Testing Rebalancing Strategies for Stock-Bond Portfolios: Where Is the Value Added of Rebalancing?

Hubert Dichtl[a], Wolfgang Drobetz[b] and Martin Wambach[c,*]

[a] Hubert Dichtl, alpha portfolio advisors GmbH, Wiesbadener Weg 2a, 65812 Bad Soden/Ts., Germany, Mail: dichtl@alphaport.de.

[b] Wolfgang Drobetz, Institute of Finance, University of Hamburg, Von-Melle-Park 5, 20146 Hamburg, Germany, Mail: wolfgang.drobetz@wiso.uni-hamburg.de.

[c] Martin Wambach, Institute of Finance, University of Hamburg, Von-Melle-Park 5, 20146 Hamburg, Germany, Mail: martin.wambach@wiso.uni-hamburg.de.

* Corresponding author: Mail: martin.wambach@wiso.uni-hamburg.de, Phone: 040-42838-6442.

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Abstract

A common argument is that portfolio rebalancing improves return and simultaneously reduces risk in the long-run. Previous rebalancing studies are unable to verify this proposition using statistical inference. We introduce a novel block bootstrap approach that enables us to test the value added of rebalancing strategies for stock-bond portfolios using historical data from the United States, the United Kingdom, and Germany. Analyzing the return, the volatility, and the Sharpe ratio of different rebalancing strategies, historical simulation results indicate that rebalancing strategies outperform a buy-and-hold strategy. While this outperformance is only of marginal importance in terms of average returns and net asset values (NAVs), it can be primarily attributed to a significant reduction of portfolio volatility. Based on the Sharpe ratio as a risk-adjusted performance measure, our results further indicate that the superior performance of rebalancing strategies compared with a buy-and-hold strategy is statistically significant and arises from this reduced portfolio volatility. Depending on the specific stock and bond market characteristics of the three countries under investigation, the optimal rebalancing frequency ranges between quarterly and yearly intervals.

Keywords: Portfolio rebalancing, performance measurement, block bootstrap

JEL classification: G11

1. Introduction

Having identified an investor's risk preference and regulatory environment, it is the primary objective of an institutional asset manager to implement and supervise the most suitable asset allocation for his client. Once this initial asset allocation has been implemented, the literature differentiates between three reasons for portfolio rebalancing: i) rebalancing due to a shift in an investor's risk profile and/or modified regulatory requirements; ii) rebalancing based on changes in the expectations about future returns and risks; and iii) rebalancing due to market movements. As discussed in Fabozzi, Focardi, and Kolm (2006) as well as in Leibowitz and Bova (2011), the first two reasons require the asset manager to construct a new optimal portfolio. In this study, we focus on the third reason: As different assets generate different rates of returns, a portfolio's relative asset composition will deviate from the target weights over time. In order to remain consistent with the institutional investor's initially evaluated return and risk preferences, the portfolio manager has to rebalance the assets back to their predefined target weights. However, as rebalancing strategies imply selling a fraction of the better performing assets and investing the proceeds in the worse performing, it is a highly challenging question whether rebalancing strategies generate a value added for institutional investors and – if so – what are the sources of this value added.

Evaluating the value added of portfolio rebalancing, our study makes two major contributions to the literature. The first contribution refers to the applied methodology. In contrast to all previous studies, we are able to statistically test the value added of a set of different rebalancing strategies. To the best of our knowledge, there are no studies which examine rebalancing strategies in terms of statistical inference. Previous research remains incomplete because it is based on historical analyses and merely investigates a single realization or a fairly small number of realizations of the stock and bond markets. Moreover, these studies document similar results for different variants of rebalancing strategies, making it impossible to recommend one specific strategy to institutional investors. The only systematic finding is that a buy-and-hold strategy seems to underperform rebalancing strategies when both the return and the risk of these strategies are taken into account. But even in this case, a major concern is whether these findings are statistically significant. It is possible that the return observations are more influenced by specific characteristics of the underlying sample period rather than by the properties of the rebalancing strategy under investigation. As this danger of data snooping can be severe, the empirical results of these studies do not allow reliable interpretations (Brock, Lakonishok, and LeBaron (1992)). Dividing the sample period into disjunctive sub-periods, e.g., up- and downswings of the stock market (Harjoto and Jones (2006)), does not solve this fundamental inference problem either, as this procedure cannot generate enough observations to conduct a statistical test. Monte Carlo simulations avoid this problem by deriving distributions under different economic scenarios. Nevertheless, this simulation technique generally suffers from the shortcoming that it is not based on historical financial markets' data. Instead, specific assumptions have to be made in advance which strongly predetermine the empirical outcome in many cases. Moreover, if time series characteristics of assets as well as of entire financial markets are not correctly or not completely incorporated, simulations results will be biased, making it very difficult to draw meaningful economic conclusions.

Given these shortcomings of both historical analyses and Monte Carlo simulations, we implement a novel block bootstrap approach that enables us to calculate confidence intervals for the different strategies' performance measures based on historical data. In addition to our major objective to provide statistical significance levels, two other aspects are of paramount importance for our analysis. Firstly, we focus on investment horizons of 5, 7, and 10 years, respectively, in order to model the requirements of institutional investors in a realistic setup. Secondly, applying a rolling time window technique, we exploit the information of the underlying sample period in the most efficient way. On the one hand, this procedure allows us to analyze typical investment horizons of institutional investors that have been realized in the past. On the other hand, we are able to generate sufficient "observations" in order to conduct a statistical test. In contrast to a common t-test, our test statistic is robust against time series dependencies, i.e., the high autocorrelation which is inherent in tests based on rolling time windows.

Being able to report statistical significance levels, our double block bootstrap approach enables a systematic analysis of the value added of rebalancing strategies. In particular, we are in the position to investigate whether the value added of rebalancing arises due to a return effect, a volatility effect, or both. A second contribution of our study relates to the observation that prior rebalancing studies mostly focus on the US market. While Buetow et al. (2002), Masters (2003) as well as McLellan, Kinlaw, and Abouzaid (2009) consider international equities in a multi-asset class portfolio, Plaxco and Arnott (2002) analyze an international balanced portfolio consisting of bonds and stocks of 11 countries. Nevertheless, to the best of our knowledge there are no studies that investigate rebalancing strategies with a focus on institutional investors outside the US. This is an important issue because country-specific characteristics could lead to different empirical findings. Apart from various regulatory peculiarities, each country features unique stock and bond markets properties that potentially have an impact on rebalancing strategies with regards to the asset allocation, investment horizon, and optimal rebalancing frequency. Thus, any conclusions based on the empirical findings of one specific country or financial market cannot immediately be transferred to other financial markets. Therefore, we analyze the value added of rebalancing strategies by considering the different stock and bond market characteristics of the United States, the United Kingdom, and Germany. Overall, these two contributions – deriving statistical inference and using an international dataset – constitute the novel path that our analysis takes and which separates us from previous rebalancing studies.

Our historical simulations provide results which have immediate practical implications. First of all, despite the strong performance of stocks relative to bonds during the sample period, the average return of a buy-and-hold strategy is statistically significantly lower than that of different rebalancing strategies in the United Kingdom and Germany (albeit the difference is small in magnitude). However, this result does not hold uniformly across all markets; there is no statistical significance for US data. In order to incorporate the compound interest effect, we investigate net asset values in addition to average returns and cannot uncover significant economic differences. According to Perold and Sharpe (1988), these findings indicate that neither the mean reversion nor the momentum effect in the return data is strong enough to produce superior returns of either strategy. Secondly, we report that rebalancing strategies at all trading frequencies exhibit a significant lower volatility compared to the corresponding buy-and-hold strategy due to better diversification properties. Thirdly, analyzing the Sharpe ratio as a performance measure that incorporates both the return and the volatility of an investment strategy, our simulation results reveal that all rebalancing strategies significantly outperform buy-and-hold strategies. This finding is robust against all trading frequencies, contributing to the explanation why rebalancing strategies are popular in the investment practice. Fourthly, comparing different rebalancing intervals, we document that quarterly rebalancing produces significantly higher Sharpe ratios compared to monthly rebalancing. These findings suggest that there is an optimal rebalancing frequency, with both excessive rebalancing and no rebalancing leading to lower Sharpe ratios. However, these patterns with respect to different rebalancing intervals can change when we incorporate no-trade regions around the target weights. Our simulations incorporate realistic transaction costs, and the results are qualitatively the same in all countries.

The remainder of this paper is structured as follows: Section 2 provides a literature review on the value added of rebalancing strategies. Section 3 describes the different rebalancing strategies, the data set we use in our analysis, and the test design of our new block bootstrap approach. Section 4 presents and discusses the results from our simulation analysis. The paper concludes in section 5 and points out implications for portfolio management and institutional investors.

2. Literature overview

Due to its high importance for institutional portfolio management, several aspects of rebalancing and its practical implications have been analyzed in previous studies. The focal point of most analyses is the question whether rebalancing generates a value added to institutional investors. In order to get a brief overview, we summarize the most important studies. Exhibit 1 presents their research objectives and main results.

[Insert Exhibit 1 here]

Our empirical analysis is based on the theoretical findings of Perold and Sharpe (1988), who discuss various portfolio strategies under different market scenarios. Focusing on a two asset portfolio consisting of stocks and bills, they document that buy-and-hold strategies offer a downside protection that is proportional to the amount allocated into bills, while the upside potential is proportional to the amount allocated into stocks. Representing the sale of portfolio insurance (hence buying stocks and selling bonds when stocks decreased), rebalancing strategies exhibit less downside protection compared to buy-and-hold. Moreover, facing a persistent market upswing, a frequent reallocation to the less-performing asset also leads to a lower upside potential. Rebalancing strategies perform best in relatively trendless, but volatile markets because mean-reversion is much more pronounced in this environment. These reversals could improve portfolio returns while simultaneously reducing the risk of rebalancing strategies. Overall, Perold and Sharpe (1988) show that dynamic portfolio strategies, such as buy-and-hold, CPPI, and rebalancing strategies, will produce different risk and return characteristics. They emphasize that the choice of an appropriate strategy is subject to the investor's risk preference. Therefore, not only the return of a strategy, but also its risk must be carefully taken into account.

In order to provide evidence that rebalancing strategies are able to generate a value added to institutional investors, Arnott and Lovell (1993) examine several rebalancing strategies over a 24-year sample period from 1968 to 1991.[1] The starting point is a 50/50 US stock-bond portfolio with 1% transaction costs included. Following Perold and Sharpe (1988), Arnott and Lovell (1993) investigate the average return, the volatility and the Treynor ratio of each strategy. They document that a monthly rebalancing strategy features the highest return, and the corresponding volatility is only slightly higher compared to the strategy with the lowest volatility. Using the Treynor ratio as a performance measure that incorporates both a strategy's return and systematic risk, the empirical results are weaker. Nine out of ten rebalancing strategies exhibit a higher Treynor ratio than the corresponding buy-and-hold strategy. While this finding seems to indicate that rebalancing outperforms a simple buy-and-hold strategy during the underlying sample period, all Treynor ratio values lie very close together within the interval [0.784; 0.794], and hence it is not obvious which strategies actually performs best during the long 24-year sample period. Nevertheless, Arnott and Lovell (1993) claim that a rebalancing strategy offers enhanced returns without increasing risk. They advise a monthly rebalancing strategy to investors with a long investment horizon and emphasize the increasing impact of the compound interest effect on the performance.

Tsai (2001) analyzes the value added of rebalancing over a 15-year sample period from 1986 to 2000. In contrast to other studies which mainly focus on a 60/40 stock-bond asset allocation, Tsai (2001) constructs five stock-bond portfolios with a 20%, 40%, 60%, 80%, and 98% equity allocation. These varying portfolio compositions are assumed to represent different risk profiles of institutional investors. Depending on the underlying risk preferences, both the equity and the bond sub-index also differs in the exact composition. Specifically, the equity sub-index consists of large-cap equity, small- and mid-cap equity, international equity, and real estate, whereas the bond sub-index includes domestic bonds, high yield bonds, and cash equivalents. Evaluating the performance on the basis of the Sharpe ratio, Tsai (2001) confirms the previous results of Arnott and Lovell (1993), and hence rebalancing outperforms a simple buy-and-hold strategy for each of the five different risk profiles during the observation period. Therefore, a frequent reallocation back to the target weights seems to provide some value added for institutional investors. However, it is again impossible to determine which of these rebalancing strategies performs best in her framework. Tsai (2001) argues that it does not

[1] In particular, Arnott and Lovell (1993) analyze buy-and-hold; periodic rebalancing with monthly, quarterly, and yearly rebalancing frequencies; ±1%, ±2%, and ±5% interval rebalancing to predefined target weights as well as ±1%, ±2%, and ±5% interval rebalancing to the nearest edge of the pre-specified thresholds.

matter much which rebalancing strategy is used because no strategy is consistently better across portfolios of different risk profiles. Moreover, transaction costs are omitted from her analysis, which weakens the explanatory power of the results as one would expect that the Sharpe ratios of rebalancing strategies are overestimated compared to a buy-and-hold strategy.

Donohue and Yip (2003) also investigate rebalancing strategies and compare their empirical findings with a self-developed rebalancing algorithm that is able to balance the competing transaction costs and tracking error. Both a historical analysis over a 10-year sample period from 1987 to 1996 and a Monte Carlo simulation provide evidence that buy-and-hold exhibits the lowest Sharpe ratio of all strategies. The results of Monte Carlo simulations indicate that the self-developed "optimal" rebalancing strategy ranks first in terms of the expected utility and the Sharpe ratio, whereas buy-and-hold performs worst in both categories. Finally, the simulation results reveal that optimal rebalancing provides both higher returns and a lower risk compared to common rebalancing heuristics.

Harjoto and Jones (2006) examine whether rebalancing to a specific threshold is able to enhance risk-adjusted portfolio performance. Concentrating on volatile market conditions, the sample period from 1995 to 2004 covers both the Asian financial crisis and the dot.com crisis. According to Perold and Sharpe (1988), investors should be more likely to rebalance their portfolio allocation during such volatile markets. Based on a 60/40 stock-bond asset allocation, Harjoto and Jones (2006) document that a rebalancing strategy with an incorporated no-trade interval of 15% leads both to the highest average return and to the lowest standard deviation. Therefore, this strategy also features the highest Sharpe ratio over the entire observation period. These findings are reinforced by dividing the full sample period into an upswing, a downswing and a recovery sub-sample. Again, a rebalancing strategy with an incorporated no-trade interval of 15% exhibits the highest Sharpe ratio in all market environments. Taken as a whole, investors should readjust their portfolio structure, but not too frequently. However, two potential drawbacks are worth noting. Firstly, transaction costs must be incorporated as they could have a major influence on any reallocation decisions. Secondly, it is possible that the standard deviations of the market downswing and recovery periods do not represent suitable estimators due to the fact that the volatility calculations are based on only 27 and 30 observations, respectively.

Tokat and Wicas (2007) characterize rebalancing as a powerful instrument for controlling risk. They conduct Monte Carlo simulations using a 60/40 stock-bond portfolio. The calibration of the mean, the volatility and the cross-correlation parameters is based on a historical sample of the US bond and stock markets from 1960 to 2003. A normal return distribution is assumed to represent the return generating process. Transaction costs are also included in all simulations. By changing the underlying autocorrelation structure, the Monte Carlo simulation enables Tokat and Wicas (2007) to investigate the impact of different market scenarios on monthly, quarterly, and annual rebalancing strategies with thresholds of ±1%, ±5%, and ±10%. In particular, they focus on trending and mean-reversion markets as well as a random walk environment. Overall, Tokat and Wicas (2007) conclude that rebalancing achieves minimizing risk relative to a predefined asset allocation in all market environments.

Jaconetti, Kinniry, and Zilbering (2010) also assume that the value-added of rebalancing can primarily be attributed to the reduction of risk relative to the predefined target allocation. In contrast to Tokat and Wicas (2007), they conduct a historical analysis to support their hypotheses. Applying a sample period from 1926 to 2009 for US stock and bond data, they analyze monthly, quarterly, and yearly rebalancing strategies with thresholds of 0, ±1%, ±5%, and ±10% as well as buy-and-hold. Transaction costs are included in all calculations. Starting with a 60/40 stock-bond portfolio, buy-and-hold exhibits the highest average annualized return with a value of 9.1% after an investment period of 84 years, but also the highest volatility with a value of 14.4% due to an average stock allocation of 84.1%. All remaining strategies feature an average stock allocation between 60.1% and 63.0% with average returns that differ only slightly ranging between 8.5% and 8.8%. The standard deviations lie within the narrow 11.8% and 12.3% band. While it is evident that most institutional investors cannot apply a buy-and-hold strategy on a long-term basis, it is again not obvious which rebalancing strategy leads to superior results.[2] Accordingly, Jaconetti, Kinniry, and Zilbering (2010) conclude that there is no universally optimal rebalancing strategy.

Despite many similarities, our analysis of the value added of rebalancing strategies differs from the studies presented in this section. Our first contribution is methodological and relates to the implementation of a novel double block bootstrap approach. Running historical simulations, this framework allows us to calculate confidence intervals, and hence we are able to analyze and compare the value added of any two rebalancing strategies by reporting statistical significance levels. Compared with a common t-test, this methodology delivers reliable

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[2] The possibility of facing extreme portfolio allocations prevents most institutional investors from conducting a buy-and-hold strategy for long investment horizons. For example, the original 60/40 stock-bond portfolio exhibits stock allocations that fluctuate between 36% and 99% during the 84 years investment period.

results even in the presence of pronounced time series dependencies in a rolling window approach. Our second contribution refers to the analysis of different countries. Given the country-specific characteristics of the stock and bond markets, one cannot assume that particular relationships that hold in one country are also observable for any other country. We therefore look at the stock and bond markets of the United States, the United Kingdom, and Germany in our analysis in order to check whether their country-specific characteristics have an impact on the value added of rebalancing. Finally, these two aspects enable us to draw implications with respect to an optimal rebalancing frequency. Based on Sharpe ratios, our simulation results indicate that the optimal rebalancing frequency ranges between quarterly and yearly. Several sensitivity tests verify that our findings are robust with respect to the changeable strategy parameters: the country, the investment horizon, transaction costs, and threshold levels.

3. Methodology

This section introduces our empirical methodology. We start with a description of the implemented rebalancing strategies in section 3.1, present our database in section 3.2, and proceed with an explanation of our block bootstrap approach in section 3.3.

3.1 Implemented rebalancing strategies

Academic literature as well as institutional portfolio managers differentiate between periodic and interval rebalancing strategies. Advising a periodic rebalancing mandate, a portfolio manager has to rebalance the assets to their initial target weights at the end of each predetermined period (e.g., yearly, quarterly, or monthly). In contrast, an interval rebalancing mandate requires the portfolio manager to adjust the asset allocation whenever an asset moves beyond a pre-specified threshold (e.g., ±3%, ±5%, or ±10%). Our study focuses on a mixture of both methodologies, hence periodic rebalancing with the additional option to incorporate a symmetric no-trade interval around the target weights.

Moreover, one has to distinguish between two different approaches with regards to the implementation of the symmetric no-trade interval. Specifically, when an asset exceeds the predetermined interval boundaries, either a strict adjustment to the target weights (Buetow et al. (2002), Harjoto and Jones (2006)) or a rebalancing to the corresponding interval boundaries (Leland, 1999) must be implemented. As Perold and Sharpe (1988) emphasize, different strategies can produce strongly different risk and return characteristics, and hence we implement the most common rebalancing strategies: i) buy-and-hold, ii) periodic rebalancing, iii) periodic interval rebalancing with a strict adjustment to the initial target weights ("threshold approach") and iv) periodic interval rebalancing with a reallocation to the nearest edge of the corresponding thresholds ("range approach"). Exhibit 2 presents the resulting classification of all rebalancing strategies.

A simple example demonstrates how our periodic interval rebalancing methodology works. Assume a 60% stocks and 40% bonds asset allocation with a quarterly rebalancing and a threshold of ±5% around the target weights. The portfolio strategy "quarterly rebalancing to target weights" implies a strict adjustment to the original stock allocation of 60% whenever the stock allocation exceeds the threshold of ±5% at the end of each quarter. In contrast, the portfolio strategy "quarterly rebalancing to range" requires the asset manager to check whether the weight of stocks exceeds 65% or falls below 55% of the portfolio's current market capitalization at the end of each quarter. In the first case, the manager must rebalance stocks to the upper threshold of 65%, whereas in the second case an adjustment of stocks to the lower threshold of 55% is required. In all other cases, no transactions are necessary because the stocks' target weight falls within the predetermined no-trade interval [55%; 65%]. According to Leland (1999), this approach reduces transaction costs and may potentially lead to superior portfolio performance. When no thresholds are specified, our rebalancing method reduces to the general periodic approach.

[Insert Exhibit 2 here]

Our simulation analysis focuses on the 10 different rebalancing strategies summarized in Exhibit 2. Specifically, we look at monthly, quarterly as well as yearly trading frequencies and implement optional interval boundaries. Moreover, we concentrate on a two-asset-class portfolio with an initial asset allocation of 60% stocks and 40% bonds. On the one hand, this approach best reflects common investment behavior in practice. On the other hand, it allows the comparison of our empirical findings with other rebalancing studies. Despite our focus on only two asset classes for the purpose of simplification, one should consider that each index constitutes a well-diversified representative of an entire asset class of the analyzed country. Moreover, we include 15 bps per round trip transaction costs in all our simulations. We apply 10 bps for buying/selling stocks and 5 bps for selling/buying bonds. Finally, the parameters i) transaction costs, ii) no-trade interval and iii) investment horizon are all modifiable in our extensive analyses.

3.2 Data description

In contrast to almost all previous rebalancing studies, we not only concentrate on domestic institutional investors of the US, but also on domestic institutional investors of the UK and Germany. We use monthly return data of well-diversified stock and bond market indices as well as money market rates for each country from Thomson Datastream. The sample period ranges from January 1981 to December 2010. This 30-year long time period is necessary in order to implement a statistical test. However, bond time series of this length are only available for the financial markets of the United States, the United Kingdom, and Germany.[3]

Two reasons are important for choosing a block bootstrap approach based on historical data. First, cross-sectional differences between countries require a separate analysis of each financial market. Exhibit 3 illustrates the differences in the stock, bond, and money markets characteristics (e.g., cross correlations between stocks, bonds, and money market rates as well as autocorrelation, left-skewness and fat tails). Therefore, an analysis of the US, the UK, and the German financial market can help us to check whether our empirical findings are robust in the cross-section.

[Insert Exhibit 3 here]

Secondly, the time series properties themselves can change over time. By using historical data, all times series information is fully incorporated into our simulation analysis. In order to get a detailed insight into the variation of the time series characteristics, we divide the entire 30-year sample period into 6 disjunctive 5-year sub-periods. Although the time series characteristics of the UK and Germany are slightly different compared with the USA, all three countries exhibit qualitatively similar patterns. As an example, Exhibit 4 shows the descriptive statistics of the US stock, bond, and money markets. In fact, the distributional characteristics exhibit variation over time, which would impose problems in calibrating the parameters for a Monte Carlo simulation.

[Insert Exhibit 4 here]

[3] All stock and bond market indices are on a total return basis. We use Treasury bills (United States), the LIBOR (United Kingdom), and the FIBOR (Germany) as proxies for the risk-free rates with 3-month maturities.

3.3 Implementing a double block bootstrap approach

Almost all previous rebalancing studies employ either an empirical analysis based on historical data or on a Monte Carlo simulation. However, previous historical analyses suffer from the shortcoming of being unable to provide information about statistical significance. Performing a standard t-test for differences in means, a sufficient number of independent observations is necessary in order to achieve a given level of statistical confidence. Neither the investigation of full sample periods (Jaconetti, Kinniry, and Zilbering (2010), Tsai (2001)) nor the examination of disjunctive sub-periods (Harjoto and Jones (2006)) is able to fulfill this statistical requirement.

Accordingly, many studies apply Monte Carlo simulations to evaluate rebalancing strategies. As Monte Carlo simulations enable to derive the entire return distribution under different economic scenarios, changing stock, bond, and money market characteristics and their impact on rebalancing strategies can be examined in more detail. Nevertheless, as it is difficult to appropriately incorporate all relevant information into the return-generating process, Monte Carlo simulations represent only simplified models for the time series properties of financial assets and even entire markets. Most important, Monte Carlo simulations often assume normally distributed stock returns even though stock market returns generally violate a normality assumption by exhibiting fat tails and heteroskedasticity as well as by tending to be left-skewed (Annaert, Van Osselaer, and Verstraete (2009)).[4] Moreover, De Bondt and Thaler (1985), Poterba and Summers (1988), as well as Brennan, Li, and Torous (2005) provide evidence that stock returns exhibit positive autocorrelation in the short-run and mean reversion in the long-run. Finally, asset class correlations tend to increase during recession periods (Longing and Solnik (2001)). While Monte Carlo simulations are unable to capture all return characteristics appropriately, a statistical test that is based on historical data is more suitable to incorporate all different time series properties.

Based on the shortcomings of both the historical analyses and the Monte Carlo simulations, we implement a double block bootstrap approach that uses historical data in order to test the value added of rebalancing strategies and to report statistical significance levels. We perform a historical simulation using data of the stock, bond, and money markets of the US, the UK, and Germany in order to incorporate the time-series properties of these return series. Due to the fact that our 30-year sample period does not provide sufficient observations to di-

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[4] Eraker (2004) uses a stochastic volatility process with jumps in asset values. This process has the geometric Brownian motion as a special case but allows for heavier tails in the return distribution.

vide the full sample period into disjunctive sub-periods, we apply a rolling window technique. This procedure separates our analysis of the value added of rebalancing from all previous studies. Specifically, it enables us to exploit the available information of the underlying sample period in the most efficient way. Instead of analyzing the entire sample period or a set of disjunctive sub-periods, we investigate investment horizons of 5 up to 10 years. Most important, our framework allows us to increase the number of "observations" that are necessary to conduct a statistical test. For example, analyzing the statistical properties of a 5-year investment horizon of any rebalancing strategy requires that 60 monthly return observations are included into the rolling time window. For each rolling window, we compute the strategy's annual return, its annual volatility and its resulting Sharpe ratio. We start by calculating these statistical measures during the period from January 1981 to December 1985 and move the rolling time window one month ahead in order to repeat the procedure for the period from February 1981 to January 1986, and so on. With a 30-year sample period and a 5-year investment horizon, we end up with 301 values for each statistical measure of interest. Given that this procedure is based on historical data, all time series' properties and financial markets' dependencies (such as positive autocorrelation in the short-run and negative autocorrelation in the long-run, heteroskedasticity, fat tails, left-skewed return distributions and asset class correlations) are preserved within the given investment horizon. Another advantage of the rolling window approach is that we analyze all investment horizons which have actually been realized during the underlying sample period. An important caveat is that moving the rolling time window on a monthly basis step-by-step alongside the entire sample period produces high autocorrelation in each statistical measure by construction. Therefore, a common t-test cannot be applied because it would require independent random variables.

We address this autocorrelation problem by implementing a double block bootstrap approach. Our framework is appropriate even under strong serial dependencies. By using rolling time windows, we exploit the information of the underlying return series as efficient as possible in order to derive meaningful implications for portfolio management. Nevertheless, our test procedure involves a trade-off between the length of the investment horizon, the number of generated "observations", and the serial dependencies induced by rolling windows. With longer investment horizons fewer "observations" can be generated and the resulting serial dependencies will be more pronounced as well. Within a given set of information, we are only able to vary the parameter 'length of the investment horizon' at the expense of the parameters 'number of observations' and 'serial dependencies', and vice versa. Therefore, we restrict our analysis to investment horizons of a maximum of 10 years, although investment horizons of institutional investors may range between 5 and up to 30 years.

Implementing a double block bootstrap approach, we follow earlier studies by Politis, Romano, and Wolf (1997), Davison and Hinkley (1997) and Politis (2003). In contrast to more standard Monte Carlo simulations, our historical simulation framework does not require specific assumptions with respect to the return distribution. Following Hall, Horowitz, and Jing (1995), we apply a *block* bootstrap approach in order to account for the time series properties of stocks and bonds. Specifically, drawing blocks of fixed length allows us to control for the pronounced serial autocorrelation (which is caused by the rolling window approach) when we resample the data. Hall, Horowitz, and Jing (1995) suggest that the length of the optimal block size to be sampled should be $n^{1/5}$ when calculating block bootstrap estimators of two-sided distribution functions, where n denotes the length of the original time series. Following this rule, the block length would be 3 $\lfloor = 301^{1/5} \rfloor$ for a 5-year investment horizon. Due to the high autocorrelation over our rolling windows, we instead use $n^{1/3}$ throughout our entire analysis. This alternative choice leads to a longer block length of 6 $\lfloor = 301^{1/3} \rfloor$. Longer block lengths lead to confidence intervals with a higher tendency of including 0, making it more difficult to find evidence for statistical significance and our statistical inference more conservative. In fact, even in the case of our block length $n^{1/3}$, the results are at least significant at the 5% level for the standard deviations and Sharpe ratios of different portfolio strategies. In order to check the robustness of our results, we also test longer block lengths up to a maximum of 20. Even in this extreme case, repeated simulations show that the Sharpe ratios of different rebalancing strategies are still significant at least at the 10% level.

Another aspect of our methodology is that we implement a *double* bootstrap approach. According to Politis, Romano, and Wolf (1997), a double block bootstrap approach mitigates the problem of selecting the appropriate block length. Furthermore, McCullough and Vinod (1998) document that this method features better convergence properties compared to a single bootstrap approach, and hence it produces more stable results.

The implementation of our double block bootstrap approach follows an algorithm introduced by Politis, Romano, and Wolf (1997). In order to compare different rebalancing strategies by reporting statistical significance levels, we compute asymptotic confidence intervals for the null hypothesis that the mean of a difference series is equal to zero. Any difference series is computed by subtracting the two respective raw series (i.e., return, volatility or the

Sharpe ratio) from the rolling windows of the respective strategies (e.g., buy-and-hold vs. quarterly rebalancing) from each other. Having determined the block length, the confidence level and the number of simulations, we hand this difference series over to our double block bootstrap simulator. The computation of the asymptotic confidence intervals takes place in two steps.

As an example, we describe this procedure for an investment horizon of 5 years, a block length of 6 and 4.000.000 simulations. The underlying sample period consists of 360 and each rolling window of 60 monthly return observations. Given an investment horizon of 5 years, the rolling window approach generates an 'original' difference time series consisting of 301 observations for any statistical measure under investigation;. Based on this time series, we create 2.000 new vectors V1 with a length of 300 each.[5] Each of these 2.000 new vectors V1(x), with $x \in \{1, \ldots, 2000\}$, consists of 50 blocks with length 6 that are randomly drawn with replacement from the 'original' difference time series. In a second step, we create for each vector V1(x) 2.000 new vectors V2(y), with $y \in \{1, \ldots, 2000\}$, that are based on the data of V1(x). Therefore, each of these 2.000 new vectors V2(y) also consists of 50 blocks with length 6 that are now randomly drawn with replacement from the respective vector V1(x). According to Davison and Hinkley (1997), this double bootstrap approach accounts for potential biases in the bootstrap distribution and leads to better convergence properties when calculating asymptotic confidence intervals. Our example involves 4.000.000 historical simulations, and from each of these 4.000.000 simulated series we calculate the average of the statistical measure under investigation and sort them according to their size. Based on these sorted observations, we derive a distribution of the measure of interest that enables us to calculate asymptotic confidence intervals of the underlying difference time series at predefined confidence levels. The high number of 4.000.000 simulations is necessary due to the asymptotic convergence of the calculated confidence intervals. Repeated simulations reveal that our results are stable in capturing the underlying patterns in our sample.

4. Simulation results

This section presents the results of our simulation analyses. We start with a comparison of a buy-and-hold strategy with simple periodic rebalancing strategies in terms of their mean returns and net asset values in section 4.1. In order to determine the value added of portfolio

[5] In general, the length of vector V1(x) with $x \in \{1, \ldots, 2000\}$ is b×k (with k=n/b), where n denotes the length of the original time series and b the block length.

rebalancing, section 4.2 compares these different strategies by further analyzing their standard deviations and Sharpe ratios. As a robustness check, section 4.3 repeats the analysis for two alternative rebalancing strategies, the “target approach” and the “range approach”.

4.1. Comparing average returns and net asset values (NAVs)

Any rebalancing strategy requires the selling of a fraction of the better-performing assets and investing the proceeds into the less-performing assets. Focusing on the portfolio return as the measure of interest, one would therefore expect that buy-and-hold strategies outperform rebalancing strategies with increasing investment horizons. Provided that one asset outperforms the other in every single period, this notion is always correct given the mechanics of rebalancing. In this specific case, it does not matter whether the returns are positive or negative. Exhibit 5 illustrates a simple example by analyzing the development of the net asset value (NAV) of a $100 investment. Both the stock and the bond time series are artificial. In order to focus on the isolated effect of rebalancing, return volatility is assumed to be zero. The stock and bond returns are held constant as 30-year monthly averages (11.4% and 8.3%, respectively). All strategies are based on a 60% stock and 40% bond allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip. As shown in Exhibit 5, buy-and-hold outperforms a monthly rebalancing strategy in terms of NAVs for any investment horizon, albeit at a surprisingly low margin.

[Insert Exhibit 5 here]

The assumption of constant moments is simplistic and is not reflected in real world data. In particular, stock markets are characterized by recurring up- and downswings. There are time periods in which stock market returns substantially outperform bond market returns, and vice versa. By using a 30-year historical data sample and implementing periodic rebalancing (strategies 2-4 in Exhibit 2), our analysis takes this aspect into consideration. Despite the strong performance of stocks relative to bonds during this sample period, our empirical findings in Exhibit 6 reveal that the average annual returns of a buy-and-hold strategy are lower (albeit at a small margin) than those of different rebalancing strategies for the UK and Germany. For example, while the average annual return of a buy-and-hold strategy is 11.21% in the UK over a 5-year horizon, quarterly rebalancing generates the highest average annual return of all strategies with a value of 11.34%.

Exhibit 7 reports whether these small return differences are statistically significant or whether they can simply be attributed to specific characteristics of the underlying sample period. In a first step, we compute the difference of the time series with annualized returns (derived from the rolling windows) of any two strategies that we compare (e.g., monthly rebalancing vs. buy-and-hold). In a second step, we hand this difference time series over to our block bootstrap approach in order to compute confidence intervals. These confidence intervals provide detailed information on whether a specific strategy generates a significantly higher or lower mean return. If both boundaries are positive (negative), rebalancing boasts a significantly higher (lower) return compared with a buy-and-hold strategy. Otherwise, the confidence interval includes zero, implying that the difference is lost in estimation error and that no statistical inferences can be drawn. Our findings in Exhibit 7 suggest that the return differences between rebalancing (at any frequency) and buy-and-hold are statistically significant at the 1% level in the UK and Germany for investment horizons of at least 7 years. In contrast, this finding cannot be confirmed for the US, where zero is included in the simulated confidence interval, implying that the differences in mean returns are lost in estimation error.

[Insert Exhibits 6 and 7 here]

In addition to average returns, we investigate the corresponding net asset values (NAV). Looking at investment horizons with different lengths, this alternative approach provides the advantage that the compound interest effect is accurately taken into account. If one strategy produces consistently higher returns than other strategies, this strategy also boasts a higher NAV. As a result, the difference in the performance of the NAVs increases with longer investment horizons. Exhibit 8 reports the growth rates of the NAVs, which are classified by strategy, investment horizon, and country. Although all rebalancing strategies require the selling of past winners and the buying of past losers, there are no economically relevant differences between the growth rates of the NAVs of the underlying strategies. This empirical finding is valid for all countries and for all investigated investment horizons. Although Perold und Sharpe's (1988) theoretical analysis suggests that time series properties – such as short-run momentum or long-run mean reversion – have an impact on the return of rebalancing strategies, they do not seem to be strongly pronounced in our sample. Otherwise, one would expect that a specific rebalancing frequency leads to higher NAVs compared with other rebalancing frequencies as well as a buy-and-hold strategy.

[Insert Exhibit 8 here]

Even if one asset substantially outperforms another asset over the entire sample period, one cannot necessarily conclude that a buy-and-hold strategy performs better than periodic rebalancing. As an example, Exhibit 9 illustrates the development of a $100 investment at the beginning of the year 1981. Although stocks substantially outperform bonds during the entire sample period, a buy-and-hold strategy produces the lowest NAV of all strategies after a 30-year time horizon. In contrast, looking at the $100 investment after 20 years, the buy-and-hold strategy dominates all other strategies in terms of NAVs. Accordingly, given that rebalancing is a dynamic portfolio strategy, its performance is path-dependent. The time series characteristics of the underlying assets, such as the volatility of the spread between the underlying assets (and hence the correlation between these assets), can have a substantial influence on the performance of any rebalancing strategy.

[Insert Exhibit 9 here]

Analyzing the notion of path-dependency in more detail, Panel A in Exhibit 10 presents the development of a $100 investment starting at the beginning of 2008 based on quarterly rebalancing (with a 0% threshold) as well as a buy-and-hold strategy. Panel B depicts the corresponding relative market capitalization of stocks in both strategies at the beginning of each month after the rebalancing event has taken place. As shown in panel A, quarterly rebalancing performs worse compared with buy-and-hold during the strong stock market meltdown in 2008, which caused a decline of the US stock market capitalization by almost 50%. This observation is explained by the regular reallocation at the end of each quarter to the initial 60/40 asset allocation. Accordingly, in a trending market environment with falling stock prices frequent rebalancing leads to inferior NAVs. Panel A further reveals that during the subsequent market upswing, quarterly rebalancing outperforms the buy-and-hold strategy. This finding can be traced back to the fact that the performance of an investment strategy not only depends on the return of the underlying assets, but also on their corresponding portfolio weights. In particular, during the following market recovery quarterly rebalancing produces higher NAVs compared with the buy-and-hold strategy because of its initial 60/40 stock-bond allocation at the start of the market recovery and the immediate readjustment at the end of each quarter. In contrast, the buy-and-hold strategy suffers from the decrease to a much lower stock allocation when the market recovery starts; the initial stock-bond allocation at the lower turning point is roughly 40/60 (rather than 60/40) because of the poor stock performance during the prior market crash. As shown in Panel B, the stock allocation cannot recover from this market crash within the remaining investment period. Due to its lower average stock allocation in the subsequent upside market, the buy-and-hold strategy is outperformed by a quarterly rebalancing strategy. This empirical results support the theoretical findings of Perold and Sharpe (1988). They argue that rebalancing strategies perform best during volatile sideway markets whereas buy-and-hold strategies lead to superior results in strongly pronounced market upswings and downswings, respectively.

[Insert Exhibit 10 here]

4.2. Comparing risk and Sharpe ratios

When rebalancing strategies perform only slightly better than the remaining strategies in terms of average returns and NAVs, a frequent rebalancing must offer other key benefits that explain their importance for institutional investors. In order to further analyze the value added of rebalancing, Exhibit 11 presents the average annualized portfolio standard deviations classified by strategy, investment horizon, and country. Buy-and-hold boasts the highest average annualized volatility. Moreover, in most instances, a monthly rebalancing strategy has a higher average annualized volatility compared to quarterly and yearly rebalancing strategies. As an example, analyzing a 10-year investment horizon for the US, the annual volatility of a buy-and-hold strategy is 10.38%. The monthly and quarterly rebalancing portfolios exhibit the lowest annual volatility with 9.73%. This finding is again robust for all countries and for all investment horizons.

Applying our double block bootstrap approach, we are again able to statistically evaluate these volatility differences. Our simulation results confirm that buy-and-hold exhibits the highest volatility for all investment horizons of at least 7 years in the US and UK stock and bond markets, and of at least 10 years in the German market. Repeated simulations reveal that the statistical significance is robust for all countries. As an example, analyzing the confidence intervals for a 10-year investment horizon and comparing the buy-and-hold strategy with periodic rebalancing strategies, zero is never included within the 99% confidence interval. An immediate explanation is that a buy-and-hold strategy involves an increasing relative proportion of stocks, which constitute the riskier asset class compared with bonds. With an increasing time horizon, the higher volatility of stocks more and more affects the volatility of the buy-and-hold strategy. In contrast, a periodic reallocation back to the original target weights prevents an extreme shift to riskier stocks. Against expectations, our results also indicate that quarterly (and to some extent yearly) rebalancing produces a lower volatility than monthly rebalancing. In results not shown in Exhibit 12, this pattern shows up even if we omit transaction costs.

[Insert Exhibits 11 and 12 here]

In order to appropriately evaluate portfolio performance, it is necessary to apply a performance measure that includes both the return and the volatility of the underlying strategies in a next step. Being well-established and widely used in practice, we choose the Sharpe ratio (Sharpe (1966)) as a risk-adjusted performance measure. Observing that rebalancing leads to only slightly (if any) superior mean returns, but to a significant reduction in risk, one would expect that this volatility pattern will also have an impact on the observed Sharpe ratio. Given our findings so far, we hypothesize that quarterly and/or yearly rebalancing will most likely generate the highest excess return per unit risk. Exhibit 13 reports the average annualized Sharpe ratios classified by strategy, investment horizon, and country. As expected, both quarterly and yearly rebalancing tend to exhibit higher Sharpe ratios than monthly rebalancing as well as a buy-and-hold strategy. For example, the average Sharpe ratio of a buy-and-hold strategy using German data and assuming a 10-year investment horizon is 0.252. A monthly rebalancing strategy produces an average Sharpe ratio of 0.308, and the Sharpe ratio increases on average to 0.319 and 0.323 for quarterly and yearly rebalancing, respectively.

[Insert Exhibit 13 here]

As expected, these patterns are also reflected in the statistical significance levels for differences in Sharpe ratios. Exhibit 14 shows the results of our double block bootstrap approach for all time horizons. Again, a buy-and-hold strategy produces the lowest Sharpe ratio, and differences in Sharpe ratios are significant at the 1% level over all time horizons when comparing buy-and-hold and rebalancing strategies. Moreover, quarterly (and for Germany even yearly) rebalancing strategies produce significantly higher Sharpe ratios than monthly rebalancing. Although the literature advises block lengths of $n^{1/5}$ or $n^{1/4}$, we choose to apply longer block lengths of $n^{1/3}$ throughout our entire analysis in order to account for very high serial dependencies. Our results are robust and statistically significant at the 1% level. Even when we extend the block length to 20, our main results for differences in Sharpe ratios are at least significant at the 10% level.

[Insert Exhibit 14 here]

Overall, our simulation setup allows us to determine whether a rebalancing strategy is able to generate a value added compared with a buy-and-hold strategy and to identify the source of this value added. Specifically, we document that the average returns of rebalancing strategies are only marginally (if any) higher than those of a buy-and-hold strategy. In contrast, rebalancing strategies exhibit a significantly lower volatility compared with a buy-and-hold strategy. Considering both return and risk of a given strategy, we further document that the Sharpe ratio – as a simple measure of value added – of all the different rebalancing strategies is significantly higher compared with a buy-and-hold strategy. In a nutshell, while the return effect is only marginally responsible for the superiority of the Sharpe ratio, it is the volatility effect which drives the value added of rebalancing strategies compared to a buy-and-hold strategy.

Observing that rebalancing strategies generally produce higher Sharpe ratios than a buy-and-hold strategy, an additional question is whether there is an optimal rebalancing frequency. For example, Jaconetti, Kinniry, and Zilbering (2010) conclude from their analysis that there is no universally optimal rebalancing strategy. In contrast, our results do not support this notion. Comparing different rebalancing frequencies (monthly, quarterly, and yearly) based on the simple periodic rebalancing methodology, our double bootstrap approach indicates that quarterly rebalancing produces the highest Sharpe ratio. This result suggests that both too frequent rebalancing as well as no rebalancing leads to inferior Sharpe ratios, and hence the optimal rebalancing frequency ranges between quarterly and yearly intervals.

Another noteworthy observation is that monthly rebalancing leads to significantly higher returns than quarterly rebalancing (Exhibit 7), and nevertheless it generates significantly lower Sharpe ratios (Exhibit 14). Recognizing that in this case the return effect and the volatility effect work in different directions, our results suggest that the volatility effect outweighs the return effect and represents the major source of the value added of rebalancing strategies. Accordingly, we conclude that it is primarily a risk management argument that justifies the widespread use of rebalancing strategies in the asset management practice.

4.3. Robustness tests

Our simulation results in sections 4.1 and 4.2 are based on a simple periodic rebalancing back to the target weights without a threshold. In the context of Exhibit 2, this approach refers to rebalancing strategies (2)-(4) with different rebalancing frequencies (monthly, quarterly, and yearly). Once a rebalancing threshold (hence a symmetric no-trade interval) is introduced, there are two cases to distinguish with regards to the practical implementation. In the first alternative strategy, a strict adjustment to the target weights (Buetow et al. (2002), Harjoto and Jones (2006)) is required when an asset exceeds the predetermined interval boundaries within a given interval. This "threshold approach" is captured by strategies (5)-(7) in Exhibit 2. In contrast, the second alternative rebalancing strategy requires a rebalancing back to the nearest edge of the given threshold rather than the initial portfolio weights (Leland (1999)). This "range approach" refers to strategies (8)-(10) in Exhibit 2.

As a robustness test, Exhibit 15 shows the confidence intervals for these two alternative rebalancing strategies. Specifically, we assume a threshold (or symmetric no-trade interval) of ±5% and a block length of 6. Confirming our previous results for the simpler periodic rebalancing strategy, a buy-and-hold strategy is significantly dominated by both the "threshold approach" and the "range approach" in terms of Sharpe ratios at all rebalancing frequencies; the difference is always significant al the 1% level. This result is robust when the threshold is changed to ±2% or ±10% (not tabulated). Accordingly, the dominance of rebalancing over a buy-and-hold strategy is independent of the choice of a specific rebalancing strategy. In contrast, in results not shown in Exhibit 15, our simulations are unable to uncover clear patterns with regards to a comparison of different rebalancing frequencies. While the optimal rebalancing frequency ranges between quarterly and yearly intervals under a periodic rebalancing strategy, no clear patterns emerge under the "threshold approach". Saving transaction costs by reallocating the assets back to the nearest edge of the predefined no-trade region, the "range approach" suggests monthly rebalancing as the optimal rebalancing frequency.

[Insert Exhibit 15 here]

5. Conclusions

This study addresses the question why institutional investors prefer rebalancing even though these strategies require the selling of a fraction of the better-performing assets and investing the proceeds in the less-performing assets. Analyzing the value added of rebalancing strategies for institutional investors, we document that the return effect is of only marginal importance, while it is primarily a risk management argument which justifies the widespread use of these strategies. Minimizing risk (defined as return volatility) with respect to a given asset allocation seems to be the primary objective of any rebalancing strategy.

In contrast to prior rebalancing studies, we investigate the potential risk-return-benefits of different rebalancing strategies by implementing a double block bootstrap approach. This methodology enables us to derive statistical inference. In fact, our study is the first to test the value added of rebalancing strategies based on statistical significance levels. Most important, our simulation framework is appropriate under strong serial time series dependencies. This property of the applied methodology is important because our empirical analysis involves rolling time windows. On the one hand, this procedure generates sufficient "observations" in order to implement a statistical test and achieve a given level of confidence. On the other hand, this technique leads to high time series dependencies by construction. However, by drawing data blocks of fixed lengths with replacement, our simulation methodology is robust in the presence of high time series dependencies.

Our simulations are based on data from the US, the UK, and Germany and deliver results that have immediate practical implications. First, given the strong performance of stocks relative to bonds during the 30-year sample period, rebalancing strategies hardly outperform a buy-and-hold strategy in terms of their average return and net asset value (NAV). According to Perold and Sharpe's (1988) notion, these empirical findings indicate that neither the mean reversion nor the momentum effects are strong enough in the return data to produce superior returns for either strategy. Secondly, we document that all rebalancing strategies exhibit a significantly lower volatility compared with the corresponding buy-and-hold strategy. This risk reduction can be explained by a diversification effect. Specifically, rebalancing the portfolio back to the original allocation prevents a drift away from the worse performing (but less risky) asset class towards the better-performing (but more risky) one, thereby reducing diversification and increasing risk. The reallocation to the less risky asset ultimately leads to a reduced volatility. Thirdly, analyzing the Sharpe ratio as a performance measure that incorporates both the return and the risk of any given portfolio strategy, our findings indicate that all different variants of rebalancing strategies (periodic rebalancing, threshold rebalancing and range rebalancing) significantly outperform a buy-and-hold strategy. Accordingly, risk reduction seems to be the main factor which presumably explains why rebalancing strategies are very popular in the investment practice.

Finally, for a periodic rebalancing strategy, monthly rebalancing generates significantly lower Sharpe ratios compared with quarterly rebalancing. This finding is robust for all countries and for all investment horizons of at least 5 years. It provides a hint that there may be an optimal rebalancing frequency, where too frequent rebalancing as well as no rebalancing leads to inferior results in terms of Sharpe ratios. While the optimal rebalancing frequency seems to lie between quarterly and yearly for a periodic rebalancing strategy, these data patterns do not show up for threshold rebalancing and range rebalancing strategies.

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EXHIBIT 1
Literature Overview

Authors	Topic	Data	Approach	Rebalancing Method	Asset Allocation
Year of Publication	**Investment Horizon**	**Transaction Costs**	**Report of Significance Levels**	**Key Results**	
Jaconetti Kinniry Zilbering	Best Practices for Portfolio Rebalancing	Jan 1926 - Dec 2009 US Data Monthly/Daily Data	Historical Analysis	Periodic Rebalancing Intervall Rebalancing Periodic Intervall Rebalancing to Target Weights	60% Stocks 40% Bonds
2010	Entire Sample Period Monthly: Jan 1926 -Dec 2009 Daily: 01/01/1989 -	Included	Not Reported	There Is No Universally Optimal Rebalancing Strategy. However, a Semiannual or Annual Rebalancing with a Threshold of about 5% Seems to Provide an Appropiate Risk Control.	
Tokat Wicas	Portfolio Rebalancing in Theory and Practice	Jan 1960 - Dec 2003 US Data Monthly Data	Monte Carlo Simulation	Periodic Intervall Rebalancing to Target Weights	60% Stocks 40% Bonds
2007	Classification in Trending and Mean-Reverting Markets and in Random-Walk Environment	Included	Not Reported	Rebalancing Achieves the Goal of Minimizing Risk Relative to a Target Asset Allocation in All Market Enviroments.	
Harjoto Jones	Rebalancing Strategy for Stocks and Bonds Asset Allocation	Jan 1995 - Dec 2004 US Data Monthly Frequency	Historical Analysis	Intervall Rebalancing to Target Weights	60% Stocks 40% Bonds
2006	Classification in Different Market Phases	Not Included	Not Reported	Investors Need to Rebalance, but Not Frequently. 15% Threshold Rebalancing Is Superior Compared to Other Rebalancing Strategies during All Market Phases.	
Donohue Yip	Optimal Portfolio Rebalancing with Transaction Costs	Jan 1987 - Dec 1996 US and International Data Monthly/Daily Data	Historical Analysis Monte Carlo Sim.	Periodic Rebalancing Interval Rebalancing Self-Developed Rebalancing Routine	US Stocks US Bonds Non-US Stocks
2003	10-Year Investment Horizon Monthly: Jan 1987-Dec 1996 Daily: Simulation (10 Years)	Included	Not Reported	Optimal Rebalancing Can Provide both Higher Returns and Lower Risk Than Other Common Rebalancing Heuristics.	
Tsai	Rebalancing Diversified Portfolios of Various Risk Profiles	Jan 1986 - Dec 2000 US and International Data Monthly Data	Historical Analysis	Periodic Rebalancing Periodic Interval Rebalancing to Target Weigths	US Stocks US Bonds Non-US Stocks
2001	Entire Sample Period	Not Included	Not Reported	Portfolios Should Be Periodically Rebalanced. However, No Strategy Is Consistently Better Across Portfolios of Differing Risk Profiles. Thus, It Does Not Matter Much Which Strategy Is Adopted.	
Arnott Lovell	Rebalancing: Why? When? How Often?	Jan 1968 - Dec 1991 US Data Monthly Frequency	Historical Analysis	Periodic Rebalancing Intervall Rebalancing Intervall Rebalancing to Target Weigths	50% Stocks 50% Bonds
1993	Entire Sample Period	Included	Not Reported	Efficient Rebalancing Has Enhanced Returns without Increasing Risk. These Modest Excess Returns Compound Over Time to Multimillion Dollar Gains to Any but the Smallest Funds.	
Perold Sharpe	Dynamic Strategies for Asset Allocation	Theoretical Stock Market Data	Theoretical Analysis	Periodic Rebalancing	60% Stocks 40% Bonds
1988	Theoretical Analysis of Dynamic Portfolio Strategies in Different Market Scenarios	Not Included	Not Reported	Different Strategies Produce Different Return and Risk Characteristics. An Appropiate Strategy Is Subject to the Investor's Risk Preference.	

EXHIBIT 2
Classification of Implemented Rebalancing Strategies

	Rebalancing Strategies	Frequency	Threshold	Reallocation	Classification
1	Buy-and-Hold	No Adjustments	No Threshold	No Reallocation	Buy-and-Hold
2	Yearly Rebalancing	Yearly	No Threshold	Target Weights	Periodic
3	Quarterly Rebalancing	Quarterly	No Threshold	Target Weights	Periodic
4	Monthly Rebalancing	Monthly	No Threshold	Target Weights	Periodic
5	Yearly Rebalancing to Target Weights	Yearly	Threshold	Target Weights	Threshold
6	Quarterly Rebalancing to Target Weights	Quarterly	Threshold	Target Weights	Threshold
7	Monthly Rebalancing to Target Weights	Monthly	Threshold	Target Weights	Threshold
8	Yearly Rebalancing to Range	Yearly	Threshold	Interval Boundaries	Range
9	Quarterly Rebalancing to Range	Quarterly	Threshold	Interval Boundaries	Range
10	Monthly Rebalancing to Range	Monthly	Threshold	Interval Boundaries	Range

Notes: The periodic rebalancing strategies 2, 3, and 4 are characterized by a regular reallocation to the predetermined target weights at the end of each period. Strategies 5, 6, and 7 represent periodic interval rebalancing with a strict adjustment to the target weights (threshold approach). In strategies 8, 9, and 10, the assets are rebalanced to the nearest edge of the predefined interval boundaries (range approach). A threshold of ±5% is applied to both periodic interval rebalancing to target weights and periodic interval rebalancing to range.

EXHIBIT 3
Deskriptive Statistics: January 1981 – December 2010

Asset	Measure	USA	UK	Germany
Stocks	Mean	10,79%	11,96%	9,83%
	Volatility	15,54%	16,32%	21,32%
	Skewness	-0,65	-0,85	-0,55
	Kurtosis	5,10	6,03	4,87
	Autocorrelations			
	Lag 1	0,07	0,04	0,07
	Lag 2	-0,01	-0,09	0,02
	Lag 3	0,05	-0,02	0,05
Bonds	Mean	8,57%	10,46%	7,33%
	Volatility	8,28%	8,25%	5,70%
	Skewness	0,25	0,03	-0,44
	Kurtosis	3,73	4,37	4,14
	Autocorrelations			
	Lag 1	0,09	0,05	0,11
	Lag 2	-0,04	-0,02	-0,04
	Lag 3	0,05	-0,05	0,10
Cash	Mean	5,01%	7,60%	4,89%
	Volatility	0,86%	1,01%	0,74%
	Skewness	0,62	0,27	0,87
	Kurtosis	3,66	2,34	3,52
	Autocorrelations			
	Lag 1	0,99	0,99	0,99
	Lag 2	0,98	0,98	0,98
	Lag 3	0,97	0,97	0,97
Correlations	Stocks/Bonds	0,10	0,24	-0,01
	Stocks /Cash	0,03	0,05	-0,04
	Bonds/Cash	0,06	0,12	0,05

Notes: All statistics are calculated on a monthly basis using discrete returns. The rows Mean and Volatility are the annualized mean returns and volatilities. Skewness and Kurtosis are calculated as the third and fourth normalized centered moments. Cash denotes the corresponding 3-month money market rates.

EXHIBIT 4
Deskriptive Statistics: USA

Stocks

Period	Start Date	End Date	Mean	Volatility	Skewness	Kurtosis	Min.	Max.
Subperiod 1	Jan-81	Dec-85	14,99%	13,82%	0,6755	3,4512	-5,70%	12,31%
Subperiod 2	Jan-86	Dec-90	12,73%	18,61%	-1,1960	6,7551	-21,22%	13,28%
Subperiod 3	Jan-91	Dec-95	16,95%	10,06%	0,2913	4,1653	4,58%	11,43%
Subperiod 4	Jan-96	Dec-00	18,41%	16,26%	-0,6439	3,3828	-13,90%	9,98%
Subperiod 5	Jan-01	Dec-05	0,54%	15,08%	-0,3443	3,0761	-11,28%	9,08%
Subperiod 6	Jan-05	Dec-10	2,44%	17,85%	-0,8202	4,0306	-17,10%	9,60%
1st Half	Jan-81	Dec-95	14,88%	14,51%	-0,6317	7,2798	-21,22%	13,28%
2nd Half	Jan-96	Dec-10	6,84%	16,48%	-0,6280	3,6364	-17,10%	9,98%
Full Sample	Jan-81	Dec-10	10,79%	15,54%	-0,6539	5,0970	-21,22%	13,28%

Bonds

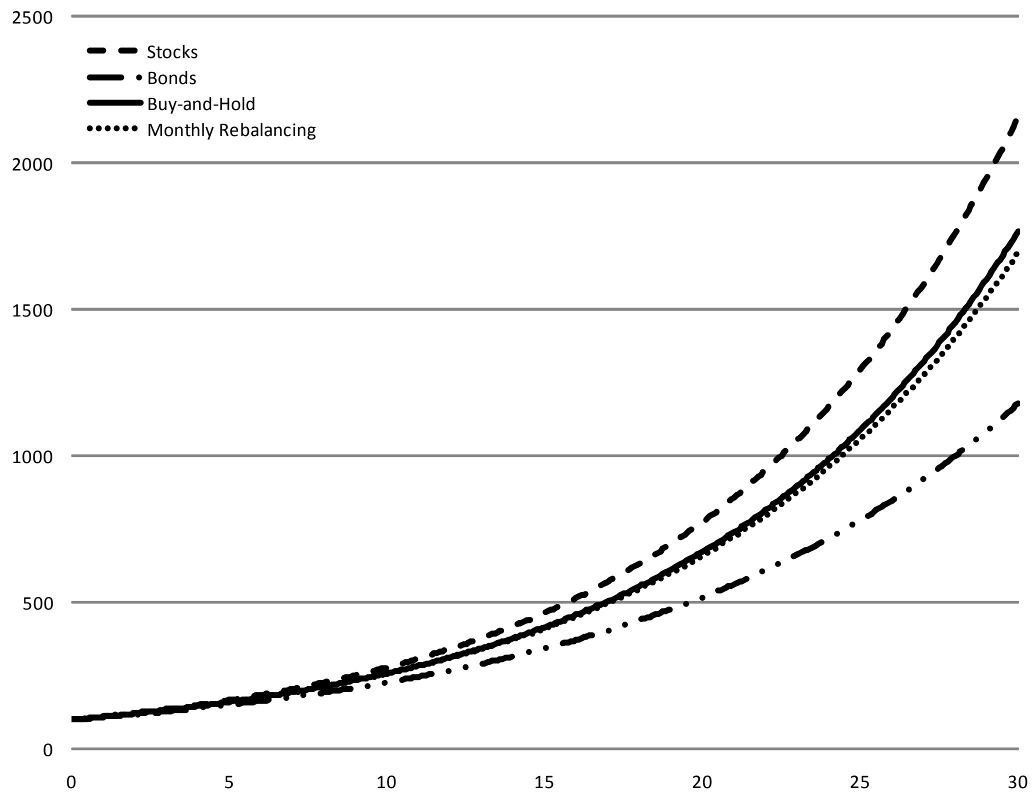
	Start Date	End Date	Mean	Volatility	Skewness	Kurtosis	Min.	Max.
Subperiod 1	Jan-81	Dec-85	16,33%	10,78%	0,1881	3,4512	-4,40%	8,36%
Subperiod 2	Jan-86	Dec-90	9,00%	8,45%	0,2647	2,6024	-3,83%	6,47%
Subperiod 3	Jan-91	Dec-95	9,75%	6,60%	-0,1675	3,5421	-3,85%	5,66%
Subperiod 4	Jan-96	Dec-00	5,70%	6,40%	0,0585	3,0728	-4,25%	5,08%
Subperiod 5	Jan-01	Dec-05	5,22%	8,07%	-0,6530	3,6747	-7,09%	4,68%
Subperiod 6	Jan-05	Dec-10	5,84%	8,58%	0,7703	5,5350	-4,88%	9,86%
1st Half	Jan-81	Dec-95	11,65%	8,77%	0,2735	2,9822	-4,40%	8,36%
2nd Half	Jan-96	Dec-10	5,59%	7,70%	0,1149	4,7409	-7,09%	9,86%
Full Sample	Jan-81	Dec-10	8,57%	8,28%	0,2464	3,7301	-7,09%	9,86%

Money Market Rates

	Start Date	End Date	Mean	Volatility	Skewness	Kurtosis	Min.	Max.
Subperiod 1	Jan-81	Dec-85	9,91%	0,66%	0,7978	2,4015	0,55%	1,21%
Subperiod 2	Jan-86	Dec-90	6,78%	0,29%	0,0676	1,7321	0,42%	0,71%
Subperiod 3	Jan-91	Dec-95	4,29%	0,31%	0,0722	1,4318	0,22%	0,50%
Subperiod 4	Jan-96	Dec-00	5,07%	0,13%	0,5521	3,1419	0,35%	0,50%
Subperiod 5	Jan-01	Dec-05	2,10%	0,32%	0,7578	2,3806	0,07%	0,40%
Subperiod 6	Jan-05	Dec-10	2,10%	0,59%	0,3192	1,3130	0,00%	0,41%
1st Half	Jan-81	Dec-95	6,97%	0,77%	0,8071	3,5617	0,22%	1,21%
2nd Half	Jan-96	Dec-10	3,08%	0,55%	-0,2404	0,1502	0,00%	0,50%
Full Sample	Jan-81	Dec-10	5,01%	0,86%	0,6225	3,6638	0,00%	1,21%

Notes: The columns Mean, Volatility, Skewness, and Kurtosis are the annualized mean return, volatility, skewness, and kurtosis, which are calculated on a monthly basis using discrete returns. Skewness and Kurtosis are calculated as the third and fourth normalized centered moments. Min and Max are the monthly minimum and maximum returns, respectively.

EXHIBIT 5
Net Asset Values of Constructed Time Series



Notes: This example shows the development of the Net Asset Values of a 100$-investment using a buy-and-hold strategy, monthly rebalancing as well as single investments in stocks and bonds. Both the stock and bond time series are artificial. They exhibit a volatility of zero in order to investigate resulting effects on the buy-and-hold and monthly rebalancing strategy. The average 30-year stock and bond return of the US financial market are calculated and then assumed to be constant during the entire investment period. Both strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 6
Average Annualized Returns

Investment Horizon	Investment Strategy	USA	UK	Germany
5	Buy-and-Hold	10,87%	11,21%	9,45%
5	Yearly Rebalancing	10,86%	11,30%	9,81%
5	Quarterly Rebalancing	10,85%	11,34%	9,72%
5	Monthly Rebalancing	10,81%	11,33%	9,59%
10	Buy-and-Hold	10,75%	10,61%	8,57%
10	Yearly Rebalancing	10,74%	10,77%	9,19%
10	Quarterly Rebalancing	10,72%	10,81%	9,13%
10	Monthly Rebalancing	10,68%	10,80%	9,00%
15	Buy-and-Hold	11,33%	10,97%	9,25%
15	Yearly Rebalancing	11,27%	11,09%	9,82%
15	Quarterly Rebalancing	11,24%	11,14%	9,76%
15	Monthly Rebalancing	11,20%	11,13%	9,64%

Notes: Classified by the underlying investment horizon, this exhibit shows the growth rates of yearly, quarterly, and monthly rebalancing strategies as well as a buy-and-hold strategy. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 7
Calculated Confidence Intervals of the Return Distribution

		USA			United Kingdom			Germany		
	Strategies	Confidence Intervalls			Confidence Intervalls			Confidence Intervalls		
5 years	M-BAH	-0.0031	0.0003		-0.0017	0.0008		-0.0009	0.0025	
	Q-BAH	-0.0027	0.0008		-0.0015	0.0010		0.0001	0.0038	*
	Y-BAH	-0.0021	0.0012		-0.0013	0.0009		0.0015	0.0053	*
	M-Q	-0.0007	-0.0002	***	-0.0003	0.0000	***	-0.0015	-0.0008	***
	M-Y	-0.0018	-0.0001	*	-0.0006	0.0003		-0.0037	-0.0014	***
	Q-Y	-0.0009	0.0001		-0.0004	0.0005		-0.0024	-0.0002	***
7 years	M-BAH	-0.0016	0.0012		0.0011	0.0026	***	0.0021	0.0056	***
	Q-BAH	-0.0012	0.0017		0.0011	0.0028	***	0.0033	0.0069	***
	Y-BAH	-0.0009	0.0021		0.0005	0.0024	***	0.0038	0.0090	***
	M-Q	-0.0006	-0.0002	***	-0.0002	0.0000		-0.0016	-0.0011	***
	M-Y	-0.0012	-0.0001	*	-0.0002	0.0009		-0.0037	-0.0014	***
	Q-Y	-0.0007	0.0003		0.0000	0.0009	*	-0.0022	-0.0001	***
10 years	M-BAH	-0.0022	0.0007		0.0014	0.0025	***	0.0029	0.0056	***
	Q-BAH	-0.0018	0.0011		0.0015	0.0026	***	0.0042	0.0068	***
	Y-BAH	-0.0016	0.0012		0.0011	0.0022	***	0.0043	0.0082	***
	M-Q	-0.0006	-0.0002	***	-0.0002	0.0000	**	-0.0015	-0.0012	***
	M-Y	-0.0012	-0.0001	**	0.0000	0.0007		-0.0031	-0.0012	***
	Q-Y	-0.0006	0.0002		0.0001	0.0008	**	-0.0015	0.0000	***

Notes: This exhibit shows the confidence intervals for Sharpe ratios for a 5, 7, and 10-year investment horizon, respectively. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip. BAH denotes buy-and-hold, Y yearly rebalancing, Q quarterly rebalancing, and M monthly rebalancing. For each two strategies that are compared, the lower and upper boundary of the corresponding confidence interval is calculated. 4 million simulations with a fixed block length of 6 are performed. Repeated simulations reveal that the results are stable. ***/**/* denotes significance at the 1%, 5%, and 10% level, respectively.

EXHIBIT 8
Average Growth of Net Asset Values

Investment Horizon	Investment Strategy	USA	UK	Germany
5	Buy-and-Hold	73,0%	75,9%	63,3%
5	Yearly Rebalancing	72,4%	76,3%	65,2%
5	Quarterly Rebalancing	72,3%	76,7%	64,6%
5	Monthly Rebalancing	72,1%	76,7%	63,7%
10	Buy-and-Hold	196,2%	194,3%	137,1%
10	Yearly Rebalancing	193,5%	197,7%	149,4%
10	Quarterly Rebalancing	193,6%	199,7%	148,3%
10	Monthly Rebalancing	192,9%	199,7%	145,5%
15	Buy-and-Hold	446,9%	426,9%	296,5%
15	Yearly Rebalancing	430,2%	430,3%	322,3%
15	Quarterly Rebalancing	429,8%	435,3%	319,8%
15	Monthly Rebalancing	427,8%	435,3%	312,9%

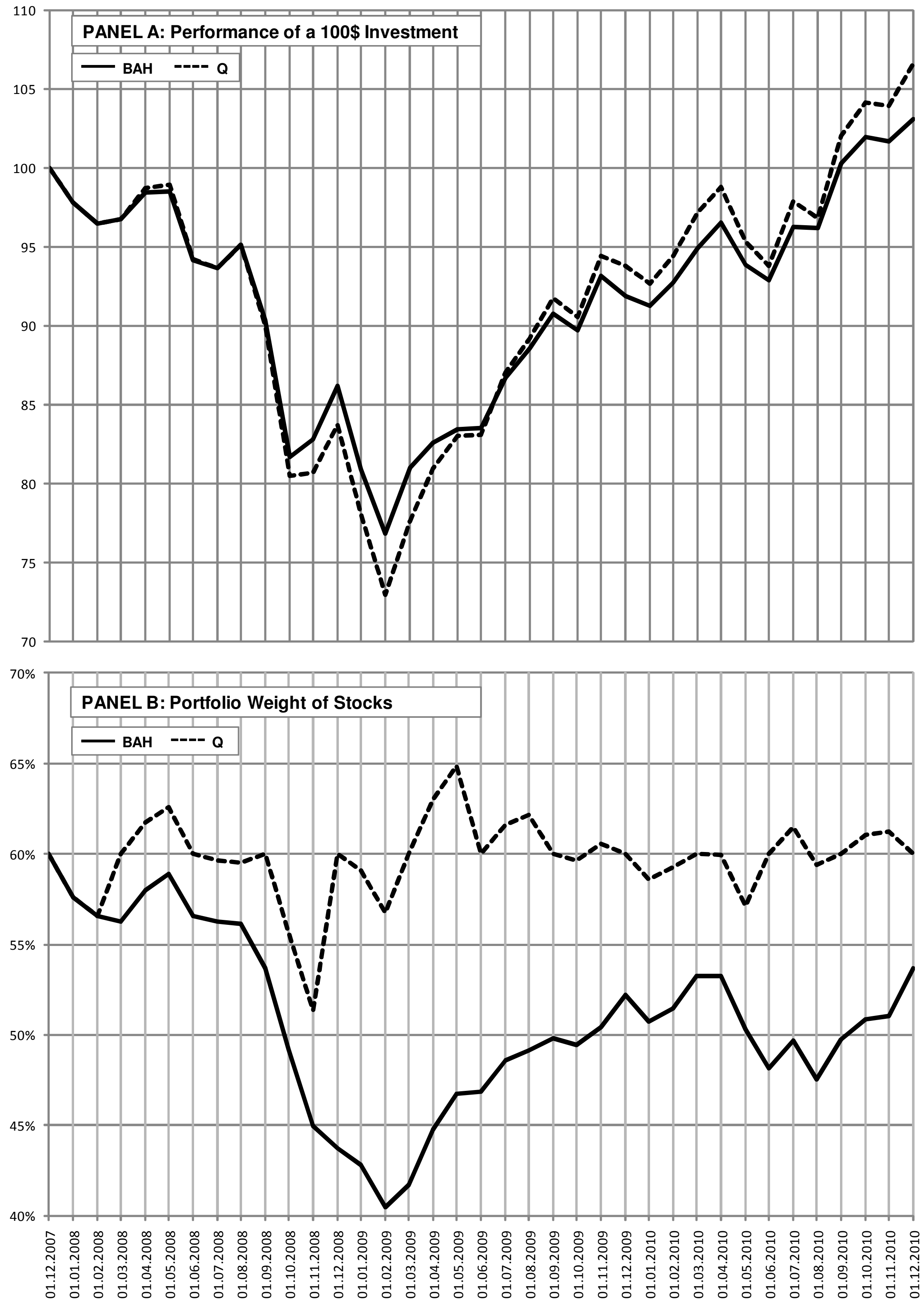
Notes: Classified by the underlying investment horizon, this exhibit shows the growth rates of yearly, quarterly, and monthly rebalancing strategies as well as a buy-and-hold strategy. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 9
Development of NAVs: USA

Investment Horizon	Period	Stocks	Bonds	BAH	Yearly Rebalancing	Quarterly Rebalancing	Monthly Rebalancing
5	01/81-12/85	201,0	213,0	205,8	207,0	207,1	207,5
10	01/81-12/90	366,0	327,8	350,7	353,8	361,3	360,6
15	01/81-12/95	800,7	522,0	689,2	684,5	698,6	697,6
20	01/81-12/00	1863,6	688,8	1393,7	1308,8	1327,3	1314,9
25	01/81-12/05	1914,9	888,5	1504,3	1516,8	1545,2	1514,8
30	01/81-12/10	2160,5	1179,9	1768,2	1956,2	1944,0	1875,0

Notes: All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 10
Panel A: Performance of a 100$ Investment
Panel B: Portfolio Weight of Stocks



Notes: Both strategies – buy-and-hold (BAH) as well as quarterly rebalancing (Q) – are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 11
Average Annualized Volatilities

Investment Horizon	Investment Strategy	USA	UK	Germany
5	Buy-and-Hold	9,88%	10,65%	13,35%
5	Yearly Rebalancing	9,63%	10,46%	12,96%
5	Quarterly Rebalancing	9,64%	10,46%	13,03%
5	Monthly Rebalancing	9,67%	10,50%	13,11%
10	Buy-and-Hold	10,38%	10,79%	14,00%
10	Yearly Rebalancing	9,73%	10,52%	13,24%
10	Quarterly Rebalancing	9,73%	10,53%	13,30%
10	Monthly Rebalancing	9,76%	10,56%	13,39%
15	Buy-and-Hold	10,92%	10,97%	14,50%
15	Yearly Rebalancing	9,67%	10,49%	13,09%
15	Quarterly Rebalancing	9,66%	10,48%	13,11%
15	Monthly Rebalancing	9,68%	10,51%	13,19%

Notes: All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip.

EXHIBIT 12
Calculated Confidence Intervals of the Volatility Distribution

		USA			United Kingdom			Germany		
	Strategies	Confidence Intervalls			Confidence Intervalls			Confidence Intervalls		
5 years	M-BAH	-0.0046	0.0008		-0.0052	0.0000		-0.0082	0.0038	
	Q-BAH	-0.0049	0.0002		-0.0054	-0.0003	*	-0.0086	0.0026	
	Y-BAH	-0.0047	-0.0008	*	-0.0052	-0.0005	**	-0.0084	0.0003	
	M-Q	0.0002	0.0006	***	0.0001	0.0005	***	0.0002	0.0014	***
	M-Y	-0.0001	0.0016		-0.0004	0.0010		-0.0003	0.0034	
	Q-Y	-0.0004	0.0011		-0.0007	0.0006		-0.0006	0.0023	
7 years	M-BAH	-0.0081	-0.0001	***	-0.0057	-0.0006	*	-0.0088	0.0022	
	Q-BAH	-0.0083	-0.0005	***	-0.0067	-0.0004	**	-0.0095	0.0010	
	Y-BAH	-0.0076	-0.0010	***	-0.0059	-0.0009	**	-0.0099	-0.0010	*
	M-Q	0.0002	0.0005	***	0.0002	0.0005	***	0.0005	0.0015	***
	M-Y	-0.0002	0.0013		-0.0003	0.0009		0.0005	0.0034	*
	Q-Y	-0.0005	0.0008		-0.0007	0.0005		-0.0002	0.0022	
10 years	M-BAH	-0.0097	-0.0017	***	-0.0043	-0.0001	*	-0.0126	0.0006	***
	Q-BAH	-0.0099	-0.0021	***	-0.0046	-0.0003	*	-0.0134	-0.0004	***
	Y-BAH	-0.0098	-0.0022	***	-0.0043	-0.0007	*	-0.0132	-0.0017	***
	M-Q	0.0002	0.0004	***	0.0003	0.0004	***	0.0006	0.0012	***
	M-Y	-0.0002	0.0010		0.0000	0.0008		0.0000	0.0033	***
	Q-Y	-0.0004	0.0006		-0.0003	0.0005		-0.0002	0.0017	

Notes: This exhibit shows the confidence intervals for Sharpe ratios for a 5, 7, and 10-year investment horizon, respectively. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip. BAH denotes buy-and-hold, Y yearly rebalancing, Q quarterly rebalancing, and M monthly rebalancing. For each two strategies that are compared, the lower and upper boundary of the corresponding confidence interval is calculated. 4 million simulations with a fixed block length of 6 are performed. Repeated simulations reveal that the results are stable. ***/**/* denotes significance at the 1%, 5%, and 10% level, respectively.

EXHIBIT 13
Average Annualized Sharpe Ratios

Investment Horizon	Investment Strategy	USA	UK	Germany
5	Buy-and-Hold	0,607	0,321	0,358
5	Yearly Rebalancing	0,636	0,347	0,422
5	Quarterly Rebalancing	0,637	0,352	0,422
5	Monthly Rebalancing	0,633	0,351	0,413
10	Buy-and-Hold	0,557	0,248	0,252
10	Yearly Rebalancing	0,603	0,277	0,323
10	Quarterly Rebalancing	0,604	0,281	0,319
10	Monthly Rebalancing	0,599	0,279	0,308
15	Buy-and-Hold	0,600	0,301	0,300
15	Yearly Rebalancing	0,667	0,331	0,379
15	Quarterly Rebalancing	0,665	0,335	0,374
15	Monthly Rebalancing	0,660	0,333	0,363

Notes: All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip. We use Treasury bills (United States), LIBOR (United Kingdom), and FIBOR (Germany) as proxies for the risk free rates with 3-month maturities.

EXHIBIT 14
Calculated Confidence Intervals of the Sharpe Ratio Distribution

		USA			United Kingdom			Germany		
	Strategies	Confidence Intervalls			Confidence Intervalls			Confidence Intervalls		
5 years	M-BAH	0.0156	0.0450	***	0.0143	0.0363	***	0.0371	0.0761	***
	Q-BAH	0.0192	0.0502	***	0.0146	0.0386	***	0.0432	0.0831	***
	Y-BAH	0.0188	0.0480	***	0.0104	0.0371	***	0.0462	0.0843	***
	M-Q	-0.0073	-0.0023	***	-0.0032	-0.0002	**	-0.0114	-0.0021	***
	M-Y	-0.0118	0.0029		-0.0046	0.0064		-0.0178	0.0025	
	Q-Y	-0.0057	0.0075		-0.0021	0.0074		-0.0087	0.0070	
7 years	M-BAH	0.0223	0.0577	***	0.0280	0.0467	***	0.0492	0.0743	***
	Q-BAH	0.0298	0.0645	***	0.0300	0.0481	***	0.0576	0.0856	***
	Y-BAH	0.0268	0.0688	***	0.0256	0.0456	***	0.0584	0.0939	***
	M-Q	-0.0080	-0.0036	***	-0.0036	-0.0001	***	-0.0137	-0.0071	***
	M-Y	-0.0149	0.0008		-0.0038	0.0080		-0.0317	-0.0037	***
	Q-Y	-0.0074	0.0060		-0.0007	0.0089		-0.0136	0.0002	
10 years	M-BAH	0.0234	0.0623	***	0.0262	0.0380	***	0.0459	0.0659	***
	Q-BAH	0.0293	0.0678	***	0.0282	0.0391	***	0.0562	0.0782	***
	Y-BAH	0.0231	0.0712	***	0.0222	0.0372	***	0.0578	0.0864	***
	M-Q	-0.0069	-0.0038	***	-0.0032	-0.0008	***	-0.0125	-0.0097	***
	M-Y	-0.0151	0.0078		-0.0020	0.0061		-0.0248	-0.0060	***
	Q-Y	-0.0096	0.0116		0.0001	0.0082	**	-0.0102	0.0018	

Notes: This exhibit shows the confidence intervals for Sharpe ratios for a 5, 7, and 10-year investment horizon, respectively. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 0%. Transaction costs are quoted at 15 bps per round trip. BAH denotes buy-and-hold, Y yearly rebalancing, Q quarterly rebalancing, and M monthly rebalancing. For each two strategies that are compared, the lower and upper boundary of the corresponding confidence interval is calculated. 4 million simulations with a fixed block length of 6 are performed. Repeated simulations reveal that the results are stable. ***/**/* denotes significance at the 1%, 5%, and 10% level, respectively.

EXHIBIT 15

Calculated Confidence Intervals of the Sharpe Ratio Distribution (Threshold & Range)

	Strategies	USA			United Kingdom			Germany		
		Confidence Intervalls			Confidence Intervalls			Confidence Intervalls		
Threshold										
5 years	M-BAH	0.0189	0.0482	***	0.0155	0.0425	***	0.0496	0.0831	***
	Q-BAH	0.0177	0.0498	***	0.0135	0.0445	***	0.0517	0.0949	***
	Y-BAH	0.0173	0.0511	***	0.0139	0.0419	***	0.0472	0.0884	***
7 years	M-BAH	0.0262	0.0591	***	0.0314	0.0518	***	0.0577	0.0892	***
	Q-BAH	0.0303	0.0640	***	0.0287	0.0478	***	0.0626	0.0963	***
	Y-BAH	0.0281	0.0711	***	0.0269	0.0527	***	0.0586	0.0963	***
10 years	M-BAH	0.0223	0.0727	***	0.0291	0.0440	***	0.0526	0.0847	***
	Q-BAH	0.0236	0.0705	***	0.0279	0.0427	***	0.0562	0.0818	***
	Y-BAH	0.0213	0.0774	***	0.0231	0.0418	***	0.0569	0.0876	***
Range										
5 years	M-BAH	0.0163	0.0449	***	0.0118	0.0401	***	0.0492	0.0875	***
	Q-BAH	0.0162	0.0436	***	0.0116	0.0389	***	0.0467	0.0851	***
	Y-BAH	0.0077	0.0340	***	0.0057	0.0275	***	0.0337	0.0680	***
7 years	M-BAH	0.0243	0.0634	***	0.0240	0.0495	***	0.0630	0.0989	***
	Q-BAH	0.0234	0.0625	***	0.0229	0.0485	***	0.0596	0.0978	***
	Y-BAH	0.0159	0.0556	***	0.0152	0.0365	***	0.0457	0.0892	***
10 years	M-BAH	0.0254	0.0770	***	0.0212	0.0373	***	0.0626	0.0943	***
	Q-BAH	0.0248	0.0752	***	0.0191	0.0363	***	0.0604	0.0942	***
	Y-BAH	0.0182	0.0708	***	0.0116	0.0259	***	0.0442	0.0874	***

Notes: This exhibit shows the confidence intervals for Sharpe ratios for a 5, 7, and 10-year investment horizon, respectively. All strategies are based on a 60% stocks and 40% bonds asset allocation with a threshold of 5%. Transaction costs are quoted at 15 bps per round trip. BAH denotes buy-and-hold, Y yearly rebalancing, Q quarterly rebalancing, and M monthly rebalancing. For each two strategies that are compared, the lower and upper boundary of the corresponding confidence interval is calculated. 4 million simulations with a fixed block length of 6 are performed. Repeated simulations reveal that the results are stable. ***/**/* denotes significance at the 1%, 5%, and 10% level, respectively. Threshold rebalancing involves a reallocation to the target weights, while rebalancing to range requires a reallocation to the nearest edge of the predefined no-trade region.