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Empirical Asset Pricing with Multi-Period Disaster Risk
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Abstract

We propose a simulation-based strategy to estimate and empirically assess a class of asset pricing models that account for rare but severe consumption contractions that can extend over multiple periods. Our approach expands the scope of prevalent calibration studies and tackles the inherent sample selection problem associated with measuring the effect of rare disaster risk on asset prices. An analysis based on postwar U.S. and historical multi-country panel data yields estimates of investor preference parameters that are economically plausible and robust with respect to alternative specifications. The estimated model withstands tests of validity; the model-implied key financial indicators and timing premium all have reasonable magnitudes. These findings suggest that the rare disaster hypothesis can help restore the nexus between the real economy and financial markets when allowing for multi-period disaster events. Our methodological contribution is a new econometric framework for empirical asset pricing with rare disaster risk.

Key words: empirical asset pricing, multi-period disasters, simulation-based estimation

JEL: C58, G12

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1 Introduction

According to Rietz's (1988) rare disaster hypothesis (RDH), the high risk premium for U.S. equity during the postwar period arose because investors ex ante demanded compensation for possibly disastrous but unlikely consumption risks that they never suffered from ex post. In turn, the RDH could help resolve the equity premium puzzle and explain the notoriously poor empirical performance of consumption/preference-based asset pricing models (C-CAPMs). However, the RDH is difficult to assess econometrically using data that contain very few, if any, disastrous consumption contractions. Many studies therefore resort to calibration methods. The few econometric studies that exist provide mixed evidence regarding the explanatory power of the RDH when it is possible for disastrous consumption contractions to build up over multiple time periods.

We contribute to this discussion by proposing a novel methodology to resolve the inherent sample selection problem that hampers empirical assessments of the RDH. Following a recommendation by Blanchard (2008), we identify the parameters of interest through moment conditions that are implied by the basic asset pricing equation of a disaster-including C-CAPM. By allowing for multi-period disaster events, conceived of as a marked point process (MPP), we take account of the caveat that the apparent success of the RDH may hinge on the assumption that consumption disasters unfold within a single period.

The empirical challenges call for a non-standard methodological approach. Inspired by ideas put forth by Dridi et al. (2007), our proposed simulation-based strategy combines the advantages of econometric analysis – namely, the appraisal of identifying restrictions, loss functions, and conditions for validity – with calibration practices, such as the use of different data to estimate different parts of a model and the treatment of unidentified parameters. The analysis proceeds in two steps: It uses maximum likelihood to estimate the MPP parameters based on multi-country data that contain disastrous consumption contractions, and then it undertakes a simulation-based estimation of the investor preference parameters using regular macroeconomic and financial data. The two-step approaches advanced by Christiano and Eichenbaum (1992), Cecchetti et al. (1993), and Heaton (1995) are early progenitors of such a strategy. As recommended by Dridi et al. (2007) and Hansen and Heckman (1996), who explore the link between econometric analysis and calibration practices, our empirical assessment focuses on the plausibility of the preference parameter estimates, and we test whether the estimated model can explain key economic indicators such as the market equity premium and Sharpe ratio.

By applying this new methodology to a combination of U.S. and multi-country data, we obtain economically plausible estimates of the investor preference parameters: the time discount factor, relative risk aversion (RRA), and intertemporal elasticity of substitution (IES). The estimates of the time discount factor are smaller than but close to unity, as expected of an investor with a positive rate of time preference and quarterly decision frequency. In line with experimental evidence provided by Meyer and Meyer (2005), Cochrane (2005) caps the upper bound of reasonable RRA at 5, stricter than Mehra and Prescott's (1985) often-cited upper bound of 10. With some variation due to the selection of test assets, we obtain RRA estimates in a range around 1.5. The 95% confidence bounds for the RRA coefficient also fall within the tighter range of plausibility. The estimates of the IES are significantly greater than unity and of a magnitude that is conveniently chosen for calibration studies. Moreover, the difference of the estimated IES and reciprocal of the RRA estimate is greater than 0, which indicates a preference for early resolution of uncertainty. Previous studies suggest that an IES > 1, in combination with a preference for early resolution, are necessary to obtain meaningful economic implications (e.g., Bansal and Yaron (2004); Epstein et al. (2014)).

Accordingly, key financial indicators – the market equity premium, mean T-bill return, and market Sharpe ratio – implied by these parameter estimates exhibit magnitudes that are plausible and consistent with the empirically observed counterparts. Moreover, the model-implied timing premium, defined by Epstein et al. (2014) as the fraction of lifetime consumption that one would relinquish to resolve consumption risk, is economically sensible. This result is noteworthy, because many estimated or calibrated C-CAPMs imply an implausibly high timing premium. The reported findings also are invariant to alternative model specifications (e.g., disaster definition, MPP model, data simulation procedure).

Empirical C-CAPM studies often produce implausible and/or imprecise parameter estimates that entail doubtful asset pricing implications. The results presented herein suggest instead that allowing for rare disasters within a preference-based asset pricing framework can help restore the link between financial markets and the real economy, even when allowing for multi-period disasters.

The RDH literature stream triggered by Barro (2006), to which our paper contributes and draws inspiration from, has been lucidly summarized by Tsai and Wachter (2015). Among studies that link the RDH to various aspects of financial economics,[1] some relate particularly closely to our contribution. Specifically, Barro and Ursúa (2008) collect historical consumption data to study the size and frequency of disasters. These data also enable Barro and Jin (2011) to fit power-law densities to the empirical distribution of macroeconomic disasters. The first-step estimation strategy proposed herein draws on their ideas. Nakamura et al. (2013) also consider

[1] These aspects include index options (Backus et al. (2011)), the business cycle (Gourio (2012)), the volatility puzzle (Wachter (2013)), credit spreads (Gourio (2013)), CDO prices (Seo and Wachter (2018)), the volatility skew (Seo and Wachter (2019)), the value premium (Tsai and Wachter (2016); Bai et al. (2019)), the persistence of dividend and consumption growth (Gillman et al. (2015); Barro and Jin (2016)), the exchange rate puzzle (Farhi and Gabaix (2016)), and return predictability (Marfè and Penasse (2020)).

a multi-period disaster process but work within a Bayesian framework. They show that, when calibrated with a sensible rate of time preference and IES, the U.S. equity premium can be explained with a plausible RRA. Our frequentist approach complements and extends their Bayesian analysis.

In many of the papers surveyed by Tsai and Wachter (2015), disasters occur as one-period events. This assumption is under suspicion of being a driving force of the apparent success of the RDH, as suggested by Constantinides's (2008) comment on Barro and Ursúa's (2008) work and as also argued by Julliard and Ghosh (2012). They offer one of the rare comprehensive econometric analyses (using an empirical likelihood approach) to assess the RDH. By allowing for multi-period disasters and modeling investor preferences with a time-additive power utility function, Julliard and Ghosh (2012) conclude that to rationalize the equity premium puzzle with the help of the RDH, the puzzle itself must be a rare event. Their results thus attenuate the appeal of the rare disaster explanation. Using a novel econometric methodology that employs global information about historical disasters instead of overweighting the U.S. experience, the present study re-emphasizes the explanatory power of the RDH when disasters can be multi-period events. To reach this conclusion though, time-additive power utility must be abandoned; it is necessary to allow for nonindifference to the temporal resolution of risk. The (im)possibility to account for a late resolution premium offers a deeper explanation for the apparently disparate results reported by Julliard and Ghosh (2012) on the one hand and Nakamura et al. (2013) and our study on the other.

The remainder of this paper is structured as follows: Section 2 links the basic asset pricing equation to a multi-period disaster process that is described by an MPP. This combination yields moment conditions that identify the parameters of interest and provide the foundations of a two-step estimation approach. Section 3 outlines the econometric strategy, and then Section 4 describes its implementation, taking into account the limitations of the available data. Section 5 presents the results of the empirical analysis and plausibility checks. By contrasting our results with previous literature, we also help reconcile some apparent disparities in assessments of the explanatory power of the RDH. A comparison of the proposed econometric strategy with related approaches is provided in Section 6. Section 7 concludes.

2 A C-CAPM with multi-period rare disaster risk

Blanchard (2008) criticizes Barro and Ursúa's (2008) analysis of the RDH that relies on a calibrated Lucas-tree model (also used by Barro (2006) and subsequent studies), which he calls a straitjacket for that purpose. Kim and Pagan (1999, p. 328) argue that when using such a model, "the specification errors being committed are of sufficient magnitude to make conventional estimation and testing of dubious value." This caveat is one reason for the prevalence of calibration studies and the limited amount of econometric work on the RDH. In the debate between advocates of calibration and econometricians, we concur with Dridi et al.'s (2007) view that even if a model is partially misspecified, economic reality should be captured by certain parameters of interest that one should aim to estimate consistently. Moreover, with regard to empirical assessments of the RDH, we agree with Blanchard (2008, p. 86), who sees "no reason not to go back to asset pricing formulas that rely only on the first-order intertemporal condition of consumers with no additional assumptions."

How can these ideas offered by Dridi et al. (2007) and Blanchard (2008) be operationalized though? We start by employing a discrete-time framework, a representative investor with recursive Epstein-Zin-Weil (EZW) preferences (Epstein and Zin (1989); Weil (1989)), and a constant decision frequency. As shown by Epstein and Zin (1989), the aforementioned intertemporal conditions, applied to a gross return on an asset i, R^i, imply:

$$\mathbb{E}\left[m_{t+1}(\boldsymbol{\theta}_1)R^i_{t+1}|I_t\right] = \text{price}(R^i_{t+1};\boldsymbol{\theta}_1) = 1, \tag{2.1}$$

where I_t is the investor's information set at time t, and $\boldsymbol{\theta}_1 = (\beta, \gamma, \psi)'$ contains the time discount factor β, the RRA coefficient γ, and the IES ψ. The EZW stochastic discount factor (SDF) reads:

$$m_{t+1}(\boldsymbol{\theta}_1) = \beta^\theta G_{t+1}^{-\theta/\psi}(R^a_{t+1})^{\theta-1}, \tag{2.2}$$

with $\theta = (1-\gamma)/(1-\psi^{-1})$, where G denotes gross consumption growth, and R^a is the return on aggregate wealth.

The basic asset pricing equation (2.1), suitably reformulated, provides the starting point to allow for the risk of multi-period consumption disasters. For this purpose, we first have to clarify what constitutes such an event. Barro (2006) defines a disaster as a contraction $\bar{b} \in [q, 1]$ of regular gross consumption growth G^r, where the threshold $q > 0$ differentiates regular bad times from disasters. This notion of a disaster threshold must be adapted when accounting for disasters that can span more than one period. Accordingly, we define a multi-period disaster as a succession of consumption contractions that starts in period s_1 and lasts until period s_2, such that the overall contraction exceeds the threshold value q:

$$1 - \prod_{j=s_1}^{s_2}(1 - b_j) \geq q, \tag{2.3}$$

where $b_j > 0$ is a period-specific random contraction factor. A *disaster event* thus describes a contraction in consumption at least of the size of the threshold q that may accrue over multiple *disaster periods* or come in the form of one sharp, single-period downturn. When dealing with multi-period disaster events, it is useful to introduce two binary indicators. The first, denoted by d_t, equals 1 if period t is part of a disaster event, and 0 otherwise. The second, denoted by d_t^+, equals 1 if period t belongs to a disaster event that began in period $s_1 \leq t$, but the accumulated contractions up to t have not yet reached the disaster threshold q, and 0 otherwise.

Adopting a prevalent specification from the rare disaster literature, we assume that the consumption process can be described by a regular consumption growth component G_t^r that is disturbed by a random contraction b_t in case of a disaster, such that:

$$G_t = G_t^r \cdot (1 - b_t)^{d_t}. \tag{2.4}$$

This multiplicative relation of regular and disaster growth component is a characteristic feature of the rare disaster models by Barro (2006), Barro and Ursúa (2008), Barro (2009), Barro and Jin (2011), and Wachter (2013). Analogous to (2.4), we assume that an asset return R_t^i can be conceived of as a regular gross return R_t^{ir} that is perturbed by a random factor c_t^i in case of a disaster:

$$R_t^i = R_t^{ir} \cdot (1 - c_t^i)^{d_t}. \tag{2.5}$$

This formulation is motivated by a return specification that can be found in Barro (2006). His Lucas-tree economy implies an expression for the return on the equity claim that inherits the multiplicative relationship of the regular and contraction components reflected in Eq. (2.4). Using Eq. (2.5), we extend this notion to other asset returns, allowing for asset-specific contraction factors.[2]

The sequence of disaster events with associated consumption and return contrac-

[2] For details on how Eqs. (2.4) and (2.5) relate to the work of Barro (2006) and Barro and Jin (2011), see Section O.1 of the Online Appendix. The aforementioned papers from the rare disaster literature rely on distributional assumptions about the regular and contraction components. As we outline subsequently, we use the empirical distribution of the non-disastrous consumption growth and asset returns to provide information about the distributional properties of regular components, and invoke distributional assumptions only for the contraction components.

tions can be described by an MPP observed in discrete time.[3] Discrete time means that a disaster event originates in some time period (e.g., certain quarter of a year) and lasts for at least the originating period, but possibly longer, in which case it becomes a multi-period disaster. In MPP terminology, the disaster periods (dates t for which $d_t = 1$) are called *points*. According to Hamilton and Jordà (2002), the distribution of the points can be suitably described by the discrete-time hazard rate, which gives the probability of period t being a disaster period, conditional on the information set Υ_{t-1}:

$$h_t = \Pr(d_t = 1|\Upsilon_{t-1}) = \Pr(N(t) \neq N(t-1)|\Upsilon_{t-1}), \quad t \in \{1, 2, \dots, T\}, \tag{2.6}$$

where $N(t)$ is a function that counts the number of disaster periods that occurred as of t. We assume that $I_{t-1} \subset \Upsilon_{t-1}$; in particular, while d_{t-1} and d^+_{t-1} are contained in Υ_{t-1}, they are not included in I_{t-1}. The investor does not know whether a consumption contraction (or a series of them) eventually will build up into a multi-period disaster. The variables observed in a disaster period are called *marks* in MPP terminology; they describe the disaster event. The period-specific contraction factors are important marks.

An MPP that accounts for the discrete hazard rate in (2.6), as well as the distribution of the marks conditional on Υ_{t-1} and $d_t = 1$, provides a comprehensive description of the disaster process, which facilitates a reformulation of the asset pricing equation (2.1). That is, stepping back one period, conditioning down, applying the law of total expectation (LTE), and using $p = \Pr(d_t = 1) = \mathbb{E}[d_t]$, we obtain:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R^i_t\right] = p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R^i_t\middle| d_t = 1\right] + (1-p)\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R^i_t\middle| d_t = 0\right] = 1. \tag{2.7}$$

[3] The use of MPPs stems from the modeling of natural disasters like earthquakes, which occur infrequently in time and with different magnitudes (Ogata and Katsura (1986)).

Rearranging terms yields:[4]

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^i \middle| d_t = 0\right] = \frac{1 - p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^i \middle| d_t = 1\right]}{1-p}. \tag{2.8}$$

Note that if disaster events were impossible, such that $p = 0$, then Eq. (2.8) would reduce to:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^i \middle| d_t = 0\right] = 1, \tag{2.9}$$

which provides the basis for a generalized method of moments (GMM) estimation performed on disaster-free data. However, with the possible occurrence of disaster events, the moment restriction in Eq. (2.9) is not correct; the right-hand side does not equal 1. In effect, the RDH suggests that this sample selection-induced misspecification could be the reason for the underwhelming empirical performance of C-CAPMs.

Eq. (2.8) can be further rewritten to replace the conditional expectations with unconditional moments, which has pricing implications for two particular payoffs. To demonstrate these implications, we use the LTE and reformulate the left-hand side of Eq. (2.8) as:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^i \middle| d_t = 0\right] = \frac{\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)x_t^i\right]}{\Pr(d_t = 0)} = \frac{\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)x_t^i\right]}{1-p} = \frac{\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)x_t^i\right]}{1-\mathbb{E}[d_t]}, \tag{2.10}$$

where $x_t^i = R_t^i \cdot (1 - d_t)$ can be conceived of as a payoff that equals 0 in every disaster

[4] For the sake of a concise exposition, the main text focuses on gross returns. The same reasoning can be applied to an excess return R_t^{ei} (the difference of two gross returns, as in a zero-cost portfolio), for which the reformulated basic asset pricing equation reads:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^{ei} \middle| d_t = 0\right] = -\frac{p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^{ei} \middle| d_t = 1\right]}{1-p}.$$

Appendix A.1 delineates the relevant steps. An alternative to conditioning down (2.1) would be to exploit the orthogonality conditions entailed by the basic asset pricing equation, such that for every instrument $z_{t-1} \in I_{t-1}$, $\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^{ei}z_{t-1}\right] = 0$. The analysis would then use $R_t^{ei}z_{t-1}$ instead of R_t^{ei}. As pointed out by Cochrane (1996), $R_t^{ei}z_{t-1}$ can be conceived of as the payoff of a managed portfolio, such that economically meaningful instruments should be chosen.

period and that is, for a limited liability security like a stock, non-negative in regular periods. The numerator on the right-hand side of Eq. (2.10) gives the expected value of the price of this non-disaster payoff, because:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)x_t^i\right] = \mathbb{E}\left(\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)x_t^i|I_{t-1}\right]\right) = \mathbb{E}[\text{price}(x_t^i;\boldsymbol{\theta}_1)]. \tag{2.11}$$

Correspondingly, the right-hand side of Eq. (2.8) can be written as:

$$\frac{1-p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)R_t^i|\, d_t=1\right]}{1-p} = \frac{1-\mathbb{E}\left[m_t(\boldsymbol{\theta}_1)y_t^i\right]}{1-\mathbb{E}[d_t]} = \frac{1-\mathbb{E}[\text{price}(y_t^i;\boldsymbol{\theta}_1)]}{1-\mathbb{E}[d_t]}, \tag{2.12}$$

where $y_t^i = R_t^i \cdot d_t$ can be conceived of as a payoff that is 0 in regular periods and, for a limited liability security, non-negative in disaster periods.

We can now express the reformulated asset pricing equation (2.8) in terms of an unconditional moment restriction that relates the pricing of the disaster payoff y and the pricing of the non-disaster payoff x. For that purpose, let us denote by $\boldsymbol{\theta}_1^0 = (\beta^0,\gamma^0,\psi^0)'$ the parameter values of the preference parameters that, when used for the SDF in (2.2), satisfy Eq. (2.8). This reformulated pricing equation and the subsequent manipulations conceal how the identification of the preference parameters is intimately connected to the MPP that describes the disaster process. While this MPP represents a model of the true data-generating process, we assume that the model-implied discrete-time hazard rate and the conditional probability distribution of the marks correspond to those of the true disaster process. Let $\boldsymbol{\theta}_2^0$ denote the parameters that govern this *defining MPP*. Using Eqs. (2.10), (2.11), and (2.12), an unconditional moment restriction that identifies $\boldsymbol{\theta}_1^0$ can be written comprehensively as:

$$\frac{\mathbb{E}[\text{price}(x_t^i;\boldsymbol{\theta}_1^0);\boldsymbol{\theta}_2^0]}{1-\mathbb{E}[d_t;\boldsymbol{\theta}_2^0]} = \frac{1-\mathbb{E}[\text{price}(y_t^i;\boldsymbol{\theta}_1^0);\boldsymbol{\theta}_2^0]}{1-\mathbb{E}[d_t;\boldsymbol{\theta}_2^0]}, \tag{2.13}$$

which highlights the dependence on the disaster process described by the defining

MPP. This reformulation of the basic asset pricing equation motivates an econometric strategy to replace the unconditional population moments in Eq. (2.13) by sample moments, as well as to exploit the resulting moment matches to estimate $\boldsymbol{\theta}_1^0$. As a distinctive feature, it uses actual data to compute sample moments for the left-hand side of Eq. (2.13), but to take account of the inherent sample selection problem, it relies on simulated moments for the right-hand side. This differential treatment of the left- and right-hand sides of Eq. (2.13) is the reason we do not cancel the common denominator.

Eq. (2.13), as Dridi et al. (2007) assert, intrinsically defines the preference parameters by an economic paradigm (the existence and specification of an SDF) and it highlights, through its dependence on the MPP parameters, the role of the disaster process. It thus represents an operationalization of Blanchard's (2008) suggestion to rely on the basic asset pricing equations when allowing for rare disaster risk.

We concede that there may be limited theoretical justification for the assumption of a representative agent; in this study, we require everyone to have identical EZW preferences. Nevertheless, we treat the representative agent as a real individual when assessing the plausibility of the preference parameter estimates. Put differently, we start from the premise that the EZW-SDF in Eq. (2.2), equipped with reasonable values for time discount factor, RRA, and IES, can price assets that are traded in real markets – the economic reality that Dridi et al. (2007) call on researchers to capture.[5]

[5] Our approach supports the use of other pricing kernels as well. For example, we provide a robustness check using the power utility SDF.

3 Econometric strategy

3.1 Motivating a two-step approach

In this section, we motivate and outline our econometric strategy before turning to its concrete implementation in the next section. From Dridi et al.'s (2007) econometric analysis of DSGE models, we adopt their distinction of two types of parameters. The parameters of the first type ("economic parameters of interest") account for an economic agent's preferences, like aversion to risk, and they can be interpreted accordingly. For the current study context, these type 1 parameters are those in $\boldsymbol{\theta}_1^0$. Dridi et al.'s (2007) type 2 parameters ("structural statistical parameters") are not directly associated with economic behavior, but they take account of aspects of the distribution of the model variables that are necessary to generate model-implied data in the course of a simulation-based estimation procedure. In the present context, they are the parameters that govern the defining MPP, $\boldsymbol{\theta}_2^0$.

As is a hallmark of calibration studies, but in contrast with standard econometric analysis, we rely on profoundly different data sources. The information about disaster events needed to estimate the parameters of the defining MPP can be retrieved from historical multi-country panel data. Unfortunately, these data do not contain much information about financial variables, which necessitates the calibration of some parameters. The data typically used for asset pricing studies instead are rich in terms of information about the time-series properties and the cross-section of asset prices, but they contain little information about disaster events. The two-step approach described hereafter is designed to make use of the disparate data sources in a coherent way by estimating the parameters of the defining MPP in the first step, then using the results for a simulation-based estimation of the preference parameters in the second.

3.2 First-step strategy

We have emphasized the significance of the defining MPP that, endowed with parameter values $\boldsymbol{\theta}_2^0$ and combined with the basic asset pricing equation, identifies the economic parameters of interest. The objective in the first step is thus to estimate the MPP parameters, which entails providing a specification of the defining MPP. For that purpose, one could consider Hamilton and Jordà's (2002) autoregressive conditional hazard model for h_t in Eq. (2.6) or transfer ideas from natural disaster literature to take account of the spatial distribution of consumption contractions (e.g., Ogata (1998)). The conditional distribution of the marks also can be specified in a more or less complex way. The Bayesian analysis by Nakamura et al. (2013), for example, assumes an elaborate distribution of contractions over the course of a multi-period disaster event. The frequentist approach pursued herein must take into account more prudently the information content of the available data.

In general, and adopting the notation of Hamilton and Jordà (2002), we can write the conditional joint density of the points and marks of the multi-period disaster MPP as follows:

$$\begin{aligned} f(d_t, \mathbf{y}_t^d | \Upsilon_{t-1}; \boldsymbol{\theta}_{21}, \boldsymbol{\theta}_{22}) &= f(d_t | \Upsilon_{t-1}; \boldsymbol{\theta}_{21}) \times f(\mathbf{y}_t^d | d_t, \Upsilon_{t-1}; \boldsymbol{\theta}_{22}) \\ &= [h_t(\boldsymbol{\theta}_{21})]^{d_t} \times [1 - h_t(\boldsymbol{\theta}_{21})]^{1-d_t} \times \left(f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}) \right)^{d_t}, \end{aligned} \tag{3.1}$$

where $f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22})$ denotes the density of the marks $\mathbf{y}_t^d$, conditional on Υ_{t-1} and $d_t = 1$. The $K \times 1$ vector $\boldsymbol{\theta}_{21}$ contains the parameters that govern h_t, and the $L \times 1$ vector $\boldsymbol{\theta}_{22}$ denotes parameters of the conditional distribution of the marks. We note that some marks of interest will not be available due to data limitations – financial returns during disaster events are hard to obtain – such that $\mathbf{y}_t^d$ is a subset of the marks of interest. To allow for a different distribution of marks (in particular the contractions) prior to reaching the disaster threshold q, compared with the distribution after q has been reached, we can use:

$$f(\mathbf{y}_t^d|d_t, \Upsilon_{t-1}; \boldsymbol{\theta}_{22}) = \left(f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^+)^{d_t^+} \times f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^-)^{1-d_t^+}\right)^{d_t}, \tag{3.2}$$

where $f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^+)$ denotes the density of the marks conditional on Υ_{t-1}, $d_t = 1$, and $d_t^+ = 1$, whereas $f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^-)$ is the density of the marks conditional on Υ_{t-1}, $d_t = 1$, and $d_t^+ = 0$. Moreover, $\boldsymbol{\theta}_{22} = (\boldsymbol{\theta}_{22}^-{}', \boldsymbol{\theta}_{22}^+{}')'$.

Anticipating limitations of data availability, we have to acknowledge that some MPP parameters, which describe the contemporaneous dependence of consumption and return contraction factors, are unidentified and must be obtained from calibrator's knowledge. The parameters to be calibrated are collected in the vector $\boldsymbol{\theta}_{23}$; thus, the complete vector of MPP parameters is given by $\boldsymbol{\theta}_2 = (\boldsymbol{\theta}_{21}{}', \boldsymbol{\theta}_{22}{}', \boldsymbol{\theta}_{23}{}')'$. The density $f(d_t, \mathbf{y}_t^d|\Upsilon_{t-1}; \boldsymbol{\theta}_{21}, \boldsymbol{\theta}_{22})$ in Eq. (3.1) is specified, such that it does not depend on $\boldsymbol{\theta}_{23}$, and the estimation of $\boldsymbol{\theta}_{21}^0$ and $\boldsymbol{\theta}_{22}^0$ is not affected by the choice of calibrated values $\bar{\boldsymbol{\theta}}_{23}$.[6]

If discrete-valued time series data are available, from which multi-period disaster events and the associated contractions can be identified, the estimation of $\boldsymbol{\theta}_{21}^0$ and $\boldsymbol{\theta}_{22}^0$ can be performed as described by Hamilton and Jordà (2002). The strategy to obtain the parameter estimates is maximum likelihood (ML) with the associated parametric inference and testing procedures. Eqs. (3.1) and (3.2) imply the following conditional log-likelihood function:

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta}_{21}, \boldsymbol{\theta}_{22}) = &\sum_{t=1}^{T_1} \left(d_t \ln h_t(\boldsymbol{\theta}_{21}) + (1 - d_t) \ln[1 - h_t(\boldsymbol{\theta}_{21})]\right) \\ &+ \sum_{t=1}^{T_1} d_t \left(d_t^+ \ln f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^+) + (1 - d_t^+) \ln f_{\mathrm{M}}(\mathbf{y}_t^d; \boldsymbol{\theta}_{22}^-)\right),\end{aligned} \tag{3.3}$$

where T_1 denotes the number of time periods (e.g., quarters) in the first-step data set. The maximization of the log-likelihood in Eq. (3.3) with respect to the unknown

[6] We address the calibration of $\boldsymbol{\theta}_{23}$ in detail in Section 4.2.

parameters yields the ML estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$. As noted supra, we conceive of the defining MPP as a model of the disaster process, but one that implies the same disaster probabilities and conditional distribution of the marks as the true data-generating process. Assuming that the defining MPP is correctly specified, then under standard assumptions for extremum estimators (e.g., Hansen (2012)), $\hat{\boldsymbol{\theta}}_{21} \underset{p}{\rightarrow} \boldsymbol{\theta}_{21}^{0}$ and $\hat{\boldsymbol{\theta}}_{22} \underset{p}{\rightarrow} \boldsymbol{\theta}_{22}^{0}$.[7] A specification of the defining MPP, endowed with the estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$ as well as the calibrated values $\bar{\boldsymbol{\theta}}_{23}$ is an essential input for the second estimation step.

3.3 Second-step strategy

The moment restriction in Eq. (2.13) captures the asset pricing implications of a disaster-including C-CAPM and provides the basis for a simulation-based estimation of the economic parameters of interest $\boldsymbol{\theta}_1^0$. In this section, we motivate a strategy that entails estimating $\boldsymbol{\theta}_1^0$ by matching the sample equivalents of the moments in Eq. (2.10) with simulated moments used for Eq. (2.12). The first-step estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$, together with the calibrated values $\bar{\boldsymbol{\theta}}_{23}$, serve to perform the data simulation.

Assuming that the data series used in the second step span T_2 periods and contain the necessary consumption and return information, a sample equivalent to Eq. (2.10) can be computed as:

$$H_{T_2}^{i}(\boldsymbol{\theta}_1) \equiv \frac{\frac{1}{T_2}\sum_{t=1}^{T_2} m_t(\boldsymbol{\theta}_1)x_t^i}{1-\frac{1}{T_2}\sum_{t=1}^{T_2} d_t} = \frac{1}{T_2-\sum_{t=1}^{T_2} d_t}\sum_{t=1}^{T_2} m_t(\boldsymbol{\theta}_1)x_t^i. \tag{3.4}$$

As we detail in the Online Appendix, our proposed estimation strategy requires that a uniform law of large numbers (LLN) holds, such that the sample moments

[7] If $\boldsymbol{\theta}_{21}$ and $\boldsymbol{\theta}_{22}$ have no parameter in common, it is possible to perform the parameter estimation separately. The estimate $\hat{\boldsymbol{\theta}}_{21}$ can be obtained by maximizing the first part of the log-likelihood function in Eq. (3.3) with respect to $\boldsymbol{\theta}_{21}$, whereas $\hat{\boldsymbol{\theta}}_{22}$ can be obtained by maximizing the second part with respect to $\boldsymbol{\theta}_{22}$.

in (3.4) converge to the population moments in Eq. (2.10). Asymptotic concerns aside, $H^i_{T_2}$ would be expected to provide a useful approximation of its population counterpart in finite samples, because the sample size available for application is large compared with the number of disaster periods. Disasters are by definition rare events; it may be that in the sample used for second-step estimation, $d_t = 0$ for all periods. Accordingly, we should expect that the approximation

$$H^i_{T_2}(\boldsymbol{\theta}^0_1) \approx \frac{\mathbb{E}[\text{price}(x^i_t; \boldsymbol{\theta}^0_1); \boldsymbol{\theta}^0_2]}{1 - \mathbb{E}[d_t; \boldsymbol{\theta}^0_2]} \tag{3.5}$$

works well for realistic T_2. However, the approximation of population moments by sample moments fails for Eq. (2.12). Due to the rarity of disastrous consumption contractions, the data frequently used for empirical asset pricing studies contain very few, if any, disaster observations. The sample equivalents of the moments in Eq. (2.12) therefore must be computed on the basis of a very small number of observations or will not be available in the first place, as in the case of post-WWII U.S. data.[8]

However, by using a specification for the defining MPP, we can *simulate* consumption and return data and thereby approximate the population moments in Eq. (2.12) by simulated moments. With the same intent for which Singleton (2006, p. 254) advocates using the simulated method of moments (SMM), namely, that "more fully specified models allow experimentation with alternative formulations of economies and, perhaps, analysis of processes that are more representative of history for which data are not readily available," the MPP simulation should produce consumption and return data from alternative histories that include multi-period disasters. Accordingly, we use the first-step estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$ and the calibrated MPP parameter values $\bar{\boldsymbol{\theta}}_{23}$ to provide simulated moments for an approximation of

[8] The United States faced only two major consumption downturns during the 20th century (see Figure 2). Julliard and Ghosh (2012) propose an empirical likelihood estimation strategy to take account of the problem by over-weighting such rare disaster events.

the expected values in Eq. (2.12),

$$\widetilde{H}^{i}_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2) \equiv \frac{1-\frac{1}{\mathcal{T}(T_2)}\sum\limits_{s=1}^{\mathcal{T}(T_2)}\widetilde{m}_s(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2)\tilde{y}^{i}_s(\hat{\boldsymbol{\theta}}_2)}{1-\frac{1}{\mathcal{T}(T_2)}\sum\limits_{s=1}^{\mathcal{T}(T_2)}\tilde{d}_s(\hat{\boldsymbol{\theta}}_2)}, \tag{3.6}$$

where $\hat{\boldsymbol{\theta}}_2 = (\hat{\boldsymbol{\theta}}_{21}{}', \hat{\boldsymbol{\theta}}_{22}{}', \bar{\boldsymbol{\theta}}_{23}{}')'$. The tildes indicate simulated variables, and $\mathcal{T}(T_2)$ is the simulation sample size generated for a given sample size T_2 of actual observations, with $\mathcal{T}(T_2) \to \infty$ as $T_2 \to \infty$, such that the simulation should achieve:

$$\widetilde{H}^{i}_{T_2}(\boldsymbol{\theta}^0_1,\hat{\boldsymbol{\theta}}_2) \approx \frac{1-\mathbb{E}[\text{price}(y^i_t;\boldsymbol{\theta}^0_1);\boldsymbol{\theta}^0_2]}{1-\mathbb{E}[d_t;\boldsymbol{\theta}^0_2]}. \tag{3.7}$$

Duffie and Singleton (1993) work out preconditions in which it is permissible to use simulated moments to approximate population moments in an SMM application. They refer to the non-stationarity of the simulated data and their dependence on parameters that change during the estimation procedure, which raises the question of the applicability of an LLN. In Section O.2 of the Online Appendix, we transfer their results to the present framework.

These considerations motivate the strategy to estimate the preference parameters $\boldsymbol{\theta}^0_1$ by matching the sample moments in Eq. (3.4) with the simulated moments in Eq. (3.6). For that purpose, we use the returns on $N > 2$ test assets and construct a vector that contains those moment matches:

$$G_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2) = [G^1_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2),\ldots,G^N_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2)]', \tag{3.8}$$

where

$$G^i_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2) = H^i_{T_2}(\boldsymbol{\theta}_1) - \widetilde{H}^i_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2). \tag{3.9}$$

As outlined in Appendix A.1, $G_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2)$ alternatively may consist of a combination of moment matches that involve both returns and excess returns. An estimator of $\boldsymbol{\theta}^0_1$

is then obtained by minimizing an SMM-type criterion function,

$$\hat{\boldsymbol{\theta}}_1 = \underset{\boldsymbol{\theta}_1 \in \boldsymbol{\Theta}_1}{\arg\min}\ G_{T_2}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2)' \mathbf{W}_{T_2} G_{T_2}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2), \tag{3.10}$$

where $\mathbf{W}_{T_2}$ is a symmetric and positive semi-definite distance matrix that converges in probability to a non-stochastic matrix $\mathbf{W}$ as both T_1 and T_2 tend to infinity. $\boldsymbol{\Theta}_1$ denotes the admissible parameter space for the economic parameters of interest.

In Section O.2 of the Online Appendix, we state the sufficient conditions under which $\hat{\boldsymbol{\theta}}_1 \underset{p}{\to} \boldsymbol{\theta}_1^0$ as both T_1 and T_2 tend to infinity. For that purpose, we adapt results from Duffie and Singleton (1993). First, we assume that the defining MPP is correctly specified. The parameters of the defining MPP, $\boldsymbol{\theta}_{21}^0$ and $\boldsymbol{\theta}_{22}^0$, are assumed to be consistently estimated, while the unidentified MPP parameters are assumed to be obtained from calibrator's knowledge, such that $\bar{\boldsymbol{\theta}}_{23} = \boldsymbol{\theta}_{23}^0$. Second, $\boldsymbol{\theta}_1^0$ and $\boldsymbol{\theta}_2^0$ must uniquely satisfy the reformulated pricing equation (2.13). Further assumptions pertain to the stationarity and ergodicity of the variables that appear in Eq. (2.13), as well as technical assumptions that allow the application of uniform LLNs to the simulated and sample moments.

These assumptions are quite general. Their verification in particular situations would require endowing the disaster-including C-CAPM with more structure. However, because our strategy to assess the RDH empirically heeds Blanchard's (2008) advice to focus on the basic asset pricing equation without further elaborations, the verification of these high-level assumptions is, to some extent, inapplicable and beyond the scope of this study.

Usual asymptotic inference can be used for the first step, but inference about the second-step estimates must take account of several peculiarities of the empirical strategy. First, it is a sequential approach in which the first-step estimates affect the distribution of the second-step estimates. Second, the preference parameter estimates are obtained by a simulation-based estimation procedure, for which standard SMM or indirect inference results do not apply. Third, the data used in the two steps have not only different lengths but also diverse origins (historical multi-country panel data and country-specific financial and economic data).

Each of these aspects itself poses a methodological challenge that has been addressed independently, yet not jointly, in previous literature.[9] To characterize the limit distribution of the second-step estimates, we note that both steps involve an extremum estimator, and we can stack the first-order conditions of the first- and second-step optimization problems such that

$$\begin{bmatrix} \mathbf{0} & \frac{\partial G_{T_2}(\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2)'}{\partial \boldsymbol{\theta}_1} \mathbf{W}_{T_2} \\ \mathbf{I}_{L+K} & \mathbf{0} \end{bmatrix} \times \begin{bmatrix} F_{T_1}(\hat{\boldsymbol{\theta}}_{21}, \hat{\boldsymbol{\theta}}_{22}) \\ G_{T_2}(\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2) \end{bmatrix} = \mathbf{0}, \tag{3.11}$$

where

$$F_{T_1}(\hat{\boldsymbol{\theta}}_{21}, \hat{\boldsymbol{\theta}}_{22}) = \frac{1}{T_1} \sum_{t=1}^{T_1} \frac{\partial l_t(\hat{\boldsymbol{\theta}}_{21}, \hat{\boldsymbol{\theta}}_{22})}{\partial (\boldsymbol{\theta}_{21}', \boldsymbol{\theta}_{22}')'}, \tag{3.12}$$

with $\mathbf{I}_{L+K}$ as the identity matrix of order $L+K$, and l_t as the period t contribution to the log-likelihood function in Eq. (3.3). With an intermediate-value expansion of F_{T_1} and G_{T_2} about $\boldsymbol{\theta}_1^0$, $\boldsymbol{\theta}_{21}^0$, and $\boldsymbol{\theta}_{22}^0$, we can solve for the sampling error, which then enables a characterization of the limit distribution of the second-step estimates $\hat{\boldsymbol{\theta}}_1$. We outline the details of this procedure in Section O.3 of the Online Appendix. The resulting limit distribution of $\hat{\boldsymbol{\theta}}_1$ emerges as a mixture of a Gaussian and a non-standard distribution, and it is not directly usable for empirical research.

Because applicable asymptotic results are not available, we account for estimation

[9] For example, Grammig and Küchlin (2018) adopt a strategy outlined by Newey and McFadden (1994) to provide the asymptotic distribution theory for a simulation-based sequential estimation of Bansal and Yaron's (2004) long-run risk (LRR) model. The efficient use of samples from different periods for GMM estimation is addressed by Singleton (2006, Ch. 4.5) and Lynch and Wachter (2013). Nakamura et al. (2013) use multi-country data for the Bayesian estimation of the parameters of a consumption process, which serve in a second step to obtain the posterior distribution of the RRA coefficient and the equity premium by matching moments of U.S. data. We borrow elements from that literature.

uncertainty by extending the second-step data simulation procedure to repeatedly produce artificial samples, on which we perform the parameter estimation. With the resulting empirical distributions, we can compute simulation-based confidence intervals and standard errors for the model parameters and model-implied indicators of interest.

3.4 Identifying the IES: caveat and solutions

The described empirical idea can be extended and specialized. One such modification involves the estimation of the intertemporal elasticity of substitution.

The recursive utility specification with the associated SDF in Eq. (2.2) is theoretically appealing but also creates econometric challenges. Thimme (2017) points out that the joint estimation of the preference parameters β, γ, and ψ that relies exclusively on moment restrictions that result from conditioning down the asset pricing equation (2.1) may yield an unstable IES estimate. This caveat in principle applies to any empirical asset pricing study that uses the EZW-SDF in Eq. (2.2).[10]

We therefore consider a modification of the second step to reflect an alternative approach for estimating the IES.[11] This approach relies on a log-linearization of the Euler Equation (2.1) outlined by Yogo (2004), which leads to the regression equation

$$r_t = \kappa + \frac{1}{\psi} g_t + \eta_t, \tag{3.13}$$

where $r_t = \ln R_t$, $g_t = \ln G_t$, κ is a constant, and η_t is an error term. The derivation of (3.13) implies that η_t is correlated with g_t, so a linear projection of r_t on g_t and a constant does not identify the IES. Instead, κ and ψ are identified by the following

[10] For example, Bansal et al. (2007) report that the efficient method of moments objective function used to estimate the parameters of the EZW-SDF is flat in ψ, so they fix $\psi = 2$.

[11] Garcia et al. (2006) propose an interesting alternative approach to identify the IES by allowing for a reference level of consumption.

moment restrictions:

$$\mathbb{E}\left[\left(r_t - \kappa - \frac{1}{\psi} g_t\right) \mathbf{z}_{t-1}\right] = \mathbf{0}, \tag{3.14}$$

where $\mathbf{z}_{t-1}$ consists of instrumental variables that are known at $t-1$ and correlated with g_t – typically, lagged consumption growth and asset returns. Eq. (3.14) is assumed to be uniquely satisfied at the values $\psi = \psi^0$ and $\kappa = \kappa^0$. Eq. (3.13) in principle applies to any asset return, but most applications use a low-risk asset (often the T-bill) or a market portfolio proxy.

This strategy for estimating the IES is not new (see Campbell's (2003) survey). The novelty is that we allow for the possibility of rare disasters. Leveraging the idea from Section 2, we apply the LTE to Eq. (3.14) to obtain restrictions that involve moments that include only disaster-free data and moments that include disaster marks. We assume that the instruments are taken from period $t-j$, where $j \geq 1$, and we define:

$$d_t^* = \begin{cases} 0 & \text{if } d_t = 0 \text{ and } d_{t-j} = 0, \\ 1 & \text{if } d_t = 1 \text{ or } d_{t-j} = 1. \end{cases} \tag{3.15}$$

Moreover, let $p^* = \Pr(d_t^* = 1) = \mathbb{E}[d_t^*]$.[12] We can then rewrite Eq. (3.14), corresponding to Eq. (2.8), to obtain:

$$\mathbb{E}\left[\left(r_t - \kappa - \frac{1}{\psi} g_t\right) \mathbf{z}_{t-1} \middle| d_t^* = 0\right] = -\frac{p^* \mathbb{E}\left[\left(r_t - \kappa - \frac{1}{\psi} g_t\right) \mathbf{z}_{t-1} \middle| d_t^* = 1\right]}{1 - p^*}. \tag{3.16}$$

Recycling our empirical idea, the left-hand side moments in Eq. (3.16) can be approximated by sample means using regular disaster-free data, and the right-hand side moments can be simulated. Applying the reasoning that led to Eq. (3.6), we

[12] An analogous definition of d_t^* and p^* applies when instruments are taken from multiple lagged periods.

obtain additional moment matches, namely,

$$G^{+}_{T_2}(\kappa,\psi,\hat{\boldsymbol{\theta}}_2) = \left[\frac{1}{T_2 - \sum_{t=1}^{T_2} d_t^*} \sum_{t=1}^{T_2} \eta_t \mathbf{z}_{t-1}(1-d_t^*) - \left[\frac{-\frac{1}{\mathcal{T}(T_2)} \sum\limits_{s=1}^{\mathcal{T}(T_2)} \tilde{\eta}_s \tilde{\mathbf{z}}_{s-1} \tilde{d}_s^*}{1 - \frac{1}{\mathcal{T}(T_2)} \sum\limits_{s=1}^{\mathcal{T}(T_2)} \tilde{d}_s^*} \right] \right], \tag{3.17}$$

where $\tilde{\eta}_s = \tilde{r}_s - \kappa - \frac{1}{\psi}\tilde{g}_s$, using an abbreviated notation in which the dependence of the simulated variables on the defining MPP parameters $\hat{\boldsymbol{\theta}}_2$ is suppressed. To integrate this approach in the setting outlined in the previous section, the moment matches in Eq. (3.17) could augment those in Eq. (3.8). Alternatively, the IES can be estimated separately using:

$$\begin{bmatrix} \hat{\kappa} \\ \hat{\psi} \end{bmatrix} = \underset{(\psi,\kappa)' \in \boldsymbol{\Theta}^+}{\arg\min} = G^{+}_{T_2}(\kappa,\psi,\hat{\boldsymbol{\theta}}_2)' \mathbf{W}^{+}_{T_2} G^{+}_{T_2}(\kappa,\psi,\hat{\boldsymbol{\theta}}_2), \tag{3.18}$$

where $\boldsymbol{\Theta}^+$ is the admissible parameter space for $(\psi,\kappa)'$. The properties of $\mathbf{W}^{+}_{T_2}$ are analogous to those of $\mathbf{W}_{T_2}$. This estimation variant also can be embedded in the general setting of Section 3.3. For that purpose, we exploit the first-order conditions for a minimum of the criterion function in Eq. (3.18), which imply that a linear combination of the moment matches in Eq. (3.17) is set to 0, and thus:

$$A_{T_2}(\hat{\kappa},\hat{\psi},\hat{\boldsymbol{\theta}}_2) \cdot G^{+}_{T_2}(\hat{\kappa},\hat{\psi},\hat{\boldsymbol{\theta}}_2) = \mathbf{0}, \tag{3.19}$$

where $A_{T_2}(\hat{\kappa},\hat{\psi},\hat{\boldsymbol{\theta}}_2) = \frac{\partial G^{+}_{T_2}(\hat{\kappa},\hat{\psi},\hat{\boldsymbol{\theta}}_2)'}{\partial(\kappa,\psi)'} \mathbf{W}^{+}_{T_2}$. Accordingly, we augment the moment matches in Eq. (3.8), such that

$$\bar{G}_{T_2}(\kappa,\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2) = \begin{bmatrix} G_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2) \\ A_{T_2}(\kappa,\psi,\hat{\boldsymbol{\theta}}_2) \cdot G^{+}_{T_2}(\kappa,\psi,\hat{\boldsymbol{\theta}}_2) \end{bmatrix}, \tag{3.20}$$

and we use $\bar{G}_{T_2}(\kappa,\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2)$ instead of $G_{T_2}(\boldsymbol{\theta}_1,\hat{\boldsymbol{\theta}}_2)$ in Eq. (3.10). Then κ becomes an additional parameter to be estimated. Assigning large weights to the last two moment conditions via $\mathbf{W}_{T_2}$ ensures that the IES will be identified by Eq. (3.16). The claim of consistency using this alternative second-step estimator can be maintained, provided that the set of assumptions is suitably modified (see Online Appendix O.2).

4 Implementing the econometric strategy

In this section, we outline the implementation of the proposed methodology. We first describe the available data and how they are processed for our purposes in Section 4.1. In Section 4.2, we consider specifications for the defining MPP, and we identify, recognizing the limitations of the data, which of its parameters are estimable and which must be calibrated. Section 4.3 outlines the implementation of the second estimation step, and presents an approach to account for estimation uncertainty. For quick reference, Table 1 provides an synoptic overview of the implementation.

[insert Table 1 here]

4.1 Data base and data processing

Due to the nature of rare disaster events, the implementation of the proposed strategy entails a course of action that is unusual in econometrics but common in calibration studies: the use of heterogeneous data sources for the empirical analysis. To deal with the specification, estimation, and calibration of the defining MPP in the first step, we use the multi-country consumption data collected by Barro and Ursúa (2008).[13] We extract data for the same 35 countries selected by Barro (2006); Table 2 lists these countries and the years for which data are available.

[insert Table 2 here]

[13] These unbalanced panel data for 42 countries feature prominently in RDH literature. They are available at https://scholar.harvard.edu/barro/publications/macroeconomic-crises-1870-bpea, accessed 04/24/2015.

We edit the raw data in the following ways: To detect consumption disasters, we adopt Barro's (2006) identification scheme, which implies that any downturn in aggregate consumption over consecutive periods that is greater than or equal to $q = 0.145$ qualifies as a disaster event.[14] A disaster may build up over multiple periods or occur as one sharp contraction. For the purpose of disentangling contraction factors b_t from regular consumption growth G_t^r according to Eq. (2.4), we follow Barro (2006) and assume that during a disaster, G_t^r is equal to 1, such that $b_t = 1 - G_t$. We emphasize here that holding G_t^r fixed during a disaster only serves to identify the contraction components b_t in the data. The empirical analysis allows for a time-varying regular consumption growth component.[15]

To enable the first-step analysis, the data are represented in event time; that is, the country-specific time series are concatenated, and sequences of the disaster indicators d_t, d_t^+, and d_t^* are computed for every country. The disaster period counter $N(t)$ is reset to 0 whenever a country change occurs within the concatenated data. The same reset is applied to another function $M(t)$, which counts the number of disaster events that occurred as of t. This identification scheme detects 89 disaster events. Figure 1 shows their distribution across countries and over time.

[insert Figure 1 here]

Initially, Barro and Ursúa's (2008) data only permit the computation of annual contractions. To align the data with the assumed quarterly decision frequency, we generate quarterly observations by drawing from a standard uniform distribution to decide which fraction of an annual contraction is attributed to the first quarter of a disaster year. Another draw determines how much of the remainder is allocated to

[14] The same threshold is used by Barro (2009) and Barro and Jin (2011); it will be varied in our robustness checks.

[15] As an alternative identification strategy, Barro (2006) uses mean gross consumption growth for G_t^r. Using $G_t^r = 1$ produces more benign contractions, which anticipates the caveat that their sizes may be purposefully inflated.

the second quarter. The same procedure gives the third quarter contraction, and the fourth takes what is left. This algorithm implies that the contraction in the first (last) quarter would be the largest (smallest), on average. To avoid such a seasonal pattern, the four contractions are randomly re-shuffled. The described procedure applies to a "within disaster" year, one that is not the first or last year of a disaster event. When dealing with the first (last) year of a disaster, or if it consists of a single annual contraction, we determine the quarter when the contraction begins (ends) by a draw from a discrete uniform distribution, such that each quarter has a 25% probability of becoming the quarter when the disaster begins (ends). The annual contraction is then distributed across the disaster quarters by applying the procedure for a within disaster year. Subsequently, the three disaster indicators are re-computed using the quarterlized, concatenated data, for which we have $T_1 = 16,984$ country/quarters. Counting the number of quarters between disaster events gives τ_m, the duration between the mth and the $(m+1)$th disaster, and the number of quarters that the mth disaster event lasts is denoted by τ_m^*.

Figure 2 illustrates the described procedure, using the U.S. subsample within Barro and Ursúa's data. It shows the quarterly contractions associated with the two U.S. disaster events detected during 1870-2009: the Great Depression (1930-1933) and another (1918-1921) that is linked to the consequences of World War I and the Spanish influenza pandemic.

[insert Figure 2 here]

The data used for the implementation of the second step, the estimation of the economic parameters of interest, are common in asset pricing literature. In particular, we use quarterly U.S. real personal consumption expenditures per capita on services and nondurable goods in chained 2009 U.S. dollars, as provided by the

Federal Reserve Bank of Saint Louis.[16] The data span the period 1947:Q2–2014:Q4, such that $T_2 = 271$ quarters. No disaster event occurred during this time interval, which, as outlined supra, does not pose a problem for the application of the proposed methodology. Financial data covering the same period, but at a monthly frequency, are retrieved from CRSP and K. French's data library.[17] The data extracted for the empirical analysis are (1) the ex ante T-bill return; (2) the return on the CRSP market portfolio, comprised of NYSE, AMEX, and NASDAQ traded stocks (*mkt*); (3) the returns on ten size-sorted portfolios (*size dec*); and (4) the returns on the ten industry portfolios (*industry*). All portfolios are value-weighted. The gross return on the CRSP market portfolio serves as a proxy for the wealth portfolio return R^a.[18] Monthly nominal returns are converted to quarterly real returns using the growth of the consumer price index of all urban consumers.[19] Following Beeler and Campbell (2012), we approximate the ex ante T-bill return by forecasting the ex post return on the basis of the quarterly T-bill yield and the average of quarterly log inflation across the past year. The three-month nominal T-bill yield comes from CRSP. Table 3 provides descriptive statistics for these data.

[insert Table 3 about here]

There are two caveats regarding the use and combination of these disparate data sources. First, a critic might ask whether the multi-country data collected by Barro and Ursúa (2008) are appropriate to estimate the parameters of a multi-period

[16] For services, see `http://research.stlouisfed.org/fred2/series/A797RX0Q048SBEA`. For nondurable goods, see `http://research.stlouisfed.org/fred2/series/A796RX0Q048SBEA`. Both accessed 03/09/2016.

[17] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html`, accessed 03/09/2016. Due to the frequent changes in the underlying CRSP data, newer or older downloads may results in different series.

[18] The approximation of the return on the wealth portfolio by the return on the portfolio of financial assets is also employed by Weber (2000), Stock and Wright (2000), and Yogo (2006).

[19] These data are provided by the Federal Reserve Bank of Saint Louis: `http://research.stlouisfed.org/fred2/series/CPIAUCSL`, accessed 03/09/2016.

disaster MPP that will be used to simulate moments to be matched with the disaster-free U.S. data in the second step. Alternatively, U.S. studies may gauge disaster probabilities entirely on a single event: the Great Depression.[20] We concur with Barro and Ursúa's (2008, p. 275) view that the likelihood, duration, and severity of a disaster, as perceived by U.S. investors, might be better accounted for by "consulting the global experience rather than overweighting the U.S. own history, for which the few observations are likely to be dominated by luck." Second, a shortcoming of Barro and Ursúa's (2008) data is that they provide insights about historical consumption disasters, but only limited financial information. Due to low trading volumes, dried-up liquidity, or market shut-downs, asset prices are unreliable, hard to come by, or unavailable during a disaster, as noted by Blanchard (2008). Prudent specifications of the defining MPP therefore should be parsimonious, and it will be necessary to calibrate some of its parameters.

4.2 First-step implementation: MPP estimation and calibration

The specifications for the defining MPP that we consider for the first-step implementation facilitate the separate treatment of the parameters $\boldsymbol{\theta}_{21}^{0}$ (points) and $\boldsymbol{\theta}_{22}^{0}$ (marks). The alternative specifications of the discrete-time hazard rate that we consider relate to the work of Hamilton and Jordà (2002). Specifically, we rely on the time durations of and between previous disaster events, the aggregate size of the previous disaster, and the size of the contraction of the last disaster period to predict

[20] Comprehensive financial data for the U.S. did not exist at the time of the first disaster event depicted in Figure 2.

h_t. The most general specification considered can be compactly written as:

$$h_t = \Big[\big[(\mu + \alpha\tau_{M(t-1)-1} + \delta b^+_{M(t-1)})(1 - d_{t-1}) \\ + (\mu^* + \alpha^*\tau^*_{M(t-1)-1} + \delta^* b_{N(t-1)}) d_{t-1} \big] (1 - d^+_{t-1}) + d^+_{t-1} \Big]^{-1}, \tag{4.1}$$

where b_n is the contraction size associated with the nth disaster period, and b^+_m denotes the aggregate size of the mth disaster.[21] For the empirical analysis, we explore specific cases of Eq. (4.1). The most parsimonious variant, referred to as CH_0, uses $\delta = \delta^* = \alpha = \alpha^* = 0$, such that Eq. (4.1) becomes:

$$h_t = \begin{cases} \mu^{-1} & \text{if } d_{t-1} = 0, \\ \mu^{*-1} & \text{if } d_{t-1} = 1 \text{ and } d^+_{t-1} = 0, \\ 1 & \text{if } d_{t-1} = 1 \text{ and } d^+_{t-1} = 1. \end{cases} \tag{4.2}$$

The most comprehensive alternative, referred to as CH_1, estimates all parameters in Eq. (4.1). The CH_2 variant allows for an effect of the durations between disasters and the disaster length on h_t (with $\delta = \delta^* = 0$). Version CH_3 allows the magnitude of the previous disaster and the size of the contraction of the previous disaster period to affect the hazard rate (with $\alpha = \alpha^* = 0$). In the CH_4 specification, the aggregate size of the previous disaster can have an effect on h_t, but the contraction of the previous disaster period does not ($\delta^* = \alpha = \alpha^* = 0$).

Because financial returns are not contained in Barro and Ursúa's (2008) data, the implementation of the log-likelihood in Eq. (3.3) must focus on the important mark that is available: the consumption contraction factor b_t. To account for the distribution of b_t conditional on a disaster event, we draw on Barro (2006) and Barro and Jin (2011) and employ a Pareto distribution to describe the transformed

[21] While consumption growth is the process that defines a disaster event, one could also consider using past return contractions to explain the hazard rate. However, limited financial information in the Barro and Ursúa (2008) data restricts the generality of the specification of h_t.

contraction size $z_t = 1/(1 - b_t)$.[22] As allowed for in Eq. (3.2), the contractions that contribute to reaching the disaster threshold q can have a different distribution than those that add to a disaster after q has been reached. Using h_t as specified in (4.1), the implementation of the log-likelihood in Eq. (3.3) thus reads:

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta}_{21}, \boldsymbol{\theta}_{22}) = &\sum_{t=1}^{T_1} \left(d_t \ln h_t(\boldsymbol{\theta}_{21}) + (1 - d_t) \ln[1 - h_t(\boldsymbol{\theta}_{21})]\right) \\ &+ \sum_{t=1}^{T_1} d_t \left(d_t^+ \ln f_{\mathrm{P}}(z_t; \theta_{\mathrm{P}}^-) + (1 - d_t^+) \ln f_{\mathrm{P}}(z_t; \theta_{\mathrm{P}}^+)\right), \end{aligned} \tag{4.3}$$

where $f_{\mathrm{P}}(\cdot; \theta_{\mathrm{P}})$ denotes the Pareto density function with parameter θ_{P} and threshold value 1. Moreover, $\boldsymbol{\theta}_{21} = (\mu, \alpha, \delta, \mu^*, \alpha^*, \delta^*)'$ and $\boldsymbol{\theta}_{22} = (\theta_{\mathrm{P}}^-, \theta_{\mathrm{P}}^+)'$. Estimates of these MPP parameters are obtained by maximizing $\mathcal{L}(\boldsymbol{\theta}_{21}, \boldsymbol{\theta}_{22})$ based on the concatenated event time data described in Section 4.1.[23]

By proceeding in this way, we can only estimate a subset of the parameters of the defining MPP. We did not yet account for the joint distribution of the consumption and return contractions or the marginal distribution of the latter. Therefore, calibrating the remaining parameters of the defining MPP becomes inevitable. We approach this issue as follows: First, we assume that the transformed contractions z and $z^i = 1/(1 - c^i)$ are random variables that are described by same marginal c.d.f.,

$$F(\cdot; \theta_{\mathrm{P}}^+, \theta_{\mathrm{P}}^-) = F_{\mathrm{P}}(\cdot; \theta_{\mathrm{P}}^+)^{d^+} \times F_{\mathrm{P}}(\cdot; \theta_{\mathrm{P}}^-)^{1-d^+}. \tag{4.4}$$

Second, we assume that the bivariate c.d.f. of z and z^i can be written using a Gaussian

[22] Barro and Jin (2011), who assume single-period disasters, use a double power-law distribution that consists of two power-law distributions that morph into each other at a certain threshold value. We find though that the flexibility of the double power-law distribution is not required when modeling multi-period disasters.

[23] Using the hazard rate specification in Eq. (4.1), τ_0 is re-initialized to reflect the average duration between disasters (179.7 quarters), τ_0^* is reset to equal the average disaster length (13.1 quarters), and b_0^+ is reset to equal the average contraction size (0.268) whenever a country change occurs in the concatenated data. These values are also the initial values used for the optimization. They correspond to $q = 0.145$; different disaster thresholds use different initial values. This procedure is adopted from Engle and Russell (1998).

copula function with parameter ρ^i:

$$F_{z,z_i}(z, z^i) = \Phi_2\left(\Phi^{-1}[F(z;\theta_{\mathrm{P}}^+,\theta_{\mathrm{P}}^-)], \Phi^{-1}[F(z^i;\theta_{\mathrm{P}}^+,\theta_{\mathrm{P}}^-)];\rho^i\right), \tag{4.5}$$

where Φ denotes the standard normal c.d.f., and Φ_2 is the c.d.f. of the standard bivariate normal distribution. The copula correlations are treated as parameters to be calibrated.[24] For a base variant, we calibrate ρ^i to the empirical correlation between non-disastrous consumption growth and gross return, calculated on the data used for the second stage. We also consider an extreme alternative and calibrate $\rho^i = 0.99$ for all i, similar to Barro and Ursúa (2008), and drawing on Longin and Solnik's (2001) insight that the dependence between financial returns increases in the tails of the joint distribution. Another calibration assumes $\rho^i = 0$ for all i, which implies that return and consumption contractions are independently drawn from the same distribution.

Among the asset returns of interest, the T-bill return (denoted R_t^f) receives special treatment, drawing on Barro's (2006) observation that a partial government default occurs in 42% of the disasters identified in the data for the aforementioned 35 countries. To take account of this finding, we define by d_t^f a binary indicator that equals 1 if the T-bill return is affected in case of a disaster and 0 otherwise, and we assume that

$$\Pr(d_t^f = 1|d_t = 0) = 0,$$

$$\Pr(d_t^f = 1|d_t = 1, d_{t-1} = 0) = p^f, \text{ and}$$

$$\Pr(d_t^f = 1|d_t = 1, d_{t-1} = 1) = d_{t-1}^f.$$

[24] We also probe alternative copulas, in particular rotated Clayton and Frank. These copulas are convenient for our purposes because, like the Gaussian, they allow us to perform the draws from the bivariate distribution of the second random variable conditional on the draws of the first. As we outline subsequently, this feature is very useful when simulating data in the second estimation step. The results of using these alternative copulas, which are qualitatively not different from the Gaussian base case, are presented in the Online Appendix, Section O.4.

Using Barro's (2006) result, we calibrate $p^f = 0.42$ and modify Eq. (2.5), such that $R_t^f = R_t^{fr}(1 - c_t^f)^{d_t^f}$. Accordingly, whether the T-bill return contracts during a disaster is determined at the onset of the event. The treatment and calibration of the joint distribution of the contraction factors c_t^f and b_t conditional on $d_t^f = 1$ are analogous to that of the other asset returns (identical marginal distributions and copula approach). The vector of calibrated MPP parameters thus reads $\boldsymbol{\theta}_{23} = (\rho^1, \ldots, \rho^M, p^f)'$, where M denotes the number of assets considered in the analysis, including the T-bill and the market return proxy.

4.3 Second-step implementation: Moment matches and data simulation

For the implementation of the second step of the proposed methodology, we rely on the variant of the moment matching approach that combines excess returns and the return of a reference asset. We use the T-bill return as the reference return and the excess returns $R^{ei} = R^i - R^f$ on N test assets.[25] The test assets considered are, alternatively, the size-sorted portfolios (*size dec*, $N = 10$), the industry portfolios (*industry*, $N = 10$), and the CRSP market portfolio (*mkt*, $N = 1$) described in Section 4.1. Furthermore, we implement the IES identification strategy outlined in Section 3.4, using the T-bill return for the left-hand side of Eq. (3.13), and as instruments

[25] See Eq. (A-7). This combination of test assets, using a risk-free rate proxy along with a set of excess returns of stock portfolios, is common practice in GMM-based testing of asset pricing models (e.g., Cochrane (1996)).

twice-lagged log T-bill return and log consumption growth, following Yogo (2004).[26] The identity matrix is used for $\mathbf{W}^{+}_{T_2}$, and $\mathbf{W}_{T_2}$ in Eq. (3.10) is a diagonal matrix containing ones, except for the last two entries, which are large to facilitate the IES identification by the moment conditions in Eq. (3.16).

The simulation of disaster-including data is performed as follows: Using the estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$ for the assumed specification of the defining MPP, we generate a series of discrete-time hazard rates $\tilde{h}_s$, with $\mathcal{T}(T_2) = 10^7$. With this series, we simulate the disaster indicators $\tilde{d}_s, \tilde{d}^+_s$, $\tilde{d}^*_s$, and $\tilde{d}^f_s$.[27] If the simulation yields $\tilde{d}_s = 1$, or $\tilde{d}^f_s = 1$ in case of the T-bill, we simulate consumption and return contractions by drawing a random variable w_s and random variables w^i_s (one for each return of interest, conditional on the same draw of w_s) from bivariate standard normal distributions with the respective calibrated copula correlation ρ^i. Consumption growth and return contraction factors are then obtained by

$$\tilde{b}_s = 1 - \frac{1}{F^{-1}(\Phi(w_s); \hat{\theta}^+_{\mathrm{P}}, \hat{\theta}^-_{\mathrm{P}})} \quad \text{and} \quad \tilde{c}^i_s = 1 - \frac{1}{F^{-1}(\Phi(w^i_s); \hat{\theta}^+_{\mathrm{P}}, \hat{\theta}^-_{\mathrm{P}})}. \tag{4.6}$$

As of yet, we have not made distributional assumptions about the regular consumption and return components G^r_t and R^r_t. For the identification of the consumption contractions in Barro and Ursúa's data, we have fixed $G^r_t = 1$, but for the simulation of disaster-including data, we allow for time-varying regular consumption and return

[26] In principle, we could also use a stock or a portfolio return or, as in Yogo (2004), a set of returns in a GMM approach. We choose the T-bill return because of its higher predictability by instrumental variables. Vissing-Jørgensen and Attanasio (2003, p. 387) observe that using stock index returns results in smaller estimates of ψ than with T-bill returns, which they posit might be caused by "the much lower predictive power of the instruments for stock returns which could lead to poorer small-sample properties of the estimator." This argument is supported by Thimme (2017), who presents fixed income returns as the usual return proxy. Section O.5 of the Online Appendix provides an extended discussion on the choice of the reference return, an alternative specification of the Eq. (3.13), and additional empirical results obtained from alternative returns and regression specifications. These additional analyses show that the results are robust, as long as the T-bill remains part of the set of reference assets.

[27] For notational brevity, we use the shorthand notation that suppresses the dependence of the simulated values on the estimated/calibrated parameters of the defining MPP.

components. We avoid additional distributional assumptions and simulate series of length $\mathcal{T}(T_2)$ of regular consumption growth and returns, $\widetilde{G}_s^r$ and $\widetilde{R}_s^r$, by drawing with replacement from the disaster-free U.S. postwar data. The draws preserve the contemporaneous dependence between variables; to allow for serial dependence, we use a block-bootstrap simulation.[28] By combining the simulated contraction factors with the simulated regular return and growth series, we generate disaster-including series by $\widetilde{G}_s = \widetilde{G}_s^r(1-\tilde{b}_s)^{\tilde{d}_s}$ and $\widetilde{R}_s^i = \widetilde{R}_s^{ir}(1-\tilde{c}_s^i)^{\tilde{d}_s}$; the simulated T-bill return is $\widetilde{R}_s^f = \widetilde{R}_s^{fr}(1-\tilde{c}_s^f)^{\tilde{d}_s^f}$.

[insert Figure 3 here]

Figure 3 illustrates a simulated multi-period disaster event, depicting the components of simulated consumption growth: regular consumption growth and the contraction factor. Using $\widetilde{R}_s^{ei} = \widetilde{R}_s^i - \widetilde{R}_s^f$, it becomes possible to evaluate $G_{T_2}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2)$ and $\bar{G}_{T_2}(\kappa, \boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2)$ in (3.8) and (3.20), and thus to perform the second-step estimation.[29]

We have seen that the asymptotic inference for the second-step estimates $\hat{\boldsymbol{\theta}}_1$ is not readily usable. However, the data simulation procedure just described suggests a strategy to extend it for the purpose of approximating the finite sample distribution of the estimates. The implementation of this idea entails repeatedly generating artificial first- and second-step samples, and performing the parameter estimation on these data. To obtain these pseudo-samples, we combine the aforementioned bootstrap from the second-step data with a Monte Carlo simulation of the defining MPP endowed with the first-step parameter estimates. After re-estimating the MPP parameters on the simulated sample, the second-step procedure is applied as outlined supra, but on the artificial block-bootstrapped second-step sample, and with

[28] We use the automatic block-length selection proposed by Politis et al. (2009), in combination with Politis and Romano's (1994) stationary bootstrap. We are grateful to Andrew Patton for providing the Matlab code that we use for this purpose.

[29] For details on the Matlab programs developed for that purpose, see Section A.3.

a data simulation that uses the re-estimated MPP parameters. The calibrated MPP parameter values remain unchanged. Repeating this procedure on independently drawn pseudo-samples yields empirical distributions of the parameter estimates as well as other statistics of interest, like the model-implied equity premium. Confidence intervals for parameters and model-implied indicators can be computed based on the empirical quantiles. Appendix A.2 provides a step-by-step description of this approach to account for estimation uncertainty. We note that the described procedure also allows for simulation-based inference on the first-step estimates, for which usable asymptotic results are available. A comparison thus serves as a robustness check.

5 Empirical results

5.1 First-step estimation results

Table 4 reports the parameter estimates, standard errors and the Akaike (AIC) and Schwarz-Bayes (SBC) information criteria for the different specifications that emerge as special cases of Eq. (4.1).

[insert Table 4 about here]

As Table 4 shows, the SBC prefers the parsimonious CH_0, for which the baseline hazard parameter estimates $\hat{\mu}$ and $\hat{\mu}^*$ are different from 0 at conventional levels of significance. The AIC instead favors the CH_4, for which the estimates of μ^* and δ are significant at the 5% level. The baseline hazard parameter estimate $\hat{\mu}$ decreases in size and is no longer significantly different from 0 at conventional levels. The likelihood-ratio statistics indicate that the constraints implied by the SBC-preferred CH_0 are rejected at the 1% level only in the case of the AIC-preferred CH_4. The subsequent analysis therefore focuses on the CH_0 and CH_4 specifications.

The estimates of the CH_0 parameters ($\hat{\mu}^* = 178.3$ and $\hat{\mu} = 1.2$) imply a probability to enter a disaster of 0.56%, whereas the probability of remaining in a disaster is 83%. Because these estimates are important input for the second estimation step, we check their economic plausibility upfront. For that purpose, we use the CH_0 and CH_4 estimates to simulate disaster-including consumption time series with length $T_2 = 271$. The simulation is repeated 10k times, and we count the number of replications for which no disastrous contraction occurs. The CH_0 simulation yields 21.9%, and the CH_4 simulation 14.1% disaster-free histories. The estimated MPPs thus imply that the U.S. postwar period represents a lucky but not unlikely path, and the model-implied disaster probabilities are not implausibly large.

Table 4 also shows that the estimates of the Pareto coefficients θ_{P}^{-} and θ_{P}^{+} are similar, indicating that the distribution of contractions that occur before reaching the disaster threshold q is not much different from that of the contractions that occur past q. Moreover, the standard errors of $\hat{\theta}_{\mathrm{P}}^{-}$ and $\hat{\theta}_{\mathrm{P}}^{+}$ are small. Figure 4 compares the Pareto distribution functions (c.d.f.s) evaluated at these estimates, along with the empirical counterparts. Figure 4a uses $\hat{\theta}_{\mathrm{P}}^{+}$ and illustrates the goodness-of-fit for contractions that contribute to reaching the disaster threshold; Figure 4b uses $\hat{\theta}_{\mathrm{P}}^{-}$ and refers to contractions beyond the disaster threshold. In both cases, the fit is quite good.

[insert Figure 4 about here]

We also observe in Table 4 that the asymptotic and simulation-based standard errors are similar. This correspondence indicates the correct implementation and sensibility of the simulation-based procedure to account for parameter estimation uncertainty.

5.2 Second-step estimation results

The three panels in Table 5 show the second-step estimation results broken down by the calibration of the copula correlations ρ^i. Within each panel, we report results by the set of test assets (column-wise) and MPP specification (row-wise). For each specification variant, we provide the point estimates, along with 95% confidence bounds and simulation-based standard errors. The IES identification strategy implies that the estimate of ψ is the same across test assets. Accordingly, there is only one IES result per row.

[insert Table 5 about here]

Let us first compare the results obtained by a variation of the calibrated copula correlations. Recall that the claim of consistency of the second-step estimates is based on the assumption that the calibrated values correspond to the true MPP parameters, $\bar{\boldsymbol{\theta}}_{23} = \boldsymbol{\theta}^0_{23}$. This theoretical result is not very useful for empirical purposes though. From an applied perspective, a more interesting question is how the calibration of the unidentified MPP parameters affects the preference parameter estimates. In Dridi et al.'s (2007) sequential partial indirect inference (SPII) framework, this issue is addressed by the so-called partial encompassing condition (PEC). Relating this notion to the present context, an equivalent of the PEC would imply that the preference parameters can be consistently estimated, irrespective of the calibration of the unidentified MPP parameters. However, our econometric strategy is not a special case of SPII, such that Dridi et al.'s (2007) test of the PEC is not directly applicable.[30] Still, our variation of the calibrated values may be used and interpreted as an informal check: If an analogy of the PEC applies, then the estimates of the

[30] In particular, there is no auxiliary model and no structural misspecified model being simulated. Nor is there any binding function, for which the partial encompassing condition is formally defined.

preference parameters should not be affected in an economically meaningful way by the alternative calibration of the copula correlations. And indeed, contrasting the panels of Table 5, we observe that the different calibrations yield similar results. The point estimates and confidence intervals do not, to a large extent, depend on the calibrated copula correlations ρ^i.

Row-wise comparisons reveal that whether the AIC-preferred CH_4 or the SBC-preferred CH_0 process is used for data simulation does not affect the results qualitatively. Column-wise comparisons show that the results remain comparable when we use different sets of test assets. Recall that each set of test assets includes the T-bill return. Because the respective moment match should be particularly useful to identify the time discount factor, it is not surprising that the variation of $\hat{\beta}$ across test asset sets is small. The size of the RRA estimates is more affected by the choice of test assets, but the differences are not substantial.

The key insight from Table 5 is that all specification variants yield economically plausible estimates of the preference parameters. The time discount factor estimates are smaller than but close to 1, as would be expected for an investor with a positive rate of time preference and a quarterly decision frequency. The RRA estimates fall well within the aforementioned strict plausibility interval; with a value of about 1.5, they also are similar to the implicit RRA estimate obtained when conceiving of the Fama-French three-factor model as an instance of an intertemporal CAPM (see Grammig and Jank (2016)). The confidence bounds for the RRA coefficient also lie within the strict plausibility range. Meeting implicit desiderata of preference-based asset pricing, the IES estimates are significantly larger than unity and similar to the values chosen in calibration studies. Moreover, the inverse of the estimated IES is smaller than the RRA estimate, which indicates a preference for early resolution of uncertainty. We discuss the economic implications of this result subsequently.

Econometric C-CAPM studies have notoriously yielded implausible and imprecise preference parameter estimates, calling into question fundamental principles of financial economics. Not surprisingly, there thus have been prominent attempts to vindicate the consumption/preference-based asset pricing approach.[31] Although these efforts can claim some success, the estimation results in Table 5 lend more obvious support to the C-CAPM paradigm.

We also conduct a series of robustness checks, the detailed results of which appear in the Online Appendix. For example, we compute bias-corrected parameter estimates and confidence bounds, and we find that the resulting adjustments are benign. Moreover, we explore the effect of choosing a different disaster threshold. In accordance with Barro and Jin (2011), we consider $q = 0.095$ and $q = 0.195$, two values that feature prominently in prior literature. The choice of the disaster threshold does not change the results substantially.

5.3 Asset pricing implications

Dridi et al. (2007, p. 401) advise econometricians to "think as calibrators that the specification tests should only be focused on the reproduction of stylized facts [...] under the constraint that some structural parameters of interest have been consistently estimated." We adopt this recommendation, and in Table 6, we combine several model-implied financial indicators, computed using the first- and second-step estimates, with their real-world equivalents. We focus on the specification variant in which the data simulation is based on the CH_0 process and empirical correlations are used to calibrate ρ^i (referred to as the base variant).

[31] Julliard and Parker (2005), for example, aggregate consumption over multiple periods and report $\hat{\gamma}$=9.1 with moderate estimation precision (s.e.=17.2). By measuring consumption with waste, Savov (2011) obtains $\hat{\gamma}$=17.0 (s.e.=9.0). In both studies, time-additive power utility is assumed (such that $\gamma = \psi^{-1}$), and β is calibrated.

[insert Table 6 about here]

When assessing whether a calibrated or estimated C-CAPM yields meaningful asset pricing implications, the magnitudes of the preference parameters all have important roles. The relative size of RRA and IES is reflected in the coefficient θ, which appears in the EZ-SDF in Eq. (2.2). If $\gamma = \psi^{-1}$, such that $\theta = 1$, an investor is indifferent to the temporal resolution of risk; the case of expected utility obtains. Early resolution is preferred if $\gamma > \psi^{-1}$. Preference-based asset pricing models characteristically require an IES greater than unity and an early resolution preference to account for key features of asset prices (e.g., Bansal and Yaron (2004); Huang and Shaliastovich (2015)).[32] Accordingly, when the RRA is greater than unity, θ should be less than 0.

Table 6 shows that the estimated θ is always negative, regardless of the set of test assets. The confidence bounds indicate that the hypothesis that $\theta < 0$ cannot be rejected at conventional levels of significance. Therefore, the estimated disaster-including C-CAPM should reveal meaningful asset pricing implications. To test this conjecture, we use the preference parameter estimates and assess the plausibility of the model-implied expected market portfolio and T-bill return, equity premium, and market Sharpe ratio. We approximate population moments by averaging over $\mathcal{T}$ simulated observations to estimate model-implied expected returns. For that purpose, we employ the data simulation procedure we used for the second-step estimation, such that the model-implied expected value of an asset return R can be approximated

[32] Long-run consumption and rare disaster risks alike are not resolved until far in the future, and a high IES penalizes these risks more heavily than current risks. An IES < 1 would result in an implausibly high and volatile risk-free rate in Bansal and Yaron's (2004) LRR model, as well as entail a positive correlation between economic uncertainty and price-dividend ratios in Barro's (2009) rare disaster model. Calibrators therefore select conforming IES values to illustrate the explanatory power of preference-based asset pricing models. For example, Bansal and Yaron (2004) choose ψ=1.5, and Barro (2009) calibrates ψ=2. The IES estimates using disaster-free data are typically much smaller (see Havránek (2015); Thimme (2017)).

by

$$\widehat{\mathbb{E}}(R) = \frac{1 - \left[\mathbb{E}_{\mathcal{T}}[\widetilde{m}\widetilde{R}] - \mathbb{E}_{\mathcal{T}}[\widetilde{m}]\mathbb{E}_{\mathcal{T}}[\widetilde{R}]\right]}{\mathbb{E}_{\mathcal{T}}[\widetilde{m}]}, \tag{5.1}$$

where we use Hansen's (1982) notation, $\mathbb{E}_{\mathcal{T}}(x) = \frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}} x_s$. The returns of primary interest are the T-bill return R^f and the market return R^a. The model-implied equity premium can then be estimated by $\widehat{\mathbb{E}}(R^a) - \widehat{\mathbb{E}}(R^f)$ and the model-implied market Sharpe ratio by

$$\frac{\widehat{\mathbb{E}}(R^a) - \widehat{\mathbb{E}}(R^f)}{\sqrt{\mathbb{E}_{\mathcal{T}}[(\widetilde{R}^a - \widetilde{R}^f)^2] - \mathbb{E}_{\mathcal{T}}(\widetilde{R}^a - \widetilde{R}^f)^2}}.$$

As can be seen in Table 6, the magnitudes of these model-implied indicators are economically sensible and comparable to their sample counterparts. The observed mean T-bill, mean market return, and equity premium fall within the 95% confidence bounds for the model-implied equivalents. As we detail in the Online Appendix, the results in Table 6 are robust to alternative calibrations of the copula correlation coefficients, copula functions, and the MPP used for the simulation.

Are these RDH-supportive findings an inevitable consequence of the estimation procedure? When using *mkt*, the number of moment matches equals the number of estimated second-step parameters, which seemingly could drive the results. However, even though in this case $G_{T_2}(\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2) = \mathbf{0}$, the estimation procedure does not imply that the empirical mean market return and mean T-bill return must match with their model-implied counterparts. Moreover, using the *size dec* or *industry* portfolios, the market portfolio is not among the set of test assets, and the plausible size of the implied financial indicators offers an out-of-sample reality check.

[insert Figure 5 about here]

Figure 5 provides a visual assessment of how well the disaster C-CAPM accounts for the cross-sectional variation of returns, by plotting the observed mean returns against the model-implied mean returns computed using Eq. (5.3). The mean absolute errors (MAE) reported in Table 6 summarize the cross-sectional fit for each set of test assets. Compared with previous results reported for preference-based asset pricing models, the explanatory power concerning the cross-section of stocks can be considered quite good. We also observe the well-known result that explaining the cross-sectional variation of industry portfolio returns is more difficult than that of size-sorted portfolios.

5.4 Model-implied timing premium

Epstein et al. (2014) note that though some prominent C-CAPMs, calibrated with sensible preference parameters, may account for empirical asset pricing puzzles, an ensuing logical inconsistency often gets overlooked. They thus call for a reality check, by assessing the magnitude of the model-implied timing premium.[33] The timing premium they define is the fraction of lifetime consumption that an agent would be willing to relinquish if all future consumption risk were resolved (albeit not removed) in the next period. The conditions to achieve meaningful asset pricing implications ($\gamma > \psi^{-1}$ and $\psi > 1$) can be met with economically sensible preference parameter values, yet the model-implied timing premium may be implausibly high.

Epstein et al. (2014) show how the timing premium can be computed when the model's endowment/consumption process and the values of the time discount factor, RRA, and IES are known, and they provide a comparison of the timing premia implied by some prominent preference-based asset pricing models.

[insert Table 7 about here]

Table 7 displays some of their results. For example, in Barro's (2009) single-period disaster model with $\gamma = 4$ and $\psi = 2$ (a calibration with meaningful asset pricing

[33] We are grateful to Marcel Rindisbacher, who suggested this model specification test.

implications), the representative agent would be willing to renounce 18% of her lifetime consumption to suspend uncertainty about future consumption. For models that feature a higher persistence of the endowment process, the timing premia become even higher: 23% for Bansal and Yaron's (2004) LRR model, and 42% for the persistent rare disaster model of Wachter (2013). These calculations are based on the calibrated parameter values reported by the authors. Arguably, these premia are implausibly high. Epstein et al. (2014) emphasize that the timing premium is concerned with resolving consumption risk, not income risk. If income risks were removed, the resulting planning advantage could justify a premium. But if the entire consumption path became known, planning would be obsolete. They argue that there is a purely visceral value associated with the information; there is no rational justification for a notable timing premium. Epstein et al. (2014) note that this argument is based on introspection. Meissner and Pfeiffer (2018) provide experimental evidence that the timing premium is indeed small.

We accordingly extend Epstein et al.'s (2014) analysis and compute the timing premium implied by the multi-period disaster consumption process, using the estimates reported in Table 5 (base variant).[34] In this case, the timing premium attains a considerably smaller value of only 0.9%. Although the nonindifference to temporal resolution of risk matters – especially to obtain meaningful asset pricing implications – it does not imply an implausible timing premium. For a further comparison, we calculate the timing premium implied by Nakamura et al.'s (2013) Bayesian RDH analysis and report the result in Table 7. The details of this analysis are provided in the Online Appendix. Their paper relates closely to ours, though the empirical methodologies are very different. With a size of 11.3%, Nakamura et al.'s (2013) implied timing premium is notably smaller than the others reported in Table 7, but

[34] We gratefully acknowledge the provision of the Matlab program by Epstein et al. (2014), which we use for this purpose.

arguably, it is still implausibly high.

5.5 Contrasting results

As mentioned in the introduction, two prior studies, by Julliard and Ghosh (2012) (henceforth J&G) and Nakamura et al. (2013) (henceforth NSBU), also conduct econometric analyses of the rare disaster hypothesis, but they vastly differ in their conclusions. Therefore, we contrast their methodologies and findings against our own to derive potential explanations for the disparate results.

The two previous studies share with ours the use of Barro and Ursúa's (2008) data and their effort to allow for multi-period disaster events. They also follow Blanchard's (2008) advice to base the empirical analysis of the RDH on the basic asset pricing equation. Whereas we and NSBU rely on the EZW-pricing kernel, J&G use the power utility stochastic discount factor. With power utility, $\psi = 1/\gamma$, so they do not have to worry about estimating the IES.[35]

Furthermore, J&G and NSBU share our view that the nature of rare disasters calls for non-standard econometric methods that combine elements of calibration practices. As in the present study, NSBU implement a two-step approach. The first step is the Bayesian modeling of a disaster-including consumption process. In the second step, they numerically solve the basic asset pricing equation, using as input the first-step endowment process that is parameterized with draws from the posterior parameter distribution. The values of β and γ are chosen such that the series generated in the process match the equity premium and mean T-bill return. However, NSBU do not extend this calibration procedure to the IES. They circumvent the aforementioned

[35] To justify working with power utility, J&G stress that the choice of the utility function did not lead to qualitatively different equity premia in previous studies, and they cite Barro (2006) (who uses power utility) and Wachter (2013) (who uses EZW-preferences). However, these studies assume that consumption disasters occur as single-period events.

identification issues by setting $\psi = 2$, the same value chosen by Bansal et al. (2007).

Adopting an alternative methodological approach, J&G rely on (Bayesian) empirical likelihood methods. They argue that as long as the consumption data include disaster events – though less frequently than expected – the empirical likelihood can overweight the probability of these events, such that it is possible to match the equity premium with a plausible RRA value.[36] Rather than trying to model the disaster process, J&G create artificial samples from non-disastrous U.S. data and the observed consumption disasters assembled by Barro and Ursúa (2017).

J&G's assessment of the explanatory power of the RDH is different from ours and that of NSBU. When they relax the assumption that disasters occur as single-period events, the RRA estimate becomes implausibly large. To obtain a reasonable level of risk aversion, disasters would have to occur much too frequently (about once in every 10 years). As a consequence, the equity premium puzzle itself would be a rare event; the implied probability of a disaster-free period of 72 years is roughly 0.1% (assuming $\gamma = 10$). By contrast, NSBU are able to match the observed equity premium with a reasonable RRA of 6.4, and the posterior distribution of the model-implied equity premium is plausible too. According to NSBU, the implied probability of observing a period of 72 disaster-free years is 12%, which broadly corresponds to the results we report in Section 5.1.

What is the reason behind these diverging results? The data used in the studies are similar, and the basic economic paradigm is the same. The econometric methods are unorthodox but well thought out; in various ways, they take account of the sample selection problem implied by the RDH. Yet one aspect is different, namely, the pricing kernel implied by the investor preferences. Assuming EZW preferences,

[36] In an exactly identified setting, such as when estimating γ using a single moment match, the procedure is equivalent to GMM. However, the approach also allows to fix the RRA coefficient to an arbitrary value (J&G consider $\gamma = 10$), estimate the respective observation weights, and conduct a test of the identifying restrictions.

we and NSBU have to deal with the identification of the IES, whereas J&G achieve it implicitly by assuming power utility. NSBU set the IES to a value that is, like our ψ estimate, well above unity. Allowing for multi-period disasters, J&G's implied IES estimate is way below 1. We think that the differential treatment of the IES is the key to explaining the apparently disparate conclusions.

In a nutshell, allowing for rare disastrous contractions means acknowledging that the economy is more risky than reflected by, for example, postwar U.S. data. Accordingly, when matching asset return moments like the equity premium, as we, J&G, and NSBU all do in some way or another, the calibrated/estimated RRA coefficient is smaller than it would be in an analysis based on regular, disaster-free data. However, by assuming that the disastrous contraction unfolds in a single period, the severity of the event becomes exaggerated. Allowing for multi-period disasters, the total contraction can pan out over subsequent quarters, and maybe even several years.

With a power utility SDF, this mitigation of the disaster event implies that to match the equity premium, the RRA estimate will be higher than in the single-period case. But the increase of RRA inevitably entails a reduction of the IES. Conversely, setting the RRA to a plausible level evokes implausibly high disaster probabilities. Such reasoning is corroborated by an analysis in which we use our proposed approach with a power utility SDF instead of the EZW pricing kernel, which yields a γ estimate that increases to the mid-twenties to mid-forties, a range that is in line with the results reported by J&G. Details of this analysis can be found in the Online Appendix.

The story unfolds differently with EZW preferences and multi-period disasters, because this setting allows for another aspect of the equity premium. The possibility that disastrous contractions can pan out over years means that an investor faces late resolution of considerable uncertainty, which is a non-issue when the contraction unfolds instantaneously. Power utility implies indifference to the timing of the resolution of uncertainty, but late resolution is disliked by an investor with EZW preferences and IES and RRA that are both above unity. Such an investor will demand a premium to be compensated for exposure to the late resolution of uncertainty (when investing in pro-cyclical payoffs). This aspect of the equity premium is not accounted for by risk aversion, and Schlag et al. (2020) provide empirical evidence of a notable late resolution premium.[37] In the multi-period disaster framework of our study, the RRA estimate remains plausible, and the IES estimate is greater than 1, which is consistent with late resolution premia and entails plausible asset pricing implications. This interpretation also applies to NSBU's study, but here the late resolution premium is preset, by choosing a convenient IES value, thereby aiding the favorable assessment of the RDH.[38]

6 Discussion: connections to related approaches

In this section, we delineate some commonalities of extant simulation-based methods with the approach proposed herein, and we highlight its distinctive features.

Although our formal arguments draw on the foundations of SMM theory laid out by Duffie and Singleton (1993), our approach is not a special case of SMM. There are two marked differences. First, we match simulated moments with sample moments (more precisely, functions of moments), but unlike with SMM, we also express the latter as functions of the structural parameters to be estimated. By contrast, standard SMM assumes that the moments computed using real data are

[37] We note that the late resolution premium is a different notion than the timing premium, a situation in which consumption uncertainty is completely resolved. An early resolution of uncertainty means that economic planning remains possible.

[38] Although NSBU also identify the IES as the crucial factor to reconcile the RDH with multi-period disasters, their explanation is different from ours: It exploits that their consumption process implies the predictability of consumption growth at the onset of a disaster.

produced through a "simulation by nature," and thus they only implicitly depend on these parameters. The difference arises as a result of the identification of the preference parameters $\boldsymbol{\theta}_1$ through the reformulated basic asset pricing equation (2.13). The pricing kernel $m_t(\boldsymbol{\theta}_1)$ appears on both sides of this equation; its left-hand side is approximated using sample moments, whereas simulated moments are used for the right-hand side. Second, our study of the RDH exploits very heterogeneous data sources, which motivates the initial estimation step and the need to calibrate some unidentified parameters. These aspects are not taken into account by conventional SMM.

Similar conclusions apply when comparing our approach to indirect inference (II) and simulated minimum distance (SMD) methods, which may be considered as generalizations of SMM. There are variants of these methodologies with similarities to the present study. For example, Grammig and Küchlin (2018) propose a two-step II approach to estimate the parameters of economic growth processes in a first stage, then use the results to estimate the parameters of the EZW pricing kernel in a second stage. Their simulation of data based on preliminary parameter estimates (and not only calibrated values) is analogous to our approach, but it is also quite uncommon. In the context of the simulation-based estimation of measurement error models, a similar strategy has been proposed by Gospodinov et al. (2017).

Insofar as the estimation strategy focuses on capturing those features of the data deemed essential, the approach pursued herein is reminiscent of the sequential partial II framework proposed by Dridi et al. (2007). They argue that it is not expedient that the auxiliary model captures every aspect of the structural model to be simulated, because it will be at least partially misspecified. Instead, they suggest relying on economically relevant binding functions. Our use of the basic asset pricing equation to identify the preference parameters follows the same reasoning. As outlined supra, Dridi et al. (2007) distinguish parameters with economic meaning and nuisance parameters that enable the data simulation from the structural model. Some of the parameters of the second type can be estimated, others must be calibrated. These are obvious similarities with our approach. However, there are also important differences. Dridi et al. (2007) refer to the parameters of the second type as nuisance parameters, because they govern a simulated process that may be a "caricature of reality." In contrast, the parameters of the defining MPP, which are needed for the simulation in the second step, account for economically important aspects. The defining MPP is a statistical model, and thus a description of the true disaster process, but this description has to be meaningful, in the sense that the disaster probabilities and conditional distribution of disaster marks are assumed to be correctly specified.

7 Conclusion

Calibration studies prevail when assessing the empirical explanatory power of preference-based asset pricing models that allow for rare disaster risk. The nature of the theoretical frameworks used – fully parameterized structural models that are not meant to be true descriptions of economic reality – renders conventional estimation and specification testing questionable. The use of such stylized models may appear inevitable, because a strategy to "let the data speak" would be subject to a sample selection problem. The data typically used in empirical asset pricing contain few, if any, disastrous consumption contractions. But shunning classic econometric analysis has its drawbacks. Hansen and Heckman (1996) criticize certain calibration practices for lacking methodological discipline and seeking confirmatory evidence instead of trying to disprove hypotheses.

A key simplifying assumption adopted in many theoretical models is that a disaster event is instantaneous. This simplification ignores empirical evidence that shows that disastrous consumption contractions can build up over multiple time periods. The single-period disaster assumption also is not necessarily innocuous. Some scholars suspect that it is the very reason for the support that calibration studies have lent to the RDH.

These observations motivate the present study, in which we propose a novel strategy to estimate and empirically assess asset pricing models that allow for multi-period disaster risk. We have been influenced by ideas put forth by Dridi et al. (2007), who waive the elusive ideal of an efficient estimation of fully parameterized structural models. They instead advocate exploiting a model's key economic implications for parameter estimation and tests of empirical validity. Blanchard (2008) similarly argues for basing the empirical analysis of disaster-including C-CAPMs on model-implied asset pricing equations (i.e., first-order intertemporal conditions).

To transfer these ideas to an empirical methodology, we start from the basic asset pricing restrictions, assuming a representative agent with recursive preferences and an endowment process that may be prone to disastrous consumption contractions. The asset pricing equations are reformulated to allow for the possibility of such multi-period disaster events, modeled as a marked point process. This setting provides the basis for an econometric strategy that takes account of the inherent sample selection problem that hampers the empirical assessment of the rare disaster hypothesis. This strategy involves the simulation of disaster-including consumption growth and return series, which are generated by a marked point process, whose parameters are estimated in a first step or, due to data limitations, calibrated. The second step consists of a simulation-based estimation of the investor preference parameters.

Econometric analyses of preference-based asset pricing models have yielded notoriously negative results, including implausible estimates of structural parameters and doubtful asset pricing implications. But the economic paradigm should not be discarded light-heartedly. It represents a rational link between the real economy and financial markets. Applying the proposed methodology, we find that the estimate of the subjective discount factor implies a sensible, positive rate of time preference; the RRA estimates and the associated 95% confidence bounds fall within a strict plausibility range, and the IES parameter estimates are significantly greater than unity. The relative magnitudes of the estimated IES and RRA coefficients indicate a preference for an early resolution of risk, which, in conjunction with an IES > 1, is a crucial condition for obtaining meaningful asset pricing implications.

As suggested by Hansen and Heckman (1996, p. 93), we interpret tests of the validity of a model "as a barometer for measuring whether a given parametric structure captures the essential features of the data." We find that the model-implied mean market return, T-bill return, and market Sharpe ratio have economically meaningful sizes. The estimated multi-period disaster C-CAPM thus reconciles plausible investor preferences with the high U.S. postwar equity premium and the low mean T-bill return. It does so without the implication of a questionably high timing premium. With an econometric analysis, we thus corroborate the capacity of the rare disaster hypothesis to provide a sensible explanation of the equity premium puzzle, even when disasters can occur as multi-period events. We identify the differential treatment of the possibility of a premium for the late resolution of risk as an explanation for the disparate conclusions of previous studies regarding the explanatory power of the rare disaster hypothesis.

Avenues for further research stretch in various directions. As mandated by limitations of the currently available data, we have focused on rather parsimonious MPP variants. With improved data quality, and a smart treatment of missing disaster return data, e.g., by adopting ideas from Chaudhuri et al. (2018), more elaborate MPP specifications could be considered for the first estimation step. For example, using a discrete-time version of the multivariate intensity process proposed by Bowsher (2007), one could attempt to model the interdependence of regional disaster intensities. Finally, recognizing the prominent role of the IES and the aforementioned caveats related to its estimation, one could draw on Garcia et al.'s (2006) idea to identify the IES by allowing for a habit level of consumption. We leave these topics for further research.

A Appendix

A.1 Using excess returns and hybrid variants

The second-step estimation approach can easily be modified to use a combination of returns and excess returns in the objective function in Eq. (3.10). Section 2 shows that the counterpart of Eq. (2.8), as it applies to an excess return R_t^{ei}, reads:

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) R_t^{ei} \middle| d_t = 0 \right] = -\frac{p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) R_t^{ei} \middle| d_t = 1 \right]}{1-p}. \tag{A-1}$$

Proceeding as for a gross return, Eq. (A-1) can be written in terms of unconditional moments that represent expected prices of special payoffs, one of which is 0 in disaster periods,

$$\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) R_t^{ei} \middle| d_t = 0 \right] = \frac{\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) u_t^i, \right]}{1-\mathbb{E}[d_t]}, \tag{A-2}$$

where $u_t^i{=}R_t^{ei}(1-d_t)$, while the other, $v_t^i = R_t^{ei} d_t$, is 0 in non-disaster periods,

$$-\frac{p\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) R_t^{ei} \middle| d_t = 1 \right]}{1-p} = -\frac{\mathbb{E}\left[m_t(\boldsymbol{\theta}_1) v_t^i \right]}{1-\mathbb{E}[d_t]}. \tag{A-3}$$

Leveraging the idea of Section 2, the sample equivalent of Eq. (A-2) is computed using actual data:

$$H_{T_2}^{ei}(\boldsymbol{\theta}_1) \equiv \frac{\frac{1}{T_2}\sum_{t=1}^{T_2} m_t(\boldsymbol{\theta}_1) u_t^i}{1-\frac{1}{T_2}\sum_{t=1}^{T_2} d_t}, \tag{A-4}$$

while simulated moments are used to approximate Eq. (A-3):

$$\widetilde{H}_{T_2}^{ei}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) \equiv \frac{-\frac{1}{\mathcal{T}(T_2)} \sum_{s=1}^{\mathcal{T}(T_2)} \widetilde{m}_s(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) \tilde{v}_s^i(\hat{\boldsymbol{\theta}}_2)}{1-\frac{1}{\mathcal{T}(T_2)} \sum_{s=1}^{\mathcal{T}(T_2)} \tilde{d}_s(\hat{\boldsymbol{\theta}}_2)}. \tag{A-5}$$

The equivalent of the moment matches in Eq. (3.9) using excess returns reads:

$$G_{T_2}^{ei}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) = H_{T_2}^{ei}(\boldsymbol{\theta}_1) - \widetilde{H}_{T_2}^{ei}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2). \tag{A-6}$$

Testing an asset pricing model by exploiting its implications for both returns and excess returns is a common empirical practice. In that vein, it is possible to use one moment match of the type in Eq. (3.9) for a base return R_t^1 and moment matches of the type in Eq. (A-6) that involve N excess returns, such that Eq. (3.8) becomes:

$$G_{T_2}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) = [G_{T_2}^1(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2), G_{T_2}^{e1}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) \ldots, G_{T_2}^{eN}(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2)]', \tag{A-7}$$

which is then used in Eq. (3.10) to estimate the preference parameters.

A.2 Accounting for parameter estimation uncertainty

We use a simulation-based approach to provide confidence intervals for model parameters and model-implied indicators. It consists of the following steps:

1. For each of $B = 1,000$ replications, the defining MPP is simulated using the first-step estimates $\hat{\boldsymbol{\theta}}_{21}$ and $\hat{\boldsymbol{\theta}}_{22}$. Each of the generated pseudo-samples contains T_1 observations, consisting of the sequence of simulated disaster indicators $\tilde{d}_s$ and $\tilde{d}_s^+$, as well as simulated consumption contraction factors $\tilde{b}_s$, which are drawn, in the case of a simulated disaster event, from the Pareto distributions endowed with parameters $\hat{\theta}_{\mathrm{P}}^-$, and $\hat{\theta}_{\mathrm{P}}^+$, respectively. The MPP parameters are re-estimated on each pseudo-sample.

2. For each of the $B = 1,000$ replications, an artificial sample of T_2 observations is drawn that consists of series of consumption growth and returns of interest. These pseudo-samples are obtained by a block-bootstrap simulation from the second-step data, as described in Section 4.1. For each pseudo-sample, the estimation of the preference parameters is performed as described in Section 4.2, with the data simulation based on the MPP estimates that correspond to the same replication. For each of the 1,000 pseudo-samples, statistics of interest

like the model-implied equity premium, market Sharpe ratio, and MAEs are computed.

3. Taking the 0.025 and 0.975 quantiles of the empirical distributions of the $B = 1,000$ estimates and statistics of interest, 95% confidence intervals for the model parameters and model-implied indicators are computed. The empirical standard deviations provide simulation-based standard errors for the parameter estimates.

A.3 Online Appendix

The first part of the Online Appendix contains additional results. It is accessible at

https://tinyurl.com/web-appendix-GS-rare-disasters.

It contains the following sections:

[O.1] Regular and contraction components: Links to Barro (2006) and Barro and Jin (2011)

[O.2] Concerning consistency

[O.3] Characterizing the limit distribution

[O.4] Alternative copula functions

[O.5] Alternative identification of ψ

[O.6] Juxtaposition with Nakamura et al. (2013) and Julliard and Ghosh (2012)

[O.7] Timing premium calculated for Nakamura et al. (2013)

[O.8] Results assuming power utility

[O.9] Alternative disaster thresholds and further robustness checks

The second part of the Online Appendix contains programs and data that can be used to replicate the reported results. It is accessible at:

https://tinyurl.com/code-data-GS-rare-disasters.

Table 1: Implementation of the econometric strategy: synopsis

This table provides an overview of the implementation of the proposed econometric strategy.

Step	Action	Motivation	Parameters involved	Section	Robustness & plausibility checks
0 (prepare data)	prepare multi-country panel data for estimation: generate sequence of multi-period disasters	allows estimation of MPP model by ML	fixed: disaster threshold q	4.1	variation of disaster threshold
1a (estimate MPP parameters)	estimate MPP parameters using the identified disasters from step 0	allows simulation of $\{d_s\}$ and $\{d_s^+\}$; allows drawing from marginal (Pareto) distribution of consumption contractions	estimated: hazard rate parameters: $\boldsymbol{\theta}_{21} = (\mu, \alpha, \delta, \mu^*, \alpha^*, \delta^*)'$ and Pareto parameters: $\boldsymbol{\theta}_{22} = (\theta_P^+, \theta_P^-)'$	4.2	variation of the MPP specification
1b (calibrate MPP parameters)	assume that the marginal distribution of return contractions is identical to the marginal distribution of consumption contractions, specify copula function, and calibrate copula parameters	complements estimation results from step 1a and allows simulation of MPP (including drawing return and consumption contractions from their joint distribution)	calibrated: copula parameter(s) and government default probability: $\boldsymbol{\theta}_{23} = (\rho^1, \ldots, \rho^M, p^f)'$ also: choices regarding copula function and marginal distribution of return contractions	4.2	variation of copula parameters and copula functions
2 (estimate preference parameters)	simulation-based estimation of preference parameters using steps 0-1b	allows assessment of the plausibility of preference parameters	estimated: preference parameters: $\boldsymbol{\theta}_1 = (\beta, \gamma, \psi)'$ and intercept in IES identification, κ	4.3	variation of test assets and IES identification
3 (assess estimation uncertainty)	assess estimation uncertainty via repeated sampling procedure	provide standard errors and confidence intervals for preference parameters		4.3, A.2	compare with asymptotic first-step standard errors
4 (assess economic sensibility)	compute model-implied key financial indicators, MAEs, and timing premium using the results from steps 2-3	assess plausibility of model-implied indicators and timing premium		5.3-5.5	contrast results with previous studies

Table 2: Multi-country consumption data
This table lists the selected 35 countries and the time periods with available consumption data. These data have been assembled by Barro and Ursúa (2008).

country	time periods
Argentina	1875 – 2009
Australia	1901 – 2009
Austria	1913 – 1918, 1924 – 1944, 1947 – 2009
Belgium	1913 – 2009
Brazil	1901 – 2009
Canada	1871 – 2009
Chile	1900 – 2009
Colombia	1925 – 2009
Denmark	1844 – 2009
Finland	1860 – 2009
France	1824 – 2009
Germany	1851 – 2009
Greece	1938 – 2009
India	1919 – 2009
Indonesia	1960 – 2009
Italy	1861 – 2009
Japan	1874 – 2009
Malaysia	1900 – 1939, 1947 – 2009
Mexico	1900 – 2009
Netherlands	1807 – 1809, 1814 – 2009
New Zealand	1939, 1944, 1947 – 2009
Norway	1830 – 2009
Philippines	1946 – 2009
Peru	1896 – 2009
Portugal	1910 – 2009
South Korea	1911 – 2009
Spain	1850 – 2009
Sri Lanka	1960 – 2009
Sweden	1800 – 2009
Switzerland	1851 – 2009
Taiwan	1901 – 2009
UK	1830 – 2009
USA	1834 – 1859, 1869 – 2009
Uruguay	1960 – 2009
Venezuela	1923 – 2009

Table 3: U.S. consumption growth and asset returns 1947:Q2–2014:Q4

This table contains descriptive statistics of consumption growth (G) and gross returns on the three sets of test assets. Panel A: CRSP value-weighted market portfolio proxy (*Market*) and T-bill; Panel B: ten size-sorted portfolios and T-bill; Panel C: ten industry portfolios and T-bill. In Panel B, 1^{st}, 2^{nd}, and so on refer to the deciles of the ten size-sorted portfolios. The ten industry portfolios in Panel C are: non-durables (*NoDur*: food, textiles, tobacco, apparel, leather, toys), durables (*Durbl*: cars, TVs, furniture, household appliances), manufacturing (*Manuf*: machinery, trucks, planes, chemicals, paper, office furniture), energy (*Engry*: oil, gas, coal extraction and products), business equipment (*HiTec*: computers, software, and electronic equipment), telecommunication (*Telcm*: telephone and television transmission), shops (*Shops*: wholesale, retail, laundries, and repair shops), health (*Hlth*: healthcare, medical equipment, and drugs), utilities (*Utils*), and others (*Other*: transportation, entertainment, finance, and hotels). In addition, *ac* stands for the first-order autocorrelation, and *std* is the standard deviation.

Panel A: mkt

	mean	std	ac	correlations	
				G	T-bill
Market	1.0211	0.0816	0.084	0.175	0.026
T-bill	1.0017	0.0045	0.857	0.204	
G	1.0048	0.0051	0.311		

Panel B: size dec

	mean	std	ac	correlations										
				G	T-bill	10^{th}	9^{th}	8^{th}	7^{th}	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}
1^{st}	1.0290	0.1251	0.061	0.178	-0.015	0.711	0.818	0.857	0.884	0.895	0.912	0.931	0.949	0.964
2^{nd}	1.0271	0.1177	-0.001	0.172	0.005	0.781	0.871	0.915	0.933	0.947	0.961	0.974	0.982	
3^{rd}	1.0287	0.1115	-0.024	0.165	-0.001	0.818	0.907	0.943	0.956	0.968	0.976	0.985		
4^{th}	1.0270	0.1072	-0.018	0.165	0.002	0.830	0.914	0.948	0.962	0.976	0.983			
5^{th}	1.0274	0.1036	0.013	0.167	0.019	0.855	0.936	0.967	0.972	0.982				
6^{th}	1.0262	0.0971	0.019	0.143	0.001	0.868	0.946	0.970	0.977					
7^{th}	1.0262	0.0964	0.042	0.157	0.009	0.892	0.965	0.982						
8^{th}	1.0249	0.0923	0.022	0.145	0.019	0.906	0.975							
9^{th}	1.0237	0.0841	0.068	0.148	0.021	0.935								
10^{th}	1.0198	0.0767	0.119	0.178	0.043									

Panel C: industry

	mean	std	ac	correlations										
				G	T-bill	Other	Utils	Hlth	Shops	Telcm	HiTec	Engry	Manuf	Durbl
NoDur	1.0238	0.0811	0.047	0.090	0.105	0.838	0.674	0.800	0.871	0.656	0.642	0.445	0.829	0.685
Durbl	1.0236	0.1156	0.103	0.190	0.009	0.801	0.484	0.520	0.773	0.581	0.690	0.490	0.832	
Manuf	1.0229	0.0899	0.082	0.173	0.014	0.901	0.580	0.745	0.825	0.647	0.807	0.635		
Engry	1.0253	0.0888	0.041	0.163	-0.039	0.592	0.534	0.423	0.422	0.432	0.497			
HiTec	1.0258	0.1159	0.070	0.167	-0.000	0.758	0.470	0.663	0.733	0.659				
Telcm	1.0187	0.0805	0.148	0.099	0.104	0.695	0.627	0.568	0.668					
Shops	1.0238	0.0957	0.039	0.158	0.044	0.837	0.557	0.704						
Hlth	1.0271	0.0909	0.054	0.092	0.085	0.726	0.542							
Utils	1.0195	0.0711	0.080	0.069	0.071	0.655								
Other	1.0217	0.0982	0.078	0.159	0.034									

Table 4: First-step estimation results

This table displays the estimates of the MPP parameters, along with asymptotic standard errors (in parentheses) and simulation-based standard errors (in brackets). Asymptotic standard errors are Hessian-based; the simulation-based standard errors are computed as described in Section A.2. The number of replications is B=1k. Here, $\mathcal{L}$ is the log-likelihood value at the maximum; AIC = $2K - 2\ln\mathcal{L}$ and SBC = $K\ln T_1 - 2\ln\mathcal{L}$, where K is the number of parameters in the vector $\boldsymbol{\theta}_{21}$. $\mathcal{LR}$ gives the p-values (in percent) of the likelihood ratio tests of the null hypothesis that the parameter restrictions implied by the CH_0 specification are correct. Specification variants are reported in ascending order by SBC.

	θ_P^+	θ_P^-	μ	μ^*	α	α^*	δ	δ^*	$\mathcal{L}$	AIC	SBC	$\mathcal{LR}$
	37.255	35.687										
	(1.478)	(1.696)										
	[1.844]	[1.261]										
CH_0			178.3	1.201					-790.3	1584.7	**1600.1**	
			(18.8)	(0.023)								
			[18.7]	[0.024]								
CH_4			64.9	1.201			441.1		-787.0	**1580.0**	1603.2	<1.0
			(49.3)	(0.023)			(211.5)					
			[55.9]	[0.025]			[228.6]					
CH_3			64.9	1.214			441.1	-0.375	-786.8	1581.5	1612.5	2.9
			(49.3)	(0.032)			(211.5)	(0.537)				
CH_2			198.9	1.220	-0.146	-0.002			-789.9	1587.7	1618.7	62.9
			(30.8)	(0.050)	(0.153)	(0.004)						
CH_1			71.7	1.236	-0.032	-0.002	430.5	-0.398	-786.6	1585.3	1631.7	11.7
			(59.2)	(0.056)	(0.145)	(0.004)	(216.1)	(0.541)				

Table 5: Second-step estimation results

This table displays the second-step estimation results. Panels A-C break down the results by the calibration of the copula correlations ρ. Each panel reports the results by set of test assets (columns) and MPP specification (rows). The bold numbers are point estimates of the preference parameters. Estimates of κ are not reported, for brevity. The numbers in parentheses are simulation-based standard errors, and the numbers in brackets are the bounds of the 95% confidence intervals, using B=1k.

Panel A: $\rho = \mathrm{Corr}(G_t^r, R_t^r)$

	mkt			size dec		industry	
	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
CH_0	**1.445** (0.160)	**0.99163** (0.00218)	**1.469** (0.276)	**0.99402** (0.00458)	**1.561** (0.261)	**0.99450** (0.00519)	**1.578** (0.287)
	[1.235 1.858]	[0.98734 0.99584]	[1.080 2.183]	[0.98683 1.00455]	[1.231 2.327]	[0.98918 1.00257]	[1.185 2.332]
CH_4	**1.438** (0.164)	**0.99164** (0.00236)	**1.465** (0.299)	**0.99402** (0.01381)	**1.556** (0.281)	**0.99450** (0.00497)	**1.573** (0.310)
	[1.233 1.861]	[0.98690 0.99614]	[1.040 2.260]	[0.98614 1.00547]	[1.209 2.349]	[0.98858 1.00445]	[1.161 2.424]

Panel B: $\rho = 0.99$

	mkt			size dec		industry	
	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
CH_0	**1.452** (0.160)	**0.99161** (0.00219)	**1.477** (0.277)	**0.99388** (0.00460)	**1.565** (0.260)	**0.99435** (0.00544)	**1.583** (0.286)
	[1.237 1.858]	[0.98732 0.99584]	[1.081 2.191]	[0.98668 1.00446]	[1.231 2.320]	[0.98897 1.00247]	[1.183 2.334]
CH_4	**1.441** (0.164)	**0.99163** (0.00237)	**1.470** (0.301)	**0.99389** (0.02293)	**1.557** (0.280)	**0.99437** (0.00506)	**1.574** (0.309)
	[1.232 1.864]	[0.98689 0.99615]	[1.042 2.271]	[0.98596 1.00538]	[1.209 2.348]	[0.98842 1.00434]	[1.159 2.423]

Panel C: $\rho = 0$

	mkt			size dec		industry	
	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
CH_0	**1.443** (0.160)	**0.99163** (0.00218)	**1.467** (0.276)	**0.99403** (0.00458)	**1.559** (0.261)	**0.99451** (0.00518)	**1.576** (0.287)
	[1.234 1.858]	[0.98734 0.99585]	[1.080 2.183]	[0.98684 1.00456]	[1.231 2.329]	[0.98918 1.00258]	[1.185 2.331]
CH_4	**1.437** (0.165)	**0.99164** (0.00236)	**1.464** (0.299)	**0.99403** (0.01360)	**1.555** (0.281)	**0.99451** (0.00496)	**1.572** (0.310)
	[1.234 1.861]	[0.98691 0.99614]	[1.040 2.260]	[0.98616 1.00548]	[1.209 2.350]	[0.98859 1.00446]	[1.161 2.425]

Table 6: Model-implied indicators

The table displays the model-implied equity premium, mean market return, mean T-bill return, market Sharpe ratio, MAE, and the estimates-implied value of $\theta = (1-\gamma)/(1-\psi^{-1})$. The computations use the base specification variant, such that preference parameter estimates rely on the CH_0, and ρ is calibrated to the empirical correlation between regular returns and consumption growth. The column labeled *data* displays the empirical values of the indicators (1947:Q2–2014:Q4 U.S. data). Results are reported by the set of test assets. The numbers in brackets are the lower and upper bounds of the 95% confidence intervals, using B=1k.

	data	**mkt**	**size dec**	**industry**
equity premium	*1.94*	**1.84**	**2.08**	**2.12**
(% per qtr)		[0.96 2.69]	[1.35 2.77]	[1.26 2.97]
mean market return	*2.11*	**1.95**	**2.19**	**2.25**
(% per qtr)		[1.08 2.71]	[1.49 2.82]	[1.37 3.05]
mean T-bill return	*0.17*	**0.11**	**0.11**	**0.12**
(% per qtr)		[-0.10 0.30]	[-0.10 0.31]	[-0.08 0.33]
Sharpe ratio	*0.237*	**0.225**	**0.254**	**0.259**
(market)		[0.111 0.373]	[0.156 0.382]	[0.144 0.406]
MAE		**0.11**	**0.16**	**0.33**
(% per qtr)		[0.10 0.16]	[0.15 0.33]	[0.24 0.58]
$\hat{\theta} = (1-\hat{\gamma})/(1-\hat{\psi}^{-1})$		**-1.52**	**-1.82**	**-1.88**
		[-3.48 -0.21]	[-3.57 -0.66]	[-3.88 -0.52]

Table 7: Implied timing premium

The table displays the implied timing premia computed for the asset pricing models proposed by Wachter (2013), Barro (2009), and Bansal and Yaron (2004) using the preference parameter values calibrated in each respective study. The timing premia implied by these models are taken from Epstein et al. (2014). The model-implied timing premium for the estimated disaster-including C-CAPM in this study (base variant) is computed using the program supplied by Epstein et al. (2014) with 10k simulated consumption growth series of 1k observations. The simulation of the timing premium in the Nakamura et al. (2013) setting also uses 10k simulated consumption growth series with length 200.

	γ	ψ	β	timing premium (%)	decision frequency
Bansal and Yaron (2004)	7.50	1.50	0.998	**23**	monthly
Barro (2009)	4.00	2.00	0.951	**18**	annual
Wachter (2013)	3.00	1.00	0.988	**42**	quarterly
Nakamura et al. (2013)	6.40	2.00	0.967	**11.3**	annual
This study (base variant)	1.47	1.44	0.992	**0.9**	quarterly

Figure 1: Distribution of disaster events

This figure depicts the 89 disaster events identified from Barro and Ursúa's (2008) (updated) multi-country panel data using q=0.145. Black lines denote European countries, red lines South American countries and Mexico, golden lines Western offshores (Australia, Canada, New Zealand, and U.S.A.), and blue lines represent Asian countries. The dotted horizontal line indicates the average contraction size.

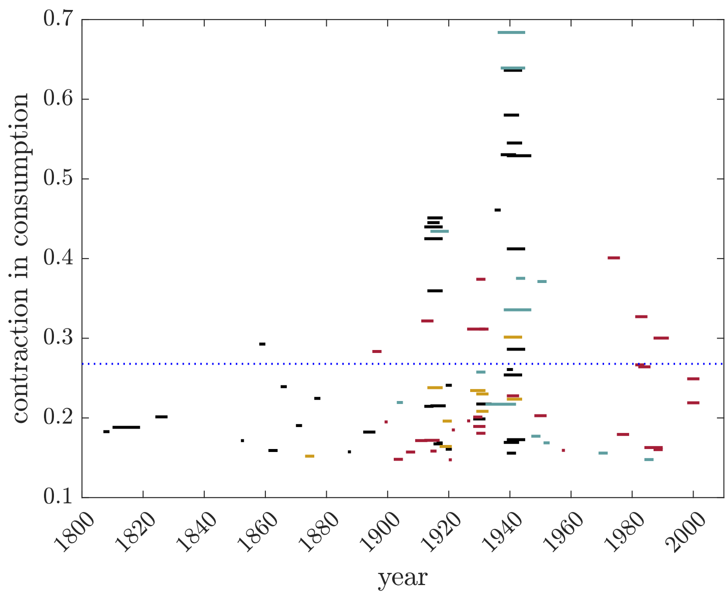
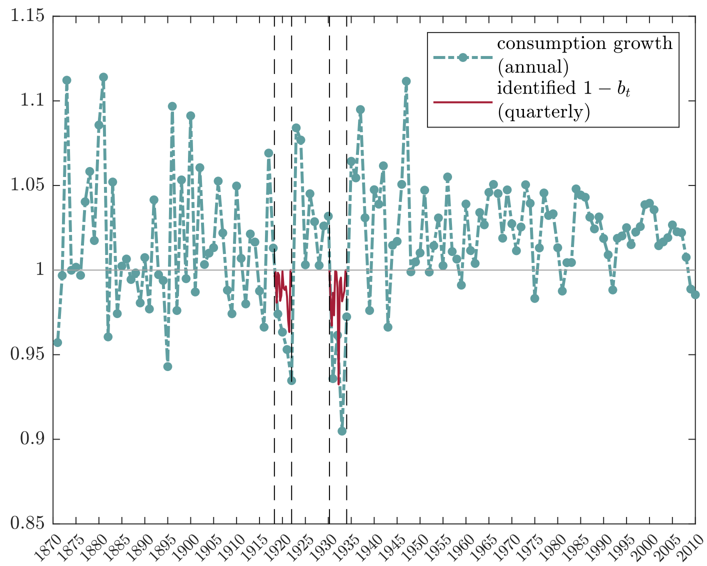
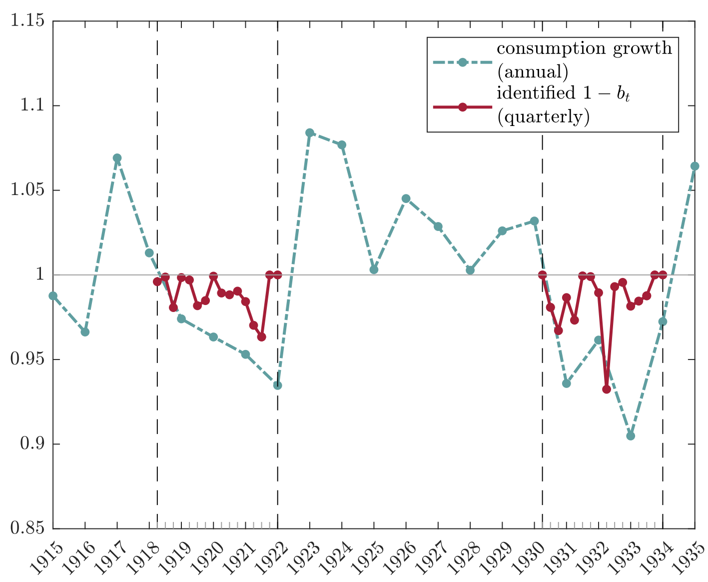


Figure 2: Identification of consumption disasters and quarterly contraction factors: The U.S. example

The dashed (blue) line in Panel 2a depicts annual U.S. consumption growth during 1870-2009 using Barro and Ursúa's (2008) data. Assuming a threshold of q=14.5%, and implementing Barro's (2006) disaster detection scheme, two disaster events are identified. The first happened during 1918–1921, when aggregate consumption dropped (peak-to-trough) by 16.4%, and the second during 1930–1933 shows a drop in aggregate consumption of 20.8%. Panel 2b zooms into a period that contains the two disaster events. The solid (red) line indicates (one minus) the quarterlized contraction factors obtained by the procedure described in Section 4.1. The vertical dashed lines highlight the beginning and end of the two disaster events.



(a) Annual U.S. consumption growth and identified quarterly contractions



(b) Zooming in 1915–1935

Figure 3: Illustration of the simulation of a multi-period consumption disaster
This figure illustrates the simulation of consumption growth before, during, and after a multi-period disaster event using the described procedure. The disaster threshold is q=14.5%. The MPP simulation uses the CH_0 variant with $\hat{\mu}$=178.3 and $\hat{\mu}^*$=1.2, along with Pareto parameters $\hat{\theta}_{\text{P}}^{-}$=35.7 and $\hat{\theta}_{\text{P}}^{+}$=37.3 (estimates reported in Table 4). The graph displays the components of simulated consumption growth $\widetilde{G}_s$: the simulated regular growth component $\widetilde{G}_s^r$ and the scaling factor $1-\tilde{b}_s$. The simulated disaster event reaches the threshold after 9 quarters, lasts 11 quarters, and implies an overall contraction of 20.6%. The sharpest quarterly contraction is 4.9%.

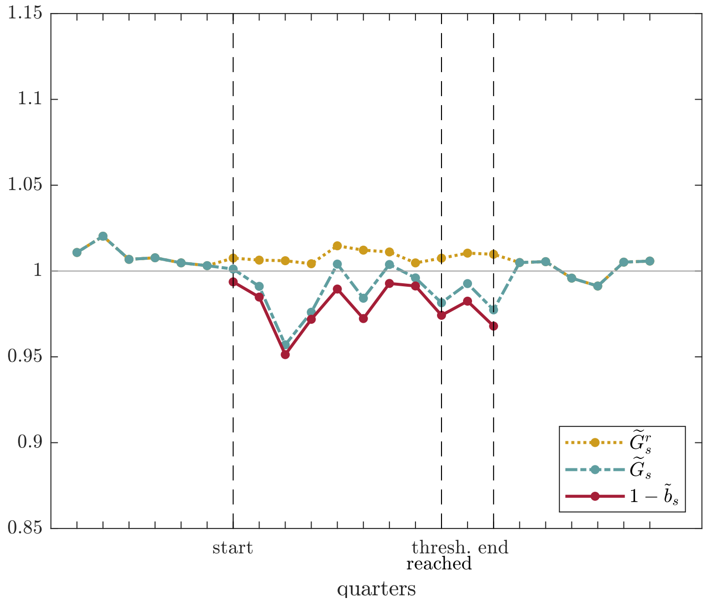
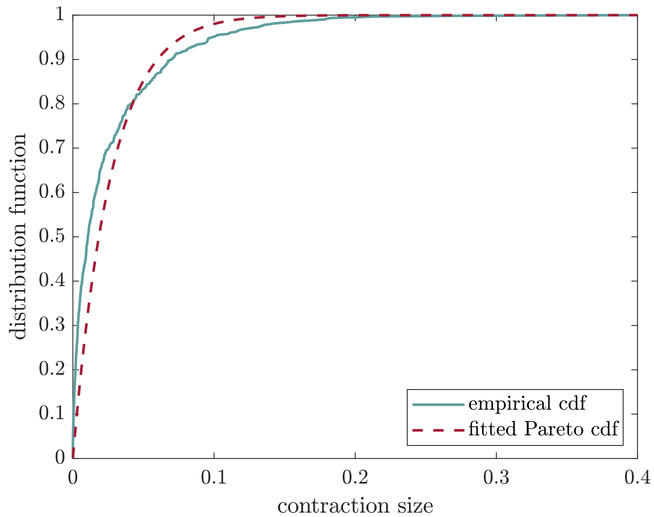
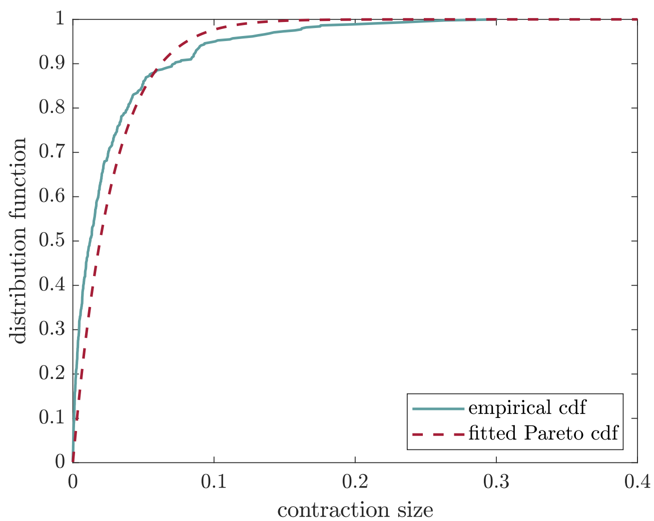


Figure 4: Fitted Pareto vs. empirical c.d.f.
This figure illustrates the empirical c.d.f.s (solid lines) and the fitted c.d.f.s (dashed lines) of the contractions identified from the Barro and Ursúa (2008) data, using a disaster threshold of q=0.145. Panel (a) captures the distribution of contractions that occur at the beginning of a disaster and contribute to reaching the disaster threshold. Panel (b) refers to contractions that extend beyond the disaster threshold. The fitted c.d.f.s use the parameter estimates from Table 4.

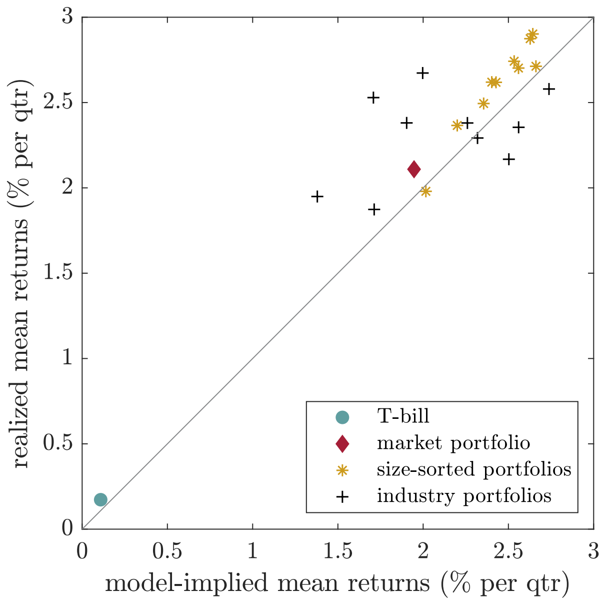


(a) c.d.f. fit for contractions that contribute to reaching q



(b) c.d.f. fit for contractions that extend beyond q

Figure 5: Cross-sectional fit
This figure compares the mean realized returns of the test assets with the model-implied mean returns computed according to Eq. (5.1). The model-implied computations use the base specification variant, such that preference parameter estimates rely on the CH_0, and ρ is calibrated to the empirical correlation between regular returns and consumption growth.



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