in that position. If the investor is not endowed with unrealized gains in some portfolio position, any deviation in that position is accepted to avoid tax-payments on other positions. This heuristic strategy is meant to make rebalancing a function of embedded capital gains. For large embedded capital gains, a larger portfolio weight deviation is allowed. Results are presented when $X = 0.1$ and 0.2.

[7]We also studied a modification of this strategy where gains are first realized in portfolio positions where the level of unrealized gains per unit of equity is smallest. Because empirically this strategy is dominated by the ORL strategy, we do not report those results here.

All of these heuristic trading strategies, summarized in Table 8, are applied to the 1/N, VW, NC, and PPN trading strategies. Since the buy-and-hold strategy BH does not involve an optimal set of portfolio weights, the tax heuristics cannot be applied to this strategy. Instead, the BH strategy should simply be viewed as an extreme from of a tax heuristic that solely focuses on tax considerations but entirely ignores risk versus return considerations.

Under all heuristic portfolio trading strategies described above, the investor realizes all capital losses immediately under the 0 basis point transaction cost case or only realizes losses when the benefit of offsetting capital gains dominates the transaction cost paid. This implies that after tax-loss selling and after adjusting portfolio weights to fulfill the rebalancing constraints outlined above, the portfolio weights no longer necessarily sum to one, requiring an adjustment to some of the weights. Our method of rebalancing the portfolio so that all funds are still invested is as follows:

1. Unrealized capital losses are realized for the 0 basis point transaction cost case. When transaction costs are paid, losses are realized only when the immediate benefit outweighs the transaction costs paid.

2. For all securities where the current equity exposure is smaller than the minimum or larger than the maximum equity exposure based on the above heuristic strategies, the portfolio weights are set to the minimum or maximum equity exposure and are no longer changed to make sure the heuristic trading strategy is followed.

3. The difference between one and the current sum of portfolio weights is computed. The securities that can be feasibly traded without violating a constraint are identified. The portfolio weights of these securities are adjusted such that the maximum absolute deviation of the heuristic portfolio weights from their corresponding benchmark portfolio weights is minimized subject to the requirement that all portfolio weights sum to one.

A simple example illustrates how this rebalancing works. Consider a portfolio with four securities and the benchmark portfolio strategy is 1/N. Assume the heuristic tax trading strategy PP20 is implemented in a setting with four assets with current equity weights of $(0.35, 0.27, 0.2, 0.18)$. The first two positions have unrealized capital gains. The last two positions have unrealized capital losses. The portfolio rebalancing would proceed as follows:

| Heuristic Trading Strategy | Abbreviation |
|---|---|
| The investor never realizes capital gains. | NRG |
| The investor only realizes gains when endowed with tax loss carry-forward, positions with the highest deviation from the benchmark portfolio weight are realized first. | ORL |
| The investor accepts a deviation from the benchmark portfolio weights that does not exceed a multiple of X percentage points. | PX |
| The investor accepts deviation from the benchmark portfolio weights that does not exceed X divided by the number of assets considered in percentage points. | PPX |
| Investor accepts deviation from benchmark portfolio weight which is a constant multiple X of average unrealized capital gains above benchmark portfolio weight | GX |

Table 8: **Heuristic Trading Strategies**.

1. First, the last two positions are tax loss sold, giving the portfolio weights $(0.35, 0.27, 0, 0)$.

2. According to the PP20 strategy, the maximum and minimum portfolio weights for all assets are $0.25 + 0.05 = 0.3$ and $0.25 - 0.05 = 0.2$, respectively. The first portfolio weight is set to the maximum level. The third and fourth portfolio weighs are set to the minimum level. The second portfolio weight is kept constant. The resulting vector of portfolio weights is then $(0.3, 0.27, 0.2, 0.2)$. The first portfolio weight cannot be increased, while the third and fourth portfolio weights cannot be decreased.

3. As the sum of the portfolio weights is now 0.97, there is still a portfolio weight of 0.03 to be distributed. Securities three and four can be moved closer to the benchmark strategy with no tax costs. Minimizing the maximum absolute deviation of these two heuristic portfolio weights from the benchmark portfolio weights adds 0.015 to the weights of both securities. The resulting heuristic portfolio becomes $(0.3, 0.27, 0.2015, 0.2015)$.

## 4.2 Results

With these heuristic modifications to our portfolio strategies, we ask how each performs by asking two different questions. First, do the heuristic modifications lead to certainty equivalent improvements relative to the non-tax-optimized strategies? In other words, do the tax heuristic strategy modifications actually add value. Second, if the tax heuristics do add value, which combination of trading strategy plus tax heuristic leads to the greatest certainty equivalent improvement?

We turn first to answering the first question. Tables 9, 10, and 11 summarize how the tax heuristic modifications impact the original 1/N, VW, NC, and PPN trading strategies under 0, 50, and 150 basis point transaction costs. In these tables, positive certainty equivalents imply that the tax heuristic modification adds value over the original trading strategy.

Under no transaction costs, Table 9 shows that the tax heuristic modifications generally improve investor welfare for the 1/N and norm-constrained mean-variance NC strategies. In particular, the portfolio weight percentage point deviation PP heuristics generally improve the 1/N strategy's performance, while the norm-constrained mean-variance strategy is most improved by the G0.2 strategy that minimizes rebalancing when embedded capital gains are large. The improvement in the 1/N strategy is as large as 3.34% of wealth, while the improvement in the NC strategy is as large as 11.50% of wealth. The value-weighted VW and parametric portfolio PPN strategies are generally not improved however. For the PPN strategy, the tax heuristic can actually lead to a significant drop in the wealth equivalents as it limits the extent to which the investor can exploit information in the fundamental data.

However, introducing transaction costs improves the performance of the tax heuristics for all trading strategies as can be seen in Tables 10 (50 basis point transaction cost) and 11 (150 basis point transaction cost). Since the tax heuristics considered also reduce transactions, we should expect to see an improvement in portfolio performance once transaction costs are imposed. Turning to the 50 basis point transaction cost case in Table 10, the portfolio weight percentage point PP heuristic improves even more for the 1/N strategy. For example, the welfare gains more than triple for the CRSP dataset rising above 6.5% in some cases. The welfare gains for the NC trading strategy also significantly improve. In contrast to the no transaction cost case, the VW and PPN strategies now can be improved with tax heuristics. For example, the the G0.1 strategy that minimizes rebalancing when embedded capital gains are large for the PPN strategy increases welfare by 17.53% for the CRSP dataset of 50 stocks. When transaction costs jump to 150 basis points, all tax heuristics improve portfolio performance as can be seen in Table 11.

Having established that the tax heuristics can improve each of the trading strategies, we know ask which combination of trading strategy plus tax heuristic leads to the greatest welfare increase for an investor. We do this by comparing each trading strategy for a particular tax heuristic to the 1/N trading strategy with the same tax heuristic imposed. Tables 12, 13, and 14 report these certainty equivalents for transaction costs of 0, 50, and 150 basis points respectively. Comparisons are made against the value-weighted VW, the buy-and-hold BH, the norm-constrained minimum-variance NC, and the parametric PPN strategies.

At a 0 basis point transaction cost, Table 12 shows that the 1/N strategy modified with a tax heuristic is not dominated by the VW and NC trading strategies. The BH strategy generally does not dominate the 1/N strategy, suggesting that an optimal portfolio strategy trades risk versus return considerations off against tax concerns. Likewise, the PPN strategy with a tax heuristic overlayed still largely dominates 1/N with a tax heuristic. In particular, the IN30 and CRSP datasets exhibit large welfare gains for the PX, PPX, and GX tax heuristics. However, from Table 9 we showed that the tax heuristics do not seem to improve the PPN strategy when there are no transaction costs.

This changes however once one turns to a non-zero transaction cost setting. At a 50 basis point transaction cost in Table 13, still the VW and NC trading strategies are dominated by 1/N with the same corresponding tax heuristic. Likewise, the BH strategy does generally not dominate 1/N. Turning to the parametric portfolio strategy PPN, its performance degrades relative to 1/N. However, the PPN strategy augmented with the GX tax heuristic still dominates 1/N for both the IN30 and CRSP datasets. Turning to the 150 basis point transaction cost in Table 14, the PPN strategy with the GX tax heuristic continues to dominate the 1/N strategy under the same heuristic.

This improvement in the PPN strategy is driven by the tax heuristic imposing a much lower trading volume as can be seen in Tables 15, 16, and 17. The 'BM' column in these tables represents the no-tax trading volume benchmark. As can be seen, the tax heuristics in general greatly reduce trading volume contributing to the increase in welfare.

Overall, these tables have demonstrated that first overlaying a tax heuristic on an existing trading strategy can improve portfolio performance as measured by the investor's certainty equivalent. Once transaction costs are incorporated, it is not possible to find trading strategies with overlayed tax heuristics that systematically dominate the corresponding tax optimized 1/N strategy. Simultaneously, the BH strategy, that solely focuses on tax optimization is outperformed by the 1/N strategy augmented with an overlayed tax heuristic for the industry portfolios and the CRSP data set. That is, optimal investment strategies should neither solely focus on pre-tax risk versus return considerations nor tax optimization. Instead, optimal portfolio strategies trade these two incentives off against each other.

## 5 Conclusion

This paper studies the out-of-sample performance of portfolio trading strategies when an investor faces capital gain taxation and proportional transaction costs. Reconfirming the results of DeMiguel et al. (2009b) and Brandt, Santa-Clara, and Valkanov (2009), we show that, under no capital gain taxation and no transaction costs, a simple 1/N trading strategy is not dominated out-of-sample by a variety of optimizing trading strategies with the notable exception being the parametric portfolios of Brandt, Santa-Clara, and Valkanov (2009). With dividend and realization-based capital gain taxes the welfare costs of the taxes are large with the cost being as large as 30% of wealth in some cases. Overlaying simple tax trading heuristics on these trading strategies can improve performance. In particular, the 1/N trading strategy's welfare gains improve with a variety of tax trading heuristics for medium to large transaction costs. In contrast to DeMiguel, Garlappi, and Uppal (2009b), trading strategies with overlayed tax trading heuristics can outperform 1/N. In particular, a 1/N trading strategy with overlayed tax trading heuristics can outperform 1/N. For medium to large transaction costs no trading strategy can outperform a 1/N trading strategy augmented with a tax heuristic, not even the most tax- and transaction cost efficient buy-and-hold strategy. Our results thus show that optimal trading strategies trade risk and return considerations off against tax considerations and neither solely focus on any of the two.

# A Data Details

### Data Selection Procedure for Industry Portfolios and CRSP Data

Our procedure for the computation of the size and book-to-market factors as well as the construction of the industry portfolios closely follows the procedure described on Kenneth French's website and in Davis, Fama, and French (2000). We use data from Compustat, CRSP, and Moody's.

The Compustat items (identifiers in parenthesis) used are total assets (AT), total liabilities (LT), preferred stock value (PSTKRV, PSTKL, UPSTK, in that order), balance sheet deferred taxes and investment tax credits (TXDITC, otherwise zero), price per share (PRC from CRSP data base), number of shares outstanding (CSHO, otherwise SHROUT from CRSP data base), delisting code (DLSTCD from the CRSP data base), fiscal year end (FYR),an identifier that allow us to follow mergers or asset exchanges (NWPERM from CRSP data base), total return (RET from CRSP data base), ex-dividend return (RETX from CRSP data base), exchange code (EXCHCD from CRSP), share code (SHRCD from CRSP), SIC code (SIC), and SICCD code (SICCD from CRSP).

We compute market equity (ME) as price times shares outstanding. Price is from CRSP; shares outstanding are from Compustat (if available) or otherwise CRSP. Book equity (BE) is constructed from Compustat data or collected from the Moody's Industrial, Financial, and Utilities manuals. BE is the book value of stockholders' equity plus balance sheet deferred taxes and investment tax credit (if available) minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) to estimate the book value of preferred stock. Stockholders' equity is the value reported by Compustat, if it is available. If not, we measure stockholders' equity as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities (in that order).

Following the procedure described in Fama and French (1997) and on Kenneth French's website, we compute the book-to-market ratio (BE/ME) for the Industry portfolios at the end of June of year $t$ as book equity for the fiscal year ending in calendar year $t-1$, divided by market equity at the end of December of $t-1$. We follow the standard convention of leaving a a lag of at least 6 months to ensure that the information from the firm's annual reports is publicly available. We assign each NYSE, AMEX and NASDAQ stock to an industry portfolio at the end of June of year $t$ based on its four-digit SIC code at that time. We use Compustat SIC codes for the fiscal year ending in calendar year $t-1$. If Compustat SIC codes are not available, we use CRSP SIC codes for June of year $t$. We eliminate observations from our data set where we do not have sufficient data to compute ME for December of year $t-1$ and June of year $t$ or BE for the fiscal year ending in year $t-1$. In order to be consistent with the construction of the other data sets considered, we further exclude observations having negative book equity for the fiscal year ending in year $t-1$.

For the randomly chosen assets from CRSP, we eliminate assets with missing data on ME or BE/ME in their time series. In line with our procedure for the Fama-French and the Industry portfolios, we allow for a lag of at least six months after the end of the fiscal year for BE/ME.

To avoid a potential survivorship bias, we do not eliminate firms that delist during the sample. We only exclude time series of assets ending without information on delisting. We further exclude assets where the delisting code DLSTCD is different from 231 (merger, where shareholders primarily receive common stocks) or 331 (asset exchanged, primarily for another class of common stock) and no information about the acquiring firm is available (i.e. where no new PERMNO (NWPERM) of the acquiring firm is available). For those time series where this information is available, we continue our time series with that of the new asset. For estimating portfolio weights, we use historical data from the new time series from that point in time on. Furthermore, we exclude assets where we cannot precisely determine in which way investors were compensated. That is, we exclude assets where the delisting code was not 231, 331, 233 (merger where shareholders are primarily compensated in cash), 333 (issue exchanged, primarily for cash), or beginning with 4 (liquidation). For assets whose time series end with a valid delisting code and could not be continued with another time series, for example due to liquidation, we randomly add new assets under the same criteria to our data. In case of a delisting, liquidation, or a compensation in cash, we assume that the investor's compensation is equal to the latest available market price. The investor is subject to the full immediate tax consequences of this realization. In the case when two companies in our sample merge, we randomly draw a new asset to make sure that the number of assets the investor can invest in does not decrease.

Finally, following Brandt, Santa-Clara, and Valkanov (2009), we eliminate the smallest 20% of stocks by market capitalization on January 1963. From the remaining stocks, we randomly draw 50 stocks to build our CRSP data set. Whenever a stock's time series ends due to reasons other than mergers, we randomly replace that stock with a new stock in our data set.

# B Treatment of Unrealized Losses with Transaction Costs

From Constantinides (1983) an investor should realize all losses immediately when trading in a setting where no transaction costs apply and realized capital gains and losses are subject to the same tax treatment. Gallmeyer and Srivastava (2011) and Marekwica (2010) generalize this result to tax systems with limited use of losses. Therefore, in our base case parameter setting where no transaction costs apply our investor realizes all losses immediately.

Once we allow for transaction costs the incentive to realize losses trades off against the incentive to avoid the trading costs from such trades. In particular, investors might want to avoid selling an asset to realize the losses and repurchasing that asset immediately. In general, determining the optimal loss realization strategy is of similar difficulty as determining the optimal timing of the realization of unrealized gains. This prevents us from determining the optimal loss realization strategy once trading costs are accounted for. Instead, we consider three heuristic strategies:

1. Losses are realized in all portfolio positions where present and potential future tax savings the realized tax loss allows for are at least as big as the present transaction costs for selling and repurchasing the assets in which a loss is realized.

2. Losses are only realized in portfolio positions where present tax savings the realized loss allows for are at least as big as the present transaction costs for selling and repurchasing the assets. In case that more portfolio positions fulfill this condition than are required for generating the desired realized loss, to minimize the transaction costs burden from trading, only the portfolio positions with the highest level of unrealized loss per unit of equity are traded.

3. Losses are not realized before the end of the investment horizon.

It turns our that the two later strategies result in rather similar welfare results, suggesting that the exact treatment of unrealized losses might be a second-order effect once a reasonable way of dealing with unrealized losses is allowed for. The first strategy tends to perform slightly poorer, suggesting that it puts a somewhat too high transaction costs burden on the investor compared to the other two strategies. Our results presented are for the second strategy in the presence of transaction costs.

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| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | FF06 | 1.13 | 0.28 | 1.59 | 1.59 | 1.17 | 1.13 | 1.13 | 1.27 | 1.15 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 0.88 | 0.25 | 1.16 | 1.49 | 1.24 | 0.88 | 0.88 | 0.87 | 0.87 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| 1/N | IN10 | 0.11 | **-0.03** | 0.49 | 0.98 | 1.02 | 0.64 | 0.31 | 0.88 | 0.24 |
| | | (0.21) | (0.91) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) | (0.00) | (0.03) |
| | IN30 | 2.09 | 0.16 | 1.66 | 2.79 | 3.33 | 3.34 | 3.15 | 2.52 | 2.12 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | **-5.03** | **-0.16** | **-1.20** | 0.25 | 1.81 | 1.81 | 1.30 | **-3.87** | **-5.03** |
| | | (1.00) | (0.89) | (1.00) | (0.07) | (0.00) | (0.00) | (0.00) | (1.00) | (1.00) |
| | FF06 | 1.07 | 0.32 | **-0.08** | 0.48 | 0.57 | 0.97 | 1.06 | 0.58 | 0.71 |
| | | (0.00) | (0.00) | (1.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 1.69 | 0.48 | 0.18 | 0.49 | 0.50 | 0.81 | 0.95 | 0.72 | 0.89 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| VW | IN10 | **-0.93** | **-0.05** | **-0.60** | **-0.78** | **-0.95** | **-0.95** | **-0.93** | **-0.73** | **-0.87** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-1.22** | **-0.12** | **-0.54** | **-0.61** | **-0.70** | **-1.05** | **-1.21** | **-1.03** | **-1.15** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-2.91** | 0.11 | **-0.22** | **-0.04** | **-0.60** | **-0.86** | **-0.92** | **-1.49** | **-2.02** |
| | | (1.00) | (0.00) | (1.00) | (0.78) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 0.83 | 1.09 | 0.34 | 2.17 | 2.19 | 1.24 | 0.28 | 1.55 | 1.98 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.02) | (0.00) | (0.00) |
| | FF25 | 0.71 | 2.13 | 0.16 | 0.41 | 0.62 | 1.21 | 1.81 | 0.96 | 1.60 |
| | | (0.10) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| NC | IN10 | **-3.24** | 2.23 | **-0.07** | **-0.07** | 0.03 | 0.62 | 0.03 | 1.98 | 1.40 |
| | | (1.00) | (0.00) | (0.99) | (0.88) | (0.43) | (0.00) | (0.49) | (0.00) | (0.00) |
| | IN30 | 3.46 | 2.96 | **-0.05** | 0.31 | 1.03 | 2.09 | 3.58 | 5.42 | 6.35 |
| | | (0.00) | (0.00) | (0.88) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 5.27 | 6.49 | **-1.41** | 0.27 | 0.29 | 1.85 | 2.76 | 7.44 | 11.50 |
| | | (0.00) | (0.00) | (1.00) | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF06 | **-2.87** | **-0.35** | **-0.19** | **-0.40** | **-0.47** | **-1.44** | **-2.56** | 0.21 | **-0.05** |
| | | (1.00) | (0.77) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (0.01) | (0.65) |
| | FF25 | **-3.61** | **-1.33** | **-0.41** | **-0.07** | **-0.70** | **-0.91** | **-0.59** | **-0.31** | 0.12 |
| | | (1.00) | (1.00) | (1.00) | (0.75) | (1.00) | (1.00) | (1.00) | (0.98) | (0.32) |
| PPN | IN10 | **-10.43** | **-0.78** | **-0.33** | 0.71 | **-0.38** | **-2.50** | **-5.96** | 0.46 | **-0.75** |
| | | (1.00) | (0.89) | (1.00) | (0.00) | (0.99) | (1.00) | (1.00) | (0.00) | (1.00) |
| | IN30 | **-25.43** | **-14.38** | 0.73 | **-0.12** | **-1.22** | **-4.47** | **-7.89** | **-6.18** | **-14.70** |
| | | (1.00) | (1.00) | (0.00) | (0.78) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-38.25** | **-19.11** | 0.14 | **-0.88** | **-0.95** | **-2.79** | **-5.70** | **-12.32** | **-20.38** |
| | | (1.00) | (1.00) | (0.03) | (1.00) | (0.99) | (1.00) | (1.00) | (1.00) | (1.00) |

Table 9: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Non-Tax-Optimized Strategy for 1/N, VW, NC, and PPN with a 0 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the non-tax-optimized strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps for the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1/N | FF06 | 2.23 | 2.00 | 2.41 | 2.70 | 2.24 | 2.23 | 2.23 | 2.09 | 2.11 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 2.37 | 2.19 | 2.20 | 2.98 | 2.71 | 2.36 | 2.37 | 2.06 | 2.09 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN10 | 1.22 | 1.37 | 1.49 | 2.20 | 2.33 | 1.87 | 1.42 | 2.21 | 1.50 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 3.84 | 3.64 | 3.40 | 5.03 | 5.54 | 5.50 | 5.17 | 3.99 | 3.66 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | **-2.00** | **-1.99** | 1.27 | 4.48 | 6.58 | 6.48 | 5.31 | **-0.72** | **-1.84** |
| | | (1.00) | (1.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.82) | (1.00) |
| VW | FF06 | 4.76 | 1.67 | 0.50 | 1.47 | 2.76 | 4.47 | 4.75 | 1.39 | 1.52 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 3.97 | 2.15 | 0.45 | 1.12 | 1.23 | 1.65 | 2.14 | 1.76 | 1.89 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN10 | 0.26 | 0.23 | 0.11 | **-0.01** | **-0.02** | 0.21 | 0.26 | 0.10 | **-0.03** |
| | | (0.00) | (0.00) | (0.00) | (0.59) | (0.62) | (0.02) | (0.00) | (0.01) | (0.71) |
| | IN30 | 0.40 | 0.46 | **-0.03** | 0.28 | 0.33 | **-0.14** | **-0.02** | 0.24 | 0.19 |
| | | (0.00) | (0.00) | (0.92) | (0.00) | (0.00) | (0.99) | (0.67) | (0.00) | (0.00) |
| | CRSP | 2.47 | 1.35 | 0.48 | 0.58 | 0.83 | 1.05 | 1.18 | 1.96 | 2.30 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| NC | FF06 | 7.05 | 3.38 | 1.28 | 3.53 | 6.19 | 9.44 | 9.03 | 3.81 | 4.17 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 11.45 | 6.92 | 1.57 | 1.55 | 2.49 | 4.36 | 5.35 | 3.09 | 3.96 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN10 | 3.39 | 3.82 | 1.30 | 1.91 | 2.54 | 2.09 | 3.34 | 3.15 | 3.77 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 13.11 | 8.06 | 1.65 | 2.02 | 3.56 | 4.87 | 6.21 | 8.34 | 9.54 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 21.53 | 17.95 | 1.20 | 3.06 | 4.55 | 6.92 | 9.80 | 11.25 | 15.25 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| PPN | FF06 | 5.62 | 5.38 | 0.53 | 1.73 | 2.42 | 4.40 | 2.61 | **-3.56** | **-1.24** |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (1.00) | (0.97) |
| | FF25 | 9.74 | 5.57 | 1.02 | **-0.76** | **-2.19** | **-2.95** | 5.01 | 5.91 | 8.46 |
| | | (0.00) | (0.00) | (0.00) | (0.97) | (1.00) | (1.00) | (0.00) | (0.00) | (0.00) |
| | IN10 | 4.58 | 2.09 | 0.51 | 3.04 | 1.75 | 3.54 | 3.34 | 2.77 | 4.58 |
| | | (0.00) | (0.13) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | **-12.67** | **-4.79** | **-2.48** | **-5.23** | **-10.14** | **-6.47** | **-5.17** | 3.15 | 0.96 |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (0.00) | (0.20) |
| | CRSP | **-11.39** | **-5.80** | **-8.17** | **-14.14** | **-15.21** | **-8.45** | **-2.83** | 17.53 | 13.32 |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (0.00) | (0.00) |

Table 10: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Non-Tax-Optimized Strategy for 1/N, VW, NC, and PPN with a 50 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the non-tax-optimized strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps for the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | FF06 | 4.00 | 3.32 | 3.85 | 4.40 | 4.01 | 4.00 | 4.00 | 3.53 | 3.64 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 4.34 | 3.62 | 3.92 | 4.87 | 4.68 | 4.34 | 4.34 | 3.80 | 3.86 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| 1/N | IN10 | 3.63 | 3.34 | 3.29 | 4.26 | 4.58 | 4.26 | 3.83 | 4.21 | 3.66 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 7.00 | 6.52 | 5.64 | 7.73 | 8.52 | 8.55 | 8.28 | 6.95 | 6.68 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 5.04 | 5.01 | 5.79 | 10.25 | 13.04 | 13.36 | 12.30 | 6.21 | 5.11 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF06 | 6.15 | 2.22 | 1.09 | 2.33 | 3.92 | 5.84 | 6.14 | 1.76 | 1.97 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 6.00 | 2.89 | 1.28 | 1.98 | 2.40 | 3.17 | 3.78 | 2.45 | 2.68 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| VW | IN10 | 1.97 | 0.96 | 1.41 | 1.49 | 1.62 | 1.91 | 1.97 | 1.11 | 1.12 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 2.65 | 1.53 | 1.51 | 1.96 | 2.26 | 1.97 | 2.16 | 1.77 | 1.82 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 6.03 | 2.53 | 1.33 | 1.43 | 2.20 | 3.15 | 3.82 | 4.07 | 4.67 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF06 | 13.79 | 6.98 | 3.59 | 6.66 | 10.48 | 14.83 | 15.24 | 7.26 | 7.86 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 25.74 | 16.27 | 4.84 | 4.90 | 7.95 | 11.81 | 14.04 | 11.83 | 13.66 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| NC | IN10 | 12.74 | 10.84 | 4.44 | 6.39 | 8.45 | 9.12 | 11.17 | 9.46 | 10.67 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 27.03 | 17.96 | 6.00 | 6.48 | 10.04 | 13.23 | 15.69 | 18.29 | 20.20 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 43.03 | 33.52 | 6.71 | 8.73 | 13.30 | 19.01 | 23.65 | 27.83 | 33.46 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF06 | 38.75 | 24.94 | 5.67 | 12.01 | 18.30 | 25.99 | 27.85 | 6.43 | 12.07 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 39.53 | 19.54 | 8.16 | 11.85 | 17.61 | 23.75 | 29.21 | 24.63 | 30.97 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| PPN | IN10 | 41.94 | 20.58 | 6.26 | 11.69 | 17.41 | 25.56 | 29.73 | 19.74 | 24.51 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | IN30 | 16.57 | 14.83 | 10.26 | 14.83 | 22.96 | 29.45 | 31.10 | 29.07 | 28.87 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | 25.78 | 27.65 | 9.45 | 14.50 | 26.60 | 42.42 | 45.87 | 56.31 | 54.69 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

Table 11: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Non-Tax-Optimized Strategy for 1/N, VW, NC, and PPN with a 150 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the non-tax-optimized strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps for the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | FF06 | **-17.94** | **-17.86** | **-19.24** | **-18.79** | **-18.38** | **-18.02** | **-17.95** | **-18.45** | **-18.25** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-17.12** | **-17.59** | **-18.57** | **-18.58** | **-18.37** | **-17.83** | **-17.72** | **-17.89** | **-17.75** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| VW | IN10 | **-4.54** | **-3.56** | **-4.59** | **-5.23** | **-5.42** | **-5.06** | **-4.74** | **-5.08** | **-4.60** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-12.10** | **-9.41** | **-11.12** | **-12.16** | **-12.70** | **-13.01** | **-12.99** | **-12.30** | **-12.06** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-22.60** | **-24.09** | **-23.53** | **-24.50** | **-26.08** | **-26.27** | **-25.95** | **-22.41** | **-21.88** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 0.40 | 1.24 | **-0.06** | **-0.06** | 0.36 | 0.40 | 0.40 | 0.25 | 0.37 |
| | | (0.00) | (0.00) | (0.72) | (0.74) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 1.02 | 1.66 | 0.74 | 0.42 | 0.67 | 1.02 | 1.02 | 1.04 | 1.04 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| BH | IN10 | **-0.63** | **-0.49** | **-1.01** | **-1.49** | **-1.53** | **-1.15** | **-0.83** | **-1.39** | **-0.76** |
| | | (1.00) | (0.99) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-1.33** | 0.57 | **-0.92** | **-2.00** | **-2.51** | **-2.52** | **-2.34** | **-1.74** | **-1.35** |
| | | (1.00) | (0.01) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-3.95** | **-8.64** | **-7.67** | **-9.01** | **-10.41** | **-10.41** | **-9.96** | **-5.11** | **-3.95** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | **-23.10** | **-22.25** | **-23.82** | **-22.43** | **-22.09** | **-22.78** | **-23.51** | **-22.66** | **-22.24** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-18.52** | **-16.85** | **-19.19** | **-19.25** | **-18.88** | **-18.11** | **-17.63** | **-18.30** | **-17.79** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| NC | IN10 | **-16.68** | **-11.84** | **-14.28** | **-14.70** | **-14.64** | **-13.81** | **-14.04** | **-12.86** | **-12.80** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-17.82** | **-16.63** | **-20.27** | **-20.86** | **-20.70** | **-19.88** | **-18.55** | **-16.60** | **-15.54** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-12.15** | **-15.47** | **-20.91** | **-20.73** | **-21.93** | **-20.72** | **-19.61** | **-11.42** | **-6.95** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 7.46 | 11.19 | 9.93 | 9.70 | 10.08 | 9.04 | 7.81 | 10.72 | 10.56 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 6.61 | 9.81 | 9.84 | 9.86 | 9.44 | 9.59 | 9.95 | 10.27 | 10.75 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| PPN | IN10 | **-9.58** | 0.31 | 0.24 | 0.79 | **-0.34** | **-2.08** | **-5.26** | 0.64 | 0.07 |
| | | (1.00) | (0.37) | (0.37) | (0.14) | (0.68) | (1.00) | (1.00) | (0.19) | (0.46) |
| | IN30 | 5.12 | 23.03 | 42.60 | 39.85 | 37.59 | 33.05 | 28.53 | 31.70 | 20.22 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | **-1.88** | 22.26 | 52.96 | 49.20 | 46.82 | 44.08 | 40.47 | 37.64 | 26.52 |
| | | (1.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

Table 12: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Corresponding Heuristic 1/N Strategy for VW, BH, NC, and PPN with a 0 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the corresponding heuristic 1/N strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | FF06 | **-15.65** | **-17.94** | **-19.21** | **-18.67** | **-17.27** | **-15.88** | **-15.65** | **-18.25** | **-18.16** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-16.52** | **-17.85** | **-19.21** | **-19.30** | **-18.99** | **-18.39** | **-17.99** | **-18.05** | **-17.97** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| VW | IN10 | **-4.04** | **-4.20** | **-4.44** | **-5.21** | **-5.34** | **-4.70** | **-4.22** | **-5.12** | **-4.58** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-11.38** | **-11.16** | **-11.39** | **-12.49** | **-12.88** | **-13.24** | **-12.88** | **-11.65** | **-11.42** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-18.59** | **-19.49** | **-22.76** | **-25.06** | **-26.35** | **-26.11** | **-25.20** | **-20.04** | **-18.87** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 0.38 | 0.61 | 0.21 | **-0.08** | 0.37 | 0.38 | 0.38 | 0.52 | 0.50 |
| | | (0.00) | (0.00) | (0.01) | (0.82) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 0.84 | 1.01 | 1.00 | 0.23 | 0.50 | 0.84 | 0.84 | 1.14 | 1.10 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| BH | IN10 | **-0.39** | **-0.54** | **-0.66** | **-1.35** | **-1.47** | **-1.03** | **-0.59** | **-1.35** | **-0.67** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-0.90** | **-0.71** | **-0.48** | **-2.02** | **-2.50** | **-2.46** | **-2.16** | **-1.04** | **-0.73** |
| | | (1.00) | (1.00) | (0.98) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-3.00** | **-3.01** | **-6.14** | **-9.02** | **-10.82** | **-10.73** | **-9.74** | **-4.25** | **-3.17** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | **-20.80** | **-23.34** | **-25.21** | **-23.76** | **-21.45** | **-19.04** | **-19.34** | **-23.10** | **-22.85** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-15.07** | **-18.38** | **-22.47** | **-23.08** | **-22.16** | **-20.47** | **-19.72** | **-21.21** | **-20.57** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| NC | IN10 | **-13.88** | **-13.65** | **-15.84** | **-15.92** | **-15.51** | **-15.50** | **-14.09** | **-14.91** | **-13.80** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-14.14** | **-17.81** | **-22.51** | **-23.44** | **-22.66** | **-21.65** | **-20.41** | **-17.88** | **-16.71** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-5.30** | **-8.10** | **-23.70** | **-24.68** | **-25.09** | **-23.33** | **-20.38** | **-14.43** | **-10.35** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 2.44 | 2.45 | **-2.67** | **-1.78** | **-0.67** | 1.25 | **-0.48** | **-6.33** | **-4.10** |
| | | (0.05) | (0.00) | (1.00) | (0.98) | (0.80) | (0.02) | (0.77) | (1.00) | (1.00) |
| | FF25 | 5.99 | 2.15 | **-2.27** | **-4.72** | **-5.84** | **-6.26** | 1.43 | 2.61 | 5.04 |
| | | (0.00) | (0.03) | (1.00) | (1.00) | (1.00) | (1.00) | (0.03) | (0.00) | (0.00) |
| PPN | IN10 | **-9.25** | **-11.54** | **-13.01** | **-11.44** | **-12.65** | **-10.72** | **-10.50** | **-11.68** | **-9.49** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-2.82** | 6.15 | 8.98 | 4.27 | **-1.62** | 2.44 | 4.19 | 14.62 | 12.54 |
| | | (1.00) | (0.00) | (0.00) | (0.00) | (0.91) | (0.02) | (0.00) | (0.00) | (0.00) |
| | CRSP | **-3.28** | 2.81 | **-3.02** | **-12.10** | **-14.91** | **-8.04** | **-1.30** | 26.63 | 23.47 |
| | | (1.00) | (0.01) | (0.95) | (1.00) | (1.00) | (1.00) | (0.74) | (0.00) | (0.00) |

Table 13: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Corresponding Heuristic 1/N Strategy for VW, BH, NC, and PPN with a 50 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the corresponding heuristic 1/N strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| | FF06 | **-15.62** | **-18.21** | **-19.53** | **-18.97** | **-17.40** | **-15.87** | **-15.63** | **-18.75** | **-18.66** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-16.49** | **-18.38** | **-19.88** | **-20.06** | **-19.59** | **-18.72** | **-18.24** | **-18.87** | **-18.74** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| VW | IN10 | **-4.02** | **-4.70** | **-4.23** | **-5.05** | **-5.22** | **-4.65** | **-4.20** | **-5.35** | **-4.84** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-11.36** | **-11.94** | **-11.23** | **-12.55** | **-12.94** | **-13.21** | **-12.83** | **-12.09** | **-11.81** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-18.56** | **-21.23** | **-22.73** | **-25.78** | **-27.07** | **-26.59** | **-25.42** | **-20.95** | **-19.67** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 0.38 | 1.04 | 0.53 | 0.00 | 0.38 | 0.38 | 0.38 | 0.84 | 0.73 |
| | | (0.00) | (0.00) | (0.00) | (0.48) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | FF25 | 0.83 | 1.53 | 1.24 | 0.33 | 0.51 | 0.83 | 0.83 | 1.36 | 1.29 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| BH | IN10 | **-0.39** | **-0.11** | **-0.06** | **-0.99** | **-1.29** | **-0.98** | **-0.58** | **-0.94** | **-0.42** |
| | | (1.00) | (0.88) | (0.64) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-0.89** | **-0.43** | 0.39 | **-1.55** | **-2.28** | **-2.30** | **-2.05** | **-0.84** | **-0.58** |
| | | (1.00) | (1.00) | (0.04) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-3.07** | **-3.05** | **-3.76** | **-7.65** | **-9.93** | **-10.19** | **-9.34** | **-4.14** | **-3.14** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | **-20.81** | **-25.06** | **-27.80** | **-26.05** | **-23.11** | **-20.08** | **-19.79** | **-25.01** | **-24.67** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | **-15.08** | **-20.93** | **-28.91** | **-29.50** | **-27.33** | **-24.48** | **-22.98** | **-24.08** | **-22.88** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| NC | IN10 | **-13.91** | **-15.12** | **-19.99** | **-19.25** | **-17.93** | **-17.17** | **-15.27** | **-16.87** | **-15.51** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-14.18** | **-19.94** | **-27.47** | **-28.55** | **-26.70** | **-24.59** | **-22.76** | **-20.05** | **-18.55** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | CRSP | **-5.54** | **-11.79** | **-30.02** | **-31.59** | **-30.47** | **-27.18** | **-23.62** | **-16.51** | **-11.92** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF06 | 2.41 | **-7.17** | **-21.88** | **-17.64** | **-12.68** | **-7.00** | **-5.63** | **-21.09** | **-17.00** |
| | | (0.05) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | FF25 | 5.97 | **-8.58** | **-17.52** | **-15.47** | **-10.96** | **-6.01** | **-1.87** | **-4.85** | **-0.07** |
| | | (0.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (0.54) |
| PPN | IN10 | **-9.21** | **-22.66** | **-31.81** | **-28.99** | **-25.58** | **-20.17** | **-17.18** | **-23.84** | **-20.38** |
| | | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) | (1.00) |
| | IN30 | **-2.83** | **-3.84** | **-6.91** | **-4.92** | 1.06 | 6.38 | 8.00 | 7.64 | 7.75 |
| | | (1.00) | (0.98) | (1.00) | (1.00) | (0.26) | (0.00) | (0.00) | (0.00) | (0.00) |
| | CRSP | **-3.43** | **-1.98** | **-16.56** | **-16.25** | **-9.69** | 1.31 | 4.75 | 18.68 | 18.68 |
| | | (1.00) | (0.93) | (1.00) | (1.00) | (1.00) | (0.18) | (0.00) | (0.00) | (0.00) |

Table 14: **Certainty Equivalent Wealth Gain/Loss of Each Heuristic Tax Trading Strategy Relative to the Corresponding Heuristic 1/N Strategy for VW, BH, NC, and PPN with a 150 Basis Point Transaction Cost.** This table reports the certainty equivalent gain/loss in percent of initial wealth a particular heuristic tax trading strategy earns relative to the corresponding heuristic 1/N strategy under each data set. The values in parenthesis are p-values from 1,000 bootstraps under the null hypothesis that the certainty equivalent gain is non-positive.

| Model | Data | BM | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/N | FF06 | 9.86 | 6.96 | 8.89 | 7.67 | 7.16 | 6.98 | 6.96 | 6.96 | 7.04 | 6.98 |
| | FF25 | 10.49 | 7.06 | 9.30 | 7.89 | 7.36 | 7.15 | 7.06 | 7.06 | 7.07 | 7.07 |
| | IN10 | 11.96 | 7.13 | 10.04 | 8.54 | 7.79 | 7.46 | 7.25 | 7.17 | 7.36 | 7.16 |
| | IN30 | 13.54 | 7.21 | 11.20 | 9.39 | 8.32 | 7.80 | 7.47 | 7.32 | 7.24 | 7.22 |
| | CRSP | 20.60 | 7.43 | 14.38 | 13.43 | 10.87 | 9.53 | 8.61 | 8.24 | 7.51 | 7.43 |
| VW | FF06 | 9.64 | 7.14 | 8.92 | 8.52 | 8.04 | 7.55 | 7.25 | 7.15 | 8.22 | 7.76 |
| | FF25 | 10.96 | 6.83 | 9.35 | 9.51 | 9.28 | 8.69 | 8.04 | 7.67 | 8.18 | 7.61 |
| | IN10 | 10.55 | 7.19 | 9.34 | 8.42 | 7.93 | 7.44 | 7.23 | 7.19 | 7.41 | 7.28 |
| | IN30 | 11.74 | 7.38 | 10.14 | 9.25 | 8.98 | 8.37 | 7.82 | 7.57 | 7.60 | 7.47 |
| | CRSP | 15.64 | 7.81 | 12.09 | 13.91 | 13.79 | 12.79 | 11.35 | 10.27 | 8.81 | 8.32 |
| NC | FF06 | 21.59 | 8.01 | 11.41 | 18.08 | 15.98 | 13.66 | 11.58 | 10.08 | 12.97 | 11.63 |
| | FF25 | 29.84 | 7.38 | 12.66 | 25.26 | 25.67 | 23.08 | 19.96 | 18.04 | 16.67 | 13.97 |
| | IN10 | 25.38 | 9.54 | 14.28 | 21.67 | 19.33 | 16.82 | 14.38 | 12.91 | 14.73 | 13.04 |
| | IN30 | 28.65 | 9.45 | 15.81 | 24.04 | 23.76 | 21.03 | 18.12 | 16.27 | 14.30 | 12.30 |
| | CRSP | 36.96 | 7.73 | 17.17 | 29.67 | 29.41 | 25.10 | 20.20 | 17.50 | 12.75 | 10.18 |
| PPN | FF06 | 41.84 | 6.80 | 13.96 | 37.76 | 32.71 | 27.04 | 20.00 | 16.16 | 35.05 | 29.16 |
| | FF25 | 34.52 | 6.60 | 13.11 | 29.64 | 26.52 | 22.06 | 16.34 | 12.96 | 16.92 | 11.20 |
| | IN10 | 48.62 | 8.37 | 17.22 | 43.83 | 40.97 | 34.83 | 26.77 | 21.68 | 38.75 | 29.92 |
| | IN30 | 48.11 | 6.42 | 14.61 | 41.89 | 38.20 | 31.71 | 23.91 | 19.05 | 21.09 | 13.10 |
| | CRSP | 65.13 | 6.36 | 15.35 | 58.04 | 51.01 | 41.41 | 29.71 | 23.06 | 16.06 | 9.87 |

Table 15: **Trading Volume under the Tax Heuristic Strategies and a 0 Basis Point Transaction Cost.** This table reports the trading volume as a percentage of wealth under the not tax benchmark strategy (BM) and the tax heuristic strategies.

| Model | Data | BM | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/N | FF06 | 1.61 | 0.20 | 0.39 | 0.53 | 0.29 | 0.21 | 0.20 | 0.20 | 0.34 | 0.30 |
| | FF25 | 1.97 | 0.18 | 0.37 | 0.61 | 0.35 | 0.24 | 0.19 | 0.18 | 0.26 | 0.25 |
| | IN10 | 2.59 | 0.20 | 0.38 | 0.90 | 0.59 | 0.40 | 0.26 | 0.22 | 0.42 | 0.30 |
| | IN30 | 3.45 | 0.19 | 0.26 | 1.38 | 0.87 | 0.58 | 0.36 | 0.27 | 0.28 | 0.25 |
| | CRSP | 8.35 | 0.57 | 0.58 | 3.91 | 2.54 | 1.86 | 1.28 | 1.01 | 0.64 | 0.61 |
| VW | FF06 | 1.19 | 0.20 | 0.55 | 0.70 | 0.54 | 0.33 | 0.24 | 0.21 | 0.79 | 0.69 |
| | FF25 | 1.85 | 0.19 | 0.70 | 1.22 | 1.12 | 0.89 | 0.63 | 0.46 | 0.97 | 0.81 |
| | IN10 | 1.97 | 0.20 | 0.62 | 0.76 | 0.55 | 0.33 | 0.22 | 0.20 | 0.61 | 0.50 |
| | IN30 | 2.50 | 0.20 | 0.68 | 1.14 | 1.01 | 0.71 | 0.44 | 0.33 | 0.63 | 0.53 |
| | CRSP | 4.56 | 0.38 | 1.28 | 3.75 | 3.63 | 3.08 | 2.28 | 1.69 | 1.38 | 1.11 |
| NC | FF06 | 6.48 | 0.23 | 0.91 | 4.28 | 3.22 | 2.27 | 1.49 | 0.89 | 2.59 | 2.06 |
| | FF25 | 11.48 | 0.22 | 1.44 | 8.31 | 8.63 | 7.07 | 5.39 | 4.49 | 4.37 | 3.25 |
| | IN10 | 6.69 | 0.25 | 1.00 | 4.64 | 3.54 | 2.76 | 1.97 | 1.46 | 2.42 | 1.90 |
| | IN30 | 9.18 | 0.24 | 1.35 | 6.52 | 6.18 | 4.89 | 3.69 | 3.04 | 2.61 | 1.90 |
| | CRSP | 15.96 | 0.53 | 1.34 | 11.58 | 11.43 | 9.19 | 6.84 | 5.57 | 3.58 | 2.27 |
| PPN | FF06 | 21.47 | 0.22 | 3.48 | 18.09 | 14.83 | 11.11 | 6.83 | 4.45 | 15.27 | 12.32 |
| | FF25 | 17.80 | 0.21 | 3.32 | 13.81 | 11.93 | 9.16 | 5.88 | 3.60 | 5.84 | 3.37 |
| | IN10 | 26.46 | 0.21 | 4.70 | 22.32 | 19.90 | 16.04 | 10.71 | 7.40 | 15.20 | 11.03 |
| | IN30 | 29.21 | 0.19 | 4.28 | 24.34 | 22.10 | 17.94 | 12.42 | 9.08 | 8.07 | 4.84 |
| | CRSP | 44.41 | 0.63 | 3.39 | 38.86 | 35.20 | 28.24 | 19.36 | 14.02 | 6.35 | 3.58 |

Table 16: **Trading Volume under the Tax Heuristic Strategies and a 50 Basis Point Transaction Cost.** This table reports the trading volume as a percentage of wealth under the not tax benchmark strategy (BM) and the tax heuristic strategies.

| Model | Data | BM | NRG | ORL | P10 | PP25 | PP50 | PP100 | PP150 | G0.1 | G0.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/N | FF06 | 1.47 | 0.20 | 0.38 | 0.43 | 0.26 | 0.20 | 0.20 | 0.20 | 0.33 | 0.30 |
| | FF25 | 1.74 | 0.18 | 0.36 | 0.48 | 0.29 | 0.22 | 0.18 | 0.18 | 0.26 | 0.25 |
| | IN10 | 2.17 | 0.20 | 0.37 | 0.67 | 0.45 | 0.33 | 0.24 | 0.21 | 0.39 | 0.30 |
| | IN30 | 2.88 | 0.19 | 0.27 | 1.05 | 0.67 | 0.46 | 0.31 | 0.25 | 0.27 | 0.24 |
| | CRSP | 6.75 | 0.50 | 0.50 | 3.07 | 2.04 | 1.49 | 1.05 | 0.86 | 0.57 | 0.53 |
| VW | FF06 | 1.13 | 0.20 | 0.54 | 0.66 | 0.51 | 0.32 | 0.23 | 0.20 | 0.76 | 0.66 |
| | FF25 | 1.76 | 0.19 | 0.68 | 1.14 | 1.07 | 0.84 | 0.59 | 0.44 | 0.93 | 0.78 |
| | IN10 | 1.86 | 0.20 | 0.60 | 0.72 | 0.52 | 0.32 | 0.21 | 0.20 | 0.59 | 0.49 |
| | IN30 | 2.34 | 0.20 | 0.66 | 1.04 | 0.94 | 0.67 | 0.43 | 0.32 | 0.61 | 0.52 |
| | CRSP | 4.18 | 0.34 | 1.21 | 3.42 | 3.35 | 2.84 | 2.08 | 1.52 | 1.28 | 1.03 |
| NC | FF06 | 6.17 | 0.23 | 0.89 | 4.02 | 2.99 | 2.06 | 1.29 | 0.79 | 2.48 | 1.97 |
| | FF25 | 10.92 | 0.22 | 1.41 | 7.73 | 8.06 | 6.51 | 4.89 | 4.00 | 4.08 | 3.02 |
| | IN10 | 6.17 | 0.25 | 0.96 | 4.08 | 3.05 | 2.34 | 1.61 | 1.19 | 2.05 | 1.64 |
| | IN30 | 8.52 | 0.24 | 1.31 | 5.77 | 5.52 | 4.30 | 3.11 | 2.49 | 2.26 | 1.69 |
| | CRSP | 14.17 | 0.47 | 1.31 | 9.99 | 9.73 | 7.64 | 5.59 | 4.60 | 3.05 | 2.02 |
| PPN | FF06 | 20.93 | 0.22 | 3.56 | 17.40 | 14.25 | 10.57 | 6.41 | 4.17 | 14.89 | 11.90 |
| | FF25 | 17.25 | 0.20 | 3.31 | 13.10 | 11.19 | 8.32 | 5.21 | 3.29 | 5.79 | 3.29 |
| | IN10 | 25.79 | 0.20 | 4.66 | 21.57 | 19.04 | 14.96 | 9.89 | 6.82 | 15.31 | 11.14 |
| | IN30 | 27.62 | 0.19 | 4.37 | 22.31 | 19.89 | 15.51 | 10.54 | 7.64 | 8.58 | 4.88 |
| | CRSP | 39.44 | 0.55 | 3.28 | 33.23 | 29.70 | 23.49 | 15.81 | 11.63 | 6.25 | 3.44 |

Table 17: **Trading Volume under the Tax Heuristic Strategies and a 150 Basis Point Transaction Cost.** This table reports the trading volume as a percentage of wealth under the not tax benchmark strategy (BM) and the tax heuristic strategies.