Strategic Mutual Fund Tournaments

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ABSTRACT

This paper characterizes the optimal strategies of mutual fund managers competing in a multi-period winner-take-all tournament. Taking account of both multiple periods and competition between more than two managers, the optimal strategies are contingent on the state at the interim date. In the final period all managers maximize the amount of risk that they add to their portfolios with the exception of the leading fund. This fund locks in its advantage by reducing risk only if it has a sufficiently large lead. Empirically, we find that consistent with the theory, funds with larger leads decrease risk; however trailing funds do not increase risks. These results are robust to using different ways of controlling for systematic risk exposures.

This paper has been presented in seminars at Arizona State University, Claremont McKenna College, Melbourne Business School, the University of Iceland, the University of Oregon and UNSW. We appreciate the seminar audiences for their comments. We are responsible for all errors.

1. Introduction

An economic tournament is said to exist whenever there is a contest between economic agents and the outcome results in a clear `winner', whose prize is greater than that of the `losers.' In finance, tournaments are most often associated with mutual funds. This is because of the well-documented fund flows effect in which investors flock to the fund with the highest relative performance within a calendar year. For example, the relationship between past returns and inflows at the end of the year is demonstrated to be convex (Chevalier and Ellison 1997). Moreover, fund ranking organizations such as Morningstar have devised a system referred to as the `star' system. There are substantial fund flows into those mutual funds that are classified in the highest (five star) category.

An important strand of empirical research on mutual fund tournaments has investigated the proposition that because fund inflows have been shown to be convex with respect to fund ranks, mutual fund managers have an incentive to increase risk, particularly late in the calendar year. This risk seeking maximizes managers' chance of winning the mutual fund tournament and hence dramatically increasing the expected size of their funds, which maximizes future fees. In an important paper (Brown, Harlow and Starks 1996) they proposed that because funds essentially compete in a tournament ending at the calendar year, funds ranked in the lower half during the first part of the year would attempt to increase their risks in the second half of the year in order to come out ahead at the end. They therefore tested for a difference in second period to first period variance ratios between the highest and lowest ranked funds and found that both losers and winners increased risk in the latter part of the year, and that losers increased risk more than winners. The difference in increased risk-taking between winners and losers was only manifest in the latter part of the sample period, however. However using a smaller sample of daily data from a different (although overlapping) period, that there was no significant impact of first half performance on second half risk taking (Busse 2001). Opposite effects have also been found on different data samples (Qiu 2003), where loser funds take less risk as the year goes on and some winner funds increase risks. Theory and empirical work (Chen and Pennachi 2009) has been conducted in the case where fund managers are compensated based on performance relative to a passive index. They found that risk taking increases only when measured as deviations relative to the index (tracking errors).

Hence despite the extensive empirical literature in mutual fund tournaments, no clear consensus has emerged for whether tournament behavior exists. One limitation of the empirical literature is in its characterization of tournament behavior. Much of the subsequent literature follows the original work (Brown, Harlow and Starks 1996) who asserted that losing funds would increase risk but assumed that winning funds would stand pat. However given that losing funds increase risk, winning funds might conceivably follow suit. Further, whether or not winning funds increased risk or stood pat could be determined by the size of their lead midway through the year.

The purpose of our paper is to develop and test a theory of tournament behavior that can reconcile the disparate empirical findings that have been observed. Our model accounts not only for the possibility that fund managers change their risk-taking behavior throughout the year, but also allows for the heterogeneity of optimal choices by funds with different intermediate rankings. Importantly, not only the number of competing funds but also their relative performances have significant consequences for the nature of the resultant equilibrium for a given tournament. In particular, equilibrium outcomes and the set of empirical predictions associated with them are very different in a tournament with two funds from those in a tournament with three or more, so that intuition based on models with only two competitors (Taylor 2003), (Lóránth and Sciubba 2006) and (Basak and Makarov 2009) may not be generic. We show however, that the general structure of the equilibrium with more than three funds can be explained in a model with three funds.

We begin with a single period winner-take-all tournament where managers have different abilities, or equivalently, where some managers have higher year-to-date returns than others. In order to win the tournament, trailing funds, or funds with less-able managers, take excessive risks in order to maximize their chances of winning. Indeed, tournament concerns cause trailing funds to optimally maximize the amount of risk they take. We also show that if the leading fund's lead is not `too big,' it also takes this same amount of excessive risk. For funds with larger leads, it is optimal to 'lock in,' that is, to dramatically reduce their risks when the final rankings begin to emerge. These funds have leads substantial enough to play safe even though their opponents are still taking risks. How large `substantial' is depends on the number of competitors, how long opponents have to catch up and the maximum amount of risk they can take to do so. The greater the number of competitors, longer the time and the greater the risk, the less safe a lead is. Finally, only for the special case of two competitors, is it *always* optimal for the leading fund to play safe, regardless of the size of her lead.

Next, we allow for multi-period contests in order to understand how risk-taking behavior changes over the year and is affected by the relative returns-to-date of the competing mutual funds. Applying the single period analysis to the second period of a two period model, we explore how the levels of risk taken in the end game compare with risks taken earlier in the tournament. This is motivated by the frequent assertion in the literature that trailing funds increase risk later on in the year to catch up with the leaders. If fund managers are equally able, trailing funds never reduce the levels of excessive risk that they take over time. In contrast, leading funds reduce risk *only if* their lead is large enough. Otherwise, they face exactly the same concerns as trailing funds. If, however, some managers possess sufficiently high ability, they may take little risk initially, because ability is isomorphic to having a lead. However, if an able fund has sufficiently lucky competitors early, it might then be forced to take risks later on to overcome a lead. It is only in this situation that fund managers increase risk over time.

The main testable implications of the tournament theory are then that provided that fund managers are equally able, all funds take excessive risk initially. Later on in the year, funds trailing in the tournament should continue to take the same amount of excessive risk. Leading funds reduce risk only if their lead is large enough. This implies that for different years, or for different mutual fund tournaments (e.g. by style box) within the same year, leading funds might or might not reduce risk, which could potentially explain the differences between Brown Harlow and Starks results for different time periods, as well as the difference between Brown, Harlow and Starks' results and Busse's.

In the empirical section of our paper, we test our theory by applying it to a large sample of US mutual funds from 1999-2007, in a sequence of annual style-adjusted tournaments. To our knowledge, this is the first large-scale study of mutual fund tournaments using daily data. Consistent with our tournament theory, we find that among the funds in the top 20% mid-year quintile, the funds with the largest leads reduce the amount of risk they take relative to trailing funds to lock in their leads. This lock-in effect is stronger as the ranking formation period is extended toward the end of the year, as even funds with smaller leads lock in their lead by reducing their risk. In a way, our results are roughly consistent with Brown, Harlow, and Starks' in that they find that leading funds take less risk than trailing funds in the latter part of the year. Their interpretation is that trailers increase risk relative to leaders – ours is that leaders reduce risk relative to trailers. Our more precise test shows that the risk reduction is confined to the funds at the very top of the returns distribution, as our theory predicts.

Given our empirical results, it is possible that Brown, Harlow and Starks have the correct interpretation, but it is difficult to formulate a model where trailing funds increase risk later in the year. This is because in most cases, funds should be taking an equal amount of risk early in the year as well. Two conditions that can result in equilibria with increased risk-taking by trailing funds are (1) fund managers have differential ability and in equilibrium, high ability managers take little excess risk early on, only to be forced to take more risk later when some less-able managers get lucky early in the year or (2) excessive risk-taking is associated with a negative expected return so that managers are wary of taking excessive risk early on and only do so at the end of a tournament to increase their chances of winning. An analogy to case (2) would be that of a golfer taking a risky shot on a par 5 hole to reach the green in two strokes late in a tournament to increase his (her) chance of winning even though the chance of a bogey is significantly increased.

As a footnote, we also find that using monthly data rather than daily data on the same sample of funds is likely to produce different results – it biases the results in *favor* of finding risk reduction, which is why it is important to use daily data to test our hypotheses. This bias, however, does not affect the conclusions in Brown, Harlow, and Starks (1996). If anything, it makes them stronger.

Although the study of tournaments in economics has a long and storied history, most theoretical models of tournament behavior in the mutual fund literature involve just two competing funds.[1] In (Taylor 2003), there is a very simple game with two funds and two binary strategies (either a risky or riskless alternative). This paper also finds results opposite to standard tournament theory. Using a small sample and measuring risk in terms of market betas, Taylor finds that higher performing funds take more risks than lower performing funds – in essence a *reverse tournament* effect. Some recent papers find that tournament effects show up in some years and reverse tournament effects in other years. One contribution of our paper therefore is to show that this kind of dichotomy can be consistent with the original notions of tournament behavior, it just depends on which historical path dependency obtains. Another paper (Lóránth and Sciubba 2006) considers a two fund tournament with discrete payoffs and correlated assets in which a third fund can enter at the interim date. A recent working paper (Basak and Makarov 2009) considers two fund tournaments in which managers have a relative performance component to their utility functions. They find a multiplicity of equilibria in which there can be either returns chasing behavior or contrarian strategies. Importantly their paper is conducted under a strong informational environment in which it is possible for each manager to base her payoffs on those of the other manager *state by state*.

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[1] [1] The seminal works in the theory of economic tournaments (Lazear and Rosen 1981) and (Nalebuff and Stiglitz 1983) investigate the impact of dichotomous reward structures on individual effort and competitive behavior. Typical agency questions deal with whether the contest can be optimally designed to elicit effort or to efficiently separate types in an adverse selection context (Bhattacharya and Guasch 1988). The major contribution of the theoretical literature in agency tournaments has been the optimal design of reward incentives (Akerlof and Holden 2010). Empirical work in the economics literature has largely concentrated in a labor markets context (O'Keefe, Viscusi and Zeckhauser 1984) and (Bognanno 2001).

Nonlinearity in reward schemes to funds management can be an implication of a tournament amongst managers (Bhattacharya and Pfleiderer 1985). In further developments (Stoughton 1993) the same nonlinear contract was used to explore the moral hazard issue. Optimal contract design in a multi-period game in which performance in one period would result in implications for fund flows in subsequent periods has also been considered (Heinkel and Stoughton 1994). The idea behind these papers is that there is an implicit relation that gives rewards to fund managers from increasing scale due to fund flows.

The rest of the paper is organized as follows. We develop the model in section 2, where a simple game of two players is also analyzed. The main part of the theory appears in section 3 which studies the two periods, three funds game. Empirical implications are summarized in section 4 and the empirical results are found in section 5. Section 6 concludes the paper.

2. Model

We consider a competitive tournament between N competitors, with $N \geq 2$. It will be shown that consideration of at least three players ($N = 3$) gives essentially enough generality to illustrate our main results. The tournament we consider is broken up into two periods. Initially there is a performance formation period where rankings amongst funds are established. Then there is an endgame where funds compete for a `winner take all' prize given at the end of the second period. In the case of a tie, the prize is split equally between the winners.

It is important to note that there is only a single winner and there is no difference in payoffs to any of the losing funds. In reality of course, mutual fund managers are typically compensated by a proportionate fee schedule based on assets under management at the end of the year as well as (implicitly) by an expected increase in fund flows based on relative performance. For the purposes of the model, and to focus specifically on the tournament aspect we do not consider the linear part of the compensation schedule and instead focus only on the race to be the winner. We also do not consider differential fund flows or termination outcomes for managers of non-winning funds. The tournament payoff structure we investigate is therefore an approximation of the well-known `convexity' of fund flows. Evidence (Kim 2010) shows that the convexity in fund flows as a function of end of year rankings is more apparent than the convexity as a function of end of year returns.

We use the following specification of fund return distributions for each period, t:

$$R_{it} = \sigma_{it}\epsilon_{it}, \tag{1}$$

where fund manager i selects a risk level $\sigma_{it} \in [0, \bar{\sigma}]$ with ϵ_{it} a standard zero mean, unit variance normally distributed random variable that is independent and identically distributed. This specification is general enough to allow for benchmarking in terms of performance, in which case the risks on the right hand side of (1) are benchmark-adjusted residuals. The assumption of normality is not completely without loss of generality. Some of our proofs rely on properties of order statistics for normal random variables.

In our basic specification all managers are assumed to start with equal abilities and so their relative performances are due entirely to the risk choices they make and the uncertain outcomes of the random processes. There is also no risk-return tradeoff as it is assumed that there exists a costless risk-taking technology that is available to all fund managers. This is realistic in view of the fact that some common methods of adding risk like holding futures are available to all managers and requires lower capital than investing in equities themselves, for instance.[2] We also assume that the amount of risk that any manager may be willing to take is bounded, although the bound may be arbitrarily high.

Denote the first period performance of each fund as $a_i = \sigma_{1t}\epsilon_{1t}$. Then the total return (without adjusting for compounding of returns) is given by

$$R_i = a_i + \sigma_{i2}\epsilon_{i2}.$$

The tournament structure is embodied in the following payoff for manager i, who chooses her risk level while taking other managers choices as given (Nash equilibrium):

$$U_i\left(R_i|\sigma_{)i(}\right) = \begin{cases} 1 \text{ if } a_i + \sigma_{i2}\epsilon_{i2} \geq \text{Max } a_j + \sigma_{j2}\epsilon_{j2}; j \neq i \\ 0 \text{ if } a_i + \sigma_{i2}\epsilon_{i2} < \text{Max } a_j + \sigma_{j2}\epsilon_{j2}; j \neq i\text{'} \end{cases} \tag{2}$$

[2] For instance the May 6, 2010 `flash crash' is alleged to have been caused by one mutual fund selling a large number of futures contracts (SEC 2010).

where $\sigma_{)i(}$ denotes the vector of risk choices for managers other than i. Each manager maximizes her expected utility, $E(U_i)$:

$$\text{Max}_{\sigma_i} E(U_i) = \text{Prob}(R_i \geq \text{max}_{j \neq i}\{R_j\}). \tag{3}$$

Two Fund Results

Infinite Risk Limits

We being with a simple case where there are only two funds, $N = 2$, and the tournament takes place over a single period. To begin with suppose that $\bar{\sigma} = \infty$. Let manager 1 have a lead over manager 2, i.e., $a_1 > a_2$. Suppose that manager 1 is currently choosing finite variance so that $\sigma_1 < \infty$ (we neglect the time subscript). If manager 2 were to choose the same variance, then her probability of winning would be less than one-half. However if manager 2 selects $\sigma_2 = \infty$ then she can increase her probability of winning to one-half. Choosing any finite variance strategy gives a probability of winning less than that. Therefore it is a best response for player 2 to select infinite variance whenever player 1 chooses finite variance. Now notice that since player 2 wins with probability one-half, then player 1 is indifferent to each risk choice. Hence the equilibrium features manager 2 selecting infinite variance while player 1 chooses any risk level $\sigma_1 \leq \infty$. Each fund wins with probability one-half.

Conversely if manager 1 forms a portfolio with infinite variance, then manager 2 is indifferent between all actions. Therefore the equilibrium always features one manager choosing $\sigma_i = \infty$, while the other manager selects $\sigma_j \leq \infty$. Importantly we cannot predict whether the manager with higher or lower ability chooses more risk than the other manager, and neither manager wins with a higher probability than the other. Based on the simplest possible single period model, we can see that there really are no implications of the theory. Almost anything can happen.

In order to focus on the dynamics of the model, let us now consider the first period. Now when we add an initial period to our single period result, we find that leading the tournament going into the second period does not matter. Since both the leader and trailer have equal chances of winning after one period, there is no particular first period advantage to any level of risk taking. Hence any level of risk in the first period can be a Nash equilibrium. Interestingly, this simple case leads to basically no prediction on respective risk-taking either in the initial or second period.

Risk Limits

Since taking infinite risk is rather unrealistic, we now consider bounding the maximal variance each manager is allowed to take at the same level, $\bar{\sigma} < \infty$. We start with the analysis after the first period. Again suppose that manager one comes in with a lead, $a_1 > a_2$. Since player 2 is now selecting a finite level of variance, if player 1 chooses zero variance, $\sigma_1 = 0$, then she wins with probability greater than one-half. Conversely by adding variance player 2 increases her probability of winning. Therefore there is now an unique Nash equilibrium in which manager 1 selects zero risk, $\sigma_1 = 0$, while manager 2 selects the maximal level of risk, $\sigma_2 = \bar{\sigma}$. The leading manager has a probability of winning greater than one-half and the trailing manager has a probability of winning less than one-half. Figure 1 illustrates the form of the equilibrium.

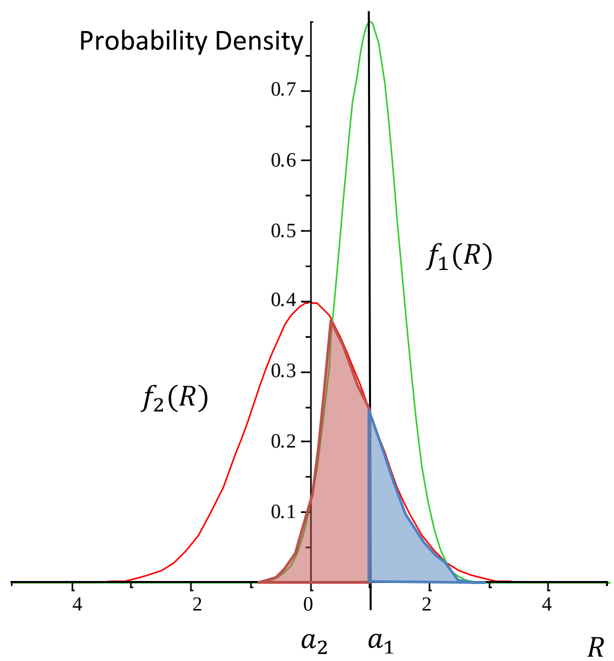

Figure 1: Originally player 1 wins with the area below player 2's probability density function to the left of a_1. By increasing risk, player 1 now wins in the blue shaded area above $a_1$but loses in the red shaded area below a_1. The former, however has lower probability mass than the latter. Hence player one should never increase risk above zero when player 2 has finite variance.

Using the consequence of this behavior in the second period, we find that it now pays to be in the lead after one period. If each fund starts off the tournament without a head start then no matter what initial risk strategy is employed, each fund will be in the lead with probability one-half at the interim stage. This means that any amount of risk is a Nash equilibrium for the first period. By bounding the risk limit in the second period we get a sharper prediction than with unbounded risk limits. Only the leading fund necessarily reduces risk from the first period to the second. The trailing fund may or may not increase risk. For instance if the risk limit in the first period is greater than the second (perhaps because of a longer time period), then risk could actually decrease. On average we would expect leading funds to reduce risk more than trailing funds.

3. Multiplayer Tournaments

We now consider the game when there are three players. It will be apparent that what matters for our tournament results is that each competitor faces multiple contenders. Further, although our formal results are only derived for three funds, they could be easily extended to more. We begin with a characterization of the objective function for a single period game. As with the two-fund tournament, we begin by analyzing second-period behavior.

For fund i to win the tournament, it must be that the other two other funds, j and k, have lower realized returns. If we define $F_i(R_i)$ as the marginal cumulative distribution function for fund $i's$ rate of return, then the probability that fund i wins with a return of R_i is $F_j(R_i)F_k(R_i)$. The expression, $F_j(R_i)F_k(R_i)$ is the cumulative distribution function of the maximum of the returns of the other two funds, evaluated at fund $i's$ realized return, R_i. In general this expression is referred to as the CDF of the highest *order statistic* of the other two funds. The event of player i winning is equivalent to the event that player i's return exceeds that of the highest order statistic of the other two fund returns. This implies that the objective function can be written as

$$E(U_i) = \int_{-\infty}^{\infty} F_j(R_i)F_k(R_i)f_i(R_i)dR_i, \tag{4}$$

where $f_i(R_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(R_i-a_i)^2}{2\sigma_i^2}}$ is the probability density function of a normal random variable. In order to analyze the impact of changes in the level of idiosyncratic volatility, one can differentiate equation (4) with respect to σ_i. This yields the following (after some simplifications):

$$\frac{dE(U_i)}{d\sigma_i} = \int_{-\infty}^{\infty} \left(-\frac{1}{\sigma_i} + \frac{(R_i - a_i)^2}{\sigma_i^3}\right) F_j(R_i)F_k(R_i)f_i(R_i)dR_i. \tag{5}$$

This expression reveals that there are two effects resulting when a fund manager increases the level of risk. The first effect is negative and occurs when returns are near their initial (expected) values and the level of risk is low. The second is positive and occurs when returns deviate from their initial (expected) values. Which of these effects dominates is very specific to the relative position of the fund coming into the final period of the tournament and the potential for risk increases. We now analyze the various cases.

Suppose initially that all funds can adopt an unlimited amount of risk: $\bar{\sigma} = \infty$. Then it is possible to show that all mutual funds will adopt identical policies of choosing infinite risk amounts.

Proposition 1. *Suppose that all mutual funds can accept unlimited amounts of risk and that they have different initial positions in a single period tournament:* $\mathrm{a}_1 > \mathrm{a}_2 > \mathrm{a}_3$. *Then each fund* i *selects* $\sigma_\mathrm{i} = \infty$.

Proof: First consider the action of the last place fund (number 3). Suppose that both fund 1 and fund 2 select finite variance strategies. Then the manager of fund three can increase the probability of winning to one-half by increasing her variance to infinity. Any lower variance will lead to a smaller probability of winning. Consider now manager 2. This manager is facing fund 1 choosing a finite variance strategy and fund 3 that is selecting an infinite variance strategy. If this manager does in fact select a finite variance strategy, her probability of winning is less than one-fourth, since she is beaten one-half the time by fund 3 and at least one-fourth by fund 1. If manager two instead increases her risk without bound, by copying the strategy of manager 3, then either manager 2 or 3 beats manager 1 three-fourths of the time. Since manager two wins with equal probability to manager 3, this means that she increases her probability of winning from less than one-fourth to $(1/2)(3/4) = 3/8^{\text{ths}}$ of the time. This is better than any finite risk strategy. Finally consider manager 1. With a finite strategy, against two infinite strategies, manager 1 can only win one-fourth of the time, while using also an infinite strategy will lead to victory with one-third probability. Therefore all managers choose infinite variances. □

Our conclusion therefore is that when unlimited risks are allowed, all managers are forced to choose identical risky strategies. No initial lead is safe. No matter how big manager one's lead is, it can be overcome sufficiently by one of the other two managers. Hence, manager one is also forced to take massive risks just to maintain an equal probability of winning.

Of course the infinite risk limit case is not very realistic. Therefore we now impose a finite risk limit, $\bar{\sigma}$. To win the tournament, each fund must have the highest return – it must beat the maximum of the return of the other two funds. The distribution of this maximum is the distribution of the highest (or second) order statistic of the other strategies. We record in the following Lemma the essential properties we require for the distribution of the highest order statistic.

Lemma. *The distribution of the maximum order statistic for two independent normal random variables is unimodal and positively skewed. Further the mode is greater than the maximum of the two constituent means.*

We begin by considering the best response strategies of managers 2 and 3, who arrive into the final period trailing manager 1.

Proposition 2. *Suppose all managers risk choices are restricted to* $\sigma_i \leq \bar{\sigma}$. *Then in a single period tournament, managers 2 and 3 choose maximal variance,* $\sigma_2 = \sigma_3 = \bar{\sigma}$.

Proof: For either manager 2 or manager 3, the normal distribution of returns lies below the mode of the highest order statistic between manager 1 and the other manager, since the mode of the order statistic is at least equal to a_1, which is greater than both a_2 and a_3. Figure 2 illustrates the situation of either manager 2 or 3 selecting a risk level strictly less than $\bar{\sigma}$. The same argument applies to each. For simplicity consider manager 3. By increasing risk slightly, it is easy to see that (because the density of the order statistic is increasing) in the region of the mean, there is a higher probability that the maximum is above rather than below. This implies that there is a benefit to trading off some downside risk against some upside gain. This shows that expanding the variance slightly can increase the probability associated with the shaded region above the mean as compared to the region below. The argument can be extended to all possible values of σ_3. Consider the density of the order statistic, $f_{12}^{\max}(R) = d(F_1(R)F_2(R))/dR$, which satisfies the property $f_{12}^{\max}(a_3 + y) > f_{12}^{\max}(a_3 - y)$ for all $y > 0$. Since the normal density of the returns of the 3rd player is symmetric, this means that any increase in risk is beneficial. □

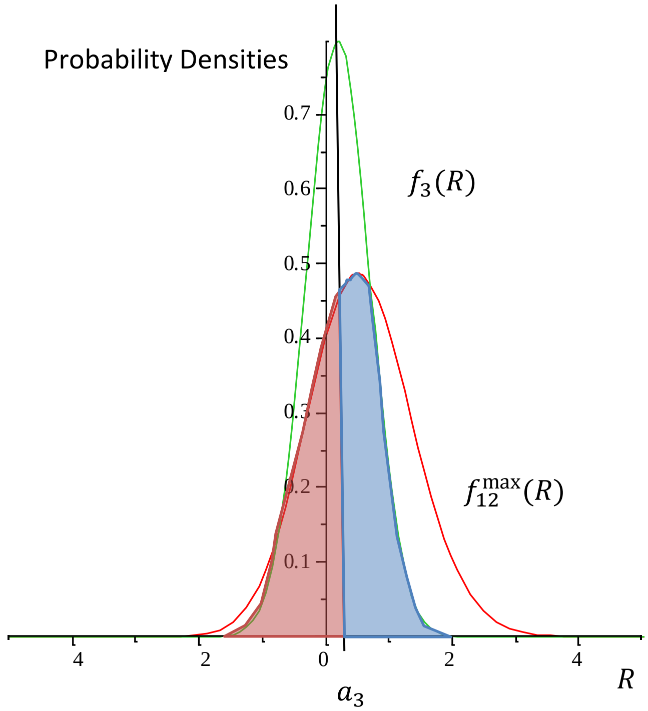

Figure 2: This depicts the situation of player 3 whose mean return distribution lies below the mode of the maximal order statistic distribution between players 1 and 2. If player 3 increases the variance of her distribution slightly the area of the shaded region above a_3 increases by more than the shaded region below. This argument can be extended to all possible values of σ_3.

Proposition 2 implies that, as in the case with infinite risk limits, the two trailing managers take as much risk as possible. We now show that *unlike* in the case of infinite risk, manager one may *not* increase risk to a maximum. From Proposition 1, we already know that if the risk limit is sufficiently high, then manager 1 will also take maximal variance. However if the risk limit is much lower, manager 1's lead might be safe enough for her to reduce risk. We first consider what happens when manager 1's position is `slightly' above that of manager 2 and 3.

Proposition 3. *Suppose that the initial return for fund manager 1, a_1, is less than the mode of the maximal order statistic density between managers 2 and 3. Then increasing the idiosyncratic variance of return always increases player 1's probability of winning. Hence the optimal response is to set $\sigma_1 = \overline{\sigma}$.*

Proof: The same argument used above for proposition 2 applies here. Increasing variance always adds more probabilistic mass when it is most useful. □

This result implies the following corollary.

Corollary 1. *If the lead of manager 1 is sufficiently small – below the mode of the maximum order statistic density of managers 2 and 3, then the unique Nash equilibrium in the single period tournament is for each manager to choose maximal variance,* $\sigma_1 = \sigma_2 = \sigma_3 = \overline{\sigma}$.

We have therefore found an equilibrium with finite variance strategies that resembles that of infinite variance strategies. But this requires a very small lead for the top performing fund at the start of the single period tournament. Notice that this result with three players is completely different from that with two players, where the top performing fund always sits on a lead, no matter how close to the other fund. The intuition for this result lies in the numbers of competitors. Here because there are two other players rather than just one to worry about, it is more likely that one of them will achieve a high outcome from a high risk strategy. As a result the leader has to mimic both of the followers.

However, if manager 1 has a large lead, as defined by her relation to the mode of the second order statistic density, then there can be an asymmetric equilibrium in which the top manager reduces risk as much as possible. This is depicted in the next proposition and illustrated in Figure 3.

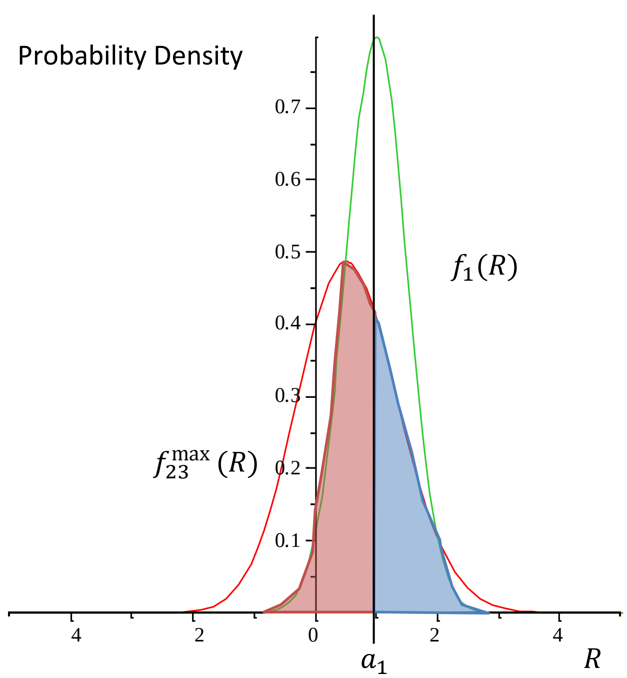

Figure 3: The leader finds it counterproductive to add some amount of risk. The density, $f_{23}^{\max}(R)$, is the second order statistic of the two trailing funds. With a zero risk, the probability that fund 1 wins is the area to the left of a_1. With risk-taking, the density, $f_1(R)$, is that of the leader causes a higher probability in red of losing versus a lower probability in blue of winning.

Proposition 4. *Suppose that the mean of player 1s return distribution,* a_1, *lies above that of the mode of the maximal order statistic density between players 2 and 3. Then player 1 always chooses either zero* *risk ,$\sigma_1 = 0$, or maximum risk, $\sigma_1 = \overline{\sigma}$, depending on the magnitude of $\overline{\sigma}$. If player 1s probability of winning the tournament with $\sigma_1 = 0$ is greater than $\frac{1}{2}$, the optimal level of risk is always $\sigma_1 = 0$. If player 1s probability of winning is less than $\frac{1}{2}$, then the optimal strategy is to select $\sigma_1 = 0$ if $\overline{\sigma} < \widehat{\sigma}$, for some critical value $\widehat{\sigma}$. On the other hand for $\overline{\sigma} \geq \widehat{\sigma}$, the optimal strategy is to set $\sigma_1 = \overline{\sigma}$.*

Proof: See the illustration of Figure 3. In this case, if the value a_1 lies above the mode of the density of the order statistic, $f_{23}^{\max}$, then increasing variance by a small amount is counterproductive at first. This is because the shaded area to the left of a_1 is greater than the shaded area to the right. However, because of the positive skewness of the order statistic density, this argument cannot be extended to the case of global optimality. This is because there are values, $y > 0$, sufficiently large such that the density in the left tail, $f_{23}^{\max}(a_1 - y) < f_{23}^{\max}(a_1 + y)$. As a result of this the marginal benefit of increasing risk can eventually become positive. However even at the upper risk bound, the probability of winning must lie below 0.5. Hence if the probability of winning is $1/2$ at $\sigma_1 = 0$, then this is the global maximum. On the other hand if the probability of winning is strictly less than $1/2$ at $\sigma_1 = 0$, then since the probability of winning must approach $1/2$ if the upper bound, $\bar{\sigma}$, is sufficiently high, then the global optimum will be at $\sigma_1 = \bar{\sigma}$ in this case. □

We can therefore derive the following corollary concerning the existence of an equilibrium in the single period tournament game.

Corollary 2. *If player 1 has a lead such that it is greater than the mode of the order statistic for the maximum of players 2 and 3, then there exists an unique Nash equilibrium in which either: (1) player 1 chooses zero variance while players 2 and 3 choose maximum variance; or (2) if the upper bound on feasible risk limits is sufficiently large then players 1, 2 and 3 all choose the highest permissible levels of risk.*

We have therefore found a specific condition identifying whether the top-performing fund will sit on a lead in the final period in order to maximize the probability of winning the tournament. It involves the joint determination of the relative position of the leading fund coming into the period *vis a vis* the most likely value of the order statistic of the trailing funds. Whenever the lead is scant, and the scope for risk-taking great, then the leader is forced to mimic the followers in terms of the level of idiosyncratic risk taken. However with a sufficient lead and limits on feasible risk-taking then the leader is safe in minimizing her risk. Note that in this simple model presentation there is never an interior optimal level of risk-taking. It's either all or nothing.

Value of the Lock-in Option

The previous subsection has fully considered all possible equilibria for the continuation game of the two period problem. The possibility to lock-in a lead after one period has implications for the first period play amongst the funds. As before, we suppose that funds start off with equal abilities at time 0, with no ability to have a `head start.'

We have demonstrated that if the leader has a sufficient lead, she will lock in her lead by reducing risk to a minimum; otherwise it will always be optimal to increase risk to the maximum. Because a manager only reduces risk when her lead is safe, we can define the value of this option to be the difference in probability of winning between reducing risk or setting it to the maximum level. If we let $F_2(a_1)$ be the cumulative probability distribution function over one period for the middle player, evaluated at the outcome for the top player and similarly $F_3(a_1)$ be the corresponding cumulative probability distribution for the bottom player then the probability that both the middle and bottom players lose if player 1 takes no risk is $F_2(a_1)F_3(a_1)$. On the other hand if player 1 takes maximum risk then the probability of winning is $\int_{-\infty}^{\infty} F_2(R)F_3(R)f_1(R)dR$, where $f_1$denotes the probability density function of the top player with mean a_1 and standard deviation $\bar{\sigma}_t$. This means that the option value of locking in can be defined as

$$\text{Lock-in Option Value} = \max\left(F_2(a_1)F_3(a_1) - \int_{-\infty}^{\infty} F_2(R)F_3(R)f_1(R)dR, 0\right). \qquad (6)$$

Figure 4 illustrates the value of the lock-in option. As is indicated there, unless the first-period return is sufficiently far above both of the competitors, the value of the option is zero. However, the option becomes `in the money' as the first period outcome of the highest player increases. The value of the option has a maximum which is where the top player gains greatest advantage over taking risks in the second period. As is indicated by Figure 3, this relative advantage is maximized at an interior point. If the lead of the top player is huge, then his position is located in the right tail of the maximum order statistic of the other two players. As a result there is less of an advantage (although still a positive one) of locking in a lead.

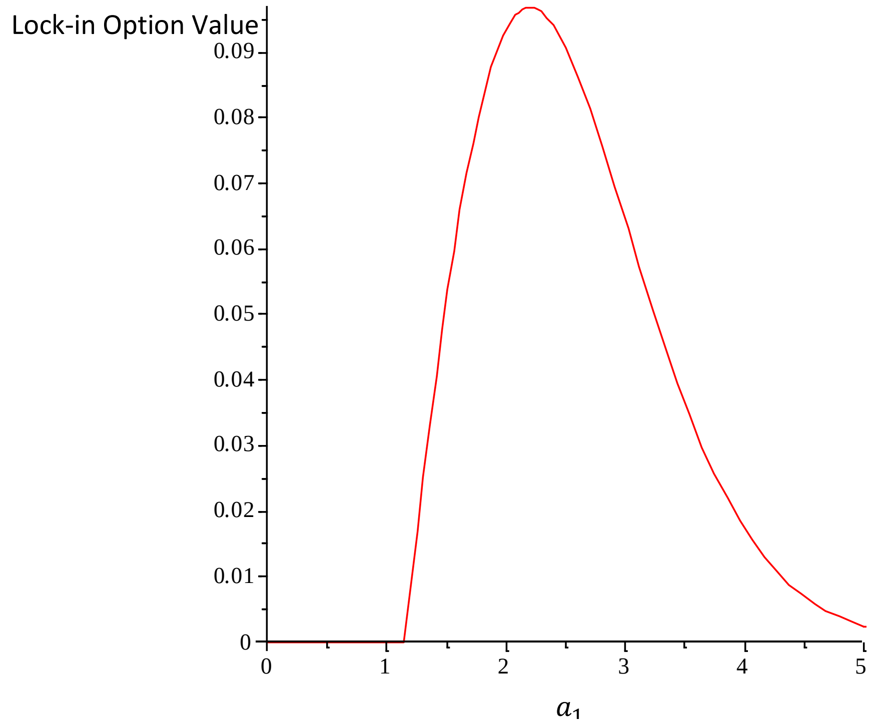

Figure 4: The value of the lock-in option in terms of increased probability of winning for fixed realization of the bottom two funds. This simulation is calibrated using the parameters $a_3 = 0$, $a_2 = 1$, and $\bar{\sigma} = \sigma_2 = \sigma_3 = 1$. The value a_1 represents the first periods outcome of the highest ranking fund.

We shall now explore the consequences of this lock-in option on first period risk-taking behavior.

First Period Equilibrium

The two period optimal risk-taking decision can be thought of as that of a single period problem in which the outcome is the combination of taking a lead at the end of the first period and then allowing for what happens in the continuation game. To gain an understanding of the intuition for the form of the equilibrium, suppose first that managers enter the tournament with first-period ability equal to zero. Further, suppose that there is a difference in the risk limits in the two periods. Namely denote the initial limit on risk-taking, $\bar{\sigma}_0$, to be infinite, while the second period limit, $\bar{\sigma}$, is finite. Then if two of the three funds were to take a finite risk in the first period, the third fund – by taking infinite risk – could gain a lead with probability ½. Since the size of the lead after one period is essentially unbounded, if the second period risk is bounded, the lead will be preserved almost surely. The implication of this is that with unbounded first period risk limits, all funds will choose infinite risk, just as in the single period case.

Suppose now that the risk limits are finite in both periods. However suppose that the leading fund never exercises the lock-in option in the second period. While this is a suboptimal strategy in the continuation game, we can still determine that the initial period optimum involves setting risk equal to its upper limit. Because all funds initially begin with identical positions, we are in the situation of Corollary 1 with $a_1 = a_2 = a_3 = 0$. Note that with finite risk taking in the second period and each firm taking the maximum the probability of winning is increasing in the ex post position of the leader. Therefore each fund in the initial period will maximize its risk in order to achieve the highest probability of being in the lead. For these reasons, *neglecting the lock-in option*, every fund would set its risk equal to $\bar{\sigma}_0$.

The complication in the multiperiod game occurs because the lock-in option is not monotonic in the amount of the lead (Figure 4). This means that it could conceivably be counterproductive to attain too large a lead by virtue of the fact that the lock in option decreases. However this is counteracted by the fact that without the lock-in option the probability of wining increases in the lead. Since the lock-in option value is not in closed form, we have resorted to numerical techniques to determine which of these two effects dominates.

Figure 5 illustrates one of the numerical simulations in which manager *j*'s first period risk varies while fixing the risks of managers *i* and *j* at $\sigma_i = \sigma_k = 1$. All managers have second period risk limits equal to $\bar{\sigma} = 1$. The full value of the lock-in option is taken into account for the second period for all players. Notice that the ex ante probability of winning is monotonically increasing for player *j*. If that player takes zero risk in the initial period, then the probability of winning is below 0.3. Obviously if the player takes equal risk to the other players, $\sigma_j = 1$, then the probability of winning for all players equals one third. By taking an even higher first period risk, player *j* improves her probability of winning. For instance with twice as much risk as the others, the player wins with probability more than 0.4.

Our multiperiod problem with symmetric mutual funds and independent outcomes has only two parameters, the first and second period risk limits, $\bar{\sigma}_0$ and $\bar{\sigma}$. All other parameters are endogenously determined in the model. Therefore it is relatively easy to establish the generality of the result. We record this as proposition 5.

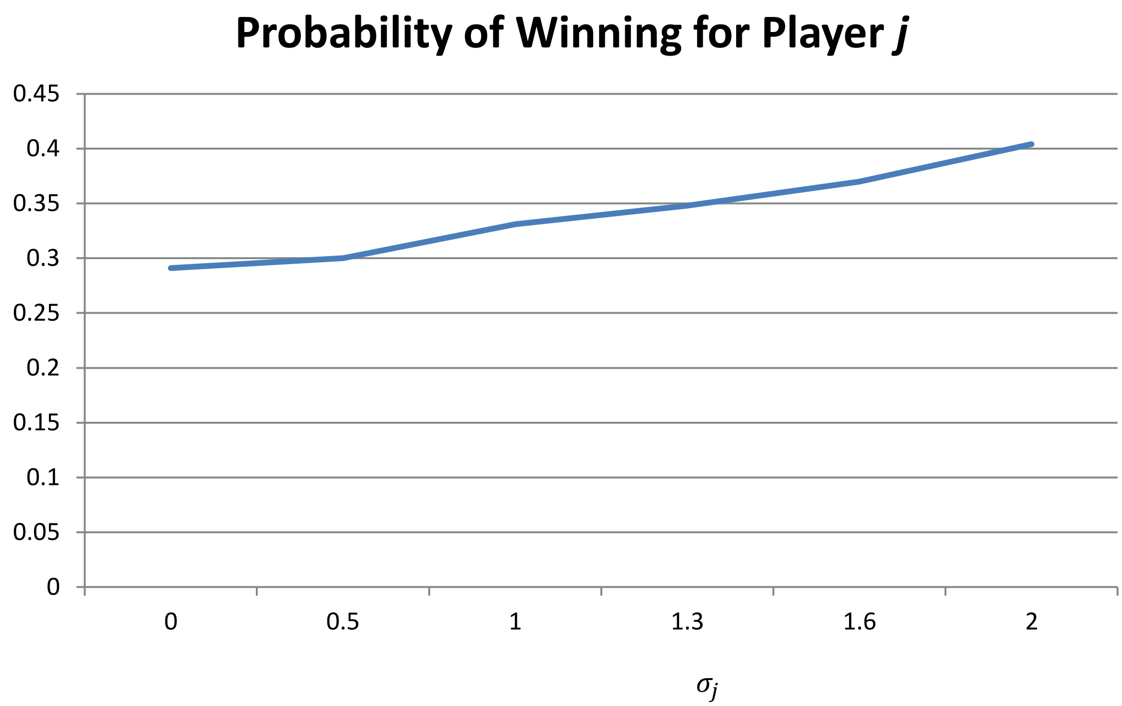

Figure 5: The probability of winning for player j as a function of the risk taken in the first period, σ_j. The other two players have their first period risk fixed at $\sigma_i = \sigma_k = 1$. The second period risk limit for all players is $\overline{\sigma} = 1$. This probability is monotonically increasing in first period risk.

Proposition 5. *Consider the case with equal fund managerial abilities and identical starting positions. Suppose that the upper bound on first period risk,* $\overline{\sigma}_0$ *, is finite. Then there exists an unique symmetric equilibrium in which each mutual fund selects an identical level of risk* $\sigma_i = \sigma_j = \sigma_k = \overline{\sigma}_0$ *in the first period.*

Note that in our model, when funds start on an equal footing, there is no situation where managers increase risk in the second period relative to the first. This is not necessarily true when we allow for differential *ability* in the initial period. In that case, were one manager sufficiently more able than the others, that manager might well *not* add volatility to her strategy. In this case, if the other managers were sufficiently lucky, they could overcome the lead and hence force the more able manager to add volatility later on. Indeed, it is only the case that *more-able* managers could conceivably increase volatility in period two. This is the only case that comports with the intuition in the original literature.

Fund Flow Convexity

One issue is how convexity in fund flows can be interpreted in the context of the model. This can be seen by looking at the reward to managers as a function of returns rather than ranks. Because the rewards funds receive equal one if and only if they win the tournament, this question can be analyzed by determining whether the probability of winning is a convex function of returns.

For notational simplicity, let the CDF for fund $i's$ two-period return be given by $F_i(R)$. Then, the probability that return R wins the tournament is given by the likelihood that it is the highest return, which is given by the CDF of the maximal order statistic, or $F_1(R)F_2(R)F_3(R)$. It can be shown that this order statistic is a CDF that is convex up to a point and then concave above that. Hence, it is possible that this relation may be convex in a relevant range, but it cannot be convex over the entire space of returns. Therefore we do not necessarily get unbounded risk-taking in our model.

4. Discussion and Empirical Implications

The main consequence of the theory of multi-period, multi-player tournaments is that optimal levels of risk taking are not only dynamic, but they are conditioned on interim performance. Although it is not possible to provide unequivocal tests, the ability to refine anticipated behavior offers the promise of more precise testing of the theory.

One possible criticism could be that it relies on very specific distributional forms. We have therefore analyzed an alternative model, based on a multi-period problem with discrete (two-point support) distributions and equal abilities. We find exactly the same behavior: leading funds lock in after one period and trailing funds maintain high variance levels.

As mentioned in the introduction, we believe it is most realistic to define the end of a calendar year as the conclusion of the tournament and the beginning of the calendar year as the start. One issue to be contended with in empirical testing is the fact that tournaments generally exist amongst similar funds within specific style categories. However there are many more than three funds in each category. In addition, it is generally some top percentile of funds that `win' rather than simply the absolute number one fund out of hundreds. Hence we will describe our results in terms of top, middle and bottom performing funds, with the top funds all winning the tournament with equal prizes.

To start off, assume for simplicity that the maximal risk each fund can add is the same in both periods: $\overline{\sigma}_0 = \overline{\sigma}$. If $\overline{\sigma}$ is sufficiently high, then in period 1, all funds take an identical level of risk that is less than the upper bound. In the second period, all funds with the exception of the leader take maximal risk, $\overline{\sigma}$. Depending on the size of the lead, as characterized in Corollary 2, the leading funds either take zero excess risk, if the lead is sufficiently large, or maximizes it, setting a level of risk equal to that of the middle and bottom category. Hence, for sufficiently high $\overline{\sigma}$, we expect the vast majority to increase risk over time, although a minority will actually decrease risk. One way to test for a high an equal level of maximal risk is to break the annual period into two equal periods: the first half up to June 30 and the second half, to December 31. In years in which there is a greater dispersion of outcomes over the first six months, one could conclude therefore that the upper bound on risk is greater.

If the level of maximal risk is sufficiently low, then Proposition 5 shows that all funds take the maximal level of risk in the first period and the trailing funds continue to do so in the second period. As above, the leading funds take no additional risk in the second period if and only if its lead is large enough. Such years may be identified with six month returns amongst the whole population that are more clustered.

In the theory we analyzed there were only three funds with only one fund winning. One question is: what happens if there are more than three competing funds, $N > 3$? It is clear that all of the qualitative results go through. However, the more competitors there are, the larger a lead must be in order to be \`safe.' This follows because adding additional competitors unambiguously increases the mode of the second highest order statistic distribution (the maximum of the funds, $2, \ldots N$). Hence, the more competitors there are, the more likely the leader would also increase risk to a maximum in period 2.

One of the problems with taking the predictions of the model too seriously is that when the number of competitors is large, then it is almost impossible to sit on a lead. This follows from the winner-take-all payoff structure assumed in our tournament. Empirically, this is unrealistic. It is probably 'good enough' to end up in the top group of funds. This implies that a larger number of funds should be able to sit on a lead and reduce volatility in the second period. Determining exactly what fraction of funds reduce variance depends on the exact nature of the tournament payoffs, as well as the distribution of returns the funds achieve in the first period. While, analyzing these questions theoretically is beyond the scope of this paper, we can still address this empirically.

When the time periods for the interim performance results and the continuation game are different, e.g., the formation period is 9 months to September 30 and the final period is the last quarter of the year, the issue of maximal risk levels must be confronted. That is, how to characterize the maximal level of risk that funds can take in any period? Above, we called it simply $\overline{\sigma}$. However, from an empirical perspective, it is probably better to define the maximal risk in terms of risk per unit time. Then, if maximal risk per unit time is constant, the total amount of idiosyncratic volatility a fund can take in any period is proportional to the period length (assuming prices following a Geometric Brownian motion).

Hence if, for example, the first period is nine months and the second period is three, and we assume that the maximal idiosyncratic volatility per unit time is identical, then this means the upper bound on end of period volatility that a fund can take in the first period is $\sqrt{3}$ times what it can take in the second.

Hence, with a relatively longer first period, two things happen. First, the value of the lock-in option increases, because the upper bound on volatility over the period decreases monotonically with the length of the second period. (It is easier to hold on to a lead in the second period.) However, smaller leads in the second period are also safe, which implies that the total amount of risk taken by all funds in the first period is likely to be interior – less than the upper bound. This implies that the larger the difference in length between the first and second periods, *ceteris paribus,* both the risk per unit time taken by trailing funds is more likely to increase over time, and it is more likely that leaders reduce their risk per unit time.

We therefore summarize the major empirical implications as follows:

- Risk taken per unit time of trailing funds can increase over time but will never decrease.
    - An increase is more likely when
        - the first period is longer relative to the second or
        - the maximum volatility a fund can add per unit time is high.
- When the distribution is more tightly clustered after the first period, leading funds behave exactly like the trailing funds.
- When leads are large (Corollary 2), top funds reduce their variances in the second period.
    - Leaders are more likely to reduce risk when the second period is short relative to the first.
    - Leaders are more likely to reduce risk when there are fewer competitors.
    - The very top funds among the leaders are more likely to reduce risk than the ones near the margin with the middle group
    - When the number of competitors is large, the value of the lock-in option is reduced, but the profile of the lock-in option is flatter. In such circumstances funds are more likely to maximize variance in both periods.

One question is how these implications differ from what might be obtained in a different kind of tournament where each fund wins by beating a static benchmark as opposed to beating all other funds. In this case, assume that the composition of the benchmark (e.g., an index) is known to all funds at all times while any risks taken are unobservable *ex ante*. Because the composition of the benchmark is known, to maximize the probability of beating the benchmark, in the final period, funds with year-to-date returns above the benchmark lock in their leads by mimicking it. By contrast to the previous model, such behavior ensures certain victory. On the other hand, trailing funds optimally maximize risk in the second period. In the first period, because the margin of victory is irrelevant, it is optimal for all funds to take a tiny amount of idiosyncratic risk. Taking more than an arbitrarily small amount does not increase utility (which is independent of victory margin) but reduces the probability of victory for negative realizations of idiosyncratic risk. Hence, it is essentially the case that all trailing funds dramatically increase risk over time and leading funds leave risk essentially unchanged – a very different prediction from what obtains in the tournament with strategic competition between many funds.

5. Empirical Study

The data used in our empirical study comes from the CRSP Mutual Fund Database. We use data on equity mutual funds starting in 1999 when fund performance data first became available at a daily frequency for an entire calendar year. While most prior research on tournament effects in mutual funds uses monthly data, monthly data does not allow us to reliably investigate changes in risk-taking behavior across time within a calendar year. For each mutual fund, we collect daily fund returns net of fees and expenses and the style category to which the fund belongs to. We merge our data with the MFLINKS database (Wermers 2000) to eliminate redundant mutual fund share classes. We end our sample in 2007, and avoid using data covering the financial crisis period of 2008. Our general approach is to compare how mutual funds change their risk-taking behavior in a later period vis-à-vis an earlier period. Thus, our approach is akin to using fixed-effects for each mutual fund and takes into consideration any fund characteristic that does not vary over time.

Table 1 shows the distribution of mutual funds used in our study. We limit our sample to funds in one of twelve equity fund categories as classified by Lipper.[3] This is a reasonable classification scheme since publications, such as Barron's, use rankings based on Lipper, rather than on Morningstar. Overall, we have 11,229 fund-year observations over 9 separate years in 12 fund style categories. Each category is well-represented and ranges from an average of 51.9 funds per year (Mid-Cap Value Funds) to an

[3] Lipper classifies equity mutual funds into twelve categories as the intersections of four size categories (Large-Cap, Mid-Cap, Small-Cap and Multi-Cap) and three valuation categories (Value, Core and Growth).

average of 179.0 funds per year (Large-Cap Core Funds). For our empirical analysis, we consider each tournament to be a distinct tournament that occurs each calendar year ending in December within each of the 12 Lipper classifications. Thus in our sample, we have 108 distinct tournaments. We assume that the tournaments are based on cumulative raw returns over a calendar year, without adjustments for risk.

We compute risk-taking behavior by mutual funds using four metrics. Risks taken by mutual funds can be systematic or idiosyncratic. However, if a tournament is based on cumulative returns without adjustment for systematic risk, it's not clear which risk-taking metric should be used. Therefore, following the standard approaches (Fama and French 1993) and (Carhart 1997) we consider up to four potential systematic risk factors when measuring fund risk-taking behavior. The four factors are the market factor, size factor (SMB), value factor (HML), and momentum factor (UMD).[4] Mutual fund returns are calculated in excess of the risk-free rate, as residual of CAPM (market factor only), as residual of a 3-factor model (market, SMB and HML), and residual of a 4-factor model (3-factors plus UMD). Within each fund-year, we compute risk-adjusted returns by regressing daily mutual fund returns on daily factor returns over the entire calendar year and taking the residuals plus the intercept term. We measure mutual fund risk-taking using the annualized standard deviation (volatility) of these risk-adjusted returns over different periods within a calendar year.

Panel A of Table 2 reports the summary statistics of mutual fund risk-taking behavior for each of the four metrics used. Risk-taking measures reported here are measured over the entire calendar year. Average volatility of mutual fund returns in our sample is 18.1%, while the median is 16.5%. The range of volatility from the 5th percentile to the 95th percentile is 9.8% to 31.6%, suggesting that there is a wide heterogeneity in risk-taking behavior. As we add adjustments for systematic risk factors, we note that average idiosyncratic volatility falls since these are factor models are nested within one another.

We provide some sense of cross-sectional variation in Panel B of Table 2. Within each fund category/year, we rank mutual funds according to their cumulative percentage returns over the first half of the calendar year (January to June). We sort funds according to quintiles of first half returns and show means of risk-taking measures over the entire calendar year in Panel B. Results based on medians are similar. Across all risk-taking measures, it naturally appears that funds that take more risk are more

[4] Data on factors used in our study is from Kenneth French's website at Dartmouth College.

likely to appear in the top quintile group (leader group) or the bottom quintile group (trailing group). There appears to be a slight tendency for funds in the trailing group to be the ones that take greater risk. The main objective of our empirical investigation is to see how these risk-taking measures change within each year for the funds in the leader group relative to those in the trailing group.

To measure changes in risk-taking behavior across time, we employ two types of measures of changes. One measure we use is the ratio of annualized fund return volatility over the second period divided by volatility over the first half of the calendar year (VOL_RATIO). This measure is commonly used in the literature, but recently it has been pointed out that this measure is problematic (Schwartz 2009). As shown earlier, funds in both leader group and trailing group tend to exhibit higher risk-taking which would bias VOL_RATIO upward. Hence our preferred measure of changes in risk-taking behavior is VOL_DIFF, which is the difference between annualized fund return volatility in the second period relative to the first half of the calendar year. We express VOL_DIFF as annualized percentage. Nonetheless, we use VOL_RATIO as a robustness check.

According to our theory, where a fund's performance stands relative to its peers during a calendar year is a key determinant of how funds alter their risk-taking behavior. We employ two types of location measures. The first type consists of indicator functions that equal one if a fund is within the leader group, within one of the three middle quintiles (middle group), or within the trailing group. We omit the indicator function for the trailing group in our estimation so the coefficient on the other indicators can be interpreted as changes in fund risk-taking behavior for a group relative to the changes in the trailing group. The second type of location measure takes the indicators for the leader group and the trailing group, and interacts them with the absolute difference between a fund's cumulative past return and that of the fund at the top or bottom quintile break points (distance). For the funds in the leader group, this measures how far ahead it is. For the funds in the trailing group, it measures how far behind the fund is from the breakpoint. A typical fund in the leader group is ahead of the 80^{th} percentile fund by 2.90% in the middle of the year, while the 95^{th} percentile fund is ahead by 9.83%.

Table 3 shows results from our base-case specifications, where the first-period is assumed to be January to June of each calendar year and we consider changes in risk-taking behavior over July to December. Our basic regression specification is a pooled-panel regression of changes in fund risk-taking behavior on measures of where a fund stands relative to its peers at a point in time. Our regressions also include fixed-effects for each fund category by year, which would absorb any common volatility changes. All standard errors reported in our tables are clustered by fund to control for any seasonal pattern experienced by a fund across years.

Panel A considers the basic tournament effect without taking into account leader and trailing distances. The first column does not use any adjustments for risk factors. The coefficients on the indicators can be interpreted as the increase in percentage volatility by the funds in the leader group (0.118) and the middle group relative to the increase in volatility by the funds in the trailing group (0.243). That is, a typical mutual fund in the leader group increases volatility by 0.118% more than what a typical mutual fund in the trailing group does, but this difference is not statistically significant. The intercept term of 0.257% can be interpreted as the increase in percentage volatility by funds in the trailing group. On average, we find that funds increase their risk later in the year.

We include controls for factor risk exposures as we move across columns. The R-squared of the regressions decreases dramatically as more factors are included because the variation in the left hand side variable decreases with the successive risk measures. We expect this since funds in different categories sorted by size and value differ most in their exposures to market, size and valuation factors. Interestingly, we do not observe that funds increase their idiosyncratic risk later in the year. This suggests that for a typical fund in our sample, they increase risk later in the year by increasing their exposures to common factors. For instance, value funds tend to hold more value stocks later in the year. However, this does not explain how the very top fund reacts.

Panel B of Table 3 shows our main result. We are most interested in the coefficient on leader distance. In the first column, without any factor risk adjustments, we find a coefficient of -0.083, which is statistically significant. This indicates that funds with significant leads reduce volatility. The coefficient implies that a fund that is ahead relative to the 80^{th} percentile fund by 10% cumulative return as of June, reduces risk by 0.083*10%=0.83%. Since the cross-sectional standard deviation across all funds is 7.8%, this is a small but not insignificant change. Reversing this procedure, since the coefficient on the leader group indicator is 0.294%, our point estimate suggests that a fund that is ahead by 0.294/0.083 = 3.54% cumulative return begins to reduce absolute risk. We consider changes in risk-taking behavior with factor risk adjustments in other columns. Overall, the results are similar, which suggests that when funds change risk-taking behavior, they are not doing so by changing their exposures to the market, Fama-French factors, or momentum.

One potential concern is that are results are due to mechanical bias. Our earlier summary statistic indicated that funds in both the leader group and the trailing group tend to be funds that take high risk. Hence it is possible that a fund randomly takes on high risk in the first period (rather than deliberately takes on high risk). If the decision was random, then we would expect to see volatility revert back to normal levels and our change in risk-taking measures would pick this up as reduction in risk. However, this mechanism would affect leaders and trailers equally and we investigate this possibility by looking at the coefficient on trailing distance. Without factor risk-adjustment, this coefficient is -0.025 but it is statistically insignificant. If anything, we find that this coefficient turns positive with factor risk-adjustments. That is, controlling for systematic risk factors, among the funds in the following group, those that are further behind tend to increase risk at the midyear mark, rather than decrease risk. This tends to indicate that there are some deliberate choices being made in risk-taking behavior.

Another way to check whether mechanical bias is responsible for our results is to repeat our analysis on simulated data under the null hypothesis that there is no change in mutual fund volatility over the year. We therefore simulated data on 1000 mutual funds where for each fund, 240 normally distributed daily returns are drawn such that mean annual fund returns are zero and the annual fund volatility is 18%, about the same as the excess return volatility in our mutual fund sample. Return draws are independent across time and funds. Volatility in the first (last) half of the year is computed over the first (second) 120 days of data. If estimation error would be responsible for our results, we would expect realized returns and volatility estimates to be correlated in-sample and we would expect reversion in volatility for top-performing funds.

The results of our simulation are given in Table 4, which is the counterpart to Table 3. Note first that the R^2 from the regressions are close to zero, and none of the coefficient estimates are statistically significant. Hence, even though the sign of the coefficient on the top 20% of funds is slightly negative (panel A) and the coefficient on the interaction between the top 20% and distance from threshold is also negative (panel B), the estimates are too small for mechanical bias to have been responsible for our results. Intuitively, estimating volatilty using 120 daily observations has low estimation error. We do find that there is a small positive correlation of about 0.10 between first half net returns and volatility estimates over the same period. The problem of estimation error is exacerbated using monthly data. To verify this, we created 12 monthly observations for each fund by compounding each month of daily data and reestimated changes in volatilities using the first and last six monthly observations.

Reestimating the same regressions we now find that the coefficient for the top 20% (the analog to panel A in Table 4) becomes -2.083% with a standard error of 0.800%, which *is* statistically significant. This follows because the standard error estimates using 6 monthly observations are much noisier and hence, realized volatilities tend to be high when returns are also high. Therefore, using monthly data to investigate volatiltiy changes over time can be problematic.

As another robustness check, we consider using VOL_RATIO as an alternative measure of changes in risk-taking behavior. It is important to recognize that this measure is biased, but it is useful to demonstrate consistency with prior literature. Table 5 reports results based on VOL_RATIO. In Panel A, we find that funds in the leader group increase volatility more than funds in the trailing group, which is consistent with what has been found using daily data (Busse 2001). We also find that since the intercepts are all significantly different from one, typical funds increase risk according to this measure. Panel B adds in controls for leader and trailing distance. With these controls, we find that leader funds tend to decrease risk rather than increase risk, though the statistical significance is weaker than before. Interestingly, we find stronger evidence for increases in idiosyncratic risk, since the results based on excess returns are not significant.

We now investigate how this tournament behavior changes during calendar time by measuring mutual fund risk-taking and fund location at different points in time. Rather than using the six month volatility difference by comparing January to June with July to December, we now compare the January to June volatility with the volatility between the starting months July, August, September, October, November, and December with the end of the year. We maintain annualized volatility so that the risk measures are in the same unit of time. Funds are ranked according to cumulative returns ending the day before each starting month.

The results from changing the performance ranking time are presented in Table 6, with different panels making factor risk adjustments using different models. In Panel A, we consider volatilities of excess returns across time. The coefficient on leader distance decreases as we move closer to the end of the calendar year. As we get near the end, the leading funds do not decrease risk much more than the fund that is just at the break-point. The coefficient on the indicator for being in leader group turns negative in October, which suggests that during the last quarter, even the fund that is just at the break-point is starting to reduce risk. This is consistent with our empirical prediction that more funds should exercise their lock-in options as the tournament nears the end. We consider how our results change with factor risk adjustments in different panels, and find consistent results throughout.

However, we obtain a picture that differs from the prediction of our model when we consider funds in the trailing group. Across all risk-taking measures, we find a statistically significant negative coefficient on the interaction between whether a fund is in the trailing group, and the distance that fund is from the lower breakpoint after October. That is, funds reduce risk the further behind that a fund falls behind others. This is opposite to the prediction of our model and the order of magnitude is up to three times that of the leading group. This effect exists only in the last quarter of the calendar year, however. Clearly, our theoretical model does not encompass all features of mutual fund incentives and behavior, and this effect may reflect an additional mechanism missing in our theoretical model.

6. Conclusion

Tournament behavior has been known to have important implications for mutual funds and the way in which they are managed. Theoretically, we have shown that risk-taking strategies by the player in the lead can be very different depending on the size of the lead, the relative position of other competitors and the amount of time left to compete for the top prize. In the context of mutual funds, in the final period before evaluation, all managers maximize the amount of idiosyncratic risk that they add to the portfolios with the exception of the fund with the highest returns to date. This fund reduces risk if and only if it has a sufficiently large lead. Otherwise, this fund too maximizes risk – there is no interior optimum.

We also characterize fund managers' optimal strategies in a multi-period setting. Initially, if all managers are equally able, all funds maximize risk. This implies that it will be optimal for only fund managers with the highest returns to date to reduce risk over time. One way to obtain the prediction of Brown, Harlow and Starks (1996) that trailing managers increase risk to catch up, is for some managers be more able than others. Only if some fund managers are sufficiently able is there a possibility that such managers will initially take little risk, and increase risk later on if they fall behind. Another way for it to be optimal for trailing funds to increase risk is if risk-taking has negative returns, so that initial risk-taking may not be optimal. This paradigm explains why in sports such as golf, players take excessive risks late in the tournament when trailing.

Empirically, we analyze multiple tournaments with heterogeneous characteristics. Our main prediction is that the very top funds reduce risk in order to lock-in their lead, which we verify empirically by relating risk reduction to the magnitude of the lead. This result is robust to different ways of controlling for systematic risk exposures. We find evidence that there is a greater lock-in effect closer to the end of the calendar year. We also find some supporting evidence suggesting that funds increase market risk exposures in the later period, but decrease idiosyncratic risks. However, contrary to our model, we find that funds at the very bottom also tend to decrease risk, particularly in the last quarter. This suggests there are mechanisms other than incentives provided by tournaments that our model does not yet incorporate.

One potential criticism of our model is that there is that given the common knowledge about abilities in our core model, there is no reason why funds should flow to the highest performers. That is true if there are no differential abilities. If however, we generalize our model slightly so that funds have differential ability then on average, funds with higher performance will have more able managers. Then, in equilibrium, ability will be positively related to returns. Most of our qualitative results about tournament behavior will be unaffected, with the exception that a fund with an extraordinarily able manager might start locking in at the beginning of the tournament.

Finally, it is worth noting that these kinds of tournament effects can have implications for optimal portfolio construction by investment advisers such as those affiliated with pension or endowment funds. One method of counteracting excessive risk taking is to hold diversified portfolios *ex ante*. Since tournament incentives encourage excessive risk-taking at the beginning of the period, it would appear that funds of funds should optimally hold a larger number of funds in their portfolio if those funds are all subject to high powered tournament incentives. Therefore our model can potentially explain why it sometimes appears that mutual fund investors hold more funds in their overall portfolio than would be optimal if simple linear reward structures applied.

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Table 1: Distribution of Mutual Funds

This table reports the distributions of mutual funds in our sample. Mutual funds are divided into categories accordings their Lipper classifications. The sample is from 1999 to 2007 with 11,229 fund-year observations.

Lipper Classification	Average number of funds per Year	Percentage of Total
Large-Cap Value Funds	78.2	6.3%
Large-Cap Core Funds	179.0	14.3%
Large-Cap Growth Funds	141.3	11.3%
Mid-Cap Value Funds	51.9	4.2%
Mid-Cap Core Funds	70.6	5.7%
Mid-Cap Growth Funds	97.0	7.8%
Small-Cap Value Funds	60.7	4.9%
Small-Cap Core Funds	101.6	8.1%
Small-Cap Growth Funds	97.0	7.8%
Multi-Cap Value Funds	110.8	8.9%
Multi-Cap Core Funds	160.7	12.9%
Multi-Cap Growth Funds	99.0	7.9%
Total	1247.7	100%

Table 2: Mutual Fund Risk-Taking

This table reports summary statistics of mutual fund risk-taking behavior. For each mutual fund, we compute the annualized volatility of daily mutual fund returns for each year in our sample. Mutual fund returns are calculated in excess of the risk-free rate, residual of CAPM, residual of 3-Factor Model and residual of 4-Factor Model. We report summary statistics in Panel A. We also rank mutual funds relative to their peers in the same Lipper classification category according to their cumulative percentage returns over January to June of a year. In Panel B, we report averages of mutual fund return volatility within each cumulative return quintile. The sample is from 1999 to 2007 with 11,148 fund-year observations.

Panel A: Summary statistics

Risk Measures	Average	Std Dev	5%-tile	Median	95%-tile
Excess Ret	18.1%	7.8%	9.8%	16.5%	31.6%
CAPM	6.9%	5.1%	2.3%	5.7%	15.8%
3-Factor	5.7%	4.3%	2.0%	4.6%	13.0%
4-Factor	5.5%	4.1%	1.9%	4.4%	12.6%

Panel B: Means by first-half return quintiles

Risk Measures	Quintile 1 (Trailers)	Quintile 2	Quintile 3	Quintile 4	Quintile 5 (Leaders)
Excess Ret	19.1%	17.8%	17.7%	17.7%	18.2%
CAPM	8.2%	6.4%	6.2%	6.3%	7.5%
3-Factor	6.9%	5.2%	5.0%	5.1%	6.3%
4-Factor	6.6%	5.0%	4.9%	5.0%	6.1%

Table 3: Second Half Tournament Effect

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over July to December of each year, minus the annualized percentage volatility of daily mutual fund returns over January to June of the same year. Mutual fund returns are calculated in excess of the risk-free rate, residual of CAPM, residual of 3-Factor Model and residual of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20%, middle 60% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative percentage returns over January to June of a year. In the second panel, we interact the indicator functions with the absolute value of the difference between a fund's cumulative percentage returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2007 with 11,148 fund-year observations. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, * at the 5% level and + at the 10% level.

Panel A: Without control for leader or trailing distance

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	0.118	0.020	0.041	0.114
	(0.106)	(0.088)	(0.081)	(0.080)
Ind(middle 60%)	0.243**	0.088	0.052	0.121+
	(0.088)	(0.072)	(0.067)	(0.068)
Intercept	0.257**	-0.267**	-0.054	-0.172**
	(0.082)	(0.069)	(0.065)	(0.065)
R-squared	0.814	0.416	0.224	0.197

Panel B: Base case

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	0.294	0.455	0.444	0.388
	(0.418)	(0.375)	(0.364)	(0.376)
Ind(middle 60%)	0.170	0.288	0.241	0.204
	(0.419)	(0.378)	(0.369)	(0.381)
Ind(top20%)*	-0.083**	-0.072**	-0.063**	-0.057**
distance above cut-off	(0.027)	(0.016)	(0.014)	(0.014)
Ind(bottom20%)*	-0.025	0.069	0.065	0.029
distance below cut-off	(0.162)	(0.146)	(0.143)	(0.148)
Intercept	0.331	-0.466	-0.243	-0.255
	(0.418)	(0.378)	(0.368)	(0.381)
R-squared	0.816	0.426	0.234	0.204

Table 4: Second Half Tournament Effect: Robustness Check Using Simulated Data

This table reports regression estimates of changes in mutual fund volatility on rankings of mutual fund performance on data for 1000 simulated funds wtih no change in volatility over the year. For each fund, 240 independent daily returns are drawn such that the annual fund volatility is 18%. Mean daily fund returns are zero, and returns are independent across funds. Each trading month equals 20 trading days. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over the last 120 returns minus the annualized percentage volatility of daily mutual fund returns over the first 120 returns of the simulated year. The independent variables are indicator functions that equal one if a mutual fund is within the top 20%, middle 60% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative percentage returns over January to June of a year. In the second panel, we interact the indicator functions with the absolute value of the difference between a fund's cumulative percentage returns and the 20th (or 80th) percentile breakpoint. Significance is indicated by ** at the 1% level, * at the 5% level and + at the 10% level.

Panel A: Without control for leader or trailing distance

Risk Measures:	Returns
Ind(top 20%)	-0.169
	(0.165)
Ind(middle 60%)	-0.0471
	(0.135)
Intercept	0.0270
	(0.116)
R-squared	0.0011

Panel B: Base case

Risk Measures:	Returns
Ind(top 20%)	0.217
	(0.188)
Ind(middle 60%)	0.159
	(0.200)
Ind(top20%)*	-0.0212
distance above cut-off	(0.1011)
Ind(bottom20%)*	0.0351
distance below cut-off	(0.0252)
Intercept	-0.179
	(0.188)
R-squared	0.0049

Table 5: Alternative Volatility Measure

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_RATIO, which is the annualized volatility of daily mutual fund returns over July to December of each year, divided by the annualized volatility of daily mutual fund returns over January to June of the same year. Mutual fund returns are calculated in excess of the risk-free rate, residual of CAPM, residual of 3-Factor Model and residual of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20%, middle 60% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative percentage returns over January to June of a year. In the second panel, we interact the indicator functions with the absolute value of the difference between a fund's cumulative returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2007 with 11,148 fund-year observations. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, * at the 5% level and + at the 10% level.

Panel A: Without control for leader or trailing distance

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	0.021**	-0.003	0.004	0.010+
	(0.004)	(0.006)	(0.006)	(0.006)
Ind(middle 60%)	0.014**	0.005	0.007	0.011*
	(0.003)	(0.005)	(0.005)	(0.005)
Intercept	1.061**	1.038**	1.043**	1.025**
	(0.003)	(0.004)	(0.004)	(0.004)
R-squared	0.899	0.574	0.496	0.450

Panel B: Base case

Risk Measures:	Excess Ret.	CAPM	3-Factor	4-Factor
Ind(top 20%)	0.011	-0.000	0.004	0.005
	(0.009)	(0.009)	(0.009)	(0.009)
Ind(middle 60%)	0.003	0.003	0.001	0.002
	(0.008)	(0.008)	(0.008)	(0.008)
Ind(top20%)*	-0.036	-0.163*	-0.187*	-0.140+
distance above cut-off	(0.087)	(0.082)	(0.076)	(0.077)
Ind(bottom20%)*	-0.363	-0.088	-0.189	-0.310
distance below cut-off	(0.284)	(0.250)	(0.246)	(0.262)
Intercept	1.072**	1.041**	1.049**	1.034**
	(0.008)	(0.008)	(0.007)	(0.008)
R-squared	0.900	0.575	0.498	0.452

Table 6: Tournament Effect across Time

This table reports pooled-panel regression estimates of changes in mutual fund volatility on rankings of mutual fund performance. The dependent variable is VOL_DIFF, which is the annualized percentage volatility of daily mutual fund returns over starting month to December of each year, minus the annualized percentage volatility of daily mutual fund returns over January to June of the same year. We vary the starting month across specifications. Mutual fund returns are calculated in excess of the risk-free rate, residual residual of CAPM, residual of 3-Factor Model and residual of 4-Factor Model. The independent variables are indicator functions that equal one if a mutual fund is within the top 20%, middle 60% or bottom 20% rankings among its peers in the same Lipper classification category according to its cumulative percentage returns over January to the end of the month before. In the second panel, we interact the indicator functions with the absolute value of the difference between a fund's cumulative returns and the 20th (or 80th) percentile breakpoint. The sample is from 1999 to 2007. All regressions include fixed-effects for each fund category by year. Standard errors are clustered by fund and are shown in parenthesis. Significance is indicated by ** at the 1% level, * at the 5% level and + at the 10% level.

Panel A: Excess Return

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	0.294	0.226	0.646	-0.069	-0.005	-0.008
	(0.418)	(0.448)	(0.578)	(0.270)	(0.278)	(0.312)
Ind(middle 60%)	0.170	0.073	0.308	-0.417+	-0.363	-0.123
	(0.419)	(0.448)	(0.581)	(0.248)	(0.260)	(0.303)
Ind(top20%)*	-0.083**	-0.077**	-0.049*	-0.050*	-0.046*	-0.064**
distance above cut-off	(0.027)	(0.023)	(0.024)	(0.022)	(0.018)	(0.016)
Ind(bottom20%)*	-0.025	-0.053	-0.003	-0.179**	-0.148*	-0.061
distance below cut-off	(0.162)	(0.158)	(0.178)	(0.066)	(0.062)	(0.068)
Intercept	0.331	0.134	-0.040	0.403	-0.820**	-1.945**
	(0.418)	(0.449)	(0.580)	(0.246)	(0.260)	(0.301)
R-squared	0.816	0.775	0.710	0.752	0.735	0.625

Panel B: Residual of CAPM

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	0.455	0.434	0.584	-0.125	-0.039	-0.040
	(0.375)	(0.412)	(0.532)	(0.245)	(0.250)	(0.270)
Ind(middle 60%)	0.288	0.279	0.374	-0.423+	-0.311	-0.118
	(0.378)	(0.414)	(0.539)	(0.233)	(0.238)	(0.262)
Ind(top20%)*	-0.072**	-0.059**	-0.045**	-0.042**	-0.045**	-0.046**
distance above cut-off	(0.016)	(0.015)	(0.015)	(0.015)	(0.014)	(0.012)
Ind(bottom20%)*	0.069	0.042	0.092	-0.164*	-0.158**	-0.115+
distance below cut-off	(0.146)	(0.145)	(0.163)	(0.064)	(0.057)	(0.059)
Intercept	-0.466	-0.511	-0.554	0.226	-0.161	-0.473+
	(0.378)	(0.414)	(0.537)	(0.233)	(0.238)	(0.260)
R-squared	0.426	0.372	0.266	0.348	0.343	0.267

Panel C: Residual of 3-Factor model

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	0.444	0.379	0.508	-0.130	-0.073	-0.132
	(0.364)	(0.403)	(0.526)	(0.246)	(0.246)	(0.268)
Ind(middle 60%)	0.241	0.211	0.313	-0.406+	-0.340	-0.190
	(0.369)	(0.407)	(0.533)	(0.239)	(0.239)	(0.263)
Ind(top20%)*	-0.063**	-0.052**	-0.035**	-0.032*	-0.040**	-0.044**
distance above cut-off	(0.014)	(0.013)	(0.013)	(0.013)	(0.011)	(0.009)
Ind(bottom20%)*	0.065	0.033	0.077	-0.137*	-0.130*	-0.081
distance below cut-off	(0.143)	(0.143)	(0.162)	(0.066)	(0.057)	(0.060)
Intercept	-0.243	-0.258	-0.315	0.437+	0.157	-0.137
	(0.368)	(0.407)	(0.531)	(0.239)	(0.239)	(0.261)
R-squared	0.234	0.201	0.142	0.235	0.237	0.211

Panel D: Residual of 4-Factor model

Risk Measures:	July	August	September	October	November	December
Ind(top 20%)	0.388	0.324	0.456	-0.195	-0.130	-0.131
	(0.376)	(0.410)	(0.534)	(0.229)	(0.232)	(0.261)
Ind(middle 60%)	0.204	0.183	0.285	-0.429+	-0.344	-0.176
	(0.381)	(0.414)	(0.542)	(0.223)	(0.225)	(0.258)
Ind(top20%)*	-0.057**	-0.046**	-0.031*	-0.030*	-0.036**	-0.041**
distance above cut-off	(0.014)	(0.013)	(0.013)	(0.013)	(0.010)	(0.009)
Ind(bottom20%)*	0.029	0.009	0.052	-0.161**	-0.151**	-0.087
distance below cut-off	(0.148)	(0.146)	(0.165)	(0.061)	(0.053)	(0.059)
Intercept	-0.255	-0.271	-0.319	0.430+	0.132	-0.126
	(0.381)	(0.415)	(0.541)	(0.223)	(0.225)	(0.257)
R-squared	0.204	0.177	0.124	0.220	0.220	0.192