Managerial Compensation Contracts and Overconfidence*

Karl Ludwig Keiber†

September, 2005

*The comments by participants of the 3rd Symposium of the German Economic Association of Business Administration 2002, of the 9th Annual Meeting of the German Finance Association 2002, of the 9th Symposium on Finance, Banking, and Insurance 2002, of the 65th Annual Meeting of the Verband der Hochschullehrer für Betriebswirtschaft e.V. 2003, and of the 14th Annual Meeting of the European Financial Management Association 2005 are gratefully acknowledged. Comments are welcome. Any errors are my responsibility.

†Address: WHU Otto Beisheim Graduate School of Management, Chair of Finance, Burgplatz 2, D–56179 Vallendar, Germany. Phone: +49–261–6509–422. Fax: +49–261–6509–409. E–mail: kkeiber@whu.edu. Karl Ludwig Keiber is a lecturer of Finance at the WHU Otto Beisheim Graduate School of Management.

Managerial Compensation Contracts and Overconfidence

Abstract

This paper analyzes the impact of overconfidence on the principal–agent relationship. We study the effects of this psychological bias on both the compensation contract which the principal offers to the agent and the severity of the moral hazard problem. The comparative static analysis reveals that a more pronounced overconfidence bias generally reduces the agency costs but enhances the incentive component of the compensation contract as well as the agent's effort. We conclude that overconfidence plays a crucial role in the design of incentive compatible compensation contracts. Furthermore, we report that from the principal's perspective overconfidence is advantageous if favorable information about the future state of nature is available. If poor signals are available the overconfidence bias is shown to be detrimental to the principal.

JEL classification: D82, G34, J33.

1 Introduction

Entrepreneurs and managers are reported to exhibit overconfidence. Evidence concerning the former group can be found in Cooper, Dunkelberg and Woo (1988) whereas managerial overconfidence is documented by Russo and Schoemaker (1992), and Malmendier and Tate (2005). Busenitz and Barney (1997) report that both entrepreneurs and managers are subject to this psychological bias.

Generally, an entrepreneur employs a manager who is instructed to perform a certain task on behalf of the employer. The entrepreneur — hereafter referred to as the principal — offers a remuneration contract which guarantees the manager — henceforth called the agent — a compensation for the accomplished effort in performing the delegated task once the agent has signed on the labor contract. The assumption that the agent's effort is unobservable on the part of the principal has two consequences. First, the agent's effort cannot be used for contracting. Second, the agent has discretion on the exerted effort. Hence, the unobservability of the agent's effort gives rise to moral hazard. Given the above evidence on entrepreneurial and managerial overconfidence, the purpose of this paper is to shed light on the impact of the overconfidence bias on both the compensation contract and the severity of the moral hazard problem.

This paper adds to the emerging body of research on behavioral corporate finance by merging an empirically well–documented aspect of human behavior — namely overconfidence — with the principal–agent paradigm of corporate finance explicitly. A number of studies already exists which are concerned with the analysis of behavioral aspects of corporate finance.

Shefrin (2001) stresses that incentive compatibility is a necessity for value maximization but incentive effects alone cannot overcome the impact of behavioral obstacles which are internal to the firm such as for example overconfidence. Heaton (2002) provides explanations for a variety of corporate finance phenomena even in the absence of both asymmetric information and moral hazard by solely relying on managerial optimism as behavioral bias. Additionally, managerial overconfidence is taken into account by Gervais, Heaton and Odean (2003) who find that both behavioral biases — overconfidence and optimism — can increase the value of the firm. Thus, the decisions of overconfident and optimistic managers align better with the interest of the shareholders than those of rational managers. Furthermore, they make the case for hiring an overconfident manager instead of realigning the decisions of a rational manager with the shareholders' interest by employing convex compensation schemes. In Goel and Thakor (2002) the competition among managers for leadership is identified as mechanism which fosters overconfidence among managers since a stronger overconfidence bias increases the manager's probability to become leader. The persistence of entrepreneurial overconfidence is discussed in Bernardo and Welch (2001). An equilibrium proportion of overconfident individuals is derived in a group selection framework. It is shown that the overconfident behavior on the part of entrepreneurs provides a positive externality with respect to information aggregation compared to an otherwise herding behavior. Thus, the persistence of overconfidence is justified.[1]

The seminal paper on the principal–agent problem is due to Holmström (1979). We perform the analysis as regards the impact of the overconfidence bias on both the compensation contract and the severity of the moral hazard problem by effecting a variation compared to Holmström's (1979) seminal approach. The information structure is modified by introducing a noisy signal on the future state of nature. This noisy signal is assumed to be available to both the principal and the agent. Since the future state of nature partially determines the monetary outcome which is to be shared between the principal and the agent ultimately, the noisy signal also provides to some extent information about the monetary outcome. However, the parties to the contract are assumed to be overconfident with respect to the noisy signal's quality. Accordingly, subject to the overconfidence bias the players judge the quality of the signal to be higher than it really is. Hence, the overconfidence bias implies that in an inference process — that is by conditioning on the

[1]Another strand of behavioral finance focusses on the analysis of the overconfidence bias' impact on financial markets. For example, the studies of Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (1998), Odean (1998), Hirshleifer and Luo (2001), Gervais and Odean (2001), and Daniel, Hirshleifer and Subrahmanyam (2001) all belong to that strand.

noisy signal — the players put more weight on the available information than they would do absent this bias.

To motivate the above modification, interpret the common noisy signal as a forecast of the future state of nature. In a broader sense one might think of the noisy signal as a forecast of the business cycle. Good future states of the economy are associated with a higher signal which in turn indicates a favorable impact of the economic environment on the final monetary outcome and vice versa. However, since the signal is noisy the forecast is not perfect. Hence, the signal noise represents the forecast error. Now, the application of the model is straightforward. Think of a company whose shareholders hire a highly specialized manager to run the business on their behalf. The manager's expertise makes him believe to have an above average ability to forecast the future state of nature. Thus, the manager is overconfident with respect to the noisy signal's quality. Finally, since it is unlikely that the shareholders hire a manager who differs with respect to the assessment of outcome relevant information, the shareholders and the manager are supposed to exhibit the same degree of overconfidence.

The behaviorally–based variation of the principal–agent problem in this paper allows us to address the question of how the noisy signal on the future state of nature and the overconfidence bias affect the principal–agent relationship. Therefore, the present paper provides novel insights on incentive compatible contracting in the principal–agent relationship in the presence of both (i.) imperfect information about the future state of nature and (ii.) managerial and entrepreneurial overconfidence. For the sake of tractability and the ability to provide closed–form solutions as well as analytical comparative static results we restrict our analysis to the class of principal–agent problems with linear sharing rules, exponential preference representation, and normally distributed uncertainty as regards the state of nature.

The results which we obtain from the analysis of the second–best problem are manifold. The compensation contract is shown to depend on both the extent of overconfidence and the common noisy signal. Consequently, different combinations of (a.) the information about the future state of nature and (b.) the severity of the overconfidence bias with respect to the quality of that information result in different sharing rules. This observation might explain the variety of compensation arrangements which can be observed for performing the same task. More precisely, the fixed compensation component depends on both the extent of overconfidence and the common noisy signal. Contrary, the incentive component, the agent's effort and the agency costs solely depend on the severity of the overconfidence bias.

The comparative static results indicate that irrespective of the available information about the future state of nature a more pronounced overconfidence reduces the agency costs. Anyway, both the incentive component of the sharing rule and the agent's effort are generally increased by a stronger overconfidence bias. The impact of the overconfidence bias on the fixed compensation depends on the noisy signal. If favorable information is observed then a more pronounced overconfidence bias decreases the fixed remuneration. For poor signals the impact of a stronger overconfidence bias on the fixed compensation is reversed. Finally, the fixed compensation ceteris paribus is found to be decreasing in the available signal whereas the variable compensation as well as the agency costs are not affected by the common signal at all. These insights let us conclude that the overconfidence bias plays a crucial role in the design of incentive compatible compensation contracts.

In addition to the common analysis of the principal–agent relationship we study the dependence of the principal's expected utility on the severity of the overconfidence bias. Thus, we extend the usual discussion by analyzing the impact of the overconfidence bias from the shareholders' perspective. The comparative static analysis shows that the principal's expected utility is increased (reduced) by a more pronounced overconfidence bias if favorable (poor) information about the future state of nature is available and vice versa. Most strikingly, we identify ranges for the common noisy signal where a stronger overconfidence bias generally is detrimental to or advantageous for the shareholders. Since this observation is true irrespective of the actual level of overconfidence we can formulate general policy implications. In essence, we report that if good (bad) signals are observed then being more (less) overconfident is advantageous for the shareholders. Put differently, overestimating the quality of good signals does not harm the shareholders whereas the shareholders suffer from the overestimation of the quality of bad signals.[2]

The remainder of the paper proceeds as follows. Section 2 summarizes the basic structure of the principal–agent relationship. In section 3 both the first–best and the second–best compensation contracts as well as the agency costs are derived. Additionally, the principal's second–best expected utility is determined. The comparative static analysis with respect to the major ingredients of the model — the noisy signal and the overconfidence bias — are provided in section 4. Section 5 concludes and outlines further research avenues both empirical and theoretical. All proofs are collected in the appendix.

2 Setup of the Principal–Agent Relationship

In this section we outline the basic structure of the principal–agent relationship. A description of how the uncertainty about the future state of nature affects the principal–agent relationship is provided. We specify the structure of the compensation contract as well as both the principal's and the agent's preferences and characterize how the overconfidence bias enters the principal–agent relationship. Completing the outline of the setup, the information structure of the principal–agent relationship is summarized.

The principal–agent relationship is subject to state uncertainty since the monetary outcome which the principal and the agent share in finally is affected by the unobservable future state of nature $\tilde{\theta}$. We assume that the monetary outcome $\tilde{x}$ results as

$$\tilde{x} = e + \tilde{\theta}, \tag{1}$$

where e denotes the agent's effort. The future state of nature $\tilde{\theta}$ is supposed to be normally distributed with law $\mathcal{N}(0, \sigma_\theta^2)$. Consequently, the monetary outcome $\tilde{x}$ also is normally distributed. The future state of nature $\tilde{\theta}$ comprises the random impact of the economic environment on the monetary outcome. More precisely, the random impact of the economic environment subsumes

[2] The meaning of "favorable/poor information" and "good/bad signals" is made concrete in the propositions 6 and 9 as well as in the corollary 5.

that part of the monetary outcome resulting from economic conditions which are beyond the control of both the agent and the principal.

The sharing rule determines how the monetary outcome is apportioned to the parties to the contract finally and aims at aligning the agent's action with the principal's interest. The sharing rule $r(\tilde{x})$ is assumed to be a linear function of the monetary outcome $\tilde{x}$ that is

$$r(\tilde{x}) = \gamma + \delta \cdot \tilde{x}, \tag{2}$$

where γ and δ are some real numbers which determine the components of the compensation contract. The agent's fixed compensation amounts to γ whereas δ defines the agent's variable compensation by the product with the monetary outcome $\tilde{x}$. Henceforth, we refer to δ itself as the variable compensation.

The principal and the agent are supposed to maximize expected utility of final wealth. The principal is assumed to be risk neutral with preference representation by the utility function V. The agent is assumed to be risk averse having preferences which are represented by the utility function U. Particularly, the agent's preferences are represented by negative exponential utility. The coefficient of absolute risk aversion is denoted by $a > 0$.

The principal's final wealth depends on both the monetary outcome $\tilde{x}$ and the sharing rule $r(\tilde{x})$. Basically, the principal's end of period wealth amounts to $\tilde{x} - r(\tilde{x})$. Thus, the principal claims the residual monetary outcome after the agent is compensated according to the contracted remuneration $r(\tilde{x})$.

The final wealth of the agent depends on both the sharing rule $r(\tilde{x})$ and the accomplished effort e. The agent's final wealth is supposed to be given by $r(\tilde{x}) - \frac{1}{2}e^2$. This definition accounts for the fact that exerting effort produces effort costs which are subtracted from the remuneration. Thus, this effort–based decrease of final wealth generates some kind of disutility and captures the notion that the agent suffers from exerting effort. Furthermore, the agent has a reservation level of wealth m which he requires at least from accepting the contract. The wealth level m produces the agent the reservation utility $U(m)$. One might think of m as being the agent's final wealth from choosing an outside option.

Since the principal cannot observe the agent's effort, the sharing rule is based on the monetary outcome $\tilde{x}$ which is observable ultimately. Therefore, the agent's compensation depends to some extent also on the random impact of the economic environment $\tilde{\theta}$. Although the future state of nature cannot be controlled neither by the principal nor by the agent we assume that a noisy signal $\tilde{s}$ on the future state of nature becomes available initially. The noisy signal is given by

$$\tilde{s} = \tilde{\theta} + \tilde{\varepsilon}, \tag{3}$$

where $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ denotes the signal noise which is assumed to be uncorrelated with the future state of nature. By construction, the noisy signal provides some information about the future state of nature. Therefore, the noisy signal is also informative for the monetary outcome at least to some extent. Thus, the signal should not be ignored in contracting, although it is less than perfect.

The overconfidence bias is introduced with respect to the quality of the signal $\tilde{s}$. Subject to the overconfidence bias the variance of the signal noise is judged to be $\kappa\sigma_\varepsilon^2$ with $0 < \kappa < 1$. The biased assessment of the signal noise's variance implies that the signal is judged to be more informative for the future state of nature than it really is. An unbiased assessment of the signal's quality obtains for $\kappa = 1$. Note that a smaller coefficient of overconfidence κ corresponds to a more severe overconfidence bias. The most extreme overconfidence bias obtains for κ converging to zero. In this limiting case, the signal is judged to be perfect. Put differently, the parties to the contract belief the forecast of the future state of nature to be perfectly accurate. Any forecast error is neglected completely. In general, the overconfidence bias effects that the forecast error is scaled down.

The model's information structure obeys to the above description. In particular, all features mentioned above are presumed to be common knowledge. Thus, the information structure is symmetric and the players are subject to the overconfidence bias in the manner described previously.

3 Sharing Rules and Agency Costs

Drawing on the setup of the principal–agent relationship this section discusses the optimal design of the compensation contract in the presence of overconfidence with respect to the quality of the common signal on the future state of nature. First, we determine the compensation contract which corresponds to the situation in which the principal lacks perfect monitoring as regards the agent's effort. This compensation contract is referred to as the second–best sharing rule. Second, we focus on the case in which the principal chooses the agent's effort level in addition to the compensation contract which is referred to as the first–best sharing rule. Third, exploiting the first–best and the second–best solution we determine the agency costs. Finally, the principal's second–best expected utility is derived.

3.1 Second–Best Compensation Contract

The determination of the optimal second–best sharing rule $r_2(\tilde{x})$ boils down to the principal's choice of γ_2 and δ_2 which represent the second–best compensation contract's components. The timing of the players' actions and events in the second–best case is depicted in figure 1. At $t = 0$ both the principal and the agent observe the common signal $\tilde{s}$. The principal offers the sharing rule γ_2 and δ_2 to the agent at $t = 1$. At $t = 2$ the agent chooses the effort e_2. The resolution of uncertainty occurs at $t = 3$ and the monetary outcome $\tilde{x}$ is observed.

The second–best compensation contract results as solution to the constrained program

$$\max_{\gamma_2,\delta_2} \quad \mathrm{E}_\kappa[V(\tilde{x} - r_2(\tilde{x}))|\tilde{s}]$$

$$\text{s.t.} \quad \mathrm{E}_\kappa[U(r_2(\tilde{x}), e_2)|\tilde{s}] \geq U(m) \tag{4a}$$

$$e_2 \in \underset{e}{\operatorname{argmax}}\, \mathrm{E}_\kappa[U(r_2(\tilde{x}), e)|\tilde{s}], \tag{4b}$$

where the subscript κ of the expectation operators is reminiscent of the parties' overconfidence bias with respect to the quality of the noisy signal. The constraint (4a) ensures that entering the principal–agent relationship is rational from the agent's perspective as the employment produces at least the reservation utility. Therefore, this constraint is referred to as the individual rationality constraint, the participation constraint or the reservation utility constraint equivalently. The constraint (4b) guarantees that the agent chooses that effort which maximizes his expected utility and is referred to as the incentive compatibility constraint. The second–best contract is given in proposition 1.

Proposition 1 *The second–best compensation contract* $r_2(\tilde{x})$ *which the principal offers to the agent is given by*

$$\gamma_2 = m - \frac{\mu_{\theta,\kappa}}{1 + a\sigma^2_{\theta,\kappa}} - \frac{1 - a\sigma^2_{\theta,\kappa}}{2\left(1 + a\sigma^2_{\theta,\kappa}\right)^2}$$

and

$$\delta_2 = \frac{1}{1 + a\sigma^2_{\theta,\kappa}},$$

where $\mu_{\theta,\kappa}$ *and* $\sigma^2_{\theta,\kappa}$ *are given in lemma 1.*

The inspection of proposition 1 yields that the second–best compensation contract depends on the conditional moments of the future state of nature. Thus, besides the coefficient of risk aversion a and the reservation level of wealth m, both the common signal $\tilde{s}$ and the coefficient of overconfidence κ are determinants of the second–best sharing rule. Since $0 < \delta_2 < 1$ the agent truly shares in the final monetary outcome. Hence, an incentive exists on the part of the agent to exert an effort which increases the final monetary outcome. The proof of proposition 1 delivers corollary 1 immediately.

Corollary 1 *The agent's second–best effort amounts to* $e_2 = \delta_2$*.*

Combining proposition 1 and corollary 1 allows to express the second–best fixed compensation as

$$\gamma_2 = m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa} - \delta_2\left(e_2 + \mu_{\theta,\kappa}\right) \tag{5}$$

equivalently. Hence, the agent's second–best fixed compensation consists of the reservation level of wealth m, the compensation of his effort costs $\frac{1}{2}e_2^2$ and a risk premium $\frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}$ which compensates for the perceived remaining uncertainty $\sigma^2_{\theta,\kappa}$ about the future state of nature. Finally, the agent's conditional expected variable compensation $\delta_2 \left(e_2 + \mu_{\theta,\kappa}\right)$ is subtracted from those three components. By exploiting (5) the agent's second–best compensation can be written as

$$r_2(\tilde{x}) = m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa} + \delta_2 \left(\tilde{\theta} - \mu_{\theta,\kappa}\right) \tag{6}$$

alternatively. Hence, the agent's second–best compensation (6) is subject to uncertainty due to the dependency on the unobservable future state of nature $\tilde{\theta}$. However, the agent's compensation risk only consists of the conditional unexpected innovation in the future state of nature, that is the deviation of the state of nature from its conditional expectation. For positive surprises in the random impact of the economic environment compared to the conditional expectation, that is for $\tilde{\theta} - \mu_{\theta,\kappa} > 0$, the agent's second–best compensation increases and vice versa. According to the agent's sensitivity to the conditional unexpected innovation, this surprise risk is compensated by the risk premium.[3]

3.2 First–Best Compensation Contract

The first–best compensation contract $r_1(\tilde{x})$ requires the determination of γ_1 and δ_1 on the part of the principal. In contrast to the second–best case the principal perfectly controls and decides on the agent's effort level e_1. Figure 2 shows the timing of the players' actions and events in the first–best case. At $t = 0$ both the principal and the agent observe the common signal $\tilde{s}$. The principal decides on the optimal compensation contract γ_1 and δ_1 as well as on the agent's effort e_1 at $t = 1$. At $t = 2$ the state uncertainty is resolved and the principal observes the monetary outcome $\tilde{x}$. Note that all decisions are taken by the principal. The agent does not take any action at all. Consequently, the principal solely is subject to the uncertainty about the future state of nature but is not anymore subject to the agents discretion as

[3]Note that by corollary 1 we calculate

$$\mathrm{E}_\kappa\left[\left(\delta_2\left(\tilde{\theta} - \mu_{\theta,\kappa}\right)\right)^2 \mid \tilde{s}\,\right] = \delta_2^2\mathrm{E}_\kappa\left[\left(\tilde{\theta} - \mu_{\theta,\kappa}\right)^2 \mid \tilde{s}\,\right] = e_2^2\sigma^2_{\theta,\kappa}.$$

regards the choice of the effort level. This ensures the first–best result for the principal.

The first–best compensation contract is obtained as solution of the following constrained programm

$$\begin{aligned} \max_{\gamma_1,\delta_1,e_1} \quad & \mathrm{E}_\kappa[V(\tilde{x} - r_1(\tilde{x}))|\tilde{s}] \\ \text{s.t.} \quad & \mathrm{E}_\kappa[U(r_1(\tilde{x}), e_1)|\tilde{s}] \geq U(m), \end{aligned} \tag{7}$$

where the subscript κ is reminiscent of both the principal's and the agent's overconfidence bias with respect to the quality of the common signal $\tilde{s}$. In the first–best case the principal's choice of the agent's effort only has to ensure that the agent accepts the offered sharing rule. Therefore, the principal's optimization solely is subject to the agent's participation constraint (7). The first–best compensation contract is given in proposition 2.

Proposition 2 *The first–best compensation contract $r_1(\tilde{x})$ which the principal offers to the agent is given by*

$$\gamma_1 = m + \frac{1}{2}$$

and

$$\delta_1 = 0.$$

Most notably, the first– best compensation contract is independent of the conditional moments of the future state of nature. Hence, it does not depend on neither the common signal nor the coefficient of overconfidence. Solely the agent's reservation level of wealth m enters the fixed component of the sharing rule. Since the principal determines the agent's effort the principal does not need to provide the agent with any incentive at all. Consequently, the variable compensation amounts to zero. The proof of proposition 2 yields corollary 2.

Corollary 2 *The agent's first–best effort amounts to $e_1 = 1$.*

Having established both proposition 2 and corollary 2 the interpretation of these results is straightforward. The agent's first–best remuneration collapses to $r_1(\tilde{x}) = \gamma_1$ and guarantees the reservation utility $U(m)$ to the agent as it pays the reservation level of wealth m and compensates the effort costs $\frac{1}{2}e_1^2 = \frac{1}{2}$. Since the agent is not subject to any compensation risk, the agent does not command a risk premium. Consequently, in the first–best case the uncertainty about the future state of nature solely is carried by the principal who does not care about that risk due to the assumption of risk neutrality.

As the agent is not subject to compensation risk and the principal does not care about the uncertainty by assumption, the perceived remaining uncertainty about the future state of nature conditional on the common signal does not affect the first–best contract at all. Therefore, the overconfidence bias with respect to the common signal's quality does not affect contracting in the first–best case.

3.3 Agency Costs

The difference between the first–best program and the second–best program is that in the latter the agent chooses his effort in order to maximize the own expected utility according to the incentive compatibility constraint whereas in the first–best case the principal determines the agent's effort.

In the second–best case the principal suffers from the agent's lower effort $\delta_2 = e_2 < e_1 = 1$, the payment of a risk premium, and the fact that the agent shares in the monetary outcome, but profits from the lower effort costs compensation. The overall impact of the agent's optimization according to the incentive compatibility constraint on the principal's expected utility is quantified by the agency costs. Thus, the agency costs represent losses in the principal's expected utility from hiring a selfish agent which the principal cannot monitor perfectly. Hence, the agency costs as given in proposition 3 quantify the severity of the moral hazard problem.

Proposition 3 *The agency costs which the principal suffers are given by*

$$\frac{a\sigma_{\theta,\kappa}^2}{2\left(1 + a\sigma_{\theta,\kappa}^2\right)},$$

where $\sigma_{\theta,\kappa}^2$ is given in lemma 1.

Note that the agency costs only are affected by the severity of the overconfidence bias through the perceived remaining state uncertainty $\sigma^2_{\theta,\kappa}$. The actual common noisy signal on the future state of nature is irrelevant for the severity of the moral hazard problem. The agency costs' independence of the noisy signal originates from the fact that the conditional expectation of the economic environment's impact symmetrically affects the principal's expected utility in both cases. Proposition 3 delivers corollary 3 immediately.

Corollary 3 *The agency costs which the principal suffers are positive.*

Corollary 3 restates the well–known result of the principal–agent theory. Namely, the moral hazard problem strictly produces disutility for the principal. That is, the principal suffers from the agency relationship. Thus, the decrease of the agent's effort costs compensation does not outweigh the detrimental impact of the other effects.

3.4 Principal's Second–Best Expected Utility

Although the severity of the moral hazard problem is judged by the magnitude of the agency costs we study the principal's expected utility in the second–best case in more detail. This allows for studying the impact of the overconfidence bias — if any — from the shareholders' perspective. Proposition 4 reports the shareholders' expected utility.

Proposition 4 *The principal's second–best expected utility amounts to*

$$-m + \mu_{\theta,\kappa} + \frac{1}{2\left(1 + a\sigma^2_{\theta,\kappa}\right)},$$

where $\mu_{\theta,\kappa}$ and $\sigma^2_{\theta,\kappa}$ are given in lemma 1.

First of all, one realizes the dependency of the principal's second–best expected utility from both the common noisy signal $\tilde{s}$ and the coefficient of overconfidence κ. Alternatively, the principal's second–best expected utility can be expressed as

$$e_2 + \mu_{\theta,\kappa} - \left(m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}\right). \tag{8}$$

Due to the principal's risk neutrality the second–best expected utility (8) simply results as the conditional expected monetary outcome less the agent's conditional expected second–best compensation. The latter can be calculated as the expectation of the right hand side of equation (6) conditional on the common signal $\tilde{s}$.

4 Comparative Static Analysis

In this section we carry out a comparative static analysis with respect to both the common noisy signal and the severity of the overconfidence bias. More precisely, we study the impact of the common noisy signal $\tilde{s}$ and the coefficient of overconfidence κ which quantifies the severity the overconfidence bias on the second–best sharing rule, the agency costs, and the principal's second–best expected utility. The comparative static analysis allow us to formulate some implications as regards the relevance of the overconfidence bias for the principal–agent relationship, finally.

As regards the interpretation of the comparative statics we emphasize that a decreasing coefficient of overconfidence κ actually comes along with a more pronounced overconfidence bias. Hence, the lower is the coefficient of overconfidence κ the more precise the parties to the contract judge the common signal on the future state of nature. Note that the below comparative static results are derived analytically. Nevertheless, we visualize them in the figures 3–7 which rely on the parameters in table 1.

Recall that the second–best sharing rule is represented by γ_2 and δ_2 as given in proposition 1. Proposition 5 pools the comparative statics of the second–best contract's variable compensation δ_2.

Proposition 5 *The variable compensation* δ_2

- *decreases in the coefficient of overconfidence* κ *and*
- *is independent of the signal* $\tilde{s}$*.*

Note that corollary 1 in turn implies corollary 4 immediately.

Corollary 4 *The proposition 5 also applies to the agents effort* e_2*.*

The graph in figure 3 depicts the comparative static results of the proposition 5 and the corollary 4. If the parties to the contract ceteris paribus are subject to a more pronounced overconfidence bias then they generally contract a higher variable compensation. Note that a higher variable compensation enhances the agent's incentive. Consistently, the agent's effort increases in parallel to the variable compensation. However, due to the agent's discretion the second–best effort is less that the first–best counterpart, that is $e_2 < e_1 = 1$. This effort reduction compared to the first–best case reflects the imperfect monitoring on the part of the principal in the second–best case. If the coefficient of overconfidence κ converges to zero the second–best effort approaches the first–best effort.

The comparative statics of the second–best contract's fixed component γ_2 are summarized in proposition 6.

Proposition 6 *The fixed compensation* γ_2

- *increases for* $\tilde{s} > s_1$ *and decreases for* $\tilde{s} < s_1$ *in the coefficient of overconfidence* κ *and*
- *decreases in the signal* $\tilde{s}$*.*

The signal s_1 *is given in the proof.*

The comparative static analysis of the second–best sharing rule's fixed component as reported in the proposition 6 is illustrated in figure 4. The shape of the surface indicates the strict monotonicity of the fixed compensation in the noisy signal. Generally, a lower signal comes along with a higher fixed compensation and vice versa. Hence, the availability of a bad (good) forecast about the future state of nature implies a premium (discount) in the fixed compensation.

This spread in the fixed compensation is amplified by the severity of the overconfidence bias. Since the agent's exposure to the impact of the economic environment — that is the variable component δ_2 — increases in the strength of the overconfidence bias, the parties contract in the presence of favorable (poor) information $\tilde{s} > s_1$ ($\tilde{s} < s_1$) an even lower (higher) fixed compensation the more overconfident they are.

In proposition 7 we present the comparative statics of the contracted risk premium $\frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}$. Since the risk premium represents a component of the fixed compensation which is exclusive to the second–best case and provides insights on the interplay of risk perception and incentives we discuss it separately.

Proposition 7 *The risk premium*

- *increases in the coefficient of overconfidence κ if $a < \sigma_\theta^{-2} + \sigma_\varepsilon^{-2}$ but*
- *increases (decreases) for $\kappa < (>) \kappa_0$ in the coefficient of overconfidence κ if $a > \sigma_\theta^{-2} + \sigma_\varepsilon^{-2}$ and*
- *is independent of the signal $\tilde{s}$.*

The coefficient of overconfidence κ_0 is given in the proof.

The proposition 7 reports that the monotonicity of the risk premium in the coefficient of overconfidence depends on the agent's risk aversion. If the agent is sufficiently risk averse the risk premium peaks at κ_0. Put differently, in that case the risk premium first increases the more overconfident the parties to the contract become, peaks and decreases in the severity of the overconfidence bias afterwards. That is, the risk premium might be hump–shaped in the coefficient of overconfidence.

The mechanics which produce this hump–shaped pattern are as follows. The more pronounced becomes the overconfidence bias, the higher the agent's effort and the lower the perceived remaining uncertainty about the future state of nature become. Initially, the increase of the effort dominates the reduction of the perceived remaining state uncertainty. Hence, the agent's increased exposure to the impact of the economic environment outweighs the reduction of the perceived remaining state uncertainty and therefore the contracted risk premium increases. Since the increase of the agents exposure is bounded from above by the first–best effort the reduction of the perceived remaining state uncertainty dominates for extreme levels of overconfidence and the risk premium decreases, ultimately.

Consistent to proposition 7 and according to the parameters in table 1, the graph in figure 5 shows that the contracted risk premium strictly monotonically decreases in the severity of the overconfidence bias. Note that $1 = a < \sigma_\theta^{-2} + \sigma_\varepsilon^{-2} = 2$. Since the agent is not quite risk averse — in the sense of proposition 7 — the reduction of the perceived remaining state uncertainty dominates the increase of the exposure to the economic environment's impact and the risk premium does not peak.

To summarize, the comparative static analysis of the second–best sharing rule's components indicate that (i.) the available information about the future state of nature and (ii.) the perception of that information's precision substantially affect contracting in the principal–agent relationship. Thus, the variety of compensation contracts is spanned by the combinations of the common noisy signal $\tilde{s}$ and the severity κ of the parties' overconfidence bias. Hence, our approach serves as explanation for the multitude of labor contracts which are observed for performing the same task depending on the forecast of the future state of nature and the perception of that forecast's precision.

The comparative static results of the agency costs are collected in proposition 8.

Proposition 8 *The agency costs which the principal suffers*

- *increase in the coefficient of overconfidence κ and*
- *are independent of the signal $\tilde{s}$.*

The graph in figure 6 confirms the agency costs' comparative statics. Generally, the more pronounced is the overconfidence bias of the parties to the contract the lower the agency costs are. Thus, the miscalibration according to the overconfidence bias reduces the severity of the moral hazard problem. Put differently, the wedge between the principal's first–best expected utility and second–best expected utility becomes smaller the more severe the overconfidence bias becomes.

In the limiting case when κ converges to zero the agency costs vanish completely. This observation is driven by two effects. First, the agent's second–best effort converges to the first–best counterpart. Second, the parties do not contract a risk premium anymore. Consequently, the first–best situation obtains. Alternatively, one might explain this observation as follows. Since in the limiting case the noisy signal is judged to be perfect, both parties do not perceive any uncertainty about the future state of nature anymore. Hence, the agent feels to be perfectly monitored by the principal and the parties agree on the first–best contract.

The previous discussion of the agency costs highlights the impact of the overconfidence bias on the principal's second–best expected utility in comparison to the corresponding first–best expected utility. However, we extend the usual discussion of the principal–agent relationship by studying the impact of both the overconfidence bias and the common signal on the principal's second–best expected utility. This allows us to assess the impact of the psychological bias and the common piece of information from the shareholders' perspective. The comparative static results of the principal's second–best expected utility are collected in proposition 9.

Proposition 9 *The principal's second–best expected utility*

- *increases for $\tilde{s} < s_2$ and decreases for $\tilde{s} > s_2$ in the coefficient of overconfidence κ and*
- *increases in the signal $\tilde{s}$.*

The signal s_2 is given in the proof.

The comparative static results of the principal's second–best expected utility are illustrated in figure 7. The shape of the surface reveals that the principal's second–best expected utility is strictly monotonically increasing in the noisy signal. Thus, the availability of a good (bad) forecast about the impact of the economic environment comes along with a high (low) second–best expected utility of the principal.

The impact of different forecasts on the principal's second–best expected utility is amplified by the severity of the overconfidence bias. That is, a more pronounced overconfidence bias boosts (depresses) the second–best expected utility of the principal more strongly in the presence of favorable (poor) information $\tilde{s} > s_2$ ($\tilde{s} < s_2$) about the future state of nature. In parallel, the principal profits (suffers) from the low (high) fixed compensation which is contracted; cf. proposition 6.

The proposition 9 allows us to derive general implications from the shareholders' perspective irrespective of the actual severity of the overconfidence bias. Corollary 5 collects these implications.

Corollary 5 *The principal's second–best expected utility increases for* $\tilde{s} < s_3$ *and decreases for* $\tilde{s} > s_4$ *in the coefficient of overconfidence* κ *with* $s_3 < s_4$. *The signals* s_3 *and* s_4 *are independent of the overconfidence bias and are given in the proof.*

In order to better grasp the intuition of corollary 5 one should realize that the forecasts s_3 and s_4 represent the endpoints of the curved line on the surface in figure 7. Note that corollary 5 identifies ranges for the noisy signal in which it is advantageous or detrimental for the principal to exhibit a stronger overconfidence. If good (bad) information $\tilde{s} > s_4$ ($\tilde{s} < s_3$) about the future state of nature becomes available then being more (less) overconfident with respect to the forecast's quality — and consequently hiring an agent which also is subject to a higher (lower) degree of overconfidence — is advantageous from the principal's perspective. Since the signals s_3 and s_4 are independent of the actual severity of the overconfidence bias, this observation generally holds.

Put differently, overestimating the quality of good forecasts is beneficial whereas it is advisable to process bad forecasts as unbiased as possible. Hence, depending on the forecast of the future state of nature different information processing capabilities are preferable. This finding has direct implications for the job market of managers. When the available information about the future state of nature shifts from good to bad it is favorable for the shareholders to recruit and employ a less overconfident management and vice versa. Simultaneously, if the shareholders pursue such an employment policy the management's compensation schedules also are affected. Hence, the variable compensation is increased if a good forecast of the future state of nature is available whereas if a poor forecast of the future state of nature is observed a lower incentive component of the sharing rule is contracted.

5 Conclusion

By nature, companies are affected by behavior internally. In this paper we addressed the question of how a well–documented psychological trait — namely the overconfidence bias — affects the principal–agent relationship arising in companies with delegated management. More precisely, we analyzed thoroughly the design of incentive compatible compensation contracts in the presence of both overconfidence and moral hazard. Hence, this paper adds to the body of research on behavioral corporate finance which makes the affirmative case for analyzing the internal effects of behavioral traits on companies.

We suggested a variant of the standard principal–agent problem by modeling explicitly an additional stage of information collection before the compensation contract is written. Basically, we assumed that both the principal and the agent observe a common noisy signal on the future state of nature determining partially the monetary outcome which the parties to the contract share in ultimately. The principal's and agent's overconfidence bias was introduced with respect to the quality of the common noisy signal on the future state of nature. Our modeling of the overconfidence bias is quite common and can be found for instance in Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (1998), and Daniel, Hirshleifer and Subrahmanyam (2001) among others.

The strength of our approach consists in allowing us to study how incentive compatible contracting is affected by the principal's and agent's belief about the future state of nature in much the same way as price formation in securities markets is affected by the investors' expectations about the future prospects of the investments. Hence, we focus on the impact of belief miscalibration on incentive compatible contracting subject to the overconfidence bias, similar to behavioral finance models which study the impact of the investors' overconfidence on securities prices.

The present paper's major contribution is to emphasize that not only the available information about the future state of nature but also the perception of that information's precision are crucial elements in the design of incentive compatible compensation contracts. Put differently, the overconfidence bias of the parties to the contract substantially affects the principal–agent relationship.

We have shown that the variety of compensation contracts is spanned by both the common noisy signal and the severity of the parties' overconfidence bias. Hence, our approach provides an explanation for the multitude of labor contracts which are observed for performing the same task depending on both the forecast of the future state of nature and the perception of that forecast's precision. The variation of the parties' sentiment — for instance during the business cycle — with respect to the assessment of the available information's quality implies different contracting schemes ceteris paribus. Briefly, our approach is suggestive for different compensation contracts in different phases of the business cycle although the delegated task remains unchanged.

The comparative static analysis produced various insights concerning the impact of the overconfidence bias on the principal–agent relationship. First, in the presence of managerial and entrepreneurial overconfidence the agent both exerts a stronger effort and a larger variable compensation is contracted. That is, an overconfident management is more active and is provided with larger incentives.

Second, overconfident shareholders perceive a reduced severity of the moral hazard problem. From their perspective the overconfidence bias mitigates the moral hazard problem. Most notably, in the event of the most severe overconfidence — that is, if the parties to the contract believe the common noisy signal to be perfect — the second–best case and the first–best case collapse and the agency costs vanish. Therefore, we conclude that the overconfidence bias plays a positive role in the principal–agent relationship.

Third, the spreading of the management's fixed compensation is reported to be increased by the overconfidence bias. Generally, the management contracts a discount (premium) in the fixed compensation upon availability of a good (bad) forecast about the future state of nature. However, the contracted discount or premium is amplified by the overconfidence bias. Put differently, believing more strongly in good (bad) news about the future lets the managers accept a lower (claim a higher) fixed compensation. Hence, besides the forecast of the future state of nature itself, it is the perceived precision of that forecast which determines the fixed compensation.

Fourth, the contracted risk premium was shown to potentially peak as function of the overconfidence bias depending on the management's risk aversion but to converge to zero in the limiting case of the most severe overconfidence. The hump–shaped risk premium is generated by two effects of the overconfidence bias which point in the opposite direction simultaneously. On the one hand, the increase of the management's variable compensation implies the increase of the risk premium whereas on the other hand, the reduction of the perceived riskiness of the future state of nature reduces the risk premium.

Fifth, we reported that the shareholders' well–being increases in the forecast about the future state of nature generally. Again, the overconfidence bias amplifies this dependency. That is, the overconfidence bias boosts (depresses) the shareholder's well–being if good (bad) news about the future state of nature is observed. In other words, overestimating the quality of good information increases the shareholders' expected utility whereas believing more strongly in bad information harms the shareholders.

Drawing on this last observation we conclude that depending on the forecast about the future state of nature the shareholders have distinct preferences as concerns the management. That is, the shareholders prefer an overconfident management upon good news and a cautious management upon bad news. Hence, our results suggest that one can expect changes of the managerial board at the frequency of the business cycle. We hypothesize that an overconfident management is replaced by a less biased one if the information about the future state of nature shifts from good to bad. Although this employment strategy in turn aggravates the moral hazard problem by increasing the agency costs it is beneficial in terms of the shareholders well–being ultimately.

Further research comprises both empirical and theoretical issues. The formulated hypotheses and the derived comparative statics could be checked empirically. For instance, given access to real world data on managerial compensation contracts one could check the predictions with respect to the model's contracting patterns on the basis of business cycle and/or industry forecasts and sentiment indices. Theoretically, studying relaxations of the suggested model would be veritably valuable. But one should trade off these relaxations and the model's tractability. Maybe, those relaxations do not add much to the economic intuition beyond that provided in the present paper.

References

Bernardo, A. E. and Welch, I. (2001), 'On the evolution of overconfidence and entrepreneurs', *Journal of Economics and Management Strategy* **10**(3), 301–330.

Busenitz, L. W. and Barney, J. B. (1997), 'Differences between entrepreneurs and managers in large organizations: biases and heuristics in strategic decision–making', *Journal of Business Venturing* **12**(1), 9–30.

Cooper, A. C., Dunkelberg, W. C. and Woo, C. Y. (1988), 'Entrepreneurs' perceived chances for success', *Journal of Business Venturing* **3**(2), 97–108.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A. (1998), 'Investor psychology and security market under– and overreaction', *Journal of Finance* **53**(6), 1839–1885.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A. (2001), 'Overconfidence, arbitrage, and equilibrium asset pricing', *Journal of Finance* **56**(3), 921–965.

Gervais, S., Heaton, J. B. and Odean, T. (2003), Overconfidence, investment policy, and executive stock options. *Working Paper*, Duke University.

Gervais, S. and Odean, T. (2001), 'Learning to be overconfident', *Review of Financial Studies* **14**(1), 1–27.

Goel, A. M. and Thakor, A. V. (2002), Do overconfident managers make better leaders? *Working Paper*, New York University.

Heaton, J. B. (2002), 'Managerial optimism and corporate finance', *Financial Management* **31**(2), 33–45.

Hirshleifer, D. and Luo, G. Y. (2001), 'On the survival of overconfident traders in a competitive securities market', *Journal of Financial Markets* **4**(1), 73–84.

Holmström, B. (1979), 'Moral hazard and observability', *Bell Journal of Economics* **10**(1), 74–91.

Kyle, A. S. and Wang, F. A. (1997), 'Speculation duopoly with agreement to disagree: can overconfidence survive the market test', *Journal of Finance* **52**(5), 2073–2090.

Malmendier, U. and Tate, G. (2005), 'CEO overconfidence and corporate investment', *Journal of Finance* **60**(6), *forthcoming*.

Odean, T. (1998), 'Volume, volatility, price, and profit when all traders are above average', *Journal of Finance* **53**(6), 1887–1934.

Russo, J. E. and Schoemaker, P. J. H. (1992), 'Managing overconfidence', *Sloan Management Review* **33**(2), 7–17.

Shefrin, H. (2001), 'Behavioral corporate finance', *Journal of Applied Corporate Finance* **14**(3), 113–124.

A Proofs

Lemma 1 *Subject to the overconfidence bias, the conditional expectation* $\mathrm{E}_\kappa[\tilde{\theta}|\tilde{s}]$ *and the conditional variance* $\mathrm{Var}_\kappa[\tilde{\theta}|\tilde{s}]$ *of the random impact of the economic environment* $\tilde{\theta}$ *are*

$$\mu_{\theta,\kappa} \equiv \mathrm{E}_\kappa[\tilde{\theta}|\tilde{s}] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2}\tilde{s}$$

and

$$\sigma_{\theta,\kappa}^2 \equiv \mathrm{Var}_\kappa[\tilde{\theta}|\tilde{s}] = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2}.$$

Proof. For a bivariate normally distributed random vector $(\tilde{x}, \tilde{y})' \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$$

one knows

$$\mathrm{E}[\tilde{y}|\tilde{x}] = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2} \cdot (\tilde{x} - \mu_x) \tag{9}$$

and

$$\mathrm{Var}[\tilde{y}|\tilde{x}] = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2}. \tag{10}$$

Note that the random impact of the economic environment $\tilde{\theta}$ and the private signal $\tilde{s}$ are bivariate normally distributed. Straightforward application of (9) and (10) as well as taking into account the definition of the overconfidence bias yields the lemma. This completes the proof. □

A.1 Proof of Proposition 1

Proof. Using (1) and (2) the principal's expected utility results as

$$\mathrm{E}_\kappa[V(\tilde{x} - r_2(\tilde{x}))|\tilde{s}] = (1 - \delta_2)e_2 + (1 - \delta_2)\mu_{\theta,\kappa} - \gamma_2, \tag{11}$$

where $\mu_{\theta,\kappa}$ is given in lemma 1. Applying (1) and (2) yields the agent's final wealth to be $\gamma_2 + \delta_2 e_2 + \delta_2\tilde{\theta} - \frac{1}{2}e_2^2$ which has a normal distribution. The agent's constant absolute risk aversion preferences yield

$$\mathrm{E}_\kappa[U(r_2(\tilde{x}), e_2)] = U\left(\gamma_2 + \delta_2 e_2 + \delta_2\mu_{\theta,\kappa} - \frac{1}{2}e_2^2 - \frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2\right) \tag{12}$$

for the agent's expected utility where $\gamma_2+\delta_2 e_2+\delta_2\mu_{\theta,\kappa}-\frac{1}{2}e_2^2-\frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2$ denotes the agent's certainty equivalent wealth. These results allow an alternative formulation of the second–best program which becomes

$$\max_{\gamma_2,\delta_2} \quad (1-\delta_2)e_2+(1-\delta_2)\mu_{\theta,\kappa}-\gamma_2 \tag{13}$$

$$\text{s.t.} \quad \gamma_2+\delta_2 e_2+\delta_2\mu_{\theta,\kappa}-\frac{1}{2}e_2^2-\frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2 = m \tag{14}$$

$$\delta_2-e_2 = 0. \tag{15}$$

Thus the optimal compensation contract that the principal offers the agent leaves the agent exactly with the reservation level of wealth m. This is reflected in the individual rationality constraint (14). The incentive compatibility constraint (15) represents the first order condition of the agent's expected utility maximization with respect to the effort e_2. Note that (15) dictates

$$e_2=\delta_2 \tag{16}$$

for the agent's optimal effort. Plugging (16) into (14) yields

$$\gamma_2=m-\frac{\delta_2^2}{2}(1-a\sigma_{\theta,\kappa}^2)-\delta_2\mu_{\theta,\kappa} \tag{17}$$

for the optimal fixed compensation as function of δ_2. Plugging (17) into (13) allows to solve for the optimal δ_2 from the first order condition of (13) with respect to δ_2. The optimal δ_2 is given in the proposition. The optimal γ_2 as given in the proposition follows from (17) immediately. This completes the proof. □

A.2 Proof of Corollary 1

Proof. Equation (16) yields the corollary. □

A.3 Proof of Proposition 2

Proof. Along the same arguments as in the proof of proposition 1 the reformulation of the first–best program yields

$$\max_{\gamma_1,\delta_1,e_1} \quad (1-\delta_1)e_1 + (1-\delta_1)\mu_{\theta,\kappa} - \gamma_1 \tag{18}$$

$$\text{s.t.} \quad \gamma_1 + \delta_1 e_1 + \delta_1 \mu_{\theta,\kappa} - \frac{1}{2}e_1^2 - \frac{1}{2}a\delta_1^2\sigma_{\theta,\kappa}^2 = m. \tag{19}$$

From the individual rationality constraint (19) we solve for the optimal fixed compensation γ_1 as function of δ_1 and e_1 by rearranging terms. We obtain

$$\gamma_1 = m - \delta_1 e_1 - \delta_1 \mu_{\theta,\kappa} + \frac{1}{2}\left(e_1^2 + a\delta_1^2\sigma_{\theta,\kappa}^2\right). \tag{20}$$

After plugging (20) into (18) we solve for the optimal δ_1 and e_1 from the system of equations

$$\nabla\left(e_1 + \mu_{\theta,\kappa} - m - \frac{1}{2}\left(e_1^2 + a\delta_1^2\sigma_{\theta,\kappa}^2\right)\right) = \mathbf{0}, \tag{21}$$

where $\nabla(\cdot)$ represents the gradient of the principal's expected utility (18) with respect to δ_1 and e_1. The optimal effort results as

$$e_1 = 1 \tag{22}$$

and the optimal δ_1 is given in the proposition. The optimal fixed compensation γ_1 results from plugging the optimal δ_1 and the optimal e_1 into (20) and is given in the proposition too. This completes the proof. □

A.4 Proof of Corollary 2

Proof. Equation (22) yields the corollary. □

A.5 Proof of Proposition 3

Proof. Plugging γ_1 and δ_1 as given in proposition 2 as well as e_1 from (22) into (18) yields the principal's expected utility in the first–best case which amounts to

$$\frac{1}{2} - m + \mu_{\theta,\kappa}. \tag{23}$$

The principal's expected utility in the second–best case is stated in proposition 4. Subtracting the principal's second–best expected utility from the first–best counterpart and collecting terms yields the agency costs as given in the proposition. This completes the proof. □

A.6 Proof of Corollary 3

Proof. Straightforward inspection of the agency costs as given in proposition 3 yields that the agency costs are strictly positive for $0 < \kappa < 1$. This completes the proof. □

A.7 Proof of Proposition 4

Proof. The principal's expected utility in the second–best case results from applying γ_2 and δ_2 as stated in proposition 1 and e_2 from (16) to (13). The second–best expected utility is given in the proposition. This completes the proof. □

A.8 Proof of Proposition 5

Proof. Checking the signs of the partial derivatives

$$\frac{\partial \delta_2}{\partial \kappa} = -\frac{a\sigma_\varepsilon^2 \sigma_\theta^4}{\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2 \left(1 + a\sigma_\theta^2\right)\right)^2} \tag{24}$$

and

$$\frac{\partial \delta_2}{\partial \tilde{s}} = 0 \tag{25}$$

yields the proposition. This completes the proof. □

A.9 Proof of Corollary 4

Proof. The corollary in turn is a straightforward implication of corollary 1. This completes the proof. □

A.10 Proof of Proposition 6

Proof. Define

$$s_1 \equiv -\frac{a\sigma_\theta^2\left(3\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(3-a\sigma_\theta^2\right)\right)}{2\left(1+a\sigma_\theta^2\right)\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)}. \tag{26}$$

The inspection of the signs of the partial derivatives

$$\begin{aligned}\frac{\partial\gamma_2}{\partial\kappa} &= \frac{\sigma_\varepsilon^2\sigma_\theta^2}{2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^3}\\ &\cdot\left(a\sigma_\theta^2\left(3\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(3-a\sigma_\theta^2\right)\right)+2\tilde{s}\left(1+a\sigma_\theta^2\right)\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)\right)\end{aligned} \tag{27}$$

and

$$\frac{\partial\gamma_2}{\partial\tilde{s}} = -\frac{\sigma_\theta^2}{\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)} \tag{28}$$

yields the proposition. This completes the proof. □

A.11 Proof of Proposition 7

Proof. Define

$$\kappa_0 \equiv -\frac{\sigma_\theta^2}{\sigma_\varepsilon^2\left(1-a\sigma_\theta^2\right)}. \tag{29}$$

The partial derivative of the risk premium with respect to the coefficient of overconfidence κ is

$$\frac{a\sigma_\varepsilon^2\sigma_\theta^4\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1-a\sigma_\theta^2\right)\right)}{2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^3}. \tag{30}$$

Checking the sign of the term $\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1-a\sigma_\theta^2\right)$ yields the proposition. □

A.12 Proof of Proposition 8

Proof. The agency costs are given in proposition 3. The partial derivative of the agency costs with respect to the coefficient of overconfidence κ is

$$\frac{a\sigma_\varepsilon^2\sigma_\theta^4}{2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^2} \tag{31}$$

whereas the partial derivative of the agency costs with respect to the signal $\tilde{s}$ is zero. Checking the sign of the first partial derivative yields the proposition. This completes the proof. □

A.13 Proof of Proposition 9

Proof. Define

$$s_2 \equiv -\frac{a\sigma_\theta^2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\right)^2}{2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^2}. \tag{32}$$

The principal's second–best expected utility is given in proposition 4. The partial derivative of the principal's second–best expected utility with respect to the coefficient of overconfidence κ is

$$-\frac{\sigma_\varepsilon^2\sigma_\theta^2\left(a\sigma_\theta^2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\right)^2+2\tilde{s}\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^2\right)}{2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\right)^2\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^2} \tag{33}$$

and the partial derivative of the principal's second–best expected utility with respect to the signal $\tilde{s}$ is

$$\frac{\sigma_\theta^2}{\sigma_\theta^2+\kappa\sigma_\varepsilon^2}. \tag{34}$$

Checking the signs of these partial derivatives yields the proposition. This completes the proof. □

A.14 Proof of Corollary 5

Proof. The signal s_2 is given in equation (32). Define $h(\kappa) \equiv s_2$. Thus, the function h determines a threshold signal $h(\kappa)$ for each coefficient of overconfidence $0 < \kappa < 1$ in the sense of proposition 9. Since

$$\frac{\partial h(\kappa)}{\partial \kappa} = \frac{a^2\sigma_\varepsilon^2\sigma_\theta^6\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\right)}{\left(\sigma_\theta^2+\kappa\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^3} \tag{35}$$

is positive generally we define the minimum threshold signal

$$s_3 \equiv \lim_{\kappa\to 0} h(\kappa) = -\frac{1}{2}a\sigma_\theta^2 \tag{36}$$

and the maximum threshold signal

$$s_4 \equiv \lim_{\kappa\to 1} h(\kappa) = -\frac{1}{2}a\sigma_\theta^2\frac{\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^2}{\left(\sigma_\theta^2+\sigma_\varepsilon^2\left(1+a\sigma_\theta^2\right)\right)^2}. \tag{37}$$

Since $a\sigma_\theta^2 > 0$ we have $s_3 < s_4$. Equations (36) and (37) report the independence of s_3 and s_4 of the coefficient of overconfidence κ respectively. This completes the proof. □

B Tables

σ_θ^2	σ_ε^2	a	m
1	1	1	1

Table 1: The table displays the parameters which quantify the uncertainty about the future state of nature, the extent of signal noise, and the agent's characteristics. These parameters underlie the figures 3–7 which illustrate the comparative static results of the propositions 5–9 as well as of the corollaries 4 and 5.

C Figures

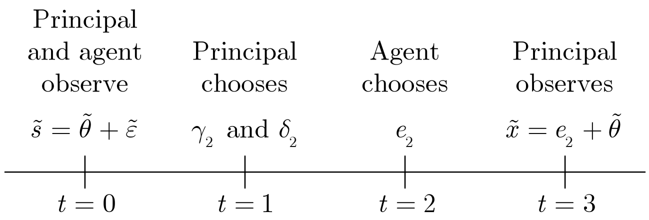

Figure 1: Timing of the players' actions and events in the second–best case

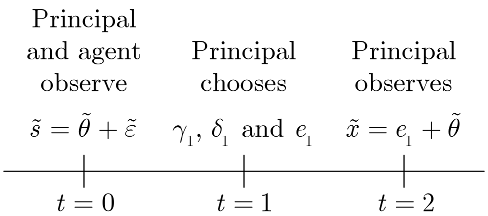

Figure 2: Timing of the players' actions and events in the first–best case

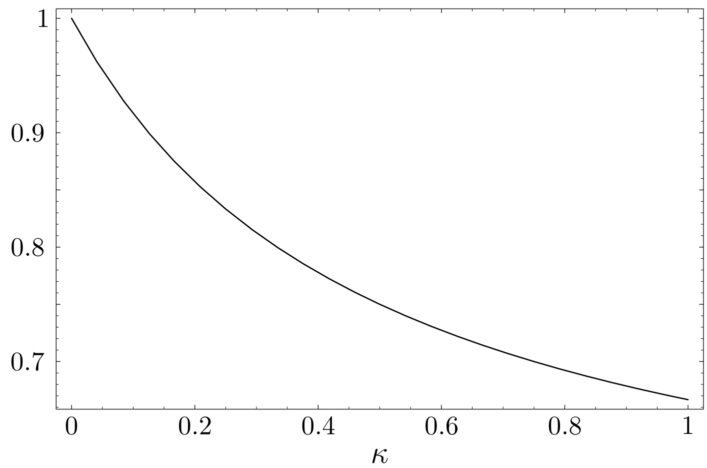

Figure 3: The graph illustrates the impact of the overconfidence bias on the second–best variable compensation component δ_2 according to proposition 6. According to corollary 4 the graph also applies to the second–best effort e_2. The shape of the graph shows that the agent exerts a higher effort and increases his share in the final monetary outcome the more pronounced is the overconfidence bias. The underlying model parameters are summarized in table 1.

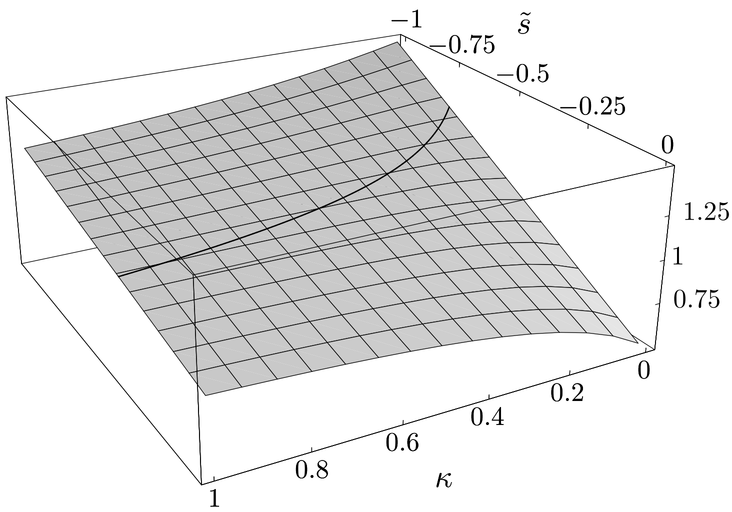

Figure 4: The surface depicts the second–best fixed compensation γ_2 for various combinations of the coefficient of overconfidence κ and the noisy signal $\tilde{s}$. The shape of the surface illustrates that γ_2 always decreases in the signal $\tilde{s}$. The curved line on the surface collects the critical signals s_1 as defined in proposition 6. For good signals — those in the front of the curved line — a stronger overconfidence bias reduces the fixed compensation and vice versa. The underlying model parameters are summarized in table 1.

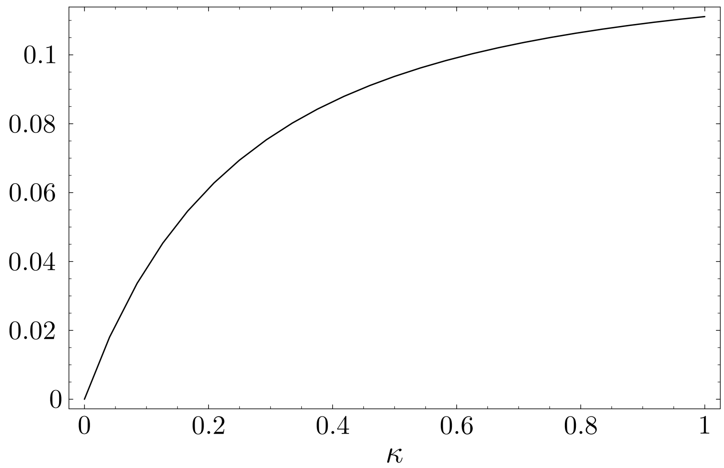

Figure 5: The graph illustrates the impact of the overconfidence bias on the risk premium $\frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}$ which compensates for the perceived remaining state uncertainty in the second–best case according to proposition 7. The underlying model parameters are summarized in table 1. Note that $1 = a < \sigma_\theta^{-2} + \sigma_\varepsilon^{-2} = 2$. Thus, the risk premium is strictly monotonically increasing in the coefficient of overconfidence.

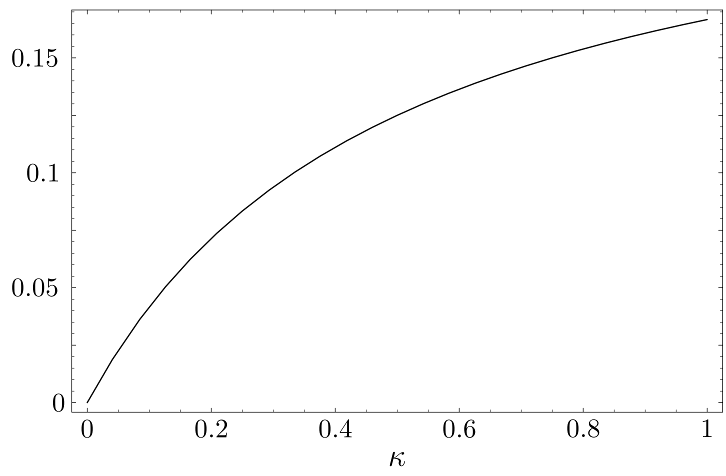

Figure 6: The graph illustrates the impact of the overconfidence bias on the agency costs according to proposition 8. The shape of the graph shows that the agency costs decrease the more pronounced is the overconfidence bias. Hence, the wedge between the principal's first–best and second–best expected utilities becomes smaller the more overconfident are the parties of the agency relationship. The underlying model parameters are summarized in table 1.

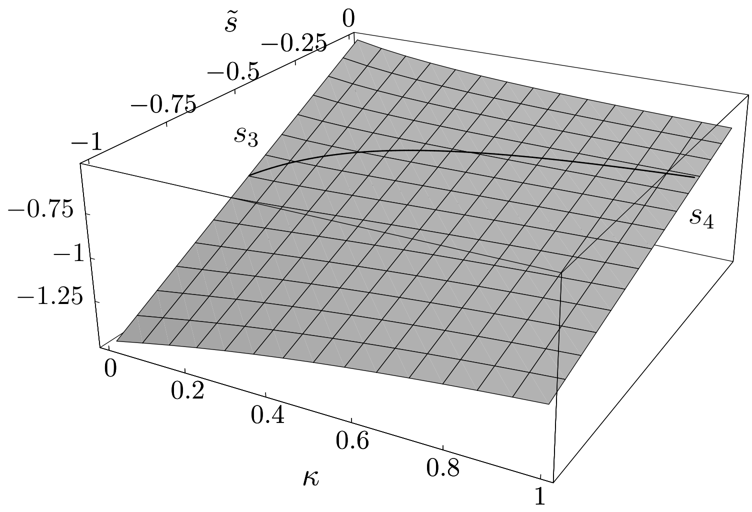

Figure 7: The surface depicts the principal's second–best expected utility for various combinations of the coefficient of overconfidence κ and the signal $\tilde{s}$. The shape of the surface illustrates that the principal's second–best expected utility generally increases in the signal $\tilde{s}$. The curved line on the surface collects the critical signals s_2 as defined in proposition 9. The endpoints of the curved line depict the signals s_3 and s_4 as defined in corollary 5. Generally, for bad (good) signals $\tilde{s} < s_3$ ($\tilde{s} > s_4$) a stronger overconfidence bias decreases (increases) the principal's second–best expected utility and vice versa. The underlying model parameters are summarized in table 1.