Is minimum-variance investing really worth the while? An analysis with robust performance inference

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Abstract

This paper examines the risk-adjusted performance of the minimum-variance equity investment strategy in the U.S. While earlier studies only relied on empirical Sharpe ratio comparisons between the (constrained) minimum-variance strategy and different benchmark portfolios, we employ bootstrap methods for statistical inference concerning the Sharpe ratio, the Sortino ratio, certainty equivalents and alpha measures based on several factor models. We confirm and provide robust inference concerning earlier findings that constrained minimum-variance portfolios do outperform a value weighted benchmark. Moreover, our findings are in line with prior research, stating that minimum-variance portfolios do not outperform a naively diversified benchmark in terms of the Sharpe ratio. Both our results are invariant to the portfolio revision frequency and may be observed in all subperiods. Nevertheless, we show the high sensitivity of the constrained minimum-variance portfolios to the revision frequency and the imposed maximum portfolio weight constraints.

1 Introduction

The foundations of modern portfolio theory go back to the seminal work of Markowitz (1952; 1959). In his framework, portfolio selection is postulated as a trade-off between risk and return, where efficient portfolios deliver a maximum return for a given level of risk or, vice versa, deliver a minimum of risk for a given level of return. Ever since then a vast amount of literature has been devoted to research about modern portfolio selection. A considerable amount of this research has focused on the evaluation of the out-of-sample performance of two portfolios on the efficient frontier, which comprises all efficient portfolios, namely the tangency and the minimum-variance portfolio.

The tangency portfolio, as the portfolio with the largest excess return per unit of risk, attracted researchers' attention due to its theoretical foundation as the optimal portfolio for an investor. Following Tobin's mutual fund separation (1958), every investor should hold (dependent on her risk aversion) a certain fraction of wealth in the tangency portfolio, but should not alter the weight of any asset in the tangency portfolio.

Stein (1956), Frost and Savarino (1986), Jorion (1986) and Black and Litterman (1992), amongst others, point out that the estimation of expected returns (from sample data), which is necessary for the calculation of the tangency portfolio, is error prone and may yield misleading results. Additionally, Goyal and Welch (2003a; 2007) and Butler et al. (2001) show that even more advanced techniques for the return prediction, based on predictive regressions, have a similarly poor out-of-sample performance. In line with this, Bloomfield et al. (1977), Jobson and Korkie (1981a) and Jagannathan and Ma (2003) find that the tangency portfolio does not outperform an equally weighted portfolio.

These sobering results concerning the performance of the tangency portfolio drew attention to the minimum-variance portfolio, the only portfolio on the efficient frontier that requires the variance-covariance matrix as input parameter for the optimization. Merton (1980), Jorion (1985), and Nelson (1992) point out that variance-covariance estimates are relatively stable over time and can, hence, be predicted more reliably than returns. Underpinning these results of lower estimation errors for the minimum-variance portfolio, Baker and Haugen (1991) and Clarke et al. (2006) find an out-of-sample outperformance of the minimum-variance portfolio relatively to a value weighted portfolio. Additionally, Chan et al. (1999), Jagannathan and Ma (2003) and DeMiguel et al. (2007) point out that the short-sale constrained minimum-variance portfolio outperforms the tangency portfolio approach.

Evidence regarding a possible outperformance of the minimum-variance portfolio relative to an equally weighted portfolio is less clear. Based on purely descriptive results, Chan et al. (1999) and Jagannathan and Ma (2003) report higher out-of-sample Sharpe ratios for the constrained minimum-variance portfolio (CMVP). Given their lack of statistical inference, a robust conclusion whether the (constrained) minimum-variance strategy offers superior risk-adjusted returns cannot be made. Employing a parametric test, DeMiguel et al. (2007) find statistically indistinguishable differences in Sharpe ratios between the minimum-variance portfolio and a value weighted and equally weighted portfolio. Given this mixed evidence, the question whether minimum-variance investing is worthwhile appears to be rather unacknowledged so far.

The aim of this paper is to assess the risk-adjusted performance of the minimum-variance strategy for equity investors using robust performance inference. Though the research question, whether minimum-variance investing can deliver a risk adjusted outperformance relatively to a value or equally weighted benchmark strategy, has already bween investigated, we offer several new results and perspectives.

First, all aforementioned papers failed to provide robust evidence whether the minimum-variance strategy is worthwhile to conduct. This paper is, to the best of our knowledge, the first to use nonparametric performance tests between the minimum-variance investment strategy employed and a naively diversified as well as a value weighted benchmark portfolio. The employed nonparametric bootstrap approach should be better suited for this kind of performance comparison since neither the strong assumption of normally distributed nor independent and identically distributed (i.i.d.) returns are required.[1] Furthermore, we provide beside the Sharpe ratio performance comparisons for the Sortino ratio, alpha measures based on diverse factor models and certainty equivalents in order to provide robust evidence of possible performance differences.

Second, we are the first to base the derived results on the whole U.S. equity universe as captured by the Center for Research in Security Prices (CRSP) stock database[2], which allows a comprehensive assessment of the considered research question. Contrary, all

[1]Evidence for non-normality of stock returns goes back to Fama (1965). Additionally, DeMiguel et al. (2007) note that "this assumption is typically violated in the data". Ledoit and Wolf (2008) shows that the commonly used parametric Sharpe ratio test by Jobson and Korkie (1981a) requires, likewise the corrected version of the test by Memmel (2003), identical and independently distributed returns.

[2]For a closer description of the dataset see section 3.

aforementioned papers base their findings on either randomly drawn samples from the CRSP database (e.g. Chan 1999 and Jagannathan and Ma 2003), stock index constituents (e.g. Baker and Haugen 1991 and Clarke et al. 2006) or sector portfolios (DeMiguel et al. 2007).

During the course of our paper, we reproduce with our dataset a part of the work by DeMiguel et al. (2007) - in particular by using a maximum portfolio weight[3] of 2% in the portfolio optimization process. Despite striking differences in the inference methodology[4] and different datasets, we find evidence in line with DeMiguel et al. (2007) that the 2% CMVP offers no outperformance compared to naively diversified portfolios. Thus, in this setup, we do not corroborate (descriptive) findings by Chan et al. (1999) and Jagannathan and Ma (2003). Nevertheless, we do find evidence, by relaxing the maximum portfolio weight constraint, that CMVPs deliver a favorable performance in terms of alpha based on every considered factor model in comparison to the equally weighted benchmark portfolio. Performance comparisons relative to the value weighted benchmark portfolio broadly show that the CMVPs outperform the value weighted benchmark portfolio.

The remainder of the paper is organized as follows: section two reviews the methodology and gives a thorough outline of the employed nonparametric performance tests. Section three describes the data used. Section four reports and discusses the empirical results whereas section five presents robustness checks. Finally, the last section concludes the findings of this paper.

2 Methodology

This section is divided into three parts. First, we describe the portfolio optimization methodology used for the derivation of the CMVPs. This is followed by a short review of the various performance measures, while the third part of this section provides a thorough outline of the employed nonparametric bootstraps for the statistical inference concerning the comparison of the considered portfolio performance metrics.

[3]In the following we use the terms "maximum portfolio weight", "upper bound" and "upper constraint" synonymously.

[4]DeMiguel et al. (2007) use the Jobson and Korkie (1981a) parametric Sharpe ratio test in its corrected version by Memmel (2003), while we employ a nonparametric bootstrap for inference concerning the equality of Sharpe ratios of the minimum-variance portfolio and the respective benchmarks.

2.1 Portfolio optimization

Despite the fact that estimates of the variance-covariance matrix are less error prone than those of expected returns, the estimation error problem is far from alleviation. Actually, Chan et al. (1999) point out that the estimation error in variance-covariance estimates from sample data is still substantial. For example, they find a correlation of only 34% (18%) between the in-sample variance-covariance matrix and the out-of-sample covariance matrix, based on a 36 (12) months realization period.

There are various ways to deal with estimation errors in order to improve the out-of-sample reliability of parameter estimates. In general, three major approaches can be distinguished: factor model approaches, Bayes-Stein shrinkage estimators and the implementation of portfolio weight constraints.[5]

Starting with the postulation of Sharpe's (1963; 1964) one factor model, the use of factor models, not only to explain and predict returns, but also to estimate the variance-covariance matrix by imposing the factor model structure on the estimator of the variance-covariance matrix, became popular. The basic assumption underlying the one factor model in the context of covariance prediction is that the mutual return correlations between all assets is attributable to a common factor - the market index. Chan et al. (1999) provide a comprehensive examination of various factor models and their capability of out-of-sample variance-covariance prediction. They show, however, that the forecasted variance-covariance matrix based on historical returns performs for the purpose of portfolio optimization about as well as factor model estimates.

The most prominent approach in dealing with the estimation error problem is probably the Bayes-Stein shrinkage estimator. Initially proposed by Stein (1956) and James and Stein (1961), Bayes-Stein shrinkage estimators have widely been applied to the estimation of both input parameters of portfolio optimization, expected returns[6] and the variance-covariance matrix[7]. However, Jobson and Korkie (1980) show that shrinkage estimators do not perform well in small samples. Additionally, Jagannathan and Ma (2003) reveal that the Bayes-Stein improvements in the variance-covariance matrix are minor relative

[5]Other solutions include equilibrium constraints (Black and Litterman 1992), optimal combinations of portfolios (Garlappi et al. 2007) and the formulation of robust portfolio optimization problems, which directly incorporate a measure of the parameter uncertainty into the optimization procedure. Fabozzi et al. (2007) provide a sound survey of the latter approach.

[6]See for instance Jobson et al. (1979), Jobson and Korkie (1981a) and Jorion (1985; 1986)

[7]We refer to Ledoit and Wolf (2003) for an application of shrinkage techniques to the variance-covariance matrix.

to estimates based on historical data. While DeMiguel et al. (2007) confirm this finding for both, the variance-covariance matrix and expected returns.

The implementation of portfolio weight constraints in order to avoid highly concentrated portfolios and extreme portfolio positions, that are frequently derived in the mean-variance optimization procedure[8], has, to the best of our knowledge, first been applied by Frost and Savarino (1988). Nevertheless, Jagannathan and Ma (2003) were the first who studied the shrinkage like effect of upper and lower (short-sale constraint) bounds on portfolio weights, preventing large exposure to single assets. Despite their promising findings only little attention has been payed to the proposed adjustments since then. However, DeMiguel et al. (2007) recently showed that the minimum-variance portfolio with the proposed portfolio weight restrictions delivers the most favorable out-of-sample performance in terms of the Sharpe ratio.

In line with DeMiguel et al. (2007) and Chan et al. (1999) we adopt the constrained myopic minimum-variance portfolio setup proposed by Jagannathan and Ma (2003) that builds on the Markowitz (1952) portfolio selection framework. Since the crucial assumption of normally distributed returns is typically violated in the data.[9] we meet the assumption that the investor's utility function is quadratic in order to keep the portfolio optimization approach valid.[10] Thus, we employ the CMVP optimization setup on a "rolling-sample" basis. Since the choice of sample size for the parameter estimation represents a delicate trade-off between an increase of statistical confidence and the incorporation of possibly irrelevant data[11], we stick to the sample size choice of Chan (1999) and Jagannathan and Ma (2003). Accordingly, the portfolio is rebalanced every three months in the base case, taking the return observations of the last 60 months as input parameter for the variance-covariance matrix estimation.[12]

Hence, we solve the standard myopic portfolio optimization problem, imposing the recommended constraints by Jagannathan and Ma (2003) in order to reduce the estimation error and to achieve a shrinkage-like effect.[13] Accordingly, we set on the portfolio weights

[8]See Green and Hollifield (1992), Chopra (1993) and Chopra and Ziemba (1993) for evidence concerning this point.

[9]For evidence concerning this point see Fama (1965) and DeMiguel et al. (2007)

[10]All portfolio optimization approaches that base on Markowitz (1952) require either normally distributed returns or quadratic utility functions of investors.

[11]See for this Jobson (1981b) and Levine (1972).

[12]Additionally, we vary the portfolio revision frequency for robustness check purposes from three months to six and twelve months. Results for this are reported in section 5.

[13]Even though the setup is multi-period in nature, Mossin (1969), Fama (1970), Hakansson (1970; 1974) and Merton (1990) show that "under several sets of reasonable assumptions, the multi-period problem

a no-short sales constraint, $w_{i,t} \geq 0$ and an upper bound w^{max}, which is varied in one percent steps in the range $\{w^{max} = 2\%, 3\%, ..., 20\%\}$, on the weight a single stock may have in the portfolio. After the optimization process the portfolio weights remain three months[14] unchanged up to the next optimization. Formally, this results in an optimization problem for each period t of the following form[15]:

$$\overset{min}{\mathbf{w_t}} \ \mathbf{w_t}'\mathbf{S}\mathbf{w_t} \text{ s.t.} \tag{1}$$

$$\sum_{i=1}^{N} \mathbf{w_{i,t}} = 1 \tag{2}$$

$$\mathbf{w_{i,t}} \geq 0, \text{ for } i = 1, 2, ..., N \tag{3}$$

$$\mathbf{w_{i,t}} \leq w^{max}, \text{ for } i = 1, 2, ..., N \tag{4}$$

The Kuhn-Tucker conditions (necessary and sufficient) are accordingly:

$$\sum_{j} \mathbf{S_{i,j}}\mathbf{w_j} - \lambda_\mathbf{i} + \delta_\mathbf{i} = \lambda_0 \geq 0, \text{ for } i = 1, 2, ..., N \tag{5}$$

$$\lambda_\mathbf{i} \geq 0 \text{ and } \lambda_\mathbf{i} = 0 \text{ if } \mathbf{w_i} > 0, \text{ for } i = 1, 2, ..., N \tag{6}$$

$$\delta_\mathbf{i} \geq 0 \text{ and } \delta_\mathbf{i} = 0 \text{ if } \mathbf{w_i} < w^{max}, \text{ for } i = 1, 2, ..., N \tag{7}$$

The notation hereby is as follows: $\mathbf{w}$ denotes the vector of portfolio weights, $\mathbf{S}$ the empirically estimated variance-covariance matrix, λ the vector of Lagrange multipliers in the non negativity constraint of portfolio weights, δ the multipliers of upper bound portfolio weight constraints and λ_0 the multiplier for the portfolio weights to sum up to one.

2.2 Performance metrics

In a second step we evaluate the out-of-sample performance of the CMVP and compare it to both, a value weighted benchmark portfolio, which serves as a market proxy, and an equally weighted portfolio, which is considered as a simple benchmark asset allocation

can be solved as a sequence of single-period problems" (Elton and Gruber 1997).

[14]The portfolio revision frequency is varied for robustness check pruposes to six and twelve months. Results for these revision frequencies are reported in section 5.2.

[15]The notation and variable explanation follows closely Jagannathan (2003). Bold factors indicate vectors and matrices, whereas e.g. $\mathbf{w_i}$ denotes the i-th element of vector $\mathbf{w}$.

strategy following for instance Bloomfield et al. (1977) and DeMiguel et al. (2007).[16] In order to provide a broad picture of risk adjusted portfolio performance, we report various performance measures which incorporate different conceptual measures of risk. Accordingly, we provide evidence based on the certainty equivalent (CEQ)[17], the Sharpe (SH) and Sortino (SR) ratio as well as on alpha measures based on a one factor model (as initially proposed by Jensen 1969), the Fama and French (1992) three factor and Carhart (1997) four factor model, which are given by

$$SH = \frac{r_p - r_f}{\sigma_p} \tag{8}$$

$$SR = \frac{r_p - r_f}{\sigma_p^d} \tag{9}$$

$$CEQ = r_p - \frac{\gamma}{2}\sigma_p^2 \tag{10}$$

$$\alpha_p = r_{p,t} - \sum_{k=1}^{K} \beta_{p,k} r_{k,t} - \epsilon_{p,t}, \text{ with } \epsilon \sim F_p(0, \sigma^2) \tag{11}$$

Thereby denotes r_p the return, σ_p the standard deviation and σ_p^d the downside deviation[18] of portfolio p, while α_p stands for the portfolio specific constant, factor independent, return, $\sum_{k=1}^{K} \beta_{p,k}$ the sensitivities of the portfolio return relative to the return of the K explanatory factors $r_{k,t}$ and $\epsilon_{p,t}$ the portfolio specific white noise error term.

2.3 Statistical performance metric inference

In order to check upon the statistical significance between the derived performance measures of the CMVPs and the considered benchmarks, we employ nonparametric bootstraps for a robust performance measure inference. This is noteworthy since none of the earlier mentioned papers employed a robust inference methodology for the comparison between the (constrained) minimum-variance strategy and alternative investment strategies. An exception is the work by DeMiguel et al. (2007), who employed a parametric Sharpe

[16]For convenience and following the common acceptance of the CRSP value and CRSP equally weighted market indices as market proxies (see Lehmann and Modest 1987) we opted to choose both as benchmark portfolios.

[17]We assume that investors have quadratic utility functions which commonly accepted (see e.g. DeMiguel et al. 2007). We choose to set the risk aversion parameter to $\gamma = 4$ in order to assess the profitability of the CMVPs from the point of view of a highly risk averse investor

[18]For the purpose of our paper, we set the shortfall threshold to zero, in order to have a common, non portfolio specific shortfall threshold for all portfolios in the analysis.

ratio test by Jobson and Korkie (1981b)[19]. Nevertheless, it is important to point out that the parametric Sharpe ratio test by Jobson and Korkie (1981b) (as well as in its corrected version by Memmel 2003) requires the strong assumption of normally and i.i.d. distributed portfolio returns.[20] The effect of non-normality of returns on this test and the resulting inappropriateness is discussed by Ledoit and Wolf (2008).

We have therefore opted to use a nonparametric bootstrap approach, which was initially proposed by Efron (1979). The superiority of the bootstrap approach vis-à-vis parametric tests is twofold. First, it does not require any assumption about the distribution of the considered performance measures nor their differences. Instead, our approach allows ous to draw inference from our sample distribution. Second, Navidi (1989) showed that the bootstrap is under all circumstances at least as good as the normal approximation which is required for the widely used test by Jobson and Korkie (1981b). The use of bootstrap methodology for robust performance metric inference has been highlighted by Morey and Vinod (1999) and Kosowski et al. (2004; 2007) with regard to the Sharpe ratio, using a simple observation bootstrap, and the factor model based alpha measures, using a residual bootstrap respectively.

2.3.1 Observation bootstrap

Our approach towards testing for the equality of performance measures incorporating a total measure of risk, namely the CEQ, the Sharpe and Sortino ratios follows the work by Morey and Vinod (1999) and can be summarized as follows: From the empirical sample comprising N monthly returns we draw a random return $r_{p,t}$ from the time series of portfolio p with replacement. It is important to note that this is done pair wise in a timely fashion, meaning that the return of the same randomly selected month is chosen from the respective time series of the benchmark portfolio returns $r_{bm,t}$. This is repeated n times, leaving a bootstrap sample of n returns for each CMVP p $\{r_{p,t}$ with $t = 1, 2, ..., n\}$ and benchmark portfolio bm $\{r_{bm,t}$ with $t = 1, 2, ..., n\}$. Repeating the prior step B times yields B bootstrap samples for every CMVP p $\{S_p^b$ with $b = 1, 2, ..., B\}$ and benchmark portfolio bm $\{S_{bm}^b$ with $b = 1, 2, ..., B\}$, each containing n returns. For each of the B bootstrap samples we compute the Sharpe and Sortino ratio differences between each

[19] Memmel (2003) proposed a minor correction for this test statistic.

[20] Evidence concerning the non-normal distribution of the CMVP returns (at least for our results) is provided in the tables 3 and 8.

particular CMVP and the (naively or the value weighted) benchmark portfolios. Sorting each of the resulting B CEQ, Sharpe and Sortino ratios differences in a vector enables us to construct confidence intervals and to conduct hypothesis testing.

Point of interest is the confidence level at which the CMVP offers a statistically significant outperformance relative to the considered benchmark portfolios. Accordingly, we compute the p-values for the one sided hypotheses that the difference between a particular portfolio performance measure of the CMVP and the benchmark portfolio (either equally or value weighted) is smaller or equal to zero:

$$H_0 : \widehat{CEQ}_{min-var} - \widehat{CEQ}_{benchmark} \leq 0 \tag{12}$$

$$H_0 : \widehat{SH}_{min-var} - \widehat{SH}_{benchmark} \leq 0 \tag{13}$$

$$H_0 : \widehat{SR}_{min-var} - \widehat{SR}_{benchmark} \leq 0 \tag{14}$$

The p-values are thereby computed by finding the element entry number of the first observation in each particular vector of sorted bootstrap differences, which has a non-negative sign. This number is in turn divided by the number of bootstrap iterations, which then yields the desired p-value for the one sided hypothesis test that the respective performance metric of the CMVP exceeds that of the benchmark portfolio. Stated differently, the p-value delivers the fraction of bootstrap iterations, and accordingly the probability, in favor of a rejection of the null hypothesis.

2.3.2 Residual bootstrap

To draw robust performance inference using factor model approaches, we adopt the residual bootstrap methodology proposed by Kosowski et al. (2004; 2007). The basic idea is that the true return data generating process P of a portfolio p is fully described by a K-factor model as given provided in equation (11). Following this, we try to capture the underlying data generating process for each portfolio P_p from sample data. This estimate, denoted by $\hat{P}_p = (\hat{\alpha}_p \ \hat{\beta}_p, F_{p,e})$, with $F_{p,e}$ being the empirical cumulative density function of the residuals $\hat{\epsilon}_p$, serves in the following as the data generating process for the creation of the B bootstrap samples.

Thus, we estimate in a first step for each of the CMVPs p the respective data generating process, $\hat{P}_p$, from the sample data. Following this, we store the estimated parameter values of $\hat{\alpha}_p$, $\hat{\beta}_p$, the t-value of $\hat{\alpha}_p$, as well as the time series of the estimated residuals $\hat{\epsilon}_{p,t}$ with t=1,2,...,T.

In a second step we construct an artificial time series of portfolio returns, r_p^b, which has an intercept term of zero by construction. This is done for any particular CMVP p by multiplying the estimated sensitivities to the considered K explanatory factors $\sum_{k=1}^{K} \beta_{p,k}$ with the respective factor returns r_k with k=1,2,...,K. Additionally, a bootstrapped residual from the empirical density function of the residuals, $\hat{\epsilon}_p^b$, is added. This operation should not alter the intercept of the regression line, since the residuals, $\hat{\epsilon}_p$, are white noise with zero mean. Accordingly, the regression line should, by construction, still pass the origin, which would be reflected by an alpha measure of zero. Formally, the constructed artificial time series for each portfolio p for which a K-factor model has been considered to capture the return data generating process is given by:

$$r_{p,t}^b = \sum_{k=1}^{K} \hat{\beta}_{p,k} r_{k,t} + \hat{\epsilon}_{p,t}^b \text{ with t=1,2,...,T} \tag{15}$$

In the following final step we re-estimate for every portfolio p all regression parameters in equation (11) from the artificial time series:

$$r_{p,t}^b = \hat{\alpha}_p^b + \sum_{k=1}^{K} \hat{\beta}_{p,k}^b r_{k,t} \tag{16}$$

The resulting intercept coefficient $\hat{\alpha}_p^b$ is now of special interest, since the sampling variation, reflected in the bootstrapped residuals, should, if the empirical parameter, $\hat{\alpha}_p$ were significant, not yield a parameter value as high as the empirical value. Repeating the prior steps $b = 1, 2, 3, ..., B$ times yields the desired number of B bootstrap samples and corresponding B parameter estimates. Sorting these parameters and applying the same test procedure as for the observation bootstrap leaves ous with the following hypothesis:

$$H_0 : \hat{\alpha} \leq \hat{\alpha}^b \tag{17}$$

Since the t-value has more favorable statistical properties, due to the additional coverage of the estimation precision of $\hat{\alpha}$, Kosowski et al. (2004; 2007) propose to bootstrap the t-value.[21] Following this argument, the resulting one sided hypothesis to test for is given by:

$$H_0 : t_{\hat{\alpha}} \leq t_{\hat{\alpha}^b} \tag{18}$$

3 Data

Our dataset comprises the entire CRSP monthly stock database from April 1964 to December 2007. From this database, we only consider stocks in the optimization that have non-zero returns and no missing values in the variance-covariance estimation period of 60 months prior to the optimization's point in time. It is important to stress the necessity of this filter since thin trading, reflected in limited or even no trading of stocks, drives the covariance of these illiquid stocks with other stocks towards zero. In consequence of the resulting downward biased covariance, foremost illiquid stocks will be selected so that the performance will likely be driven by soaking up the stock market inherent liquidity premium[22]. In order to avoid these thin trading effects, the aforementioned filter is employed.[23] In case of missing values in the out-of-sample period we opted to set the stock return to zero.

Finally, we follow the argument by Chan et al. (1999), who claim that the variance-covariance matrix becomes too noisy if very small and micro capitalized firms are included in the sample. Hence, we include in our final sample only those stocks that are in the upper 80% size percentile (according to their market capitalization) and have a stock price of more than $5.

Time series data for interest rates and factor return data for the factor models described

[21] We control for autocorrelation and heteroscedasticity using the autocorrelation and heteroscedasticity consistent covariance matrix from Newey and West (1987).

[22] For evidence concerning the liquidity premium inherent in stock markets see Pastor and Stambaugh (2003).

[23] Despite the availability of adjustment procedures for the thin trading effect concerning the covariance estimation (see e.g. Scholes and Williams 1977 and Dimson 1979), none of these adjustments has proven to account properly for the thin trading effect. Evidence for this is provided by a comprehensive study by McInish and Wood (1986).

in section 2.2 are obtained from Kenneth French's website[24], while the National Bureau of Economic Research (NBER) recession period data, which we use for the determination of U.S. recession periods[25] is taken from the corresponding NBER website[26].

4 Empirical results

The first part of this section describes the empirical performance measures of the CMVPs with a portfolio revision frequency of three months over the complete sample period from April 1968 to December 2007, while the second part provides statistical inference for the empirical results and checks upon the significance of the empirical findings.

4.1 Descriptive performance metric analysis

Since the minimum-variance approach aims at the reduction of risk it is suggestive to assess the empirical standard deviation of the CMVPs first. Quite obvious is the almost nondecreasing pattern of standard deviations for the less CMVPs, which is accompanied by a decreasing average number of stocks in the respective minimum-variance portfolios. Accordingly, the observable increase in standard deviations can be attributed to an increase in idiosyncratic risk, due to the reduced deterministic diversification of less restrictively CMVPs. This effect is reflected in the declining values of the adjusted R^2 as depicted in tables 1 and 2, which is observable for every considered benchmark portfolio and factor model. Noteworthy is however the reduced standard deviation of the CMVPs. As depicted in table 3 the CMVPs deliver, up to a portfolio weight constraint of 8% (19%), standard deviations below those of the value (equally) weighted benchmark portfolio. Nevertheless, this risk reduction bares the "cost" of lower returns.[27]

A quantification of the observable trade-off between risk reduction and return is given by the described performance metrics in section 2.2. The broad picture in table 3 shows that all CMVPs clearly outperform the value weighted benchmark based on all performance metrics that incorporate a total measure of risk. This is especially interesting for

[24] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[25] We are in line with other studies (e.g. Campbell et al. 2001 and Goyal and Santa-Clara 2003b) by defining U.S. recession periods based on the NBER recession data.

[26] http://www.nber.org/cycles/recessions.html

[27] We would like to pronounce at this point that this trade-off is ex post in nature since the "cost" of lower returns has not been considered in the optimization objective, which only aims at the reduction of risk, irrespective of the associated return.

less CMVPs with a higher standard deviation than the value weighted benchmark portfolio. It suggests that the higher return of the CMVPs overcompensates the increased standard deviation. In line with this, all CMVPs performed worse in terms of the aforementioned performance metrics compared to the equally weighted benchmark portfolio, which achieved a higher return at a higher standard deviation than (almost) every CMVP.

A completely different impression arises if one considers the factor model based alpha measures. Irrespective of the employed factor model and benchmark portfolio, all factor models achieve a (in some cases substantially) positive alpha. Though this may be encouraging at first sight, it is important to highlight the associated regression statistics as well as the behavior of the alpha measures with respect to the maximum portfolio weight constraint.

Irrespective of the considered factor model, the alpha measure follows a hump shaped pattern with respect to the maximum portfolio weight constraint, which is attributable to the annualized mean return behaivor.[28] Though this comovement may seem surprising at first sight, the effect becomes clearer by taking the fairly constant absolute values of systematic risk, as measured by the beta factors, into account. The increasing share of non-systematic risk for less CMVPs, which is at least partially accompanied by increasing returns (up to a maximum portfolio weight of 11%) casts nevertheless doubts upon the postulation that systematic risk is the only source of risk which is rewarded at the market.[29]

Concluding the empirical results, the CMVPs seem to outperform the value weighted benchmark portfolio based on all considered performance metrics that incorporate a total measure of risk. The opposite of this finding holds for the equally weighted benchmark portfolio. If one assesses the performance based on the alpha measure, all CMVPs seem to outperform both benchmark portfolios. Within the group of CMVPs, the empirical trade-off between risk and return seems to be optimal for portfolios with a maximum portfolio weight in the range between 8% - 11%.

[28]This is mirrored in a cross correlation of 0.934 (0.935) between the four factor alpha and the equally (value) weighted benchmark portfolio. The cross correlations between the other factor model based alphas and the annualized mean returns are likewise well above 0.9.

[29]In the ongoing discussion whether idiosyncratic risk has explanatory power concerning the cross section of returns (and may thus be rewarded), Malkiel and Xu (1997; 2004) and Goyal and Santa-Clara (2003b) provide evidence in favor of the paper, while Bali et al. (2005) contradict those results.

4.2 Robust performance metric inference

Inference concerning the thus far derived empirical findings of the CMVP performance is drawn by the bootstrap approaches described in section 2.3. Given the empirical evidence of the non-normality of the CMVP returns, provided by the Jarque-Berra test p-values in table 3, we stress once more the necessity of bootstrap inference. Accordingly any inference for the Sharpe ratio based on the parametric Sharpe ratio test by Jobson and Korkie (1981a) in its corrected version by Memmel (2003) would be incorrect - at least for our data. We turn in the following to the CMVP performance inference relative to the value weighted benchmark portfolio, which is followed by the assessment relative to the equally weighted benchmark portfolio. Beside the bootstrap inference concerning the performance metrics described in section 2.2, we provide inference for the annualized mean return and standard deviation of the CMVPs. This is especially important for the assessment whether the empirically observable overcompensation of risk by return, reflected in the empirical performance metric pattern, is statistically significant.[30]

The bootstrapped p-values in table 4 show that a significantly higher return compared to the value weighted benchmark portfolio, at the 5% (10%) significance level, has only been delivered by the CMVPs with a maximum portfolio weight restriction of 9% - 11% (6% - 14% and 20%). This proves that the empirical observation of higher returns for every CMVP in comparison to the value weighted benchmark portfolio is statistically not signficant for most portfolios. Broadly in line with the empirical evidence is the reduction of the standard deviation. The standard deviations of the CMVP with a maximum portfolio weigh constraint of 6% and less are accordingly significantly reduced in comparison to the value weighted benchmark portfolio.

Most interestingly is the assessment whether the empirically observable domination of the return over the risk effect is of statistical significance. Starting with the alpha measures[31], the empirically observable dominance of the return effect seems to be underpinned by the bootstrap inference. All minimum-variance portfolios in the maximum portfolio weight

[30] The p-values for the differences in annualized mean returns and standard deviations are derived by the in section 2.3.1 described observation bootstrap for the one sided hypotheses that the difference between the annualized mean return (standard deviation) of the CMVP and the benchmark portfolio (either equally or value weighted) is equal or less than zero:
$H_0 = \mu_{min-var} - \mu_{benchmark} \leq 0$
$H_0 = \sigma_{min-var} - \sigma_{benchmark} \leq 0$

[31] We base in the following all alpha measure descriptions and interpretations on the four factor model based alpha measure, since the four factor model yields the highest adjusted R^2 for all CMVP.

constraint range between 7%-17% (9%-10%) as well as the 20% maximum portfolio weight constraint minimum-variance portfolio yield significant alphas on the 5% (10%) confidence level. Nevertheless, it is important to be aware of the particularities associated with the alpha measure in the context of the minimum-variance portfolio performance mentioned in section 4.1. Taking a closer look, it shows that almost only those portfolios with a statistically significant increase in the annualized mean return achieved a significant alpha. Following this, the significance of the alpha measure for portfolios that have a significant increase in annualized mean return does not come at a surprise, since the systematic risk measure does not mirror the steady increase in total risk.

Based on the empirical results, the empirical Sharpe ratio and CEQ point estimates of every CMVP have been higher for every CMVP. Nevertheless, the bootstrap inference shows that only those portfolios with a maximum portfolio weight constraint of less than 13% (8%) for the Sharpe ratio and 12% (8%) for the CEQ delivered significantly better performance metrics on the 10% (5%) confidence level. Accordingly, the bootstrap reveals that CMVPs with a significantly reduced standard deviation (in comparison to the value weighted benchmark portfolio) deliver significantly higher risk adjusted performance metrics than the benchmark portfolio. Following this, the empirically observable domination of the return over the risk effect may not be considered to be statistically significant. Additional evidence for this is provided by the Sortino ratio. All considered minimum-variance portfolios with a maximum portfolio weight of less than 12% (14%) deliver significantly higher Sortino ratios relative to the value weighted benchmark portfolio on a 5% (10%) confidence level.

The statistical inference of the CMVP performance in comparison to the equally weighted benchmark portfolio reveals almost no surprising insights. The empirically lower performance metrics of the CMVP in comparison to the equally weighted benchmark portfolio are underpinned by the bootstrap results in table 4. Accordingly, the one sided hypothesis that the CEQ, Sharpe and Sortino ratios of the equally weighted benchmark portfolio exceed those of the CMVPs may not be rejected.

A somewhat different impression arises again from the alpha measures, which clearly points at an outperformance of the CMVPs. The problem with the alpha measure based on the equally weighted benchmark portfolio is however the same as already described.

Concluding these first results, the broad picture shows that deterministic diversification, which is achieved by restrictive maximum portfolio weight constraints, does significantly reduce the realized out-of-sample portfolio risk, as measured by the standard deviation. Nevertheless, it is empirically observable that the imposed maximum portfolio weight constraints lead for the least and most restrictively CMVPs to a decline in the annualized mean return. This is in turn reflected in the empirically declining values of risk adjusted portfolio performance metrics that incorporate a total measure of risk.

The bootstrap inference proves however, that this empirical finding is not significant for the most restrictively CMVPs. Contrary, it turns out that those portfolios with the highest deterministic diversification have the statistically most significant outperformance based on total risk incorporating performance metrics in comparison to the value weighted benchmark portfolio. A somewhat contrarian picture is presented by the systematic risk adjusted alpha measure. Due to the fairly stable share of systematic risk across all CMVPs, those portfolios with a high annualized mean return achieve a statistically significant alpha, irrespective of the considered market proxy (benchmark portfolio). Accordingly one may constitute that certain CMVPs (roughly all minimum-variance portfolios with a portfolio weight constraint of 7% or less) significantly outperform the value weighted benchmark portfolio based on risk adjusted performance.

This observation may not be confirmed in comparison to the the equally weighted benchmark portfolio. Though the most restrictively CMVPs significantly reduce the standard deviation in comparison to the equally weighted benchmark portfolio, none of the CMVPs achieves higher empirical performance metrics. This empirical finding is clearly underpinned by the bootstrap results.

5 Robustness checks

In this section we provide evidence of the constrained minimum-variance investment strategy performance in different market phases. Additionally, we check upon the sensitivity of our results with respect to the portfolio revision frequency. The section is accordingly divided into two parts, whereby we assess the performance of the constrained minimum-variance strategy over three subperiods first. The subperiods comprise in particular U.S. recession periods as well as high volatility and low volatility periods. This is followed by an assessment of the variability of our results with respect to the portfolio revision frequency, which is in the second part of this section varied from three to six and twelve months respectively.

5.1 Subsamples

The assessment of subperiods shall foremost yield insights concerning the robustness of the derived results for the complete sample period and reveal whether the constrained minimum-variance performance over the complete sample period is driven by any specific subperiod. Attention is specifically paid to the best performing CMVP in each subperiod in order to draw conclusions concerning the constancy of the optimal[32] portfolio weight constraint. Our choice of subsample periods, namely U.S. recession periods as well as high and low idiosyncratic volatility periods, bases on the work by Kosowski (2004) and Campbell et al. (2001) respectively.

Empirically recession periods are characterized by increased volatility and low (negative) annualized mean returns for the equally (value weighted) benchmark portfolio. Moreover, Kosowski (2004) argues that investors care especially about portfolio performance in recession periods due to the high marginal utility of wealth of investors over these periods. Accordingly improvements in the (risk adjusted) performance relative to the equally and value weighted benchmark portfolios seem to be particularly valuable for investors.

The segregation into high and low idiosyncratic volatility periods follows Campbell et al. (2001) who find a deterministic trend in idiosyncratic firm level volatility we choose to assess the CMVP performance in both volatility market states, with the low idiosyncratic volatility subsample from April 1968 to December 1985 and the high idiosyncratic volatility subsample from January 1986 to December 1997.[33] Campbell et al. (2001) note that the "correlation among individual stock returns declined", while idiosyncratic risk increased over their sample period. Accordingly, possible sensitivities of the CMVP performance to the development of idiosyncratic risk and return correlations are assessed over these two subperiods.

Starting with the U.S. recession periods, the big picture shows clearly the profitability of the constrained minimum-variance approach, resulting in overall higher performance metrics in comparison to both benchmark portfolios. Empirical evidence concerning the

[32]Optimal has in this context to be understood in the sense of empirically best performing.

[33]Campbell et al. (2001) define the complete low idiosyncratic volatility period from July 1962 to December 1985. Since our sample starts in April 1968, statements concerning the low idiosyncratic volatility period are accordingly based on the period from April 1968 to December 1985.

negative CEQ reveals that more risk averse investors still shy, despite the empirically favorable risk return profile of the CMVPs, investments in any of the portfolios during recession periods. Despite the overall encouraging empirical results, only few CMVPs deliver a statistically significant better portfolio performance relative to the benchmark portfolios. In comparison to the value weighted benchmark portfolio only those CMVPs with a significantly higher return than the benchmark portfolio achieved significantly higher Sharpe and Sortino ratios.[34] Accordingly all CMVPs with a maximum portfolio weight constraint of 5% (9%) and less achieved a higher Sharpe and Sortino ratios than the value weighted benchmark portfolio on a 5% (10%) significance level. The favorable results for the more restrictively CMVP are underpined by the alpha measure. Significant alphas are only generated by the CMVPs with a maximum portfolio weight constraint of 4% and less on a 10% confidence level. Similar results for the alpha measure are obtained if one considers the equally weighted benchmark portfolio as market proxy. Contrary, inference concerning the Sharpe and Sortino ratios reveals that the empirically observable outperformance of the equally weighted benchmark portfolio by the CMVPs is in no case statistically significant.

To sum up the findings of this first subperiod assessment, the more restrictively CMVPs achieve significantly higher returns at a lower risk, which is finally reflected in significantly higher performance metrics in comparison to the value weighted benchmark. In comparison to the equally weighted benchmark portfolio, almost every CMVP achieved a substantial risk reduction and an empirically higher annualized mean return. Nevertheless, this does not result in statistically higher performance metrics that base on a total measure of risk. Opposed to that, the most restrictively CMVPs deliver a substantial and significant alpha. All in all, the more restrictively CMVPs seem to perform reasonably well during recession periods.

Turning to the high and low volatility periods[35], as defined by Campbell et al. (2001), the empirical characteristics may be confusing at first sight. Both benchmark portfolios exhibit higher standard deviations in the low than in the high volatility period. This effect is attributable to the development of the correlations among stocks. As pointed out by Campbell et al. (2001), the average correlation among stocks declined over time. The

[34] We do not focus on the CEQ since risk averse investors would, according to the empirically negative CEQ measure, not have valued any of the CMVPs.

[35] We stress once more that the segregation bases upon the firm level specific and not on the market level volatility.

diversification effect of both benchmark portfolios in the low volatility period, from April 1968 to December 1985, has correspondingly been low, resulting in the comparably high standard deviation of both benchmark portfolios. Turning to the CMVP performance, the value weighted benchmark portfolio delivered in the low volatility period higher performance metrics that incorporate a total measure of risk as well as positive alphas in comparison to the value weighted benchmark portfolio. This observation may only partially be confirmed for the high volatility period, namely for the most restrictively CMVPs. Compared with the equally weighted benchmark portfolio, none of the CMVPs deliver higher performance metrics based on a total measure of risk in the low volatility period. Nevertheless, all CMVPs achieve positive alphas, which holds as well for the high volatility period. Beside the positive alphas, the most restrictively CMVPs achieve additionally Sharpe ratios in excess of the equally weighted benchmark portfolio, while almost every CMVP exhibits higher Sortino ratios than the equally weighted benchmark portfolio.

The bootstrap results in tables 6 and 7 underpin the empirical findings for the high and low volatility periods. The significantly higher returns of the CMVP in comparison to the value weighted benchmark portfolio lead in turn to significantly higher CEQ, Sharpe and Sortino ratios. Even more striking is the high statistical significance of the alpha measures for every CMVP, irrespective of the considered market proxy. Despite the significantly reduced standard deviations of the CMVPs in comparison to the equally weighted benchmark portfolio during the low volatility period, the bootstrap results point at statistically indistinguishable CEQ, Sharpe and Sortino ratios. This shows that the empirically higher performance metrics of the equally weighted benchmark portfolio are not significantly higher than those of the CMVPs.

During the high volatility period, the comparison to both benchmarks shows the benefit of deterministic diversification. The empirical observation of higher performance metrics for the most restrictively CMVPs during this period is clearly underpinned by the bootstrap results. Significantly higher alpha measures on the 5% (10%) confidence level are achieved for the minimum-variance portfolios with a maximum portfolio weight constraint of 2% - 3% (4%), irrespective of the market proxy. Significantly higher Sharpe and Sortino ratios in comparison to both benchmarks may as well be reported for the most restrictively CMVPs. The CMVPs with a maximum portfolio weight constraint of 2% - 3% (2%) achieved significantly higher Sharpe and Sortino ratios in comparison to the equally (value) weighted benchmark portfolio on a significance niveau well below the

10% level.

Concluding the findings from the considered subsamples the results show that there is not *the* optimal maximum portfolio weight constraint for all subsamples. The results from the subperiods rather convey the impression that the degree of deterministic diversification depends on the level of idiosyncratic risk. Both, the U.S. recession periods as well as the high volatility period are characterized by increased idiosyncratic volatility. Though the average correlation among stocks in these periods is likely to be contrary[36], the results show clearly that statistically significant results in both periods may only be derived for more restrictively CMVPs. This is in line with the finding by Campbell et al. (2001), who note that an increased number of stocks is necessary to diversify the high level of idiosyncratic risk.

Though the most restrictively CMVPs were not always the best performing ones, one may nevertheless conclude that the most restrictively CMVPs achieved in every subsample a statistically significant risk reduction in comparison to the equally weighted benchmark portfolio. This shows clearly that the deterministic diversification through maximum portfolio weight constraints additionally reduces the portfolio risk as measured by the standard deviation. Additionally, the result point out that there is not *the* "optimal" maximum portfolio weight constraint. Moreover, the degree of deterministic diversification seems to depend on the respective market phase.

Most results for the considered subperiods are broadly in line with the overall picture. The outperformance of the value weighted benchmark portfolio by the more restrictively CMVPs is especially in U.S. recession and high volatility period observable, while almost every CMVP outperformed the value weighted benchmark portfolio during the low volatility period. The worse performance of the CMVPs in terms of total risk incorporating performance metrics compared to the equally weighted benchmark portfolio is confirmed for almost every subperiod. An exception build the most restrictively CMVPs in the high volatility period, which offer significantly higher performance metrics. Accordingly, we do not find evidence, that the overall findings over the complete sample period depend on a specific subperiod.

[36]The average correlation among individual stocks in recession periods is usually tight, while the average correlation among individual stocks during the high volatility period is low.

5.2 Portfolio revision frequency

In order to check upon the sensitivity of our results to the portfolio revision frequency for both, the overall sample period as well as the considered subperiods, we vary in the following the revision frequency from three to six and twelve months. Assessing the results for the complete sample period first we find that the standard deviations, as depicted in table 8 show a parallel behavior of the CMVP with different revision frequencies. Remarkable is the relation between the standard deviation and the revision frequency for CMVPs with a maximum portfolio weight constraint of 6% and more. The differences in standard deviations become for those CMVP increasingly large, whereby the CMVP with the highest (lowest) revision frequency exhibit the highest (lowest) standard deviation. This observation is reflected in the empirical performance metrics which nevertheless follow closely the behavior of the annualized mean returns. Correspondingly, the CMVP with a three (twelve) months portfolio revision frequency exhibit the empirically highest (lowest) performance metric values up to the maximum portfolio weight constraint level of 10%. As already observed for the annualized returns, this observation is inverted for CMVP with a maximum portfolio weight constraint of 14% and more, leaving the CMVP with a revision frequency of 12 (3) months with the highest (lowest) performance metric values.

The inference results for the CMVP with six and twelve months revision frequency over the complete sample period yield noteworthy insights. As observed for the CMVP with a three months portfolio revision frequency, a significant reduction in the portfolio standard deviation in comparison to the value weighted benchmark portfolio is only for the most restrictively CMVPs observable. This finding holds for all CMVPs, irrespective of their revision frequency.

While the significance of the performance metrics was in case of the three months revision frequency mainly driven by the significantly lower standard deviation, one may now observe two sources that drive the significance of the performance metrics: the significantly reduced standard deviation and the significantly higher return. Following this, the most restrictively CMVPs (2%-4% maximum portfolio weight constraint) achieve for the six and twelve months revision frequency significantly higher CEQ, Sharpe and Sortino ratios in comparison to the value weighted benchmark portfolio on a siginificance level well below the 10% level. This is mainly attributable to the significantly reduced standard (and downside-) deviation. The effect of the significantly increased return is in turn reflected in the significant performance metrics for the less constrained portfolios. Accordingly, all CMVP with a six months portfolio revision frequency and maximum portfolio weight constraints between 7% - 13% deliver significantly[37] higher CEQ, Sharpe and Sortino ratios in comparison to the value weighted benchmark portfolio. The results for the CMVP with a twelve months portfolio revision frequency point in the same direction and deliver even more pronounced findings concerning the driving forces of the performance metric significance.

The factor model based alpha measures are over the complete sample period invariant to both, the market proxy and the portfolio revision frequency. Accordingly, the in section 4.1 reported alpha pattern[38] emerges for all settings, which results in the reduced importance of the alpha measure for the CMVP assessment in the complete sample period due to the in section 4.1 elaborated shortcomings associated with this pattern. Further invariance to the portfolio revision frequency is observable for the performance metrics in comparison to the equally weighted benchmark portfolio. Though all CMVP deliver significantly lower standard deviations than the equally weighted benchmark portfolio, the hypothesis of higher performance metrics for the equally weighted benchmark portfolio may in no case be rejected.

Concluding the results for the complete sample period, the qualitative findings from section 4.2 are confirmed. The constrained minimum-variance approach delivers, irrespective of the portfolio revision frequency, higher risk adjusted performance than the value weighted benchmark portfolio. Broad evidence for this finding is provided by the reported CEQ, Sharpe and Sortino ratios. The driving source for the significantly higher CEQ, Sharpe and Sortino ratios varies however with the portfolio revision frequency. The significance of the higher performance metrics of the CMVPs with a three months portfolio revision frequency in comparison to the value weighted benchmark portfolio is solely driven by the significantly reduced standard deviation. This changes for the CMVP with portfolio revision frequencies of six and twelve months. Though the standard deviation of the CMVPs is in both cases significantly reduced in comparison to the value weighted benchmark portfolio, the significance of the performance metrics is mainly driven by the significantly higher annualized return in comparison to the value weighted benchmark

[37]Mostly below the 5% but always well below 10% confidence level.

[38]Significant alphas are only achieved by less CMVPs which bare a large share of idiosyncratic risk

portfolio.[39] The reason for the significantly higher risk reduction observed for the twelve months portfolio revision frequency remains open. The explanation for the lower standard deviation of the twelve months revision frequency based CMVPs in comparison to the higher revision frequencies based CMVPs must be founded in better out-of-sample properties of variance-covariance estimate, which is estimated accoring to the portfolio revision frequency. Following this, it is suggestive that the cummulative estimation error of four quarterly variance-covariance estimates exceeds that of a single estimate for an annual period.

The results for the U.S. recession periods[40] underpin the high influence of the revision frequency on the portfolio performance. The variation of the initial portfolio revision frequency from three to six months leads to significantly higher returns for the less restrictively CMVPs. In turn, the initial outperformance of the value weighted benchmark portfolio by the CMVPs with a revision frequency of three months and maximum portfolio weight constraints of less than 10% is extended by the CMVPs with a six months portfolio revision frequency. Accordingly, the CMVPs with maximum portfolio weight constraints between 12% - 19% achieve significantly higher CEQ, Sharpe and Sortino ratios than the value weighted benchmark portfolio, which are also according to amount higher than those of the CMVPs with a three months portfolio revision frequency. The further variation of the portfolio revision frequency to twelve months changes the results dramatically. Though the most restrictively CMVPs still reduce the standard deviation substantially, none of the considered performance metrics exceeds the corresponding value weighted benchmark portfolio significantly. Compared to the equally weighted benchmark portfolio, almost[41] none of the CMVPs delivers significantly higher performance metrics.

Broadly invariant results with respect to the portfolio revision frequency are obtained for the low volatility periods, as defined by Campbell et al. (2001). Irrespective of the portfolio revision frequency and maximum portfolio weight constraint, almost every CMVP outperforms the value weighted benchmark portfolio based on every considered performance metric in the low volatility period. Additionally, all CMVP deliver significant

[39]This may be deducted from the bootstrap inference in tables 9 and 10, which show that almost all significantly higher performance metrics on the 5% level are associated with significantly higher returns, whereas the reduction of the standard deviation is not significant.

[40]The tables for the considered subperiods with different portfolio revision frequencies are not provided but are available from the authors upon request.

[41]Only limited and very weak statistical evidence of an outperformance of the equally weighted benchmark portfolio by the less CMVPs with a revision freqency of 6 months is provided by the Sortino ratio.

alphas, irrespective of the portfolio revision frequency and market proxy, while none of the CMVP outperforms the equally weighted benchmark portfolio.

Somewhat mixed results are obtained for the high volatility period. The empirical performance metric values show that the most favorable portfolio weight constraints for the CMVPs with three and six months revision frequencies, are the most restrictive ones. Contrary, the CMVPs with a twelve months revision frequency achieve the largest performance metric values for the less restrictively portfolio weight constraints. Given these empirical differences, the bootstrap reveals that the most restrictively CMVP with the three months portfolio revision frequency achieves the most significant and according to amount highest risk adjusted performance based on any of the considered performance metrics.

Evidence from the three subsamples shows that not only the choice of the maximum portfolio weight constraint but also the portfolio revision frequency has substantial influence on the portfolio performance. Moreover, it shows that the best performing revision frequency for the complete sample period is not optimal for every considered subperiod. According to our afore defined criterion for assessing the "optimal" portfolio revision frequency, we find that the six months portfolio revision frequency is the best performing. During the high (low) volatility period a portfolio revision frequency of three and six (six and twelve) months would have been optimal.

6 Conclusion

This paper analyzed the risk-adjusted performance of constrained minimum-variance portfolios in the U.S. during the period from April 1968 to December 2007. We employed the constrained minimum-variance approach by Jagannathan and Ma (2003) on a rolling sample basis and vary the maximum portfolio weight constraint in one percentage steps between 2% - 20%. The implementation of maximum portfolio weight constraints assures thereby a certain degree of diversification and achieves a shrinkage like effect. We assess the CMVP performance by several performance metrics in order to provide a comprehensive picture. In particular, we provide evidence of the constrained minimum-variance portfolio performance on the certainty equivalent, the Sharpe and Sortino ratio as well as on factor model based alpha measures. We test upon the significance of these performance metrics by employing a nonparametric bootstrap. Bootstrap methods are required for robust inference concerning the performance metrics since the CMVP returns are not normally distributed.

Since earlier papers either focused on the descriptive performance metric comparison (e.g. Bloomfield et al. 1977 and Chan et al. 1999) or employed invalid inference methodologies (DeMiguel et al. (2007), a statistically robust assessment concerning the profitability of the (constrained) minimum-variance approach has not been done. Additionally, none of the prior studies based the portfolio optimization on a complete U.S. equity universe. Our paper closes thereby the lack of robust inference concerning the profitability of the (constrained) minimum-variance approach.

Our empirical results over the complete sample period show that the imposed maximum portfolio weight constraints reduce the out-of-sample standard deviation. The resulting performance metric values of the the more restrictively CMVP exceed those of the value weighted benchmark portfolio significantly, while they are well below those of the equally weighted benchmark. The assessment of various subperiods, namely the U.S. recession periods, as well as low and high volatility periods broadly confirmed the significant outperformance of the value weighted benchmark portfolio by the more restrictively CMVPs. The broad result of the significant outperformance of the value weighted benchmark portfolio by the CMVPs holds even for different portfolio revision frequencies, whereas the lowest revision frequency yields surprisingly the most favorable results. Sobering results are obtained in comparison to the equally weighted benchmark. The CMVP achieved in no setting noteworthy statistically higher performance metrics in comparison to the equally weighted portfolio.

Nevertheless, the high sensitivity of the CMVPs to the portfolio revision frequency remains thus far unexplored. Highlighted should in this context the obvious link between the portfolio revision frequency and the behavior of the standard deviations. Last but not least remains the question for *the* optimal maximum portfolio weight constraint. Although the results of Bloomfield et al. (1977) and Campbell et al. (2001) suggest that a reasonably diversified portfolio should consist of at least 25 or - nowadays that stock markets became more volatile - 50 stocks, it seems that these guidelines are not a panacea for the choice of the "optimal" constraint for minimum-variance investing.

Table 1: Regression statistics for the constrained minimum-variance portfolios with 3 months portfolio revision frequency for the period from April 1968 to December 2007 with a value weighted market proxy

The table reports the regression statistics for the considered one, three and four factor models (as described in section 2.3.2) over the complete sample period from April 1968 to December 2007. The market factor is hereby proxied by the CRSP value weighted market index. All factor time series data are taken from Kenneth French's website: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. The provided t-values are adjusted for autocorrelation and heteroscedasticity using the autocorrelation and heteroscedasticity consistent covariance matrix from Newey and West (1987).

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
1 Factor Model																			
Market	0.706	0.667	0.655	0.651	0.650	0.643	0.616	0.634	0.656	0.635	0.632	0.591	0.642	0.643	0.621	0.622	0.597	0.601	0.669
t-Value	*16.873*	*15.209*	*14.202*	*12.979*	*12.220*	*10.834*	*10.043*	*9.841*	*9.902*	*9.080*	*8.904*	*8.428*	*9.065*	*8.788*	*8.495*	*8.247*	*7.959*	*7.135*	*7.607*
Adj. R^2	0.701	0.641	0.579	0.539	0.508	0.459	0.408	0.393	0.388	0.350	0.337	0.288	0.326	0.317	0.293	0.285	0.254	0.240	0.272
3 Factor Model																			
Market	0.768	0.724	0.702	0.694	0.692	0.686	0.651	0.668	0.688	0.663	0.665	0.615	0.675	0.673	0.644	0.642	0.637	0.618	0.696
t-Value	*19.921*	*17.606*	*16.238*	*14.329*	*13.699*	*11.945*	*10.938*	*10.668*	*10.839*	*9.950*	*9.659*	*8.892*	*9.906*	*9.604*	*9.140*	*8.805*	*8.538*	*7.473*	*8.080*
SMB	0.033	0.018	0.031	0.044	0.025	0.026	0.010	0.021	-0.004	-0.009	-0.032	0.025	-0.053	-0.058	-0.041	-0.027	-0.028	-0.003	-0.028
t-Value	*0.841*	*0.419*	*0.681*	*0.878*	*0.478*	*0.444*	*0.165*	*0.333*	*-0.060*	*-0.130*	*-0.496*	*0.387*	*-0.810*	*-0.877*	*-0.640*	*-0.413*	*-0.418*	*-0.044*	*-0.350*
HML	0.241	0.213	0.187	0.182	0.165	0.169	0.128	0.137	0.108	0.094	0.093	0.104	0.074	0.062	0.051	0.049	0.117	0.056	0.075
t-Value	*3.680*	*3.113*	*2.488*	*2.284*	*1.928*	*1.809*	*1.335*	*1.303*	*0.950*	*0.804*	*0.816*	*0.902*	*0.606*	*0.490*	*0.400*	*0.370*	*0.852*	*0.382*	*0.479*
Adj. R^2	0.728	0.662	0.594	0.552	0.517	0.468	0.412	0.397	0.389	0.350	0.337	0.288	0.326	0.317	0.292	0.283	0.255	0.238	0.271
4 Factor Model																			
Market	0.777	0.733	0.712	0.708	0.706	0.701	0.668	0.685	0.706	0.681	0.685	0.627	0.693	0.692	0.662	0.659	0.653	0.632	0.712
t-Value	*20.939*	*18.191*	*16.834*	*15.037*	*14.361*	*12.569*	*11.562*	*11.217*	*11.376*	*10.432*	*10.157*	*9.093*	*10.330*	*9.965*	*9.428*	*9.010*	*8.610*	*7.514*	*8.129*
SMB	0.034	0.019	0.032	0.045	0.026	0.027	0.011	0.022	-0.002	-0.007	-0.031	0.026	-0.051	-0.057	-0.039	-0.025	-0.027	-0.002	-0.026
t-Value	*0.813*	*0.415*	*0.692*	*0.891*	*0.496*	*0.474*	*0.193*	*0.356*	*-0.037*	*-0.109*	*-0.485*	*0.400*	*-0.787*	*-0.850*	*-0.607*	*-0.379*	*-0.392*	*-0.028*	*-0.321*
HML	0.256	0.227	0.205	0.206	0.190	0.195	0.158	0.166	0.140	0.125	0.127	0.124	0.106	0.094	0.083	0.080	0.145	0.080	0.102
t-Value	*3.728*	*3.154*	*2.626*	*2.550*	*2.165*	*2.054*	*1.643*	*1.561*	*1.207*	*1.058*	*1.112*	*1.027*	*0.843*	*0.719*	*0.620*	*0.572*	*1.014*	*0.512*	*0.619*
MOM	0.063	0.061	0.075	0.104	0.104	0.113	0.126	0.124	0.133	0.132	0.145	0.086	0.134	0.136	0.133	0.129	0.121	0.102	0.118
t-Value	*1.394*	*1.297*	*1.536*	*2.032*	*1.769*	*1.807*	*1.950*	*1.828*	*1.862*	*1.822*	*2.106*	*1.107*	*1.743*	*1.682*	*1.582*	*1.459*	*1.356*	*1.028*	*1.138*
Adj. R^2	0.732	0.666	0.599	0.563	0.527	0.478	0.425	0.408	0.401	0.361	0.350	0.291	0.336	0.327	0.302	0.292	0.262	0.242	0.276

Table 2: Regression statistics for the constrained minimum-variance portfolios with 3 months portfolio revision frequency for the period from April 1968 to December 2007 with an equally weighted market factor proxy

The table reports the regression statistics for the considered one, three and four factor models (as described in section 2.3.2) over the complete sample period from April 1968 to December 2007. The market factor is hereby proxied by the CRSP equally weighted market index. All factor time series data are taken from Kenneth French's website: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. The provided t-values are adjusted for autocorrelation and heteroscedasticity using the autocorrelation and heteroscedasticity consistent covariance matrix from Newey and West (1987).

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
1 Factor Model																			
Market	0.465	0.435	0.429	0.428	0.426	0.419	0.401	0.416	0.426	0.408	0.403	0.394	0.403	0.400	0.390	0.394	0.377	0.389	0.424
t-Value	*11.269*	*10.756*	*10.474*	*9.880*	*9.358*	*8.656*	*8.119*	*8.321*	*8.408*	*7.875*	*7.669*	*7.588*	*7.675*	*7.481*	*7.221*	*7.172*	*6.790*	*6.531*	*6.563*
Adj. R^2	0.495	0.444	0.404	0.380	0.354	0.317	0.280	0.276	0.266	0.235	0.222	0.208	0.208	0.199	0.187	0.185	0.163	0.163	0.177
3 Factor Model																			
Market	0.652	0.613	0.591	0.583	0.587	0.577	0.553	0.571	0.591	0.565	0.571	0.535	0.576	0.572	0.546	0.545	0.537	0.528	0.590
t-Value	*12.363*	*11.568*	*10.692*	*9.738*	*9.474*	*8.501*	*8.109*	*8.186*	*8.359*	*7.812*	*7.751*	*7.546*	*7.924*	*7.749*	*7.434*	*7.255*	*7.082*	*6.437*	*6.655*
SMB	-0.492	-0.476	-0.444	-0.424	-0.447	-0.438	-0.436	-0.440	-0.482	-0.465	-0.494	-0.410	-0.518	-0.519	-0.481	-0.466	-0.460	-0.430	-0.503
t-Value	*-6.288*	*-5.998*	*-5.013*	*-4.435*	*-4.553*	*-3.982*	*-3.912*	*-3.898*	*-4.143*	*-3.814*	*-4.066*	*-3.639*	*-4.287*	*-4.260*	*-4.128*	*-3.996*	*-4.005*	*-3.427*	*-3.872*
HML	-0.012	-0.026	-0.045	-0.048	-0.063	-0.058	-0.085	-0.081	-0.116	-0.124	-0.123	-0.094	-0.147	-0.159	-0.160	-0.162	-0.094	-0.146	-0.154
t-Value	*-0.152*	*-0.322*	*-0.515*	*-0.530*	*-0.661*	*-0.569*	*-0.809*	*-0.720*	*-0.948*	*-0.985*	*-1.012*	*-0.789*	*-1.124*	*-1.171*	*-1.195*	*-1.180*	*-0.680*	*-0.981*	*-0.971*
Adj. R^2	0.591	0.535	0.479	0.443	0.421	0.376	0.337	0.328	0.325	0.289	0.281	0.245	0.269	0.260	0.239	0.233	0.204	0.198	0.221
4 Factor Model																			
Market	0.694	0.652	0.634	0.633	0.637	0.630	0.608	0.627	0.650	0.623	0.633	0.579	0.635	0.631	0.604	0.601	0.590	0.576	0.644
t-Value	*17.379*	*15.677*	*14.470*	*13.250*	*12.907*	*11.311*	*10.903*	*10.678*	*10.999*	*10.015*	*9.968*	*9.363*	*10.408*	*10.104*	*9.562*	*9.190*	*8.767*	*7.847*	*8.121*
SMB	-0.531	-0.512	-0.484	-0.471	-0.495	-0.487	-0.488	-0.493	-0.538	-0.519	-0.552	-0.451	-0.573	-0.575	-0.535	-0.518	-0.510	-0.475	-0.554
t-Value	*-7.690*	*-7.215*	*-6.249*	*-5.682*	*-5.861*	*-5.134*	*-5.071*	*-5.087*	*-5.456*	*-4.934*	*-5.419*	*-4.585*	*-5.559*	*-5.472*	*-5.297*	*-5.088*	*-5.203*	*-4.183*	*-4.766*
HML	0.025	0.009	-0.007	-0.004	-0.018	-0.012	-0.037	-0.033	-0.064	-0.073	-0.069	-0.056	-0.095	-0.107	-0.110	-0.113	-0.047	-0.104	-0.107
t-Value	*0.299*	*0.103*	*-0.082*	*-0.041*	*-0.190*	*-0.117*	*-0.348*	*-0.286*	*-0.518*	*-0.580*	*-0.569*	*-0.448*	*-0.713*	*-0.769*	*-0.794*	*-0.786*	*-0.325*	*-0.656*	*-0.634*
MOM	0.156	0.148	0.160	0.189	0.190	0.198	0.209	0.210	0.223	0.217	0.233	0.166	0.222	0.222	0.216	0.212	0.201	0.181	0.205
t-Value	*2.488*	*2.358*	*2.501*	*2.878*	*2.626*	*2.590*	*2.689*	*2.594*	*2.608*	*2.535*	*2.856*	*1.860*	*2.461*	*2.372*	*2.233*	*2.102*	*1.988*	*1.644*	*1.771*
Adj. R^2	0.617	0.558	0.504	0.477	0.453	0.408	0.372	0.360	0.358	0.319	0.314	0.261	0.297	0.288	0.265	0.257	0.225	0.213	0.239

Table 3: Empirical statistics and performance metrics for the constrained minimum-variance portfolios with 3 months portfolio revision frequency over the period from April 1968 to December 2007

The table reports summarizing portfolio statistics as well as the annualized empirical performance metrics for the constrained minimum-variance portfolios with the respective maximum weight a single stock may have in the portfolio. The summary statistics show the mean number of stocks in the portfolio as well as the Jarque-Berra test p-values for the hypothesis that the portfolio returns are normally distributed. The certainty equivalent measure (CEQ) is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the in section 2.2 described factor models. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. The market factor benchmark for every alpha is reported in brackets, where EW denotes the equally weighted market benchmark portfolio (proxied by the CRSP equally weighted market index) and VW the value weighted market benchmark portfolio (proxied by the CRSP value weighted market index). The performance metrics for both benchmark portfolios are, for the purpose of vis-a-vis performance comparison, reported as well.

	Maximum Weights																			**Benchmarks**	
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**	**VW**	**EW**
Mean no. of Stocks	53	38	29	25	22	20	18	17	16	16	15	14	14	14	13	13	12	12	12		
Jarque-Berra p-values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.071	0.072	0.078	0.078	0.083	0.086	0.090	0.093	0.097	0.097	0.094	0.090	0.091	0.088	0.086	0.089	0.077	0.082	0.104	0.056	0.144
Std. Dev.	0.132	0.130	0.134	0.138	0.142	0.148	0.150	0.157	0.164	0.167	0.169	0.171	0.175	0.177	0.178	0.181	0.184	0.190	0.199	0.156	0.198
CEQ	0.037	0.039	0.042	0.040	0.042	0.043	0.045	0.044	0.043	0.041	0.037	0.031	0.030	0.025	0.023	0.024	0.010	0.010	0.025	0.008	0.065
Sharpe Ratio	0.541	0.558	0.581	0.566	0.583	0.584	0.600	0.592	0.592	0.582	0.557	0.525	0.519	0.497	0.486	0.493	0.421	0.431	0.523	0.362	0.725
Sortino Ratio	0.955	0.998	1.063	1.023	1.050	1.062	1.098	1.110	1.105	1.099	1.049	0.998	0.971	0.926	0.890	0.906	0.786	0.801	1.003	0.514	1.056
Alpha 1 Factor (EW)	0.031	0.035	0.041	0.041	0.046	0.050	0.056	0.057	0.061	0.062	0.060	0.056	0.056	0.054	0.053	0.055	0.045	0.049	0.068		
Alpha 1 Factor (VW)	0.031	0.035	0.041	0.041	0.046	0.050	0.055	0.057	0.060	0.061	0.059	0.056	0.055	0.052	0.051	0.054	0.044	0.048	0.066		
Alpha 3 Factor (EW)	0.026	0.031	0.039	0.039	0.045	0.049	0.056	0.057	0.062	0.065	0.062	0.057	0.060	0.058	0.058	0.061	0.046	0.053	0.072		
Alpha 3 Factor (VW)	0.014	0.020	0.028	0.028	0.034	0.038	0.046	0.048	0.052	0.055	0.052	0.049	0.050	0.048	0.048	0.051	0.036	0.044	0.061		
Alpha 4 Factor (EW)	0.006	0.012	0.018	0.015	0.021	0.024	0.029	0.031	0.034	0.037	0.032	0.036	0.032	0.030	0.031	0.034	0.020	0.030	0.046		
Alpha 4 Factor (VW)	0.007	0.013	0.019	0.016	0.022	0.025	0.031	0.033	0.037	0.039	0.035	0.039	0.034	0.032	0.032	0.036	0.021	0.032	0.048		

Table 4: Performance metric inference for the constrained minimum-variance portfolios with 3 months portfolio revision frequency over the period from April 1968 to December 2007

The table reports the bootstrapped p-values of the performance metric differences between the constrained minimum-variance portfolios (with the different maximum weight a single stock may have in the portfolio) and the value (equally) weighted benchmark portfolio. The p-values for the certainty equivalent measure (CEQ), Sharpe and Sortino ratios are computed for the one sided hypothesis that the particular performance metric of the benchmark portfolio exceeds that of the respective constrained minimum-variance portfolio. P-values for the alpha measures are computed for the one sided hypothesis that the bootstrapped alpha t-value exceeds the empirical t-value and thus indicates that the generated alpha is due to sampling variation. The employed bootstrap methodologies are outlined in detail in section 2.3. The difference in the CEQ is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. Panel I reports the results for the value weighted benchmark portfolio, while panel II does so for the equally weighted benchmark portfolio.

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
Panel I: Value weighted benchmark																			
Δ Mean	0.152	0.155	0.117	0.125	0.086	0.066	0.058	0.047	0.039	0.046	0.062	0.094	0.085	0.116	0.132	0.116	0.215	0.196	0.051
Δ Std. Dev.	1.000	1.000	1.000	1.000	0.997	0.938	0.837	0.461	0.150	0.104	0.066	0.046	0.020	0.008	0.008	0.004	0.003	0.001	0.001
Δ CEQ	0.026	0.027	0.028	0.044	0.038	0.046	0.047	0.052	0.055	0.076	0.109	0.188	0.182	0.230	0.270	0.267	0.473	0.465	0.278
Δ Sharpe Ratio	0.032	0.028	0.031	0.057	0.044	0.049	0.051	0.051	0.051	0.065	0.095	0.148	0.139	0.183	0.218	0.200	0.358	0.339	0.160
Δ Sortino Ratio	0.024	0.020	0.017	0.039	0.036	0.037	0.040	0.031	0.035	0.043	0.066	0.093	0.110	0.139	0.180	0.164	0.272	0.270	0.111
Alpha 1 Factor	0.014	0.004	0.005	0.002	0.005	0.000	0.000	0.001	0.002	0.002	0.003	0.007	0.008	0.016	0.017	0.017	0.046	0.047	0.009
Alpha 3 Factor	0.018	0.010	0.005	0.003	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.001	0.027	0.023	0.001
Alpha 4 Factor	0.333	0.203	0.118	0.172	0.117	0.085	0.053	0.051	0.049	0.034	0.060	0.052	0.069	0.092	0.098	0.077	0.188	0.120	0.038
Panel II: Equally weighted benchmark																			
Δ Mean	0.744	0.737	0.654	0.652	0.568	0.515	0.448	0.395	0.363	0.360	0.406	0.467	0.439	0.468	0.483	0.450	0.596	0.548	0.283
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.989	0.976	0.953	0.939	0.909	0.866	0.746	0.510
Δ CEQ	0.092	0.083	0.073	0.096	0.086	0.088	0.080	0.100	0.103	0.120	0.158	0.223	0.223	0.259	0.287	0.289	0.451	0.465	0.284
Δ Sharpe Ratio	0.170	0.159	0.147	0.177	0.139	0.151	0.136	0.154	0.149	0.175	0.217	0.292	0.287	0.323	0.355	0.344	0.517	0.507	0.277
Δ Sortino Ratio	0.187	0.162	0.128	0.162	0.145	0.148	0.131	0.133	0.130	0.140	0.186	0.235	0.251	0.282	0.326	0.307	0.456	0.435	0.240
Alpha 1 Factor	0.008	0.003	0.002	0.003	0.002	0.000	0.001	0.000	0.002	0.001	0.002	0.003	0.009	0.014	0.018	0.016	0.049	0.051	0.010
Alpha 3 Factor	0.020	0.008	0.004	0.005	0.001	0.001	0.000	0.000	0.001	0.000	0.000	0.002	0.001	0.000	0.001	0.000	0.030	0.017	0.001
Alpha 4 Factor	0.344	0.220	0.110	0.169	0.121	0.094	0.055	0.057	0.047	0.027	0.057	0.060	0.071	0.096	0.096	0.080	0.192	0.116	0.033

Table 5: Empirical performance metrics for the constrained minimum-variance portfolios with 3 months revision frequency over the selected subperiods

The table reports the annualized empirical performance metrics for the constrained minimum-variance portfolios with the respective maximum weight a single stock may have in the portfolio. The certainty equivalent measure (CEQ) is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the in section 2.2 described factor models. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. The market factor benchmark for every alpha is reported in brackets, where EW denotes the equally weighted market benchmark (proxied by the CRSP equally weighted market index) and VW the value weighted market benchmark (proxied by the CRSP value weighted market index). Panel I reports the empirical performance metrics for U.S. recession periods, panel II reports the empirical performance metrics for the low volatility period from April 1968 to December 1985 as defined by Campbell (2001), while panel III the empirical performance metrics for the low volatility period from January 1986 to December 1997 as defined by Campbell (2001). The performance metrics for both benchmarks are, for the purpose of vis-a-vis performance comparison, reported as well.

	Maximum Weights																			Benchmarks	
	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	VW	EW
Panel I: U.S. Recession Periods																					
Mean	0.084	0.092	0.100	0.098	0.092	0.097	0.090	0.096	0.079	0.080	0.082	0.085	0.095	0.088	0.090	0.090	0.183	0.108	0.112	-0.063	0.020
Std. Dev.	0.207	0.203	0.198	0.199	0.200	0.205	0.196	0.211	0.213	0.215	0.216	0.210	0.227	0.232	0.227	0.231	0.232	0.249	0.264	0.219	0.285
CEQ	-0.031	-0.020	-0.007	-0.011	-0.018	-0.016	-0.016	-0.022	-0.041	-0.042	-0.040	-0.033	-0.037	-0.049	-0.043	-0.046	0.046	-0.045	-0.056	-0.158	-0.142
Sharpe Ratio	0.265	0.306	0.358	0.344	0.312	0.331	0.310	0.316	0.231	0.234	0.245	0.264	0.290	0.251	0.264	0.261	0.661	0.314	0.313	-0.287	0.071
Sortino Ratio	0.525	0.595	0.694	0.649	0.575	0.619	0.555	0.602	0.418	0.415	0.450	0.481	0.520	0.454	0.465	0.469	1.400	0.609	0.610	-0.521	0.137
1 Factor Alpha (EW)	0.074	0.083	0.091	0.088	0.083	0.086	0.079	0.085	0.067	0.066	0.061	0.076	0.079	0.070	0.074	0.074	0.171	0.094	0.095		
1 Factor Alpha (VW)	0.072	0.082	0.090	0.088	0.083	0.086	0.077	0.084	0.066	0.066	0.062	0.075	0.080	0.072	0.074	0.074	0.173	0.096	0.096		
3 Factor Alpha (EW)	0.057	0.059	0.063	0.058	0.057	0.055	0.045	0.054	0.032	0.032	0.025	0.045	0.041	0.034	0.041	0.042	0.180	0.064	0.067		
3 Factor Alpha (VW)	0.055	0.057	0.061	0.056	0.055	0.055	0.044	0.053	0.031	0.031	0.024	0.044	0.040	0.033	0.040	0.041	0.178	0.064	0.066		
4 Factor Alpha (EW)	0.051	0.060	0.064	0.057	0.057	0.057	0.052	0.053	0.024	0.023	0.015	0.033	0.018	0.015	0.016	0.016	0.155	0.038	0.022		
4 Factor Alpha (VW)	0.046	0.054	0.061	0.052	0.053	0.055	0.048	0.050	0.022	0.021	0.014	0.030	0.016	0.013	0.013	0.014	0.150	0.037	0.019		
Panel II: Low Volatility Period																					
Mean	0.056	0.059	0.061	0.060	0.057	0.058	0.056	0.065	0.063	0.054	0.059	0.061	0.058	0.053	0.053	0.062	0.074	0.063	0.086	0.031	0.147
Std. Dev.	0.161	0.160	0.158	0.158	0.158	0.160	0.158	0.162	0.164	0.160	0.163	0.165	0.166	0.167	0.165	0.167	0.165	0.176	0.188	0.163	0.219
CEQ	0.004	0.008	0.011	0.010	0.007	0.007	0.006	0.013	0.009	0.002	0.006	0.007	0.003	-0.004	-0.001	0.007	0.020	0.001	0.015	-0.022	0.051
Sharpe Ratio	0.347	0.368	0.386	0.381	0.362	0.364	0.353	0.402	0.383	0.334	0.363	0.370	0.351	0.314	0.323	0.373	0.448	0.358	0.456	0.189	0.671
Sortino Ratio	0.648	0.680	0.717	0.697	0.649	0.654	0.628	0.736	0.679	0.575	0.636	0.675	0.610	0.545	0.548	0.646	0.841	0.655	0.885	0.307	1.100
1 Factor Alpha (EW)	0.011	0.016	0.019	0.018	0.016	0.017	0.016	0.024	0.022	0.014	0.019	0.021	0.018	0.012	0.013	0.021	0.033	0.020	0.040		
1 Factor Alpha (VW)	0.027	0.030	0.033	0.032	0.029	0.030	0.029	0.037	0.035	0.026	0.031	0.034	0.030	0.024	0.026	0.034	0.047	0.034	0.055		
3 Factor Alpha (EW)	0.032	0.038	0.041	0.039	0.040	0.040	0.043	0.047	0.047	0.040	0.046	0.046	0.050	0.045	0.043	0.052	0.053	0.051	0.073		
3 Factor Alpha (VW)	0.020	0.025	0.029	0.027	0.028	0.028	0.031	0.035	0.035	0.028	0.034	0.034	0.037	0.032	0.031	0.040	0.042	0.038	0.061		
4 Factor Alpha (EW)	0.033	0.039	0.040	0.039	0.038	0.042	0.040	0.051	0.049	0.042	0.043	0.053	0.050	0.044	0.043	0.054	0.047	0.056	0.084		
4 Factor Alpha (VW)	0.024	0.031	0.032	0.031	0.030	0.033	0.032	0.043	0.041	0.034	0.035	0.044	0.041	0.035	0.034	0.045	0.040	0.048	0.076		
Panel III: High Volatility Period																					
Mean	0.103	0.096	0.096	0.084	0.085	0.090	0.098	0.075	0.089	0.101	0.091	0.082	0.096	0.100	0.096	0.092	0.033	0.061	0.098	0.104	0.138
Std. Dev.	0.111	0.107	0.118	0.122	0.127	0.133	0.130	0.137	0.143	0.149	0.148	0.147	0.155	0.156	0.154	0.153	0.161	0.160	0.165	0.144	0.162
CEQ	0.078	0.073	0.068	0.054	0.053	0.055	0.064	0.038	0.048	0.057	0.048	0.038	0.048	0.051	0.048	0.045	-0.019	0.010	0.043	0.062	0.085
Sharpe Ratio	0.924	0.895	0.817	0.689	0.672	0.676	0.756	0.551	0.622	0.678	0.620	0.555	0.621	0.639	0.621	0.603	0.206	0.382	0.593	0.720	0.850
Sortino Ratio	1.458	1.479	1.394	1.154	1.109	1.131	1.285	0.920	1.062	1.222	1.107	0.982	1.108	1.141	1.104	1.071	0.349	0.675	1.097	0.832	0.983
Alpha 1 Factor (EW)	0.060	0.058	0.056	0.044	0.042	0.047	0.058	0.035	0.045	0.058	0.051	0.045	0.055	0.060	0.058	0.054	0.003	0.032	0.062		
Alpha 1 Factor (VW)	0.031	0.032	0.031	0.017	0.016	0.019	0.033	0.007	0.016	0.029	0.023	0.025	0.027	0.031	0.031	0.029	-0.025	0.012	0.033		
Alpha 3 Factor (EW)	0.044	0.044	0.044	0.027	0.026	0.027	0.037	0.011	0.021	0.036	0.026	0.026	0.032	0.037	0.040	0.040	-0.017	0.021	0.039		
Alpha 3 Factor (VW)	0.026	0.027	0.027	0.010	0.009	0.010	0.023	-0.004	0.006	0.020	0.013	0.017	0.019	0.023	0.026	0.026	-0.033	0.012	0.024		
Alpha 4 Factor (EW)	0.035	0.037	0.036	0.014	0.011	0.007	0.018	-0.014	-0.001	0.019	0.007	0.017	0.013	0.019	0.022	0.020	-0.013	0.014	0.022		
Alpha 4 Factor (VW)	0.023	0.025	0.025	0.003	0.001	-0.003	0.010	-0.022	-0.009	0.010	0.000	0.012	0.006	0.011	0.014	0.012	-0.025	0.010	0.014		

Table 6: Performance metric inference for the constrained minimum-variance portfolios with 3 months portfolio revision frequency over the selected subperiods with a value weighted benchmark

The table reports the bootstrapped p-values of the performance metric differences between the constrained minimum-variance portfolios (with the different maximum weight a single stock may have in the portfolio) and the value weighted benchmark portfolio. The p-values for the certainty equivalent measure (CEQ), Sharpe and Sortino ratios are computed for the one sided hypothesis that the particular performance metric of the benchmark portfolio exceeds that of the respective constrained minimum-variance portfolio. P-values for the alpha measures are computed for the one sided hypotheses that the bootstrapped alpha t-value exceeds the empirical t-value and thus indicates that the generated alpha is due to sampling variation. The employed bootstrap methodologies are outline in detail in section 2.3.

The difference in the CEQ is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. Panel I reports the results for U.S. recession periods, panel II for the low volatility period from April 1968 to December 1985 as defined by Campbell (2001), while panel III reports the results for the low volatility period from January 1986 to December 1997 as defined by Campbell (2001).

	Maximum Weights																		
	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
Panel I: U.S. Recession Periods																			
Δ Mean	0.015	0.010	0.012	0.024	0.050	0.048	0.067	0.061	0.119	0.123	0.129	0.125	0.106	0.144	0.140	0.143	0.006	0.113	0.104
Δ Std. Dev.	0.895	0.944	0.987	0.987	0.974	0.898	0.959	0.740	0.684	0.589	0.534	0.713	0.259	0.160	0.196	0.104	0.102	0.130	0.143
Δ CEQ	0.004	0.006	0.008	0.011	0.022	0.031	0.039	0.057	0.099	0.123	0.125	0.096	0.137	0.190	0.166	0.185	0.012	0.203	0.242
Δ Sharpe Ratio	0.018	0.017	0.016	0.029	0.060	0.057	0.078	0.069	0.124	0.127	0.130	0.140	0.107	0.136	0.132	0.137	0.008	0.103	0.088
Δ Sortino Ratio	0.014	0.014	0.016	0.027	0.049	0.051	0.068	0.064	0.115	0.124	0.126	0.134	0.098	0.128	0.123	0.133	0.008	0.103	0.091
Alpha 1 Factor	0.223	0.219	0.192	0.189	0.232	0.227	0.296	0.242	0.360	0.358	0.352	0.326	0.287	0.331	0.325	0.324	0.026	0.228	0.189
Alpha 3 Factor	0.029	0.058	0.034	0.054	0.096	0.096	0.097	0.138	0.222	0.202	0.222	0.246	0.152	0.176	0.191	0.208	0.030	0.142	0.150
Alpha 4 Factor	0.068	0.099	0.084	0.120	0.169	0.151	0.179	0.228	0.342	0.338	0.390	0.383	0.248	0.293	0.342	0.348	0.067	0.234	0.216
Panel II: Low Volatility Period																			
Δ Mean	0.008	0.013	0.011	0.015	0.053	0.051	0.101	0.032	0.036	0.129	0.059	0.081	0.088	0.150	0.151	0.050	0.016	0.082	0.010
Δ Std. Dev.	0.710	0.820	0.885	0.914	0.902	0.796	0.867	0.653	0.472	0.759	0.612	0.499	0.382	0.345	0.428	0.314	0.405	0.095	0.016
Δ CEQ	0.005	0.009	0.005	0.009	0.037	0.048	0.079	0.023	0.044	0.107	0.068	0.092	0.104	0.191	0.167	0.078	0.014	0.159	0.045
Δ Sharpe Ratio	0.007	0.010	0.006	0.012	0.043	0.047	0.089	0.027	0.040	0.114	0.058	0.084	0.094	0.162	0.153	0.062	0.014	0.101	0.022
Δ Sortino Ratio	0.007	0.011	0.005	0.012	0.050	0.051	0.098	0.029	0.048	0.143	0.076	0.094	0.120	0.186	0.192	0.086	0.016	0.114	0.020
Alpha 1 Factor	0.274	0.200	0.169	0.173	0.211	0.195	0.198	0.153	0.186	0.269	0.211	0.173	0.227	0.284	0.284	0.199	0.086	0.192	0.071
Alpha 3 Factor	0.015	0.017	0.010	0.010	0.010	0.012	0.008	0.001	0.002	0.021	0.005	0.008	0.009	0.016	0.024	0.007	0.000	0.012	0.001
Alpha 4 Factor	0.035	0.024	0.017	0.015	0.017	0.011	0.011	0.003	0.003	0.013	0.009	0.006	0.004	0.020	0.020	0.003	0.004	0.001	0.000
Panel III: High Volatility Period																			
Δ Mean	0.511	0.632	0.618	0.775	0.764	0.672	0.562	0.817	0.674	0.510	0.633	0.718	0.566	0.521	0.572	0.624	0.961	0.854	0.582
Δ Std. Dev.	1.000	1.000	1.000	1.000	0.972	0.911	0.938	0.793	0.565	0.318	0.399	0.409	0.206	0.185	0.248	0.285	0.163	0.191	0.132
Δ CEQ	0.189	0.328	0.420	0.617	0.632	0.590	0.456	0.756	0.661	0.542	0.660	0.722	0.638	0.621	0.630	0.664	0.966	0.878	0.686
Δ Sharpe Ratio	0.047	0.131	0.317	0.572	0.608	0.585	0.434	0.760	0.678	0.572	0.674	0.733	0.662	0.642	0.649	0.684	0.965	0.880	0.706
Δ Sortino Ratio	0.059	0.115	0.248	0.495	0.552	0.503	0.351	0.717	0.578	0.452	0.560	0.641	0.555	0.531	0.573	0.599	0.948	0.842	0.604
Alpha 1 Factor	0.000	0.000	0.010	0.033	0.066	0.045	0.030	0.134	0.074	0.038	0.067	0.102	0.043	0.047	0.049	0.060	0.440	0.187	0.036
Alpha 3 Factor	0.007	0.018	0.033	0.127	0.153	0.169	0.073	0.366	0.228	0.140	0.212	0.240	0.163	0.152	0.127	0.133	0.636	0.284	0.117
Alpha 4 Factor	0.043	0.046	0.080	0.299	0.350	0.438	0.287	0.714	0.557	0.297	0.426	0.331	0.364	0.319	0.280	0.272	0.603	0.335	0.251

Table 7: Performance metric inference for the constrained minimum-variance portfolios with 3 months portfolio revision frequency over the selected subperiods with an equally weighted benchmark

The table reports the bootstrapped p-values of the performance metric differences between the constrained minimum-variance portfolios (with the different maximum weight a single stock may have in the portfolio) and the equally weighted benchmark portfolio. The p-values for the certainty equivalent measure (CEQ), Sharpe and Sortino ratios are computed for the one sided hypothesis that the particular performance metric of the benchmark portfolio exceeds that of the respective constrained minimum-variance portfolio. P-values for the alpha measures are computed for the one sided hypotheses that the bootstrapped alpha t-value exceeds the empirical t-value and thus indicates that the generated alpha is due to sampling variation. The employed bootstrap methodologies are outline in detail in section 2.3.

The difference in the CEQ is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. Panel I reports the results for U.S. recession periods, panel II for the low volatility period from April 1968 to December 1985 as defined by Campbell (2001), while panel III reports the results for the low volatility period from January 1986 to December 1997 as defined by Campbell (2001).

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
Panel I: U.S. Recession Periods																			
Δ Mean	0.153	0.143	0.138	0.148	0.166	0.157	0.177	0.162	0.215	0.218	0.220	0.222	0.192	0.213	0.208	0.211	0.036	0.186	0.165
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.996	0.998	0.966	0.944	0.986	0.981	0.989	0.894	0.826
Δ CEQ	0.009	0.007	0.011	0.008	0.023	0.029	0.024	0.028	0.049	0.052	0.065	0.033	0.063	0.080	0.074	0.079	0.011	0.082	0.095
Δ Sharpe Ratio	0.170	0.150	0.145	0.145	0.172	0.178	0.187	0.163	0.237	0.246	0.244	0.238	0.202	0.229	0.224	0.224	0.030	0.183	0.166
Δ Sortino Ratio	0.176	0.152	0.139	0.136	0.164	0.170	0.177	0.160	0.221	0.229	0.234	0.231	0.192	0.224	0.220	0.222	0.031	0.187	0.165
Alpha 1 Factor	0.188	0.188	0.186	0.184	0.230	0.222	0.284	0.245	0.351	0.345	0.337	0.325	0.299	0.325	0.334	0.323	0.023	0.227	0.200
Alpha 3 Factor	0.029	0.036	0.024	0.047	0.088	0.084	0.094	0.130	0.209	0.199	0.214	0.236	0.145	0.168	0.190	0.202	0.025	0.144	0.135
Alpha 4 Factor	0.067	0.079	0.063	0.106	0.165	0.144	0.174	0.215	0.336	0.333	0.384	0.370	0.256	0.290	0.338	0.352	0.068	0.228	0.212
Panel II: Low Volatility Period																			
Δ Mean	0.739	0.690	0.662	0.678	0.706	0.689	0.713	0.619	0.640	0.740	0.691	0.662	0.691	0.745	0.740	0.652	0.530	0.661	0.427
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.987
Δ CEQ	0.199	0.165	0.161	0.167	0.215	0.216	0.233	0.177	0.201	0.257	0.232	0.232	0.255	0.312	0.288	0.238	0.128	0.282	0.173
Δ Sharpe Ratio	0.485	0.436	0.391	0.395	0.472	0.465	0.499	0.374	0.414	0.535	0.465	0.445	0.507	0.591	0.559	0.451	0.264	0.505	0.285
Δ Sortino Ratio	0.463	0.427	0.384	0.406	0.494	0.487	0.521	0.380	0.443	0.567	0.501	0.464	0.540	0.612	0.596	0.494	0.274	0.507	0.277
Alpha 1 Factor	0.248	0.183	0.143	0.158	0.180	0.182	0.191	0.132	0.164	0.239	0.196	0.159	0.207	0.263	0.270	0.177	0.066	0.173	0.072
Alpha 3 Factor	0.018	0.020	0.007	0.008	0.011	0.014	0.008	0.006	0.006	0.028	0.008	0.012	0.007	0.020	0.025	0.008	0.000	0.012	0.000
Alpha 4 Factor	0.028	0.023	0.015	0.017	0.016	0.014	0.012	0.006	0.004	0.020	0.013	0.011	0.006	0.032	0.023	0.002	0.005	0.004	0.000
Panel III: High Volatility Period																			
Δ Mean	0.313	0.405	0.393	0.541	0.540	0.482	0.394	0.621	0.492	0.372	0.476	0.556	0.418	0.391	0.433	0.474	0.882	0.745	0.456
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	0.990	0.995	0.971	0.908	0.771	0.841	0.820	0.663	0.612	0.654	0.673	0.498	0.532	0.439
Δ CEQ	0.102	0.159	0.197	0.306	0.313	0.322	0.240	0.472	0.392	0.303	0.402	0.480	0.380	0.354	0.395	0.423	0.860	0.727	0.467
Δ Sharpe Ratio	0.025	0.061	0.140	0.294	0.304	0.312	0.222	0.487	0.417	0.320	0.426	0.502	0.401	0.380	0.414	0.443	0.875	0.745	0.482
Δ Sortino Ratio	0.030	0.066	0.124	0.251	0.273	0.272	0.189	0.454	0.376	0.251	0.348	0.425	0.350	0.340	0.369	0.397	0.849	0.686	0.420
Alpha 1 Factor	0.000	0.001	0.009	0.035	0.056	0.047	0.021	0.116	0.070	0.043	0.079	0.096	0.058	0.050	0.052	0.046	0.436	0.178	0.028
Alpha 3 Factor	0.005	0.011	0.022	0.107	0.139	0.149	0.071	0.334	0.212	0.113	0.209	0.220	0.155	0.142	0.120	0.121	0.621	0.270	0.106
Alpha 4 Factor	0.024	0.034	0.064	0.280	0.310	0.399	0.251	0.685	0.515	0.272	0.406	0.314	0.344	0.294	0.260	0.261	0.586	0.317	0.239

Table 8: Empirical performance metrics for the constrained minimum-variance portfolios with 6 and 12 months portfolio revision frequency over the period from April 1968 to December 2007

The table reports summarizing portfolio statistics as well as the annualized empirical performance metrics for the constrained minimum-variance portfolios with the respective maximum weight a single stock may have in the portfolio. The summary statistics show the mean number of stocks in the portfolio as well as the Jarque-Berra test p-values for the hypothesis that the portfolio returns are normally distributed. The certainty equivalent measure (CEQ) is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. The market factor benchmark for every alpha is reported in brackets, where EW denotes the equally weighted market benchmark portfolio (proxied by the CRSP equally weighted market index) and VW the value weighted market benchmark portfolio (proxied by the CRSP value weighted market index). Panel I reports the results for the six months portfolio revision frequency, while panel II does so for the twelve months portfolio revision frequency. The performance metrics for both benchmark portfolios are, for the purpose of vis-a-vis performance comparison, reported as well.

	Maximum Weights																			Benchmarks	
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**	**VW**	**EW**
Panel I: 6 months adjustment frequency																					
Mean no. of Stocks	53	38	29	25	22	20	18	17	16	16	15	14	14	14	13	13	12	12	12		
Jarque-Berra p-values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.070	0.072	0.070	0.067	0.074	0.081	0.086	0.090	0.096	0.091	0.100	0.107	0.092	0.094	0.095	0.100	0.101	0.094	0.106	0.056	0.144
Std. Dev.	0.132	0.131	0.134	0.139	0.140	0.145	0.150	0.154	0.159	0.161	0.164	0.168	0.171	0.174	0.174	0.179	0.183	0.184	0.193	0.156	0.198
CEQ	0.035	0.038	0.034	0.028	0.034	0.039	0.041	0.042	0.046	0.039	0.046	0.051	0.034	0.033	0.035	0.036	0.034	0.026	0.032	0.008	0.065
Sharpe Ratio	0.530	0.555	0.523	0.482	0.526	0.557	0.571	0.583	0.605	0.563	0.609	0.639	0.538	0.539	0.548	0.557	0.550	0.509	0.551	0.362	0.725
Sortino Ratio	0.929	0.984	0.949	0.867	0.944	1.006	1.044	1.085	1.132	1.053	1.170	1.221	1.017	1.024	1.033	1.044	1.022	0.943	1.025	0.514	1.056
Alpha 1 Factor (EW)	0.030	0.035	0.034	0.031	0.038	0.046	0.050	0.055	0.061	0.056	0.066	0.074	0.058	0.059	0.062	0.066	0.068	0.060	0.074		
Alpha 1 Factor (VW)	0.030	0.034	0.033	0.030	0.037	0.045	0.050	0.054	0.060	0.055	0.065	0.074	0.056	0.058	0.061	0.064	0.067	0.060	0.071		
Alpha 3 Factor (EW)	0.025	0.031	0.031	0.029	0.037	0.046	0.049	0.055	0.062	0.057	0.068	0.073	0.059	0.062	0.063	0.068	0.069	0.060	0.078		
Alpha 3 Factor (VW)	0.013	0.020	0.020	0.018	0.027	0.035	0.040	0.045	0.051	0.047	0.058	0.065	0.050	0.052	0.054	0.058	0.059	0.051	0.067		
Alpha 4 Factor (EW)	0.007	0.013	0.011	0.006	0.014	0.023	0.024	0.029	0.035	0.031	0.038	0.048	0.031	0.034	0.036	0.039	0.042	0.036	0.050		
Alpha 4 Factor (VW)	0.008	0.014	0.012	0.006	0.015	0.024	0.027	0.031	0.037	0.033	0.041	0.052	0.034	0.037	0.039	0.042	0.044	0.039	0.051		
Panel II: 12 months adjustment frequency																					
Mean no. of Stocks	53	38	30	25	23	21	18	17	17	16	16	13	15	15	14	13	12	13	12		
Jarque-Berra p-values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.066	0.066	0.068	0.064	0.070	0.073	0.077	0.083	0.089	0.093	0.093	0.100	0.099	0.107	0.105	0.111	0.111	0.104	0.114	0.056	0.144
Std. Dev.	0.133	0.130	0.132	0.138	0.139	0.144	0.147	0.150	0.156	0.158	0.158	0.161	0.162	0.165	0.167	0.170	0.169	0.169	0.174	0.156	0.198
CEQ	0.031	0.032	0.033	0.027	0.031	0.031	0.034	0.038	0.040	0.043	0.043	0.048	0.047	0.052	0.049	0.054	0.053	0.047	0.053	0.008	0.065
Sharpe Ratio	0.500	0.509	0.513	0.469	0.503	0.506	0.522	0.554	0.571	0.589	0.585	0.622	0.612	0.645	0.629	0.655	0.654	0.617	0.654	0.362	0.725
Sortino Ratio	0.877	0.888	0.916	0.843	0.907	0.914	0.930	1.011	1.046	1.094	1.093	1.139	1.149	1.235	1.190	1.243	1.240	1.174	1.230	0.514	1.056
Alpha 1 Factor (EW)	0.026	0.027	0.030	0.027	0.034	0.036	0.042	0.049	0.054	0.058	0.059	0.069	0.065	0.072	0.072	0.078	0.080	0.072	0.081		
Alpha 1 Factor (VW)	0.026	0.027	0.030	0.027	0.033	0.035	0.040	0.047	0.053	0.057	0.057	0.066	0.063	0.070	0.070	0.076	0.077	0.070	0.080		
Alpha 3 Factor (EW)	0.019	0.023	0.026	0.023	0.029	0.032	0.038	0.047	0.052	0.058	0.059	0.071	0.065	0.072	0.073	0.080	0.082	0.072	0.082		
Alpha 3 Factor (VW)	0.008	0.013	0.015	0.011	0.018	0.021	0.026	0.035	0.041	0.047	0.048	0.060	0.054	0.062	0.063	0.070	0.072	0.062	0.072		
Alpha 4 Factor (EW)	0.006	0.008	0.011	0.004	0.009	0.011	0.011	0.022	0.025	0.032	0.031	0.041	0.033	0.039	0.041	0.048	0.052	0.042	0.054		
Alpha 4 Factor (VW)	0.007	0.010	0.012	0.004	0.010	0.012	0.013	0.023	0.026	0.034	0.032	0.040	0.035	0.042	0.043	0.050	0.054	0.045	0.056		

Table 9: Performance metric inference for the constrained minimum-variance portfolios with 6 months portfolio revision frequency over the period from April 1968 to December 2007

The table reports the bootstrapped p-values for the performance metrics for each constrained minimum-variance portfolio with the respective maximum weight a single stock may have in the portfolio over the complete sample period from April 1968 to December 2007. We use an observation bootstrap for the statistical performance metric inference concerning the Sharpe ratio, Sortino ratio and certainty equivalents; inference concerning the different alpha measures is based on an residual bootstrap. Both bootstrap methodologies are outline in detail in section 2.3. The difference in certainty equivalent measures (CEQ) is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. The market factor benchmark for every alpha is reported in brackets, where EW denotes the equally weighted market benchmark (proxied by the CRSP equally weighted market index) and VW the value weighted market benchmark (proxied by the CRSP value weighted market index). Panel I reports the results for the value weighted benchmark portfolio, while panel II does so for the equally weighted benchmark portfolio.

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
Panel I: Value weighted benchmark																			
Δ Mean	0.156	0.133	0.204	0.293	0.168	0.087	0.068	0.050	0.030	0.062	0.026	0.011	0.063	0.052	0.049	0.037	0.046	0.077	0.035
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	0.983	0.866	0.663	0.376	0.283	0.205	0.109	0.060	0.031	0.035	0.006	0.003	0.002	0.001
Δ CEQ	0.030	0.023	0.052	0.116	0.064	0.043	0.048	0.044	0.033	0.082	0.036	0.027	0.127	0.133	0.129	0.119	0.156	0.245	0.184
Δ Sharpe Ratio	0.030	0.025	0.068	0.145	0.076	0.048	0.049	0.045	0.031	0.080	0.033	0.022	0.105	0.098	0.094	0.090	0.096	0.180	0.115
Δ Sortino Ratio	0.024	0.017	0.046	0.099	0.054	0.034	0.037	0.031	0.023	0.053	0.022	0.013	0.069	0.067	0.073	0.065	0.078	0.137	0.088
Alpha 1 Factor	0.013	0.008	0.016	0.034	0.015	0.008	0.005	0.003	0.002	0.009	0.001	0.000	0.007	0.007	0.005	0.005	0.007	0.010	0.004
Alpha 3 Factor	0.016	0.010	0.018	0.027	0.011	0.004	0.003	0.000	0.000	0.003	0.000	0.000	0.002	0.002	0.002	0.001	0.002	0.005	0.001
Alpha 4 Factor	0.302	0.192	0.224	0.329	0.200	0.097	0.090	0.067	0.044	0.050	0.031	0.017	0.070	0.060	0.045	0.036	0.042	0.065	0.015
Panel II: Equally weighted benchmark																			
Δ Mean	0.789	0.751	0.786	0.816	0.725	0.609	0.525	0.461	0.374	0.461	0.335	0.243	0.431	0.407	0.382	0.329	0.326	0.406	0.285
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.989	0.975	0.968	0.942	0.874	0.861	0.680
Δ CEQ	0.107	0.090	0.134	0.201	0.135	0.104	0.096	0.092	0.078	0.126	0.083	0.063	0.178	0.187	0.165	0.163	0.179	0.268	0.226
Δ Sharpe Ratio	0.201	0.156	0.254	0.374	0.243	0.183	0.166	0.149	0.116	0.191	0.118	0.084	0.249	0.246	0.220	0.206	0.220	0.317	0.239
Δ Sortino Ratio	0.217	0.161	0.227	0.342	0.240	0.176	0.158	0.134	0.107	0.164	0.094	0.070	0.205	0.207	0.194	0.180	0.203	0.279	0.230
Alpha 1 Factor	0.014	0.009	0.014	0.024	0.012	0.007	0.004	0.003	0.000	0.009	0.000	0.000	0.005	0.006	0.007	0.006	0.006	0.010	0.003
Alpha 3 Factor	0.019	0.012	0.021	0.029	0.012	0.001	0.001	0.001	0.000	0.002	0.000	0.000	0.001	0.002	0.002	0.000	0.002	0.004	0.001
Alpha 4 Factor	0.304	0.190	0.223	0.333	0.195	0.088	0.096	0.063	0.047	0.061	0.031	0.013	0.072	0.074	0.059	0.040	0.041	0.070	0.021

Table 10: Performance metric inference for the constrained minimum-variance portfolios with 12 months portfolio revision frequency over the period from April 1968 to December 2007

The table reports the annualized empirical performance metrics for the constrained minimum-variance portfolios with the respective maximum weight a single stock may have in the portfolio over the complete sample period from April 1968 to December 2007. The certainty equivalent measure (CEQ) is reported for a risk aversion coefficient of $\gamma = 4$. Alpha measures are computed for the described factor models in section 2.2. Hereby denotes "Alpha 1 Factor" the alpha measure based on the one factor model, while "Alpha 3 Factor" and "Alpha 4 Factor" do so for the three and four factor model. The market factor benchmark for every alpha is reported in brackets, where EW denotes the equally weighted market benchmark (proxied by the CRSP equally weighted market index) and VW the value weighted market benchmark (proxied by the CRSP value weighted market index). Panel I reports the results for the value weighted benchmark portfolio, while panel II does so for the equally weighted benchmark portfolio.

	Maximum Weights																		
	2%	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**	**13%**	**14%**	**15%**	**16%**	**17%**	**18%**	**19%**	**20%**
Panel I: Value weighted benchmark																			
Δ Mean	0.240	0.275	0.273	0.375	0.245	0.188	0.153	0.099	0.059	0.048	0.053	0.039	0.034	0.014	0.020	0.014	0.017	0.028	0.010
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	0.996	0.982	0.908	0.473	0.394	0.373	0.228	0.234	0.100	0.083	0.042	0.049	0.063	0.019
Δ CEQ	0.053	0.059	0.072	0.175	0.100	0.103	0.094	0.071	0.061	0.051	0.058	0.054	0.047	0.029	0.043	0.038	0.041	0.064	0.039
Δ Sharpe Ratio	0.069	0.076	0.093	0.211	0.134	0.114	0.109	0.082	0.060	0.053	0.054	0.049	0.046	0.026	0.042	0.029	0.032	0.055	0.031
Δ Sortino Ratio	0.049	0.056	0.066	0.161	0.096	0.082	0.078	0.050	0.046	0.034	0.040	0.040	0.029	0.016	0.027	0.022	0.027	0.039	0.021
Alpha 1 Factor	0.025	0.032	0.023	0.042	0.022	0.016	0.011	0.007	0.007	0.001	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Alpha 3 Factor	0.060	0.038	0.033	0.055	0.023	0.021	0.014	0.006	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Alpha 4 Factor	0.293	0.243	0.217	0.317	0.247	0.233	0.251	0.113	0.101	0.055	0.064	0.038	0.059	0.040	0.029	0.011	0.008	0.022	0.005
Panel II: Equally weighted benchmark																			
Δ Mean	0.818	0.821	0.799	0.833	0.775	0.727	0.657	0.582	0.488	0.431	0.444	0.364	0.360	0.276	0.301	0.229	0.237	0.302	0.217
Δ Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994
Δ CEQ	0.146	0.142	0.147	0.237	0.179	0.180	0.152	0.122	0.117	0.094	0.106	0.079	0.085	0.071	0.083	0.066	0.074	0.108	0.081
Δ Sharpe Ratio	0.318	0.296	0.300	0.442	0.341	0.318	0.299	0.224	0.191	0.160	0.189	0.131	0.139	0.100	0.122	0.104	0.094	0.152	0.112
Δ Sortino Ratio	0.315	0.316	0.293	0.403	0.315	0.298	0.287	0.207	0.183	0.145	0.159	0.120	0.113	0.081	0.114	0.095	0.092	0.134	0.111
Alpha 1 Factor	0.038	0.034	0.030	0.050	0.022	0.019	0.014	0.008	0.007	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Alpha 3 Factor	0.054	0.038	0.037	0.053	0.023	0.020	0.015	0.004	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Alpha 4 Factor	0.294	0.243	0.205	0.325	0.252	0.236	0.231	0.112	0.087	0.051	0.052	0.031	0.051	0.035	0.025	0.012	0.009	0.019	0.005

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