Why Adding Firm Value With a Put Feature in Debt Contracts is Better Than Renegotiation

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Abstract

In this paper, we analyze the ability of putable debt to add firm value. To stress the impact of a put feature, we compare the resulting optimal firm values and capital structures to those of a firm with straight that can be renegotiated. For this purpose, we consider a time-independent firm value model with tax-deductibility of coupon payments, bankruptcy costs in the case of a default, and dynamic restructuring. A put right can always be designed so that a put is enforced for low asset values but the bond remains alive for high asset values. The optimal firm value arising from this type of equilibrium strategy is remarkable for several reasons: The optimal firm value under putable debt is always higher than under straight debt even under renegotiation with arbitrary negotiation power of debt and equity holders. Moreover, the optimal firm value under putable debt always benefits from higher bankruptcy costs, while the optimal firm value under straight debt suffers. Accordingly, a higher volatility of asset value returns can be favorable for a high firm value under putable debt, while it always destroys value of a firm with straight debt.

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1 Introduction

Debt is a crucial factor for firms to add firm value. The seminal papers by Fischer/Heinkel/Zechner (1989a) and Leland (1994) show how firms have to choose their capital structure to accomplish their maximum firm value. The firm value increase in their models comes from a tax advantage of coupons. The effect of coupon payments is that the taxable income on the corporate level declines, but the bondholders, who receive the coupons, suffer from an additional taxation. Since the tax reductions on the corporate level are usually higher than the additional taxes on the private level, there is an overall net tax reduction so that debt adds value. Even in the presence of bankruptcy costs, it is always optimal to lever a firm with debt and not to keep it unlevered as Leland (1994) shows.

In the standard model by Leland (1994) with straight debt, the optimal capital structure results from a tradeoff between a potentially higher present value of tax benefits and a higher present value of bankruptcy costs associated with higher leverage. In general, a higher leverage in form of a higher coupon creates higher tax benefits as long as the coupon is paid and the firm does not default. However, a higher coupon also results in an earlier default so that the tax benefits get lost earlier and the present value of bankruptcy costs increases. As a result, in order to have a high firm value the design of debt contracts should be so that for a given coupon, a default is relatively unlikely or in other words the critical asset value (default barrier), at which a default occurs, is relatively low.

The case of straight debt presented by Leland (1994) has the character of a benchmark model, because several authors introduce more sophisticated debt contracts to analyze its impact on the firm value. A commonly regarded additional feature of debt is an issuer call right (see e.g. Fischer/Heinkel/Zechner (1989b), Leland (1998), Goldstein/Ju/Leland (2001), Dangl/Zechner (2004), Titman/Tsyplakov (2005), Ross (2005)). A call right has two advantages. First, the default barrier for a contract with a given coupon is lower than without a call right. As mentioned above, the lower default barrier is a valuable factor to add firm value as bankruptcy costs are reduced and tax benefits arise for a longer time.[1] Second, after a call the firm can be restructured which creates further value. Due to the dynamic restructuring, debt with a higher coupon is issued after every call date which results in further tax benefits and therefore increases the firm value.

A model framework in that a default and the associated bankruptcy costs can be avoided with certainty is presented by Fan/Sundaresan (2000) and in an extended version also by Christensen/Flor/Lando/Miltersen (2002). The reason for why no bankruptcy costs arise is because the firm can renegotiate the terms of the debt contract.[2] Once the firm stops the coupon payments to enter into renegotiation,

[1]Koziol (2006) shows that for convertible debt, the default barrier rises through the conversion feature. Hence, convertible debt is usually less well-suited to add firm value than callable debt.

[2]Anderson/Sundaresan (1996) present the possibility of strategic debt service which can be seen as renegotiation where the equity holders have a first mover advantage, i.e. the full negotiation power. Mella-Barral/Perraudin (1997) and Mella-Barral (1999) are further examples for debt renegotiation.

the debt holders will agree to swap their debt contract into an equity contract as Fan/Sundaresan (2000) show. The terms of the new claims result from Nash bargaining between the debt and initial equity holders and primarily depend on the bargaining power of each counterparty. Since the outcome of the Nash bargaining game ensures that debt holders become equity holders, the firm will be unlevered after renegotiation. Therefore, bankruptcy costs are successfully avoided so that the firm value can be higher due to renegotiation.

A feature that has not been analyzed so far in terms of its ability to add firm value is the put feature of debt. A put feature allows the bondholders to sell the bond at a pre-specified put price during its lifetime. Although putable bonds are not as widely-used as callable bonds, there is still a proportion of about five percent among the outstanding corporate bonds in the U.S. market that contains a put right.[3] Moreover, a put right usually arises in every bond and debt contract whenever the firm violates a covenant. There are two ways to look at a put feature and its ability to add firm value. From the discussion of the call feature in a debt contract, we know that a default becomes less likely as a call feature increases the equity value. Due to the higher equity value, the firm has a higher incentive to avoid a default. Since the put right — in contrast to a call right — is an advantage for the debt holders, it is supposed to reduce the equity value which might speak for an earlier default. As long as a default of putable debt is possible, the default is supposed to take place earlier than without the put feature so that the put right reduces firm value. However, we cannot be sure whether a default of putable debt will in fact take place. In particular, an equilibrium put strategy under which a put takes place before the firm defaults seems to be highly attractive. Hence, the question is whether a design of putable bonds exists that ensures a put strategy without a default. Given that a suchlike bond design always exists, a further question is whether a put feature (without renegotiation) or a straight debt contract with renegotiation add more firm value, because this put strategy exhibits parallels to that under renegotiation.

In this paper, we analyze how a putable consol bond adds value to a firm and compare its outcome to the case with renegotiation. For this purpose, we consider a model framework with tax-deductibility of coupon payments and bankruptcy costs. In particular, we extend the Leland model by giving the bondholders the right to sell the bond to the firm at the put price. In addition, we allow for a dynamic restructuring whenever the bondholders put their bond. To better see the effects

[3]According to Bloomberg, 81,212 corporate bonds were outstanding in the U.S. market by the end of October 2005. Among those bonds, 41,006 bonds are callable and still 3,607 bonds are equipped with a put right.

caused by the put right, we abstract from other bond characteristics such as a call right and voluntary debt reductions. Suchlike features can be incorporated within our model in a straightforward way and might result in even higher firm values.

As a consequence of the put feature, debt holders cannot only put to obtain the put price but depending on the asset value a put can force the firm into default. Likewise, it is not only possible for the firm to trigger a default by stopping the coupon payments but it might otherwise enforce a put by the debt holders. Despite these complex consequences of a default announcement by the firm and a put by the debt holders, a bond design is always possible under which the debt holders will wait with a put until the firm announces a default. At the default announcement, a put is better for the debt and equity holders than a default and therefore bankruptcy costs are successfully avoided. Since a put for any given coupon under the optimal choice of the put price will occur at a relatively low barrier, the optimal firm value with putable debt is remarkably high. In particular, putable debt results in higher firm values than under straight debt with and without renegotiation. This is true for any arbitrary negotiation power of equity and debt holders. The rationale for the fact that a put feature results in higher firm values than renegotiation is as follows. If a firm has debt with a given coupon outstanding, the equity holders want to renegotiate even for asset values for which a put cannot take place. Since the firm value benefits from a lower put/renegotiation barrier, putable debt dominates debt under renegotiation.

The optimal put strategy reveals further surprising properties. The optimal firm value increases with the bankruptcy costs that would occur in the case of a default. Clearly, this finding is contrary to that for optimal firm values with straight debt. The reason for this outcome is that under putable debt, bankruptcy costs are a disciplining device for the debt holders to accept a put once the firm announces a default. Moreover, while under straight debt the optimal firm value suffers from a higher volatility of asset value returns, as a default becomes more likely, the optimal value of a firm with putable debt might increase.

The remainder of the paper is as follows: Section 2 presents the model framework. The optimal default strategy and the optimal firm value under the standard case of Leland (1994) with straight debt are described in Section 3. Section 4 derives the firm value under putable debt for an equilibrium strategy under that the firm cannot become insolvent. The optimal firm value under renegotiation based on Fan/Sundaresan (2000) is given in Section 5. A comparison of the optimal firm values and the corresponding debt structures under straight debt, renegotiation, and putable debt is in 6. Section 7 provides a comparative static analysis of the optimal firm values under straight debt, renegotiation, and putable debt. Section 8 concludes. Technical derivations are in Appendix B.

2 Model Framework

To analyze in how far a put feature in a debt contract adds value to a firm, we extend the time-independent firm value model presented by Leland (1994) for straight debt contracts by regarding debt with a put feature and accounting for dynamic restructuring. A major part of the analysis will be to compare the optimal firm value with putable debt to the values of otherwise identical firms with optimally-designed straight debt with and without renegotiation. The standard case with straight debt is by Leland (1994), while Fan/Sundaresan (2000) provide an extension with renegotiation. To have a consistent setup with both of these model frameworks, we also regard the asset value of the firm as state variable.[4] This allows us to compare the outcome for putable debt with the well-known results from Leland for straight debt and Fan/Sundaresan under renegotiation by using the same notation in an arbitrage-free framework.

We consider an entrepreneur who is endowed with initial assets having a value equal to V_0 at time $t = 0$. At this point in time, the entrepreneur has the possibility to issue a debt contract to new investors. Whether the entrepreneur holds the equity contract or sells it to new equity holders different from the debt holders is not crucial for the following analysis.[5]

[4]Goldstein/Ju/Leland (2001) provide an extensive discussion about advantages and disadvantages of the asset value models compared to models using the firm's free cash flows before interest and tax payments as state variable. However, there is still a major advantage of the asset value models. The advantage is that e.g. a low dividend can be associated with a high firm value and vice versa. In the case of the models with instantaneous cash flow as the state variable, the dividend is endogenous. Therefore, a high instantaneous free cash flow, which is related to a high firm value, results in a high dividend payment. This is the reason why very successful firms that mostly reinvest their free cash flows rather than to pay a dividend can hardly be captured by the cash flow models. To understand the equilibrium put strategies, it will be important to allow for a flexible choice of the payout rate which is accomplished by the asset value models. Moreover, both models do not allow for arbitrage opportunities as we will see further below. Ross (2005) illustrates the close relation of the cash flow models to the asset value models by showing how to derive the free cash flow process from the asset value process.

[5]Of course, at this point it is not crucial whether the entrepreneur holds the equity or the equity is sold to any other market participant different from the debt holders. The important characteristic is that equity holders can have other incentives in terms of managerial decisions such

The advantage of issuing a debt contract is that coupon payments are tax deductible on the corporate level. A tax advantage from coupon payments always arises if the tax rate of the firm exceeds the tax rate an investor has to pay for receiving the coupon payments. To keep the notation simple, we denote τ with $0 < \tau < 1$ as the corporate tax rate and consider tax exempt bondholders.[6] This means that if the firm pays a coupon equal to C to the debt holders, the firm reduces its taxable base by this amount. Under the assumption that the firm always has a positive taxable income, the additional payment resulting from the debt obligation is only $C \cdot (1 - \tau)$ rather than C. The residual component $C \cdot \tau$ indicates the size by which the firm's total tax payments decline through the coupon payment. In other words, if the firm promises a coupon payment C in form of a debt contract, it must only pay a lower amount equal to $C \cdot (1 - \tau)$ due to tax benefits.

Coupon obligations, however, can also have a drawback which is in effect if the firm is not able or does not want to fulfil the promised coupon obligations and defaults. In this case, bankruptcy costs α $(0 < \alpha < 1)$ proportional to the asset value V_t at this time arise.

To satisfy the coupon payments, the firm requires an external financing as a sale of assets is not possible. This financing is covered by equity holders. We can think of this procedure as an issuance of new shares. We note that it makes no difference whether the new shares are bought by the former or by new equity holders. Thus, we will speak in what follows from capital injections by the equity holders even though it is also thinkable that new equity holders come into the firm to finance the coupon payments.

Moreover, we consider dividend payments of the firm. In each dividend date, the firm pays a dividend proportional to the asset value V_t with a rate $\beta \cdot V_t$. This dividend payment goes on a pro rata basis to the equity holders.

Since the analysis takes place in a time-continuous setting, we have coupon and dividend dates in each instant of time. To be precise, from now on C denotes the coupon rate. Therefore, the size of the payout ratio β, the asset value V_t, and the coupon C decide whether the instantaneous net payoff $(\beta \cdot V_t - C \cdot (1 - \tau))\,dt$ to the equity holders is positive or negative.

as the optimal default point than the debt holders. Since we abstract from all agency problems between equity holders and managers, we assume that all decisions of the firm are made in the best interest of the equity holders. In the case that the entrepreneur leads the firm as manager and holds all the equity this assumption is obviously satisfied. Bank/Lawrenz (2005) address a conflict of interest between the equity holders and the manager in a related model.

[6] One can easily reformulate this model with a positive tax rate for bondholders.

A putable consol bond pays a promised coupon rate C until the debt holders put or the firm defaults. With a put the bondholders receive the put price PP if the firm is still solvent. Otherwise, if the firm is not willing to pay the put price, the firm defaults. The payment of the put price is financed by capital injections through the equity holders in the same way as the coupon payments. After a put the debt is no longer alive. Since a put without a default results in an unlevered firm, there is a potential for a firm value increase by issuing a further bond. To capture the potential firm value increase, we account for a restructuring after a put. A restructuring requires costs k proportional to the asset value V_t at the restructuring date. At this point we assume that primarily the size of the firm determines the restructuring costs. In particular, other assumptions e.g. that restructuring costs are fixed or proportional to the coupon are also possible and the results remain valid.[7]

The asset value V_t is assumed to follow a geometric Brownian motion, where the current asset value and its distribution is commonly known. To justify the Leland approach that the asset value V_t can work as a state variable in an arbitrage-free world, we can think of the asset value V_t as the value of a project, which can only be run by the entrepreneur. The financial claims such as (putable) debt and equity are traded on frictionless and arbitrage-free capital markets where market participants are risk neutral. Hence, the values of equity S and debt D, given that they are still alive, must satisfy the following set of differential equations:

$$\frac{1}{2}\sigma^2 \cdot \frac{\partial^2 S}{\partial V^2} + (r-\beta) \cdot V \cdot \frac{\partial S}{\partial V} - r \cdot S + \beta \cdot V - C \cdot (1-\tau) = 0, \tag{1}$$
$$\frac{1}{2}\sigma^2 \cdot \frac{\partial^2 D}{\partial V^2} + (r-\beta) \cdot V \cdot \frac{\partial D}{\partial V} - r \cdot D + C = 0,$$

where $r > 0$ is the time-constant interest rate for all maturities. These differential equations will allow us to determine the values of equity and the corresponding debt contracts in the following sections.

In this setup, there are three levels of optimal decision making:

First, the entrepreneur optimally determines the debt contract so that the firm value is maximized. The firm value rather than the equity value is maximized since the

[7] Under the assumption that restructuring costs are proportional to the debt value at the issuance date, we can also observe analogous results. Therefore, this assumption is not crucial for the results which we will observe. However, our assumption, that restructuring costs are proportional to the asset value, allows for an intuitive comparison of the firm value with putable debt to the firm value under renegotiation and provides meaningful insights about optimal firm values and capital structures. The notion behind our choice of restructuring costs is that the major costs from a restructuring are fixed in the sense that they do not depend on the type of issued debt.

entrepreneur wants to maximize his or her wealth. The wealth consists of the value of issued debt and equity. Hence, the firm value, which is the sum of the equity and the debt value, is the objective function of the entrepreneur. At this point, we stress that the firm value can be different from the asset value V_t.

Second, after a bond contract has been sold, the firm follows a strategy in favor of the equity holders, i.e. the firm desires to maximize the equity value rather than the total firm value. This strategy implies that the equity holders decide whether or not to inject money into the firm for coupon payments and if necessary for the payment of the put price. Since this decision is made by the firm in favor of the equity holders, we will also use the term that the equity holders default to indicate that this decision is made in their interest.

Third, the debt holders, which hold a put right, must continuously determine the optimal put strategy that maximizes the debt value.

3 Straight Debt

In this section, we repeat the standard case of a firm with straight debt which was also presented by Leland (1994). The purpose of this analysis is to provide a benchmark for optimal values of firms with putable debt which will be derived later.

3.1 Analysis of an Arbitrary Straight Debt Contract

To analyze the case of an arbitrary straight debt contract, we assume that the firm has a consol bond with a coupon C outstanding. The firm can decide how long to pay the coupon. If the asset value V hits the critical default barrier V_B, the firm stops paying the coupon or alternatively the equity holders are no longer willing to inject money into the firm. At the default barrier V_B, the following values for equity and debt arise:

$$\lim_{V \to V_B} S\left(V, V_B\right) = 0,$$

$$\lim_{V \to V_B} D\left(V, V_B\right) = (1 - \alpha) \cdot V_B.$$

These relations formally indicate that in the case of a default the equity holders are left with nothing and the debt holders obtain the asset value $(1 - \alpha) \cdot V$ after proportional bankruptcy costs α. In the case that the asset value becomes sufficiently high, the danger of a default diminishes. Therefore, the debt value equals the value

$\frac{C}{r}$ of a default-free consol bond and the equity value is equal to the tax advantage $\frac{C}{r}\tau$ arising from this bond:

$$\lim_{V\to\infty} S(V,V_B) = V + \frac{C}{r}\tau, \tag{2}$$
$$\lim_{V\to\infty} D(V,V_B) = \frac{C}{r}.$$

With these conditions, the values of equity and straight debt for a given default strategy V_B with $V > V_B$ can be obtained from (1) as

$$S(V,V_B) = V - \frac{C}{r}(1-\tau) + \left(\frac{V}{V_B}\right)^Y \cdot \left(-V_B + \frac{C}{r}(1-\tau)\right), \tag{3}$$
$$D(V,V_B) = \frac{C}{r} + \left(\frac{V}{V_B}\right)^Y \cdot \left((1-\alpha)\cdot V_B - \frac{C}{r}\right),$$

where

$$Y = \frac{1 - \frac{\sqrt{4\cdot(r-\beta)^2 + 4\cdot(r+\beta)\cdot\sigma^2 + \sigma^4}}{\sigma^2}}{2} - \frac{r-\beta}{\sigma^2}.$$

It will be helpful to see that Y is always negative. According to the representation of the equity value, we can think of this value as a portfolio comprising of the asset value V minus the present value of a default free consol bond $-\frac{C}{r}$ plus the present value of the tax benefits $\frac{C\cdot\tau}{r}$ if the coupon is paid infinitely long and a further correction. This correction ensures that for a default, i.e. $V = V_B$, the equity value equals zero. This is achieved by the component $-V_B + \frac{C}{r}(1-\tau)$ that the equity holders obtain in addition to the initial position $V - \frac{C}{r}(1-\tau)$. This component means that equity holders lose the asset value in the case of a default as well as the coupon obligation with the resulting tax benefits. Since this component is only due under a default, the component $-V_B + \frac{C}{r}(1-\tau)$ is weighted by $\left(\frac{V}{V_B}\right)^Y$. We can interpret the value $\left(\frac{V}{V_B}\right)^Y$ as the value of a claim that pays one unit if the asset value V hits the default barrier V_B.

The debt value can accordingly be understood as the value of a default-free consol bond $\frac{C}{r}$ plus a correction that ensures that in the case of a default the debt holders lose the value of a default-free consol bond but obtain the asset value minus bankruptcy costs.

The optimal default barrier V_B^* follows from the firm's objective to maximize the equity value or in other words the coupon is paid as long as the required capital injections are worthwhile for the equity holders:

$$V_B^* = \arg\max_{V_B} S(V,V_B)$$

To obtain the optimal default strategy V_B^*, it is convenient to solve the smooth-pasting condition of the equity value at V_B^*. Under the optimal choice of the default barrier, the smooth-pasting condition must be valid. The smooth-pasting condition reads:

$$\left.\frac{\partial S\left(V, V_B^*\right)}{\partial V}\right|_{V=V_B^*}=0$$

Solving for V_B^* yields

$$V_B^*=\frac{C}{r}\left(1-\tau\right)\frac{-Y}{1-Y}. \tag{4}$$

Since Y is always negative, the default barrier V_B^* is a finite positive number. Hence, we always have a region with asset values $V \leq V_B^*$ for which the firm defaults, and a region with the remaining asset values $V > V_B^*$ for which the firm is alive and fulfills its coupon obligation.

3.2 Optimal Firm Value Under Straight Debt

In the case of straight debt, the optimal characteristics of debt only concern the size of the coupon. A higher coupon creates higher tax benefits if the firm remains alive. However, with an increase of the coupon, the likelihood of a default, which is associated with a loss of future tax benefits, rises. A further disadvantage of a higher coupon is that a default takes place at a higher asset value V_B^* which results in higher bankruptcy costs $\alpha \cdot V_B^*$ in the case of default and a default is more likely. This tradeoff between tax and bankruptcy effects is revealed by the representation of the firm value

$$\begin{aligned} v\left(C\right) &= S\left(V, V_B^*\right)+D\left(V, V_B^*\right) \\ &= V+\left(1-\left(\frac{V}{V_B^*}\right)^Y\right)\cdot\frac{C}{r}\tau-\left(\frac{V}{V_B^*}\right)^Y\cdot\alpha\cdot V_B^*, \end{aligned}$$

where the default barrier is chosen according to equation (4). According to the representation of the firm value $v\left(C\right)$, we can think of $v\left(C\right)$ as the asset value V plus the present value of tax benefits minus the present value of bankruptcy costs. The present value of tax benefits are

$$\left(1-\left(\frac{V}{V_B^*}\right)^Y\right)\cdot\frac{C}{r}\tau,$$

and

$$\left(\frac{V}{V_B^*}\right)^Y\cdot\alpha\cdot V_B^*$$

indicates the present value of bankruptcy costs.

The optimal coupon size C^* formally results from the first order condition

$$\frac{\partial v(C)}{\partial C} = \frac{\tau - \left(\frac{V}{V_B^*}\right)^Y \cdot (\tau - Y \cdot (\alpha + (1-\alpha) \cdot \tau))}{r} = 0$$

and simplifies to

$$C^* = \frac{V \cdot r}{1-\tau} \frac{1-Y}{-Y} \left(\frac{\tau - Y \cdot (\alpha \cdot (1-\tau) + \tau)}{\tau} \right)^{1/Y}.$$

Since the second derivative

$$\frac{\partial^2 v(C)}{\partial C^2} = \frac{Y}{C \cdot r} \left(\frac{V}{V_B^*}\right)^Y \cdot (\tau - Y \cdot (\alpha + (1-\alpha) \cdot \tau))$$

is always negative due to $Y < 0$, the local optimum C^* is the global maximum. As C^* is always positive, a positive amount of debt can always be issued so that the firm value $v(C)$ is higher than that of an unlevered firm whenever the tax rate τ is positive.

In order to distinguish between the optimal coupon of a firm with straight debt from firms with other types of bonds, we will also use the notation C^{plain} to refer to the optimal coupon under straight debt where necessary. Accordingly, V_B^{plain} stands for the optimal default barrier of a firm with straight debt and $v^{plain}\left(C^{plain}\right)$ for the optimal firm value.

4 Putable Debt

4.1 Pricing of an Arbitrary Putable Debt Contract for a Given Strategy

In this subsection, we consider a firm that has a putable consol bond outstanding with coupon C and put price PP. The firm can follow the default strategy V_B and the debt holders decide at which critical asset value V_P to put their debt. Strictly speaking, the default strategy V_B indicates at which barrier the firm announces a default given that debt holders have not put before. Accordingly V_P is the barrier at which the debt holders put the debt given that the firm has not announced a default before. At this point it is important to note that after an announcement of a default by the firm, the debt holders can still respond with a put. Accordingly, the firm can decide whether to default or not after a put.

Figure 1: Decision Structure After a Default Announcement

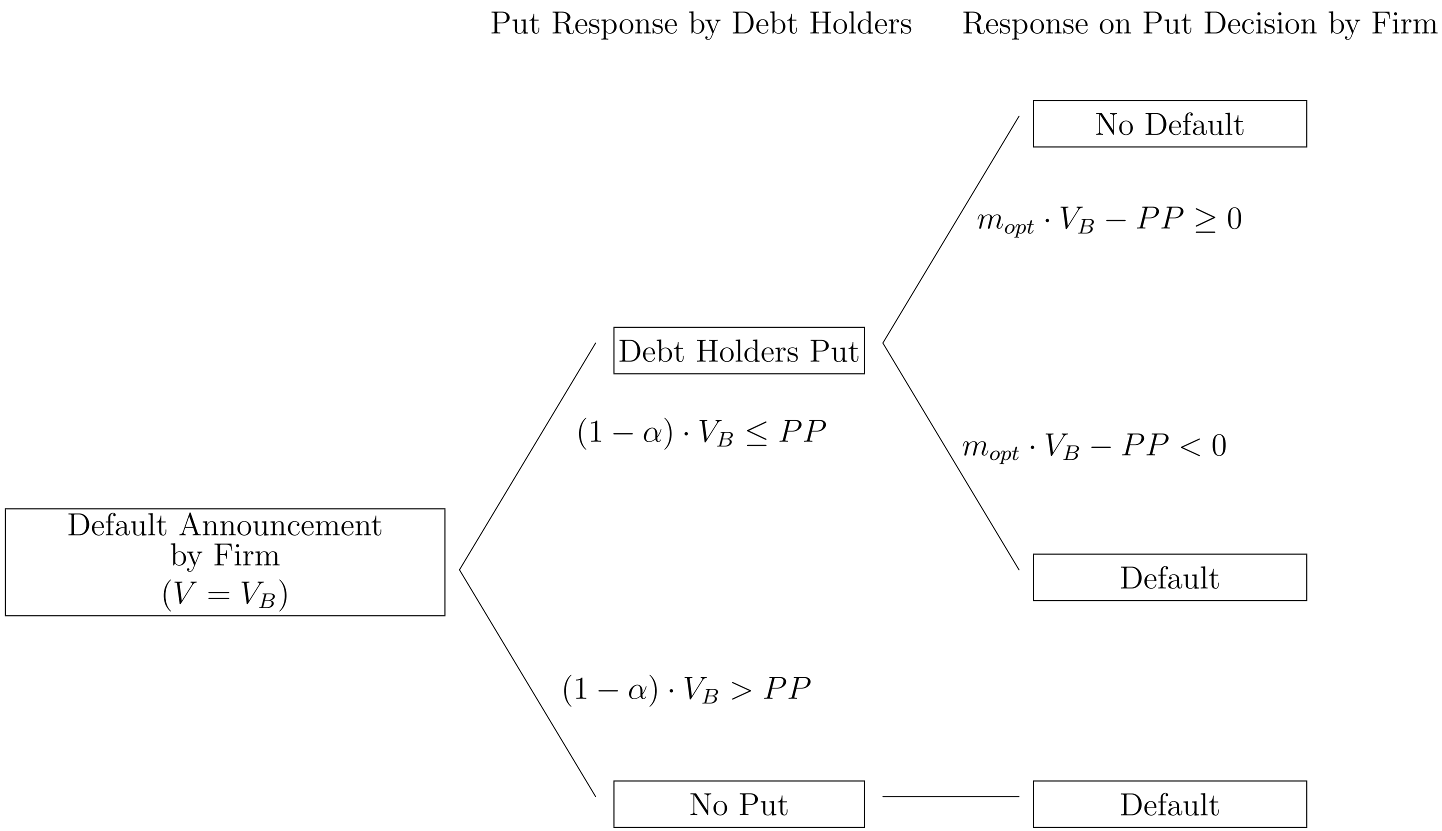


Figure 1 shows the decision process after a default announcement. If the asset value hits a default barrier V_B and the debt holders do not put, a default occurs and the equity and debt values are as for a straight debt contract. However, if the firm announces a default, $V = V_B$, the debt holders still have the right to put the debt. A put is optimal for the debt holders if the default value $(1-\alpha) \cdot V_B$ is below the put price PP. After an announcement of a default and the succeeding put, equity holders can decide whether they still want to default or whether they want to keep the firm alive by paying the put price PP to the debt holders. For the equity holders it is optimal to prevent a default and to pay the put price if and only if the firm value (after the gains from a potential restructuring) minus the put price is non-negative. This is because if the equity holders save the firm by paying the put price PP, they obtain an unlevered firm with asset value V. This unlevered firm value can potentially be further increased by a restructuring. We account for the restructuring option by a factor $m_{opt} \geq 1$ that increases the asset value V to the firm value $m_{opt} \cdot V$. We will show later how to determine the endogenous asset value multiplier m_{opt}. We note that bankruptcy costs only occur, if the ownership of the firm changes. A default announcement to enforce a put is equivalent to stopping the coupon payments which must take place before a firm enters into renegotiation in the model by Fan/Sundaresan. Since both a default announcement that results in a payment of the put price as well as a successful renegotiation of the debt contract do not result in a default and therefore do not change the ownership structure of the equity of the firm, there are no bankruptcy costs involved.[8] We will present the case of renegotiation in Section 5.

As a consequence of the strategic options of the firm and the debt holders we see that the firm can force the debt holders to an immediate put by a default announcement whenever the current default value $(1-\alpha) \cdot V$ is below the put price PP. This gives the following conditions for the equity value $S(V, V_B, V_P)$ and debt value $D(V, V_B, V_P)$ depending on the default barrier V_B followed by the firm and the put barrier V_P chosen by the debt holders. If V hits a default barrier V_B, it holds

$$\begin{aligned}
\lim_{V \to V_B} S(V, V_B, V_P) &= m_{opt} \cdot V_B - PP \\
&\qquad - 1_{\{m_{opt} \cdot V_B - PP < 0 \vee (1-\alpha) \cdot V_B > PP\}} \cdot (m_{opt} \cdot V_B - PP), \\
\lim_{V \to V_B} D(V, V_B, V_P) &= PP - 1_{\{m_{opt} \cdot V_B - PP < 0 \vee (1-\alpha) \cdot V_B > PP\}} \cdot (PP - (1-\alpha) \cdot V_B),
\end{aligned}$$

[8] At this point one can introduce costs associated with a default announcement in a straightforward way. Section 7 shows that putable debt is still an attractive debt contract even if there are extremely high (restructuring) costs involved. Moreover, the key relationship that putable debt results in a higher firm value than under renegotiation will be still valid.

where $1_{\{.\}}$ denotes the indicator function. In particular, $1_{\{m_{opt} \cdot V_B - PP < 0 \vee (1-\alpha) \cdot V_B > PP\}}$ equals one if and only if the firm finally defaults after a default announcement. This is true if either the debt holders will not put after a default announcement because the default value exceeds the put price, $(1-\alpha) \cdot V_B > PP$, or the equity holders are not willing to pay the put price due to $m_{opt} \cdot V_B - PP < 0$.

If the debt holders put the bond, which occurs for $V = V_P$, the equity holders decide whether or not to pay the put price. If they refuse the put price payment, the firm defaults and equity holders are left with nothing. As seen above, the payment of the put price is optimal rather than a default for $m_{opt} \cdot V_P \geq PP$. Therefore, the values of equity and debt for a put are given by:

$$\lim_{V \to V_P} S(V, V_B, V_P) = \max(m_{opt} \cdot V_P - PP, 0),$$
$$\lim_{V \to V_P} D(V, V_B, V_P) = PP - 1_{\{m_{opt} \cdot V_P - PP < 0\}} \cdot (PP - (1-\alpha) \cdot V_P)$$

Hence, we see that the put right can provide the debt holders with two different possibilities. First, the debt holders can terminate the debt relation by getting a payment equal to PP. Second, debt holders can force a default, if the firm value $m_{opt} \cdot V$ after restructuring is below the put price. From the boundary conditions in the case of a put and a default, condition (2) for sufficiently high asset values where required, and the pricing equation (1), we obtain a general representation for the equity and debt value. If the debt relationship is terminated either by a default or a put at a barrier $V_j^{(1)}$ below the current asset value, but the debt contract remains alive for higher asset values, the equity and debt values are:

$$S(V, V_B, V_P) = V - \frac{C}{r}(1-\tau) + \left(\frac{V}{V_j^{(1)}}\right)^Y \cdot S^{(1)}\left(V_j^{(1)}\right), \tag{5}$$
$$D(V, V_B, V_P) = \frac{C}{r} + \left(\frac{V}{V_j^{(1)}}\right)^Y \cdot D^{(1)}\left(V_j^{(1)}\right),$$

with

$$V_j^{(1)} = \max\left(V_B^{(1)}, V_P^{(1)}\right),$$

$$S^{(1)}\left(V_j^{(1)}\right) = \begin{cases} (m_{opt}-1)\cdot V_B^{(1)} - PP + \frac{C}{r}(1-\tau) & \\ -1_{\left\{m_{opt}\cdot V_B^{(1)} - PP<0 \vee (1-\alpha)\cdot V_B^{(1)} > PP\right\}} & \\ \cdot\left(m_{opt}\cdot V_B^{(1)} - PP\right), & \text{if } V_B^{(1)} > V_P^{(1)}, \\ -\min\left(PP - (m_{opt}-1)\cdot V_P^{(1)}, 0\right) + \frac{C}{r}(1-\tau), & \text{if } V_B^{(1)} \le V_P^{(1)}, \end{cases}$$

$$D^{(1)}\left(V_j^{(1)}\right) = \begin{cases} PP - \frac{C}{r} & \\ -1_{\left\{m_{opt}\cdot V_B^{(1)} - PP<0 \vee (1-\alpha)\cdot V_B^{(1)} > PP\right\}} & \\ \cdot\left(PP - (1-\alpha)\cdot V_B^{(1)}\right), & \text{if } V_B^{(1)} > V_P^{(1)}, \\ PP - \frac{C}{r} & \\ -1_{\left\{m_{opt}\cdot V_P^{(1)} - PP<0\right\}} \cdot \left(PP - (1-\alpha)\cdot V_P^{(1)}\right), & \text{if } V_B^{(1)} \le V_P^{(1)}. \end{cases}$$

The structure of these representations is as that for straight debt in (3). The equity value consists of the asset value minus a default-free consol bond. If the asset value V hits the barrier $V_j^{(1)}$ and a put optimally occurs, the equity value is corrected by an additional term $-V_j^{(1)} + \frac{C}{r}(1-\tau) - PP + m_{opt}\cdot V_j^{(1)}$ that reflects that with a put the equity holders lose the asset value and the coupon obligation. Additionally, they have to pay the put price and obtain the firm value after restructuring. If at the barrier $V_j^{(1)}$ a default occurs, the adjustment only concerns the loss of the asset value and the coupon obligation, $-V_j^{(1)} + \frac{C}{r}(1-\tau)$. Since these adjustments will only be effective if the asset value V hits the corresponding barrier, the present value of the adjustment is weighted by the factor $\left(\frac{V}{V_j^{(1)}}\right)^Y$.

Similarly, we can think of the debt value as the value of a default-free coupon bond plus an adjustment. In the case of a put without a default, the adjustment equals $PP - \frac{C}{r}$ which is the put price payment PP and a loss of the coupon claim. In the case of a default, the adjustment is the default value $(1-\alpha)\cdot V_j^{(1)} - \frac{C}{r}$ and the loss of the value of the future coupon payments. Like for the equity value the value of the corresponding adjustments is weighted by the factor $\left(\frac{V}{V_j^{(1)}}\right)^Y$.

At this point, it is not generally clear whether the default and put barriers optimally lie above and/or below the current asset value V given that the debt contract is alive for this level V. Depending on the size of the coupon and the put price, a default announcement by the equity holders or a put after an increase of the asset value might be optimal. However, these cases are not relevant for the optimal bond design presented in Section 4.2. For completeness, we present the equity and debt values for a given upper *and* lower put or default barrier in Appendix A.

4.2 Equilibrium Put and Default Strategy

In what follows we formulate desirable properties of an equilibrium put strategy and try to find a putable bond design so that these properties will be satisfied. In this context, we are particularly inspired by strategies under renegotiation where the firm remains alive until for low asset values the firm announces a default and a renegotiation takes place. The equivalent strategy for putable bonds is that a put with a payment of the put price takes place for low asset values but the bond remains alive for higher asset values. There are two reasons for why this strategy is especially beneficial. First, the bond remains alive over a broad range of asset values which creates a high present value of tax benefits. Second, a put takes place before a default occurs. Hence, bankruptcy costs are saved.

We now provide three necessary conditions for a suchlike equilibrium strategy. To ensure that the bond remains alive for high asset values and debt holders do not want to put, the coupon payment must be higher than the interest on the put price:

$$C > r \cdot PP \tag{6}$$

As a consequence of this condition, the debt holders always prefer a put to take place as late as possible. This is because the coupon payments are more favorable than receiving the put price, which only yields $r \cdot PP$ if it is invested into the riskfree asset. However, even if debt holders do not voluntarily want to put, the firm has the possibility to enforce a put by a default announcement as shown in Figure 1. Therefore, we suppose the case that a put takes place at V_B^* after a default announcement of the firm. The incentive condition of the debt holders, which ensures that at V_B^* after a default announcement in fact a put takes place and the firm does not default, is:

$$(1 - \alpha) \cdot V_B^* \leq PP. \tag{7}$$

This condition means that at the asset value V_B^* for which the firm announces a default, the debt holders are better off with a put than without a put. This is because the put price PP is higher than the default value $(1 - \alpha) \cdot V_B^*$ of debt.

To ensure that the strategy to enforce a put at V_B^* is also incentive compatible for the equity holders, the following incentive condition for the equity holders must hold

$$V_B^* \geq \overline{V}_P, \tag{8}$$

where the critical put barrier $\overline{V}_P$ is the lowest put barrier that is still incentive compatible for the equity holders. An incentive-compatible put barrier is defined as follows: If a put takes place at V_P above the critical put barrier $\overline{V}_P$ and no default occurs, the equity value $S(V, V_B, V_P)$ $(V_B \leq V_P)$ is non-negative for any asset value V above the put barrier V_P. Conversely, for a put at $V_P < \overline{V}_P$ the equity value $S(V, V_B, V_P)$ is negative for some $V > V_P$. Clearly, equity holders will not follow a strategy under which the equity value for some attainable asset values V is negative. At these asset values the firm could default to achieve an equity value equal to zero rather than to accept a negative equity value. Obviously, if a put at V_P is incentive compatible for the equity holders, every higher put barrier $V_P' > V_P$ is also incentive compatible. The necessary incentive conditions (7) and (8) simplify to

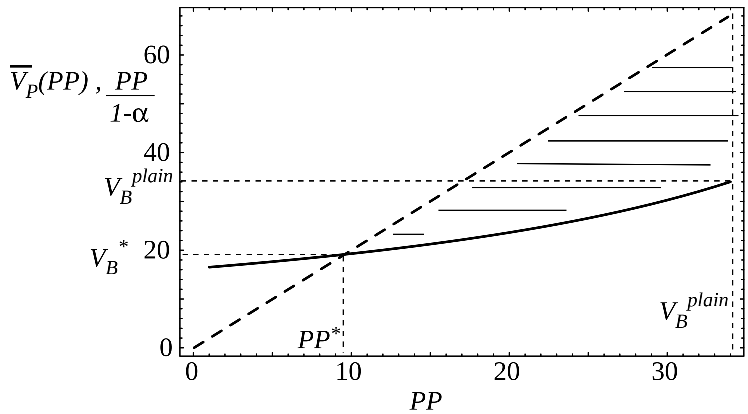
$$V_B^* \in \left[\overline{V}_P, \frac{PP}{1-\alpha}\right].$$

To illustrate under which conditions in fact an equilibrium with a put exists, we plot the lower and upper bound of the put strategy interval $\left[\overline{V}_P, \frac{PP}{1-\alpha}\right]$. Figure 2 shows the lower bound $\overline{V}_P(PP)$ and the upper bound $\frac{PP}{1-\alpha}$ of the interval as a function of the put price PP. In line with (7) and (8), only those combinations of the put price PP and the put barrier V_B that are within the dashed area of Figure 2 satisfy the incentive constraint of both the equity holders and the debt holders. The figure indicates that both bounds are increasing with PP. The reason for why this is always true is that if PP increases, equity holders must pay a higher price at the put barrier V_B^* which is less favorable for them. Therefore, they are only willing to accept the payment of the put price PP if the asset value in a put date is still high enough. Hence, the critical put barrier $\overline{V}_P(PP)$, that is still acceptable for equity holders, can be decreased with a lower put price PP. The fact that the other bound $\frac{PP}{1-\alpha}$ increases with PP is obvious. For $PP = 0$, the critical value $\overline{V}_P(PP)$ must be strictly positive. A put barrier equal to zero means that a put never occurs and the firm pays the coupon infinitely long. Since in this case the equity value would amount to $V - \frac{C(1-\tau)}{r}$, negative equity values would arise for low asset values which is in conflict to the incentive condition of the equity holders. This is the reason for why $\overline{V}_P(PP)$ always exceeds $\frac{PP}{1-\alpha}$ for put prices PP sufficiently close to zero which economically means that a put cannot be an equilibrium strategy for suchlike low put prices. In the example of Figure 2 $\overline{V}_P(PP = 0)$ equals 16.3.

In order to have an equilibrium with a put, the put price PP for a given coupon C has to be so that the lower bound $\overline{V}_P(PP)$ is below the upper bound $\frac{PP}{1-\alpha}$. We show that a put price for a given coupon C can always be obtained so that the put interval $\left[\overline{V}_P, \frac{PP}{1-\alpha}\right]$ has a positive length. This is true if the put price equals the

Figure 2: Boundaries of the Put Barrier Interval

The diagram shows the lower bound $\overline{V}_P$ of the put barrier interval (solid line) and the upper bound $\frac{PP}{1-\alpha}$ as a function of the put price. The parameter values are: $C = 5$, $m_{opt} = 1$, $\alpha = 0.5$, $\tau = 0.3$, $\sigma = 0.3$, $r = 0.05$, and $\beta = 0.02$.



default barrier of straight debt

$$PP = V_B^{plain}.$$

In the case that the restructuring multiplier m_{opt} equals one, the critical barrier $\overline{V}_P(PP)$ for this put price is equal to V_B^{plain} and therefore also below the upper bound $\frac{PP}{1-\alpha} = \frac{1}{1-\alpha} \cdot V_B^{plain}$ of the put barrier interval. The first statement is due to the fact that a put at a level of $\overline{V}_P(PP) = V_B^{plain}$ results in an equity value equal to zero which therefore coincides with the equity value of otherwise identical but straight debt. Since for all V above the put barrier V_B^{plain} the equity value is positive, a put at V_B^{plain} is an incentive-compatible put barrier for the equity holders. In the case that the restructuring multiplier m_{opt} exceeds one, the critical put barrier $\overline{V}_P(PP)$ is lower than for $m_{opt} = 1$. This is because for $m_{opt} > 1$ the equity holders have the advantage that they obtain a higher value from restructuring after a put. Therefore, $\overline{V}_P(PP)$ declines in m_{opt}. The consequence of a lower $\overline{V}_P(PP)$ is that for more put prices the lower bound $\overline{V}_P(PP)$ of the put price interval is below the upper bound $\frac{PP}{1-\alpha}$. Figure 2 illustrates this effect. As a result, the necessary condition for a put is always satisfied if the put price is high and sufficiently close to V_B^{plain}, but it is violated if the put price is low and close to zero.

At this point it is helpful to see that under the optimal design of the put feature, the put barrier V_B^* for the optimal coupon is as low as possible. A formal proof for this assertion will be presented in Section 6 where we compare the optimal firm value under putable debt and under renegotiation. The intuition for this relation is as follows. The lower the put barrier is, the longer the firm benefits from tax advantages until the debt is restructured. At a restructuring date, restructuring costs arise and the coupon declines so that future tax benefits decrease. For this reason it is optimal to have a put as late as possible.

In order to find the optimal design of the put price, the goal now is to determine the optimal put price $PP^*(C)$ for a given coupon C that results in a put at a level V_B^* as low as possible. For this purpose, it is useful to recall that both bounds $\overline{V}_P(PP)$ and $\frac{PP}{1-\alpha}$ continuously increase with PP where $\overline{V}_P(PP)$ exceeds $\frac{PP}{1-\alpha}$ for low PP, but it is lower for some high PP. In order to have a put, the put price must be so that $\overline{V}_P(PP) \leq \frac{PP}{1-\alpha}$ holds. As a consequence of these properties of $\overline{V}_P(PP)$ and $\frac{PP}{1-\alpha}$, the optimal put price is the lowest put price so that

$$\overline{V}_P(PP^*(C)) = \frac{PP^*(C)}{1-\alpha}$$

holds.

We emphasize that for a given putable bond with arbitrary coupon C and optimal put price $PP^*(C)$, the equilibrium strategy in fact satisfies the properties claimed above. This means that a put with a payment of the put price takes place at $V_B^* = \frac{PP^*(C)}{1-\alpha}$ and for higher asset values the bond remains alive. This is due to the following reasons: First of all the necessary conditions (6), (7), and (8) are all satisfied. This is obvious for the incentive conditions (7) and (8) of debt and equity holders. Condition (6), that the put price $PP^*(C) \cdot r$ times the interest rate is below the coupon, is a result of the fact that the optimal put barrier V_B^* is always below the default barrier V_B^{plain} in the case of straight debt. As argued before, the optimal put price $PP^*(C)$ for $m_{opt} = 1$ has to be below V_B^{plain} and if m_{opt} rises the optimal put price is even lower. Since V_B^{plain} is obviously below C/r due to (4), the optimal put price satisfies condition (6):

$$PP^*(C) \leq V_B^{plain} < \frac{C}{r}$$

Moreover, neither the debt nor the equity holders have an incentive to deviate from this equilibrium strategy. This assertion holds for the debt holders because if they are sure that a successful put will take place in the future they will not voluntarily put before V hits V_B^*. This is due to the fact that the coupon payment, which debt holders receive until a put is higher than investing the put price into a riskfree asset. The equity holders also have no incentive to deviate from this strategy. A default announcement at a higher asset value than V_B^* results in a default as the debt holders will not be forced to put. Since under the put strategy the equity value is always non-negative but with an earlier default it equals zero, we see why an earlier default is not worthwhile for the equity holders. This discussion yields the following proposition.

Proposition 1 (Optimal Put Price) *It is always possible for a given coupon to select a put price so that a put without a default takes place at V_B^* but the bond remains alive for all higher asset values. The lowest feasible put barrier V_B^* is lower than the default barrier V_B^{plain} in the case of straight debt.*

This proposition allows us to conclude that a straight debt contract is less favorable than a putable debt contract with optimal put price $PP^*(C)$ for a given coupon. This is due to the fact that with putable debt bankruptcy costs are prevented and the optimal put barrier $V_P^*(C) = \frac{PP^*(C)}{1-\alpha}$ is below the corresponding default barrier V_B^{plain} of straight debt as seen above so that the present value of tax benefits is higher. The reason for the relatively low put barrier is that equity holders cannot enforce a put earlier and debt holders do not want to voluntarily put.

4.3 Optimal Firm Value with Putable Debt

In the next step, we determine the characteristics $(C^{put}, PP^*(C^{put}))$ of optimally-designed putable debt so that the firm value for a given asset value V is maximized. For this purpose, we focus those strategies described in Proposition 1 where a put without a default occurs for low asset values but the bond remains alive for high asset values. We will denote this bond design as optimal putable debt. As a consequence, if C is the optimal coupon $PP^*(C)$ is the corresponding optimal put price as presented in the previous subsection. To have a more intuitive notation, we will denote the optimal barrier, at which a put takes place as V_P^*. We note that V_P^* implies a choice of the put price $PP^*(C)$ so that the put barrier for given values of V and C is as low as possible. At this point it is convenient to consider a restructuring multiplier $m_{opt}(C)$ that depends on the choice of the coupon rather than the optimal multiplier m_{opt}. A multiplier $m_{opt}(C)$ indicates that if for a current asset value V the chosen coupon is C, then in every further restructuring date the same relation between the coupon and the asset value holds as long as a restructuring is worthwhile. Thus, at the next restructuring date with an asset value equal to V_P^*, the coupon of the newly issued debt will be $C \cdot V_P^*/V$ rather than the optimal coupon or no debt will be issued if the benefits from a restructuring are below its costs. The considered equilibrium strategy results in the following firm value

$$v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right) = V + \left(1 - \left(\frac{V}{V_P^*}\right)^Y\right) \cdot \frac{C}{r}\tau + \left(\frac{V}{V_P^*}\right)^Y \cdot \left(\left(m_{opt}\left(C\right) - 1\right) \cdot V_P^*\right), \tag{9}$$

where

$$m_{opt}\left(C\right) = \max\left(\frac{V + \left(1 - \left(\frac{V}{V_P^*}\right)^Y\right) \cdot \frac{C}{r}\tau + \left(\frac{V}{V_P^*}\right)^Y \cdot \left(\left(m_{opt}\left(C\right) - 1\right) \cdot V_P^*\right)}{V} - k, 1\right) \tag{10}$$

$$= \max\left(1 + \frac{\left(1 - \left(\frac{V}{V_P^*}\right)^Y\right) \cdot \frac{\frac{C}{r}\tau}{V} - k}{1 - \left(\frac{V}{V_P^*}\right)^{Y-1}}, 1\right)$$

holds. The value $v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right)$ of a firm with putable debt and an arbitrary coupon C comprises of the asset value V, the present value of tax benefits $\left(1 - \left(\frac{V}{V_P^*}\right)^Y\right) \cdot \frac{C}{r}\tau$ until the restructuring date, and the present value of an additional firm value increase $\left(\frac{V}{V_P^*}\right)^Y \cdot \left(\left(m_{opt}\left(C\right) - 1\right) \cdot V_P^*\right)$ through restructuring at the succeeding restructuring date. A restructuring takes place when the asset value hits the put barrier V_P^*. Then, after paying the put price the firm is unlevered and can again issue putable debt. The representation for the multiplier $m_{opt}\left(C\right)$ results from the following consideration. As long as a restructuring is worthwhile for the firm, the firm value $m_{opt}\left(C\right) \cdot V$ after restructuring for a given asset value V must equal the optimal firm value $v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right) - k \cdot V$ minus restructuring costs. Thus, the asset value multiplier in every restructuring date is[9]

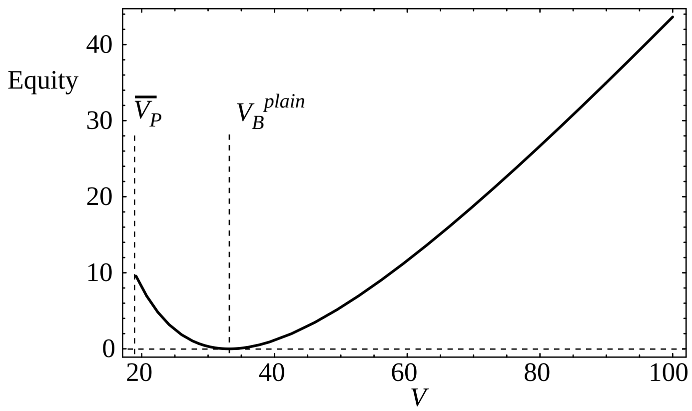
$$m_{opt}\left(C\right) = \frac{v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right) - k \cdot V}{V}. \tag{11}$$

In the opposite case that the restructuring costs do not allow for a profitable restructuring, the firm remains unlevered and $m_{opt}\left(C\right) = 1$ holds. The multiplier $m_{opt}\left(C\right)$ does not depend on the current asset value V as the terms of the debt are scaled relative to the asset value V. Using the representation of $v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right)$ according to (9) together with the condition that $m_{opt}\left(C\right)$ is at least equal to one, we can show by solving for $m_{opt}\left(C\right)$ that (11) implies the representation for the asset value multiplier (10).

[9]We assume that restructuring costs arise in every restructuring date but are not relevant for the initial capital structure decision.

Figure 3: Equity Value with Optimal Put Price

The diagram shows the equity value $S(V, V_B^*, V_P^*)$ *as a function of the asset value* V *where the optimal put price* $PP = 9.6$ *is chosen. Hence, the put strategy is* $\overline{V}_P = 19.2$*. The other parameter values are:* $C = 5$*,* m_{opt}*,* $\alpha = 0.5$*,* $\tau = 0.3$*,* $\sigma = 0.3$*,* $r = 0.05$*, and* $\beta = 0.02$*.*



To compute the put price $PP^*(C)$ for a given coupon C, we can regard the case in which the critical put barrier $\overline{V}_P(PP)$ equals $\frac{PP}{1-\alpha}$. According to Section 4.2 this approach ensures that the put barrier V_P^* is as low as possible for a given coupon. Figure 3 shows an example of the equity value under the strategy $V_P = \overline{V}_P(PP) = \frac{PP}{1-\alpha}$. The reason for why the equity value is first decreasing and then increasing in V is as follows: For low asset values a put is worthwhile for the equity holders because a put provides them with a positive value $m_{opt}(C) \cdot \overline{V}_P(PP) - PP$. For slightly higher asset values, equity holders still have to pay the coupon for a certain time until they obtain the fixed payment $m_{opt}(C) \cdot \overline{V}_P(PP) - PP$ (with a high probability). The coupon payments are the reason for an equity value below the payoff $m_{opt}(C) \cdot \overline{V}_P(PP) - PP$ at the put date. Conversely, if V is high, a higher asset value results in a higher equity value as it is usual in the case with straight debt. Hence, the equity value under this strategy must be equal to zero at a critical barrier. This barrier must be equal to the default barrier V_B^{plain} in the case of a straight bond with the same coupon. This because in both cases the equity value satisfies the smooth-pasting condition at this point and yields identical payoffs for higher asset values.

In addition, Figure 3 can provide a reasonable intuition for why the barrier V_P^* at which a put optimally occurs lies considerably below the default barrier V_B^{plain} under straight debt. Since the equity holders obtain a positive value at the put barrier V_P^*, they are willing to pay the coupon for a longer time than under straight debt. In the case of straight debt, the equity value is zero after the firm stops the coupon payments.

Since under the considered put strategy, a put takes place at $V_P^* = \overline{V}_P\left(PP^*\right) = \frac{PP^*}{1-\alpha}$, we can compute the unique local minimum of the equity value as follows:[10]

$$\frac{\partial S\left(V, \frac{PP^*}{1-\alpha}, \frac{PP^*}{1-\alpha}\right)}{\partial V} =$$

$$1 - V^{Y-1}\left(-Y\right) \cdot \left(\frac{1-\alpha}{PP^*}\right)^Y \left(\frac{C}{r}\left(1-\tau\right) - PP^* + \left(m_{opt}\left(C\right) - 1\right)\frac{PP^*}{1-\alpha}\right) = 0$$

As a result of the fact that the equity value has its local minimum equal to zero at V_B^{plain}, we can implicitly obtain $PP^*\left(C\right)$ by evaluating the condition that the asset value V with the local minimum coincides with V_B^{plain}:

$$V = \left(\frac{1-\alpha}{PP^*\left(C\right)}\right)^{\frac{Y}{1-Y}} \left(-Y \cdot \left(\frac{C}{r}\left(1-\tau\right) - PP^*\left(C\right) \cdot \left(\frac{m_{opt}\left(C\right) - 1}{1-\alpha} + 1\right)\right)\right)^{\frac{1}{1-Y}} \tag{12}$$

$$= V_B^{plain}$$

Equations (12) and (10) implicitly provide the optimal put barrier $V_P^* = \frac{PP^*(C)}{1-\alpha}$ and the corresponding restructuring multiplier $m_{opt}\left(C\right)$ for a given coupon C. The optimal coupon C^{put} maximizes the value of a firm with a putable bond $\left(C, PP^*\left(C\right)\right)$:

$$\begin{aligned} C^{put} &= \arg\max_{C \geq 0} v^{put}\left(C, V_P^*, m_{opt}\left(C\right)\right) \\ &= \arg\max_{C \geq 0} V + \left(1 - \left(\frac{V}{\frac{PP^*(C)}{1-\alpha}}\right)^Y\right) \cdot \frac{C}{r}\tau \\ &\quad + \left(\frac{V}{\frac{PP^*(C)}{1-\alpha}}\right)^Y \cdot \left(\left(m_{opt}\left(C\right) - 1\right) \cdot \frac{PP^*\left(C\right)}{1-\alpha}\right) \end{aligned}$$

5 Debt Contracts under Renegotiation

The optimal put strategy presented in the previous section has as a major advantage that no default arises and therefore bankruptcy costs do not reduce the firm value. The fact that no bankruptcy costs arise is also true for straight debt if renegotiation is possible as Fan/Sundaresan (2000) and Christensen/Flor/Lando/Miltersen (2002) show.[11] As a result of the parallels between putable debt and straight debt

[10] The uniqueness follows from the second derivative of the equity value for V which is always positive.

[11] Hackbarth/Hennessy/Leland (2006) consider the case with both a bank loan and a corporate bond where only the bank loan is subject to renegotiation.

with renegotiation, we analyze next the optimal firm value under renegotiation to see how the firm value with putable debt is relative to this related alternative. If renegotiation is possible, equity holders can enter into renegotiation by stopping the promised coupon payment or alternatively announcing a default. According to Fan/Sundaresan (2000), renegotiation results in a conversion of the debt contract into an equity contract so that after a renegotiation the firm remains unlevered. The proportions of the asset value, which are held by equity and debt holders, follow from the bargaining power modeled within a Nash bargaining game. Fan/Sundaresan provide two important results. First, bankruptcy costs are successfully avoided. Second, they state a closed-form solution for the barrier V_R^* at which the firm optimally enters into renegotiation. This barrier depends on the bargaining power of equity and debt holders.

In general, the higher the bargaining power of the equity holders is, the earlier equity holders will enter into renegotiation and therefore V_R^* is an increasing function in the bargaining power of the equity holders. In Section 6 we will see that a lower barrier V_R^* results in higher firm values. To have an especially demanding benchmark for the optimal firm value with putable debt, we consider the highest feasible firm value under renegotiation. This is accomplished by choosing the lowest possible renegotiation barrier V_R^*. Therefore, we will focus throughout the following analysis on the special case where debt holders have the full bargaining power, i.e. they can make take-it-or-leave-it offers once the equity holders stop to pay the coupon. Additionally, we will extend the renegotiation framework of Fan/Sundaresan (2000) by allowing for a restructuring in every renegotiation date like Christensen/Flor/Lando/Miltersen (2002) do. As a consequence, the firm value under renegotiation for a given coupon C and the corresponding renegotiation barrier V_R^* is given by:

$$\begin{aligned} v^{reneg}\left(C, V_R^*, m_{opt}^{reneg}(C)\right) = V &+ \left(1 - \left(\frac{V}{V_R^*}\right)^Y\right) \cdot \frac{C}{r}\tau \\ &+ \left(\frac{V}{V_R^*}\right)^Y \cdot \left(\left(m_{opt}^{reneg}(C) - 1\right) \cdot V_R^*\right) \end{aligned} \tag{13}$$

This representation is analogous to that for putable debt. The only difference is that the optimal renegotiation barrier V_R^* and the restructuring possibility $m_{opt}^{reneg}(C)$ might differ from the optimal put barrier V_P^* and the restructuring possibility $m_{opt}(C)$ under putable debt, respectively. In fact, the optimal renegotiation barrier V_R^* for a given coupon follows directly from the fact that equity holders do not obtain anything after a renegotiation. Hence, the position of the equity holders is analogous to the case without renegotiation but the whole benefits from renegotiation go to the debt holders. Since it makes no difference for the equity holders if they stop the coupon payments and a default occurs or they enter into renegotiation, the optimal point in time to start renegotiation coincides with the optimal default announcement of an otherwise identical firm with straight debt but without renegotiation. This is the reason for why the optimal renegotiation barrier is at the optimal default barrier V_B^{plain} in the case of a straight debt contract without renegotiation. Therefore, we can write for the renegotiation barrier

$$V_R^* = V_B^{plain} = \frac{C}{r}(1-\tau)\frac{-Y}{1-Y}.$$

Since V_R^* and V_B^{plain} coincide, the renegotiation barrier is higher than the corresponding optimal put barrier V_P^* of an otherwise identical firm where the consol bond with the same coupon has a put feature. The intuition for this key finding is the following: Equity holders can enter into renegotiation whenever they like, while they cannot enforce a put for any asset value. A too early put triggers a default without a put which is not worthwhile for them. Under optimally-designed putable debt, equity holders enforce a put as early as possible even though they would benefit from an earlier put by the debt holders. As a result of fewer restrictions under renegotiation, the equity holders decide for renegotiation before they can enforce a put.

In an analogous way as for putable debt, we obtain from (13) the following representation for the restructuring multiplier $m_{opt}^{reneg}(C)$, where in every future restructuring date the relation between the coupon and the asset value C/V is as at the current issuance date:

$$m_{opt}^{reneg}(C) = \max\left(\frac{V + \left(1 - \left(\frac{V}{V_R^*}\right)^Y\right)\cdot\frac{C}{r}\tau + \left(\frac{V}{V_R^*}\right)^Y \cdot \left(\left(m_{opt}^{reneg}(C) - 1\right)\cdot V_R^*\right)}{V} - k, 1\right) \tag{14}$$

$$= \max\left(1 + \frac{\left(1 - \left(\frac{V}{V_R^*}\right)^Y\right)\cdot\frac{\frac{C}{r}\tau}{V} - k}{1 - \left(\frac{V}{V_R^*}\right)^{Y-1}}, 1\right)$$

Inserting the representations for the renegotiation barrier V_R^* and the firm value multiplier $m_{opt}^{reneg}(C)$ into the firm value (13), the first order condition provides the optimal size of the coupon as:

$$C^{reneg} = \frac{V\cdot r}{1-\tau}\frac{1-Y}{-Y}\left(\frac{\tau - Y\cdot\left(1 - m_{opt}^{reneg}\cdot(1-\tau)\right)}{\tau}\right)^{1/Y}$$

6 Comparison of Optimal Firm Values and Capital Structures

In this section, we compare the optimal firm values and the corresponding capital structures under putable debt, renegotiation, and straight debt. As a result of the previous sections, we know that for a given firm with coupon C a default of straight debt without renegotiation occurs at the same barrier V_B^{plain} at which renegotiation takes place if debt holders have the full renegotiation power. In the case of putable debt, the put barrier V_P^* for every given coupon C is below the common default and renegotiation barrier V_R^*. These relations allow us to draw conclusions about the optimal firm values $v^{plain}\left(C^{plain}\right)$, $v^{reneg}\left(C^{reneg}\right)$, and $v^{put}\left(C^{put}\right)$. Obviously, the renegotiation possibility adds value to a firm with straight debt. This is due to the fact that under renegotiation bankruptcy costs do not arise at the common default and renegotiation barrier and further benefits from a restructuring are also possible. Since for every arbitrary coupon C, $v^{plain}(C) \leq v^{reneg}(C)$ holds, the optimal firm value with straight debt $v^{plain}\left(C^{plain}\right)$ is below the optimal firm value $v^{reneg}\left(C^{reneg}\right)$ under renegotiation:

$$v^{plain}\left(C^{plain}\right) \leq v^{reneg}\left(C^{reneg}\right)$$

Next, we regard the relation between the optimal firm value $v^{reneg}\left(C^{reneg}\right)$ under renegotiation and the optimal firm value $v^{put}\left(C^{put}\right)$ with putable debt. The firm value under renegotiation/putable debt is given by

$$v^{put/reneg}(C) = V + \left(1 - \left(\frac{V}{V_{P/R}^*}\right)^Y\right) \cdot \frac{C}{r}\tau + \left(\frac{V}{V_{P/R}^*}\right)^Y \cdot \left(\left(m_{opt}^{put/reneg}(C) - 1\right) \cdot V_{P/R}^*\right). \tag{15}$$

According to this representation, the primary difference between the firm values for a given coupon C comes from the difference between the put barrier V_P^* and the renegotiation barrier V_R^*. A further effect is caused by a different restructuring option $m_{opt}^{put/reneg}(C)$ in the case of a put and renegotiation. To accomplish a better comparison between the firm value with putable debt and under renegotiation, we plug in (10) and accordingly (14) for the optimal restructuring factor into the firm value $v^{put/reneg}(C)$ given that a restructuring is beneficial for the firm. This approach results in the following representation for the firm value which allows for a meaningful interpretation:

$$v^{put/reneg}(C) = V + \frac{C}{r}\tau \cdot V \cdot \frac{1 - \left(\frac{V}{V^*_{P/R}}\right)^Y}{V - \left(\frac{V}{V^*_{P/R}}\right)^Y \cdot V^*_{P/R}} - k \cdot V \cdot V^*_{P/R} \cdot \frac{\left(\frac{V}{V^*_{P/R}}\right)^Y}{V - \left(\frac{V}{V^*_{P/R}}\right)^Y \cdot V^*_{P/R}} \tag{16}$$

According to this representation, we can understand the firm value as the sum of the asset value V plus the present value of tax benefits (not only from the currently outstanding bond but also from all further issued bond) minus the present value of all restructuring costs arising in the future. The present value of all future tax benefits results from the following infinite series:

$$\begin{aligned} \frac{C \cdot V}{r}\tau \cdot \frac{1 - \left(\frac{V}{V^*_{P/R}}\right)^Y}{V - \left(\frac{V}{V^*_{P/R}}\right)^Y \cdot V^*_{P/R}} &= \frac{C}{r}\tau \cdot \left(1 - \left(\frac{V}{V^*_{P/R}}\right)^Y\right) \\ &\quad \cdot \sum_{i=0}^{\infty} \left(\left(\frac{V}{V^*_{P/R}}\right)^Y \cdot \frac{V^*_{P/R}}{V}\right)^i , \end{aligned}$$

Since the coupon reduces by the factor $\frac{V^*_{P/R}}{V}$ in each restructuring date, the coupon size after i restructuring dates is $C \cdot \left(\frac{V^*_{P/R}}{V}\right)^i$. Given that this restructuring would take place at time $t = 0$, the present value of tax benefits from this bond issue is $\frac{C \cdot \left(\frac{V^*_{P/R}}{V}\right)^i}{r}\tau \cdot \left(1 - \left(\frac{V}{V^*_{P/R}}\right)^Y\right)$. As a consequence of the fact that a future restructuring does not necessarily occur and if it occurs it takes place in the future, the present value of tax benefits from the i-th bond must additionally be discounted by the corresponding state price $\left(\left(\frac{V}{V^*_{P/R}}\right)^Y\right)^i$ for i restructuring dates. In addition, we account for the tax benefits of the initially issued bond by $i = 0$. Simplifying the infinite series yields the representation for the present value of all future tax benefits.

The present value of all future restructuring costs is given by the following infinite series

$$k \cdot V \cdot V^*_{P/R} \cdot \frac{\left(\frac{V}{V^*_{P/R}}\right)^Y}{V - \left(\frac{V}{V^*_{P/R}}\right)^Y \cdot V^*_{P/R}} = k \cdot V \cdot \sum_{i=i}^{\infty} \left(\left(\frac{V}{V^*_{P/R}}\right)^Y \cdot \frac{V^*_{P/R}}{V}\right)^i ,$$

which can be interpreted in an analogous way.

In general, a lower put barrier V^*_P for every given coupon C implies a higher optimal firm value so that the optimal firm value under putable debt is higher than in the case of renegotiation. In what follows, we provide a formal proof for this statement. To ensure a better comparison between the firm value under renegotiation and under putable debt, we determine for every arbitrary coupon C under renegotiation a corresponding coupon C' so that the renegotiation barrier $V_R^*(C)$ at C equals the put barrier $V_P^*(C')$ at C'. Then, we can argue that the firm value with putable debt for this coupon is higher than the corresponding firm value with renegotiation. As $V_P^*(C)$ increases in C, the coupon C' under putable debt is higher than C. In the case that a restructuring is not optimal under renegotiation, we can directly see that the optimal firm value under renegotiation is lower than under putable debt. This is a consequence of a higher coupon under putable debt so that the present value of tax benefits is higher. In the case that a restructuring is optimal after a renegotiation, we also consider a restructuring of the firm with putable debt. In particular, we assume that the coupon C' under putable debt in every further restructuring date is chosen so that the optimal renegotiation barrier coincides with the corresponding put barrier. Hence, the restructuring dates for both instruments coincide. This choice has the important consequence that the present value of all future restructuring costs according to representation (16) for debt under renegotiation with coupon C is as high as under the considered strategy with putable debt. Since under putable debt the coupon C' is always higher than the coupon C in the case of renegotiation, the present value of tax benefits is higher for putable debt. As a result of the fact that for every debt contract with renegotiation a corresponding bond design with putable debt exists so that the restructuring dates coincide but the coupons of putable debt are higher, the firm value of putable debt for this strategy exceeds $v^{reneg}(C)$ for every coupon C. For this reason the optimal firm value $v^{reneg}(C^{reneg})$ under renegotiation is below the optimal firm value $v^{put}(C^{put})$ under putable debt:

$$v^{reneg}(C^{reneg}) \leq v^{put}\left(C^{put}\right)$$

At this point we see why a low put barrier V_P^* relative to the renegotiation barrier V_R^* for every given coupon C is crucial for a high firm value. The lower V_P^* is, the higher is the corresponding coupon C' that results in the same restructuring dates under putable debt like under renegotiation for a given coupon C. The higher firm value of this strategy comes from the difference $C' - C$ of coupons which causes higher tax benefits in the case of putable debt.

These considerations show that the optimal firm value benefits if a lower restructuring barrier for every given coupon C can be implemented. For this reason, the considered form of renegotiation, where debt holders have the full bargaining power, is that form of renegotiation which is beneficial for the optimal firm value. Under every other assumption about the bargaining power, the renegotiation barrier is higher for a given coupon C as Fan/Sundaresan (2000) show. Therefore, the optimal firm value with a certain renegotiation power of the equity holders must be even lower than under renegotiation with full bargaining power of the debt holders. This comparison gives us the following proposition.

Proposition 2 (Firm Values) *The optimal firm value with putable debt is higher than the optimal firm value under renegotiation, where the optimal firm value under renegotiation exceeds that under straight debt.*

In the next step, we want to analyze the amount of debt, the firm assumes to implement the optimal capital structure. A straightforward way to measure the amount of debt is to regard the optimal coupons, C^{plain}, C^{reneg}, and C^{put}. We start our analysis with a comparison of C^{plain} and C^{reneg}. For a given coupon we know that the default and renegotiation barrier $V_B^*(C) = V_R^*(C)$ coincide. In general, an increase of the coupon C always results in a higher firm value increase under renegotiation than under straight debt. The main reason for this is that a higher coupon increases the present value of bankruptcy costs under straight debt, while bankruptcy costs are not relevant under renegotiation. Additionally, the firm value under renegotiation benefits from an earlier restructuring when C rises. Therefore, the firm under renegotiation optimally chooses a higher coupon than in the case without renegotiation:

$$C^{plain} \leq C^{reneg}$$

We can alternatively verify this relation by computing the ratio of optimal coupons using their closed-form representations.

A further consequence of the relation between the optimal coupons is that the renegotiation barrier $V_R^*(C^{reneg})$ under the optimal debt level is higher than the default barrier $V_B^*\left(C^{plain}\right)$ under the optimal amount of straight debt:

$$V_B^*\left(C^{plain}\right) \leq V_R^*\left(C^{reneg}\right)$$

This relation means that a restructuring of a firm under renegotiation occurs earlier than a default if an otherwise identical firm has optimally-designed straight debt outstanding.

The proof of the relation between C^{reneg} and C^{put} is more sophisticated and will be presented in Appendix B. We can show that C^{put} is higher than C^{reneg}:

$$C^{reneg} \leq C^{put}$$

The intuition for this result is that a firm with putable debt can use a higher coupon than a firm under renegotiation to benefit from the tax advantage, while the associated increase of the restructuring costs is relatively small. This is a consequence of the fact that the put barrier V_P^* for a given coupon and its increase in C is relatively low compared to V_R^*.

Moreover, it is not only possible to show that under putable debt a higher coupon will be optimally used but also that C^{put} exceeds C^{reneg} so strongly that the restructuring barrier $V_P^*\left(C^{put}\right)$ under putable debt is above the restructuring barrier $V_R^*\left(C^{reneg}\right)$ under renegotiation

$$V_R^*\left(C^{reneg}\right) \leq V_P^*\left(C^{put}\right).$$

This outcome is remarkable because for a given coupon the restructuring barrier $V_P^*(C)$ with putable debt is below $V_R^*(C)$ with renegotiation. Hence, under optimally-designed debt, a put will occur before an otherwise identical firm with optimal debt under renegotiation will enter into renegotiation. This provides us with the following proposition.

Proposition 3 (Optimal Coupons) *The optimal coupon with putable debt is higher than the optimal coupon under renegotiation, where the optimal coupon under renegotiation exceeds that under straight debt. More than that these relations even apply to the corresponding restructuring and default barriers $V_P^*\left(C^{put}\right)$, $V_R^*\left(C^{reneg}\right)$, and $V_B^*\left(C^{put}\right)$, respectively.*

7 Comparative Static Analysis

In this section, we analyze the optimal firm values under straight debt, renegotiation, and putable debt. In addition, we refer to the optimal coupon sizes. The coupons illustrate the amount of leverage the firm optimally has and indicate the tax benefits a firm obtains as long as the debt contract is still alive. For the analysis, we consider changes of those parameters that exhibit the most remarkable relationships. In particular, we look at the restructuring costs k, the bankruptcy costs α, and the volatility σ of asset value returns. The goal of the following analysis is twofold: Firstly, we show by which extent putable debt can add value relative to a firm with straight debt with and without renegotiation. Secondly, we want to see whether there are factors that have an opposite effect on the optimal firm values under straight debt and putable debt. Such an observation implies that while a change of such a factor destroys firm value under straight debt, it adds value if putable debt is used instead.

(i) Effect of Restructuring Costs k

Figure 4: Optimal Firm Values and Coupon Sizes

The left diagram shows the optimal firm values of firms with putable debt, straight debt with renegotiation, and straight debt without renegotiation as a function of the restructuring costs k. The right diagram shows the corresponding optimal coupons. The parameter values are: $V = 100$, $\alpha = 0.5$, $\tau = 0.3$, $\sigma = 0.3$, $r = 0.05$, and $\beta = 0.02$.

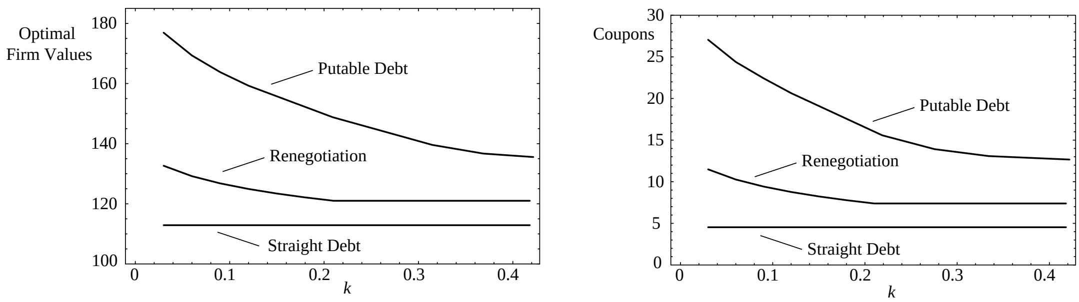


Figure 4 shows the optimal firm values for putable debt and straight debt with and without renegotiation as a function of restructuring costs k for a given asset value equal to 100. Hence, the optimal firm value minus 100 indicates in percent the feasible firm value increase through debt relative to an unlevered firm.

It is intuitive that the firm values under putable debt and under renegotiation suffer from higher restructuring costs while the firm value with straight debt does not depend on k. For putable debt and under renegotiation, this assertion is due to the fact that at the restructuring date the firm value benefits less strongly the higher the restructuring costs are. The fact that the restructuring possibility is less favorable when k is high also affects the critical put barrier $V_P^*(C)$. If the relative firm value increase m_{opt} at the restructuring date is lower due to higher k, the equity holders require a put at a higher asset value. This is the reason why the optimal default barrier $V_P^*(C)$ for a given coupon C increases with k. As explained above a higher restructuring barrier destroys firm value. Conversely, the renegotiation barrier $V_R^*(C)$ remains unaffected. As a consequence of these effects, we find that a higher k results in a less favorable restructuring and does not provide more favorable restructuring barriers V_P^* and V_R^*. For this reason, we can conclude that higher restructuring costs k result in a lower firm value.

If the restructuring costs k exceed a critical level equal to 0.36 in this example, a restructuring does not take place after a put and the firm remains unlevered. In the case of renegotiation, the critical restructuring costs are 0.21. Figure 4 shows that the restructuring option is an important source for the firm value created with putable debt. For restructuring costs equal to 0.03 the optimal firm value is 180, while for so high k for which no restructuring takes place the firm value is about 136. Under renegotiation, the optimal firm value varies less strongly from 133 to 121 for restructuring costs of this range. It is remarkable that the optimal firm value with putable debt (136) for high restructuring costs is considerably higher than both the firm value under renegotiation (121) and that for straight debt (113). This difference can be interpreted as follows: While the firm value increase of an unlevered firm through straight debt is 21 percent with renegotiation and 13 percent without renegotiation, this increase can be almost doubled by using putable debt even without the restructuring possibility. If the restructuring costs are low the increase with putable debt is even more than twice as high as that under renegotiation.

As a result, putable debt is an extremely well-suited debt contract to add firm value. In particular, we see that this increase does not only come from the possibility to restructure and to avoid bankruptcy costs as this is also true under renegotiation. The main value driver is that a restructuring under putable debt for a given coupon occurs for relatively low barriers compared to the case with renegotiation.

Furthermore, Figure 4 illustrates as stated in Proposition 3 that a firm with putable debt optimally uses a higher coupon than an otherwise identical firm with straight debt with renegotiation, where the coupon is again lower in the case without renegotiation. Moreover, Figure 4 shows that the optimal size of the coupon under putable debt and under renegotiation declines with restructuring costs k. If the restructuring costs are lower, the firm wants to restructure earlier. An earlier restructuring results from a higher restructuring barrier V_P^* and V_R^* which is accomplished by a higher optimal coupon C.

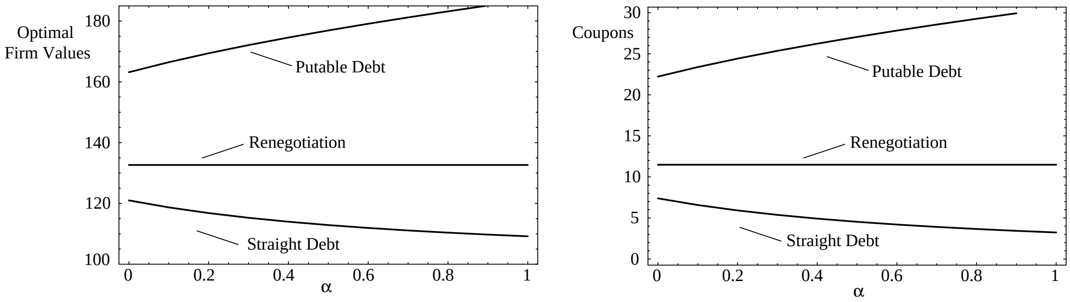
(ii) Effect of Bankruptcy Costs α

Figure 5 shows the optimal firm values if the bankruptcy costs α in the case of a default increase. It is intuitive that under straight debt without renegotiation the optimal firm value declines with α. This is because higher bankruptcy costs are a less favorable environment to issue debt as in the case of a default more value is destroyed. This is also the reason why the firm wants to avoid a default when α is high. Therefore, the optimal coupon C^{plain} declines with α.

Under renegotiation, the optimal firm value and the optimal size of the coupon do not depend on α. This is intuitive as bankruptcy costs do not arise and are therefore not relevant for the decisions of the equity holders.

Figure 5: Optimal Firm Values and Coupon Sizes

The left diagram shows the optimal firm values of firms with putable debt, straight debt with renegotiation, and straight debt without renegotiation as a function of the bankruptcy costs α. The right diagram shows the corresponding optimal coupons. The parameter values are: $V = 100$, $k = 0.03$, $\tau = 0.3$, $\sigma = 0.3$, $r = 0.05$, and $\beta = 0.02$.



Since a firm with optimally-designed putable debt cannot default, it might be at first glance surprising to see that the optimal firm value under putable debt benefits from higher bankruptcy costs α. However, α has an impact on the put barrier $V_P^*(C)$ and therefore affects the firm value. Since the optimal put barrier $V_P^*(C)$ declines in α for a given C, we can use our standard argumentation to conclude that the firm value benefits from α due to a lower $V_P^*(C)$. The fact that $V_P^*(C)$ declines with α can be illustrated by Figure 2 where $V_P^*(C)$ results from the intersection of the critical barrier $\overline{V}_P$ and $\frac{PP}{1-\alpha}$. The critical barrier $\overline{V}_P$ reflects the calculus of the equity holders only and is therefore independent of α. Conversely, a higher α results in a higher $\frac{PP}{1-\alpha}$ so that the intersection of the two curves is at a lower barrier V_P^*. The economic reasoning behind this technical argument is that bankruptcy costs act as a mechanism to discipline debt holders. We recall that a put is forced by the equity holders through announcing a default. If the bankruptcy costs are low and the firm announces a default at a given asset value V, the debt holders might not want to put as the default value is higher than the put price. However, if the bankruptcy costs are sufficiently high a default announcement forces the debt holders to put the debt. As a result, we see that higher bankruptcy costs provide better opportunities to enforce a put and therefore even lower put prices and put barriers V_P^* can be implemented that are incentive compatible not only for the equity holders but also for debt holders.

Clearly, the put strategy is especially valuable relative to the other two forms of debt if bankruptcy costs are high.[12] In the example considered in Figure 5, the firm

[12]Nevertheless, even if bankruptcy costs α become very large and tend to one, the optimal firm value converges to a finite level. This is a result of the fact that the optimal put barrier for a

value increase of an unlevered firm with putable debt is about three times as high as under renegotiation and about ten times as high as under straight debt without renegotiation if α is close to one.

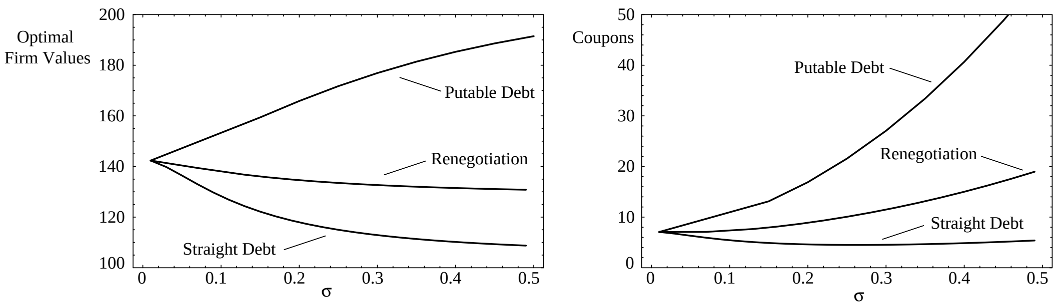
In the special case without bankruptcy costs $\alpha = 0$ and without restructuring $m_{opt} = 1$, not only the default and renegotiation barriers V_B^* and V_R^* coincide, but also the put barrier V_P^* does. As a consequence, the optimal firm values and coupon sizes are also the same in this case.

Figure 5 additionally shows that the coupon C^{put} under putable debt increases with α. Since with higher bankruptcy costs a lower put barrier V_P^* can be achieved with putable debt, we can understand why the incentive to use more debt rises with α.

(iii) Effect of Volatility σ

Figure 6: Optimal Firm Values and Coupon Sizes

The left diagram shows the optimal firm values of firms with putable debt, straight debt with renegotiation, and straight debt without renegotiation as a function of the volatility σ of asset value returns. The right diagram shows the corresponding optimal coupons. The parameter values are: $V = 100$, $k = 0.03$, $\alpha = 0.5$, $\tau = 0.3$, $r = 0.05$, and $\beta = 0.02$.



If the asset value V exhibits a higher degree of uncertainty in the form of a higher volatility σ of asset value returns, it is a well-known and intuitive finding that the optimal value of a firm with straight debt without renegotiation declines. Figure 6 illustrates this effect. The reasoning behind this observation is that a higher σ makes a costly default more likely which reduces the firm value. From a technical perspective, this effect is not obvious because the volatility σ has two effects. First, the default barrier V_B^* declines with σ. This is because the firm is willing to serve the debt even for lower asset values V, as a higher σ results in a more favorable equity value for a given V. Second, the distribution of V changes with σ so that the

given coupon cannot be lower than the barrier $\overline{V}_P\,(PP = 0)$ for a put price equal to zero. Figure 2 illustrates this property. Since $\overline{V}_P\,(PP = 0)$ is strictly positive and increases with C, the firm value cannot be infinite.

probability that V attains a given lower bound increases. Since the second effect dominates for optimal choices of the coupon, the default barrier is hit by V earlier. Therefore, the optimal firm value declines in σ. The optimal coupon under straight debt without renegotiation shows the well-known form that it first declines and then increases with σ.

Similarly, the optimal firm value under renegotiation also declines in σ as Figure 6 shows. The reasoning behind this observation is that the renegotiation barrier is attained earlier at which a lower coupon is optimally implemented and restructuring costs occur. The optimal coupon under renegotiation is — as Figure 6 shows — increasing in σ. Since the restructuring barrier V_R^* declines with σ, the firm can still use a higher coupon, that creates more tax benefits until a restructuring, without increasing the likelihood of a restructuring disproportionately.

For a firm with putable debt, we surprisingly observe the opposite relation for the optimal firm value namely that the optimal firm value benefits from a higher volatility σ. The firm value under putable debt is like the firm value under renegotiation affected by a higher σ in two ways. First, the optimal put barrier $V_P^*(C)$ for a given coupon C declines. The reason for a decline of $V_P^*(C)$ can be illustrated by Figure 2. The critical put barrier $\overline{V}_P$ so that the put strategy is still incentive compatible for the equity holders decreases with σ. This is due to the fact that a higher σ is beneficial for the equity value and therefore equity holders are willing to pay the coupon for a longer time until a put takes place. Since $V_P^*(C)$ results from the intersection of $\overline{V}_P$, which decreases with σ, and $\frac{PP}{1-\alpha}$, which is independent of σ, we see why $V_P^*(C)$ declines with σ.

The second effect for the value of a firm with putable debt is that a higher σ ensures that the likelihood for the asset value V hitting any lower bound increases. According to our preconsiderations, we know that at the restructuring barrier restructuring costs arise and a lower coupon is implemented. In this example, we observe that the decline of the restructuring barrier V_P^* when σ increases is so strong that the asset value V will hit the restructuring barrier V_P^* later. Hence, the negative effects from a restructuring play a less important role if σ is high, which increases the firm value.[13]

Without uncertainty, $\sigma = 0$, the optimal firm values under the three considered strategies coincide as the asset value V will not decline and therefore a restructuring will not take place.

[13] In other examples, e.g. for high restructuring costs k, we observe the opposite effect that the optimal firm value v^{put} declines in σ.

8 Conclusion

A put feature concerns many traded corporate bonds. Either a put right is explicitly embedded in the bond contract or a put right might arise whenever the firm violates pre-specified covenants. We analyze the consequences of a put feature within a typical time-independent firm value model with tax deductibility of coupon payments, bankruptcy costs in the case of a default, and dynamic restructuring. A put right in a debt contract requires the examination of a complex game between debt and equity holders. The complexity of this game is a result of the fact that a put by debt holders and a default announcement by the firm have various consequences. A put by the debt holders does not necessarily result in a redemption of the debt at the put price but might possibly trigger a default. Accordingly, if debt holders announce a default, either the firm in fact finally defaults and bankruptcy costs occur or a put is enforced and the firm does not default.

Fortunately, the firm can always issue putable debt so that the bond remains alive if the asset value increases and the bond will be put after a sufficient asset value decline. This strategy parallels that under renegotiation. However, the optimal firm value under putable debt is always higher than under straight debt even with renegotiation and arbitrary negotiation power of debt and equity holders. The reason for the relative high firm value under putable debt is that the optimal put barrier for a given coupon is low compared to the case with renegotiation. This is a result of the fact that under the optimal design of the put feature equity holders cannot enforce a put earlier, while equity holders can always enter into renegotiation. The low put barrier results in a high optimal coupon which yields high tax benefits. In particular, the optimal coupon under putable debt is considerably higher than under renegotiation; it is in fact so high that a restructuring under putable debt optimally occurs before a renegotiation of optimally-designed straight debt.

Moreover, there are parameter values such as bankruptcy costs and the volatility of the asset value return that have a different effect on the optimal firm value under putable debt than on the firm value under straight debt. In particular, bankruptcy costs play an important role for putable debt contracts. The higher the bankruptcy costs, the better the firm can force a put by announcing a default which results in a lower put barrier.

The major consequence from these findings is the following: In order to maximize the firm value by preventing bankruptcy costs, it is not optimal for the firm to set up a possibility to renegotiate debt contracts. It is rather optimal to use debt with a put feature. If renegotiation is possible even for putable debt, it will take place before a put optimally occurs which again destroys value. For this reason equity and debt holders should focus on the put feature and implement a mechanism that prevents renegotiation. Aghion/Dewatripont/Rey (1994) discuss devices such as penalties for the firm to implicitly affect the negotiation power.

The high optimal firm values arising from the put feature of debt highly recommend that in future studies on optimal firm values a put feature should be considered. Together with a call feature, a putable bond is supposed to result in even higher firm values than just a putable debt. Therefore, it will be interesting to analyze the optimal leverage and implications for the credit spread for a suchlike bond.

A Asset Values With Upper and Lower Barrier

In this appendix, we consider the case that the debt holders employ two put barriers, so that a put occurs for low asset values $V_P^{(1)}$ with $V_P^{(1)} < V$ and for a higher asset value $V_P^{(2)}$ with $V_P^{(2)} > V$. In this case, the put strategy V_P is no longer a scalar but a two dimensional vector $V_P = \left(V_P^{(1)}, V_P^{(2)}\right)$ with $V_P^{(1)} < V < V_P^{(2)}$. Accordingly, we can formulate a two-dimensional default strategy $V_B = \left(V_B^{(1)}, V_B^{(2)}\right)$ with $V_B^{(1)} < V < V_B^{(2)}$ with finite barriers. Then, the equity and debt values can be written as follows:

$$\begin{aligned}
S\left(V, V_B, V_P\right) &= V - \frac{C}{r}\left(1-\tau\right) \\
&\quad + P^{(1)}\left(V, V_j^{(1)}, V_j^{(2)}\right) \cdot S^{(1)}\left(V_j^{(1)}\right) \\
&\quad + P^{(2)}\left(V, V_j^{(1)}, V_j^{(2)}\right) \cdot S^{(2)}\left(V_j^{(2)}\right), \\
D\left(V, V_B, V_P\right) &= \frac{C}{r} + P^{(1)}\left(V, V_j^{(1)}, V_j^{(2)}\right) \cdot D^{(1)}\left(V_j^{(1)}\right) \\
&\quad + P^{(2)}\left(V, V_j^{(1)}, V_j^{(2)}\right) \cdot D^{(2)}\left(V_j^{(2)}\right),
\end{aligned}$$

with

$$
\begin{aligned}
V_j^{(2)} &= \min\left(V_B^{(2)}, V_P^{(2)}\right), \\
S^{(2)}\left(V_j^{(2)}\right) &= \begin{cases} (m_{opt}-1)\cdot V_B^{(2)} - PP + \frac{C}{r}(1-\tau) \\ -1_{\left\{m_{opt}\cdot V_B^{(2)} - PP < 0 \vee (1-\alpha)\cdot V_B^{(2)} > PP\right\}} \\ \cdot\left(m_{opt}\cdot V_B^{(2)} - PP\right), & \text{if } V_B^{(2)} < V_P^{(2)}, \\ -\min\left(PP - (m_{opt}-1)\cdot V_P^{(2)}, 0\right) + \frac{C}{r}(1-\tau), & \text{if } V_B^{(2)} \geq V_P^{(2)}, \end{cases} \\
D^{(2)}\left(V_j^{(2)}\right) &= \begin{cases} PP - \frac{C}{r} \\ -1_{\left\{m_{opt}\cdot V_B^{(2)} - PP < 0 \vee (1-\alpha)\cdot V_B^{(2)} > PP\right\}} \\ \cdot\left(PP - (1-\alpha)\cdot V_B^{(2)}\right), & \text{if } V_B^{(2)} < V_P^{(2)}, \\ PP - \frac{C}{r} \\ -1_{\left\{m_{opt}\cdot V_P^{(2)} - PP < 0\right\}} \cdot \left(PP - (1-\alpha)\cdot V_P^{(2)}\right), & \text{if } V_B^{(2)} \geq V_P^{(2)}, \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}\left(V, V_j^{(1)}, V_j^{(2)}\right) &= \left(\frac{V}{V_j^{(1)}}\right)^Y \frac{V^X - V_j^{(2)X}}{V_j^{(1)X} - V_j^{(2)X}}, \\
P^{(2)}\left(V, V_j^{(1)}, V_j^{(2)}\right) &= \left(\frac{V}{V_j^{(2)}}\right)^Y \frac{V^X - V_j^{(1)X}}{V_j^{(2)X} - V_j^{(1)X}}, \\
X &= \frac{\sqrt{4\cdot(r-\beta)^2 + 4\cdot(r+\beta)\cdot\sigma^2 + \sigma^4}}{\sigma^2}, \\
Y &= \frac{1-X}{2} - \frac{r-\beta}{\sigma^2}.
\end{aligned}
$$

The difference to the values in (5), where only a lower put or default barrier is in effect, is that not only an adjustment must be considered if the asset value hits the lower barrier $V_j^{(1)}$ but also a further adjustment to account for an asset value V hitting the upper barrier $V_j^{(2)}$. The adjustments for a put and a default are analogous to those in (5). However, these adjustments must be weighted by the factors $P^{(1)}\left(V, V_j^{(1)}, V_j^{(2)}\right)$ and $P^{(2)}\left(V, V_j^{(1)}, V_j^{(2)}\right)$. $P^{(1)}\left(V, V_j^{(1)}, V_j^{(2)}\right)$ is the present value of a monetary unit that is paid if the asset value V hits the lower barrier $V_j^{(1)}$ before hitting the upper barrier $V_j^{(2)}$. Accordingly, $P^{(2)}\left(V, V_j^{(1)}, V_j^{(2)}\right)$ indicates the present value of one monetary unit that is paid if V hits the upper barrier before touching the lower barrier.

In general, the bond remains alive until a default or put barrier is hit. If $\max\left(V_B^{(1)}, V_P^{(1)}\right)$ is equal to $V_P^{(1)}$ $\left(V_B^{(1)}\right)$ a put (default announcement) occurs if the asset value V declines. Accordingly, if the asset value increases and $\min\left(V_B^{(2)}, V_P^{(2)}\right)$

is equal to a finite value of $V_P^{(2)}$ $\left(V_B^{(2)}\right)$, the debt holders will put the bond (the equity holders will announce a default). Hence, the relations between $V_B^{(1)}$ and $V_P^{(1)}$ as well as between $V_B^{(2)}$ and $V_P^{(2)}$ decide whether a put or a default is announced for low and high asset values.

B Proof of Proposition 3

For the proof of Proposition 3 it is helpful to make the following remarks:

(1) The put barrier $V_P^*(C)$ as a function of the coupon C is a linear function in C through the origin. Obviously, under the presented choice of the put price for a given coupon C, the put price is chosen in a fixed relation to the coupon and is therefore proportionate to the put barrier V_P^*. In other words, if the coupon is scaled, the optimal put price and accordingly the optimal put barrier also scale with the same factor. Since the optimal renegotiation barrier $V_R^*(C)$ is also proportionate to C, where $V_R^*(C)$ for a given coupon is higher than $V_P^*(C)$, we find the following relation

$$V_P^*(C) = \frac{\partial V_P^*(C)}{\partial C} \cdot C \leq \frac{\partial V_R^*(C)}{\partial C} \cdot C = V_R^*(C).$$

(2) The value of a firm $v^{put}(C, V_P^*(C), m_{opt}(C))$ with putable debt has one local maximum in the coupon C but no further local optima. To see this assertion, we regard the second derivative of the firm value $v^{put}(C, V_P^*(C), m_{opt}(C))$ for the coupon C. As long as that the restructuring costs k are such high that $m_{opt}(C)$ equals one for all C, the second derivative reads:

$$\frac{\partial^2 v^{put}(C, V_P^*(C), m_{opt}(C))}{\partial C^2} = \frac{\left(\frac{V}{V_P^*(C)}\right)^Y (1-Y) Y \tau}{Cr} \leq 0.$$

To come to this representation, we made use of the relation $V_P^*(C) = \frac{\partial V_P^*(C)}{\partial C} \cdot C$ shown under remark (1). Since the firm value is concave in C, the firm value has one local maximum but no further optima. Clearly, for a too high C the firm value declines in C as e.g. $V_P^*(C)$ equal to the current asset value V cannot be optimal due to infinite restructuring costs.

In the opposite case that $m_{opt}(C)$ can be higher than one for some coupons C, the optimal strategy will clearly be so that $m_{opt}(C) > 1$ holds. For $m_{opt}(C) > 1$ the firm value $v^{put}(C, V_P(C), m_{opt}(C))$ and the multiplier $m_{opt}(C)$ are related by the condition:

$$v^{put}(C, V_P^*(C), m_{opt}(C)) = m_{opt}(C) \cdot V + k \cdot V. \tag{17}$$

This condition implies the following relation between the derivative of the firm value $\frac{\partial v^{put}\left(C,V_P^*(C),m_{opt}(C)\right)}{\partial C}$ and the derivative of the multiplier $\frac{\partial m_{opt}(C)}{\partial C}$:

$$\frac{\partial v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)}{\partial C}=\frac{\partial m_{opt}\left(C\right)}{\partial C}\cdot V.$$

Accordingly, the second derivative of the firm value $v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)$ for C is given by:

$$\frac{\partial^2 v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)}{\partial C^2}$$
$$=\frac{\left(\frac{V}{V_P^*(C)}\right)^Y(1-Y)\left(Y\left(\frac{\partial V_P^*(C)}{\partial C}\cdot r+\tau\right)-\frac{\partial V_P^*(C)}{\partial C}\cdot rY\cdot m_{opt}\left(C\right)+2V_P^*\left(C\right)r\frac{\partial m_{opt}(C)}{\partial C}\right)}{Cr\left(V-V_P^*\left(C\right)\cdot\left(\frac{V}{V_P^*(C)}\right)^Y\right)}$$

For all relevant coupons C^{max}, at which the firm value has a local extremum, $\frac{\partial m_{opt}(C)}{\partial C}=0$ must hold so that we can write

$$\left.\frac{\partial^2 v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)}{\partial C^2}\right|_{C=C^{max}}$$
$$=\left.\frac{\left(\frac{V}{V_P^*(C)}\right)^Y(1-Y)\left(Y\left(\frac{\partial V_P^*(C)}{\partial C}\cdot r+\tau\right)-\frac{\partial V_P^*(C)}{\partial C}\cdot rY\cdot m_{opt}\left(C\right)\right)}{Cr\left(V-V_P^*\left(C\right)\cdot\left(\frac{V}{V_P^*(C)}\right)^Y\right)}\right|_{C=C^{max}}.$$

As the current asset value V exceeds $V_P^*\left(C\right)$ by construction of the put strategy, the denominator is always positive and the sign of the second derivative can be further simplified by using remark (1):

$$sign\left(\left.\frac{\partial^2 v^{put}}{\partial C^2}\right|_{C=C^{max}}\right)=sign\left(-C^{max}\cdot\tau+V_P^*\left(C^{max}\right)\cdot r\cdot\left(m_{opt}\left(C^{max}\right)-1\right)\right).$$

Hence, as long as

$$\frac{C^{max}}{r}\tau\geq V_P^*\left(C^{max}\right)\cdot\left(m_{opt}\left(C^{max}\right)-1\right)\tag{18}$$

holds, the value $v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)$ of a firm with putable debt is concave in C at every local extremum. The reason why this condition holds for every C is as follows. $\frac{C}{r}\tau$ would be the present value of the tax benefits given that no restructuring takes place. However, a restructuring occurs at an asset value $V_P^*\left(C\right)$ below the current asset value V. At this point, the firm issues debt with a lower coupon. For this reason the value of instantaneous tax benefits that arise after a restructuring are lower than the benefits with a rate $C\cdot\tau$ before restructuring. Since the firm value increase $v^{put}\left(C,V_P^*\left(C\right),m_{opt}\left(C\right)\right)-V$ comes from the tax benefits minus restructuring costs, we see that the present value $\frac{C}{r}\tau$ of tax benefits in the case without restructuring costs and where the coupon will not be reduced in the future is higher than $v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right) - V = \left(m_{opt}\left(C\right) - 1\right) \cdot V + k \cdot V$. As in addition $\left(m_{opt}\left(C\right) - 1\right) \cdot V$ exceeds $V_P^*\left(C\right) \cdot \left(m_{opt}\left(C\right) - 1\right)$, relation (18) always holds. Since v^{put} is concave in C in every local extremum, v^{put} can only have a local maximum that is also the global maximum.

Now, we prove Proposition 3 that the optimal put barrier $V_P^*\left(C^{put}\right)$ under the optimal coupon is higher than the optimal renegotiation barrier $V_R^*\left(C^{reneg}\right)$ under the optimal coupon with renegotiation. This property implies that the optimal coupon C^{put} under the optimal put strategy exceeds the optimal coupon C^{reneg} under renegotiation because $V_P^*\left(C\right)$ is below $V_R^*\left(C\right)$ for a given coupon according to remark (1).

To accomplish the proof, we consider the coupon C' under the optimal put strategy so that $V_P^*\left(C'\right)$ is equal to the renegotiation barrier $V_R^*\left(C^{reneg}\right)$ under the optimal coupon C^{reneg}:

$$V_P^*\left(C'\right) = V_R^*\left(C^{reneg}\right)$$

The optimal coupon C^{reneg} follows from the first order condition:

$$\frac{\partial v^{reneg}\left(C, V_R^*\left(C\right), m_{opt}^{reneg}\left(C\right)\right)}{\partial C} = v_1^{reneg} + v_2^{reneg} \cdot \frac{\partial V_R^*\left(C\right)}{\partial C} + v_3^{reneg} \cdot \frac{\partial m_{opt}^{reneg}\left(C\right)}{\partial C} = 0,$$

where v_i^{reneg} denotes the partial derivative of $v^{reneg}\left(C, V_R^*\left(C\right), m_{opt}^{reneg}\left(C\right)\right)$ with respect to the i-th argument. Since under the optimal coupon C^{reneg} at which the firm value is maximized, the first order condition for the total firm value holds, the derivative $\frac{\partial m_{opt}^{reneg}(C)}{\partial C}$ must also equal zero. Hence, the first order condition simplifies to

$$\frac{\partial v^{reneg}\left(C, V_R^*\left(C\right), m_{opt}^{reneg}\left(C\right)\right)}{\partial C} = v_1^{reneg} + v_2^{reneg} \cdot \frac{\partial V_R^*\left(C\right)}{\partial C} = 0. \tag{19}$$

The derivative of the firm value $v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)$ under the optimal put strategy for the coupon is

$$\frac{\partial v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)}{\partial C} = v_1^{put} + v_2^{put} \cdot \frac{\partial V_P^*\left(C\right)}{\partial C} + v_3^{put} \cdot \frac{\partial m_{opt}\left(C\right)}{\partial C},$$

where v_i^{put} is defined in a similar way as v_i^{reneg} as the partial derivative of the firm value $v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)$ with respect to the i-th argument. Again, we distinguish between the case that $m_{opt}\left(C\right)$ equals one and that $m_{opt}\left(C\right)$ is higher than one for the relevant coupons C. In the first case the derivative simplifies to

$$\frac{\partial v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)}{\partial C} = v_1^{put} + v_2^{put} \cdot \frac{\partial V_P^*\left(C\right)}{\partial C}.$$

In the second case, we can use the relation between the firm value $v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)$ and the multiplier $m_{opt}\left(C\right)$ according to (17) for all relevant C that might be an optimal coupon. This formula yields the following relation between the derivative of the firm value $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ and of the multiplier $\frac{\partial m_{opt}(C)}{\partial C}$:

$$\frac{\partial v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)}{\partial C} = \frac{\partial m_{opt}\left(C\right)}{\partial C} \cdot V$$

This representation allows us to write for the first derivative of the firm value:

$$\frac{\partial v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)}{\partial C} = \frac{v_1^{put} + v_2^{put} \cdot \frac{\partial V_P^*(C)}{\partial C}}{1 - v_3^{put} \cdot \frac{1}{V}}$$

To show that the optimal put barrier $V_P^*\left(C^{put}\right)$ is above the optimal renegotiation barrier $V_R^*\left(C^{reneg}\right)$, we now have to show that $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ at $C = C'$ is positive. Then, we can use remark (2), which states that $v^{put}\left(C, V_P^*\left(C\right), m_{opt}\left(C\right)\right)$ has a local maximum in C but no local minimum. This remark implies that a positive derivative $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ at $C = C'$ implies that C^{put} must be even higher than C'. Then, we can conclude

$$V_P^*\left(C^{put}\right) \geq V_P^*\left(C'\right) = V_R^*\left(C^{reneg}\right).$$

The derivative $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ at $C = C'$ is positive if and only if the term $v_1^{put} + v_2^{put} \cdot \frac{\partial V_P^*(C)}{\partial C}$ is positive. For $m_{opt}\left(C\right) = 1$ this statement is obvious, while for $m_{opt}\left(C\right) > 1$ this is a result of the fact that the denominator $1 - v_3^{put} \cdot \frac{1}{V}$ is always positive. We can see this assertion from the following representation of the denominator

$$1 - v_3^{put} \cdot \frac{1}{V} = 1 - \left(\frac{V}{V_P^*\left(C'\right)}\right)^{Y-1} > 0,$$

as Y is negative. We recall that $V_P^*\left(C'\right) = V_R^*\left(C^{reneg}\right)$ is below the current asset value V.

In what follows, we compare the components of the derivative $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ of the firm value with putable debt to those under renegotiation. Clearly, at the coupon C' with $V_P^*\left(C'\right) = V_R^*\left(C^{reneg}\right)$, the first partial derivative of the value of a firm with putable debt coincides with the corresponding derivative of the value of a firm under renegotiation:

$$v_1^{put} = \frac{\tau}{r}\left(1 - \left(\frac{V}{V_P^*\left(C'\right)}\right)^Y\right) = \frac{\tau}{r}\left(1 - \left(\frac{V}{V_R^*\left(C^{reneg}\right)}\right)^Y\right) = v_1^{reneg}$$

Regarding $V_R^*(C) = \frac{\partial V_R^*(C)}{\partial C} \cdot C$ as shown in remark (1), the second term simplifies to

$$\begin{aligned}
v_2^{reneg} \cdot \frac{\partial V_R^*(C)}{\partial C} &= \left(\frac{V}{V_R^*(C)}\right)^Y \\
&\cdot \left((1-Y) \cdot \left(m_{opt}^{reneg}(C) - 1\right) + Y \frac{C\tau}{V_R^*(C) \cdot r}\right) \cdot \frac{\partial V_R^*(C)}{\partial C} \\
&= \left(\frac{V}{V_R^*(C)}\right)^Y \cdot \left((1-Y) \cdot \left(m_{opt}^{reneg}(C) - 1\right) \cdot \frac{\partial V_R^*(C)}{\partial C} + Y\frac{\tau}{r}\right)
\end{aligned}$$

The corresponding term for a firm with putable debt is

$$v_2^{put} \cdot \frac{\partial V_P^*(C)}{\partial C} = \left(\frac{V}{V_P^*(C)}\right)^Y \cdot \left((1-Y) \cdot (m_{opt}(C) - 1) \cdot \frac{\partial V_P^*(C)}{\partial C} + Y\frac{\tau}{r}\right).$$

These considerations allow us to evaluate the derivative $\frac{\partial v^{put}(C, V_P(C), m_{opt}(C))}{\partial C}$ at C', because $\frac{\partial v^{put}\left(C, V_P^*(C), m_{opt}(C)\right)}{\partial C}$ at $C = C'$ is higher than $\frac{\partial v^{reneg}\left(C, V_R^*(C), m_{opt}^{reneg}(C)\right)}{\partial C}$ at the optimal coupon C^{reneg} if and only if

$$v_1^{put} + v_2^{put} \cdot \left.\frac{\partial V_P^*(C)}{\partial C}\right|_{C=C'} \geq 0 = v_1^{reneg} + v_2^{reneg} \cdot \left.\frac{\partial V_R^*(C)}{\partial C}\right|_{C=C^{reneg}}$$

holds. Since the first partial derivatives coincide, this relation simplifies to

$$v_2^{put} \cdot \left.\frac{\partial V_P^*(C)}{\partial C}\right|_{C=C'} \geq v_2^{reneg} \cdot \left.\frac{\partial V_R^*(C)}{\partial C}\right|_{C=C^{reneg}}.$$

A comparison of these two derivatives results in the equivalent condition:

$$(m_{opt}(C') - 1) \cdot \frac{\partial V_P^*(C)}{\partial C} \geq \left(m_{opt}^{reneg}(C^{reneg}) - 1\right) \cdot \frac{\partial V_R^*(C)}{\partial C} \tag{20}$$

If $m_{opt}^{reneg}(C^{reneg}) - 1$ and $m_{opt}(C') - 1$ equal zero due to high restructuring costs, the two terms coincide. As a consequence, C' satisfies the first order condition and is therefore the optimal coupon under putable debt. Hence, the put barrier $V_P^*(C^{put})$ and the renegotiation barrier $V_R^*(C^{reneg})$ under the optimal coupons coincide. Otherwise, $m_{opt}(C')$ is higher than $m_{opt}^{reneg}(C^{reneg})$ because the value of a firm with putable debt and coupon C' is higher than the optimal firm value under renegotiation. We have shown this effect in Section 6.

If $m_{opt}(C')$ is higher than one, we can make use of the relation $m_{opt}(C') = \frac{v^{put}\left(C', V_P^*(C'), m_{opt}(C')\right)}{V} - k$ and solve for $(m_{opt}(C') - 1) \cdot \frac{\partial V_P^*(C)}{\partial C}$:

$$(m_{opt}(C') - 1) \cdot \frac{\partial V_P^*(C)}{\partial C} = \frac{\frac{V_P^*(C')}{V}\frac{\tau}{r}\left(1 - \frac{V}{V_P^*(C')}\right)^Y - k \cdot \frac{\partial V_P^*(C)}{\partial C}}{1 - \left(\frac{V}{V_P^*(C')}\right)^{Y-1}} \geq 0$$

If under renegotiation $m_{opt}^{reneg}\left(C^{reneg}\right)=1$ holds, the corresponding term under renegotiation $\left(m_{opt}^{reneg}\left(C^{reneg}\right)-1\right)\cdot\frac{\partial V_R^*(C)}{\partial C}$ obviously equals zero and is therefore lower than for putable debt. If $m_{opt}^{reneg}\left(C^{reneg}\right)>1$ holds, the corresponding equation under renegotiation at $C=C^{reneg}$ is

$$\left(m_{opt}^{reneg}\left(C^{reneg}\right)-1\right)\cdot\frac{\partial V_R^*\left(C\right)}{\partial C}=\frac{\frac{V_R^*(C^{reneg})}{V}\frac{\tau}{r}\left(1-\frac{V}{V_R^*(C^{reneg})}\right)^{Y}-k\cdot\frac{\partial V_R^*(C)}{\partial C}}{1-\left(\frac{V}{V_R^*(C^{reneg})}\right)^{Y-1}}.$$

Since C' is constructed so that the renegotiation barrier $V_R^*\left(C^{reneg}\right)$ and the put barrier $V_P^*\left(C'\right)$ coincide, we find in this case that (20) is equivalent to the much simpler relation

$$-\frac{\partial V_P^*\left(C\right)}{\partial C}\geq-\frac{\partial V_R^*\left(C\right)}{\partial C}.$$

This relation is always true as stated under remark (1). Hence, if $m_{opt}\left(C'\right)$ is higher than one, the derivative $\frac{\partial v^{put}\left(C,V_P^*(C),m_{opt}(C)\right)}{\partial C}$ at $C=C'$ is strictly positive. Therefore, the optimal coupon C^{put} must be above C'. As a consequence, the optimal put barrier $V_P^*\left(C^{put}\right)$ is higher than the optimal renegotiation barrier $V_R^*\left(C^{reneg}\right)$.

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