

Idiosyncratic Volatility, its Expected Variation, and the Cross-Section of Stock Returns

Nicole Branger* Hendrik Hülsbusch* T. Frederik Middelhoff*

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Abstract

This paper explains the negative relation between the realized idiosyncratic volatility (IVOL) and expected returns. Using implicit information from the cross-section of options we extract expectations about the volatility of idiosyncratic volatility (IVOLVOL) in an almost model-free fashion. We show that IVOL is mean-reverting and that IVOLVOL serves as proxy for the mean-reversion speed. Running double sorts on both measures reveals no differences in returns or alpha if the mean-reversion is low. However, the negative relation is amplified if IVOL shows fast mean-reversion. Both findings go along with a positive compensation for idiosyncratic risk. While for low mean-reversion, IVOL levels are expected to change little, current high (low) IVOL is expected to decrease (increase) by a larger amount for high mean-reversion. Thus, investors demand lower (higher) returns for current high (low) IVOL levels if it mean-reverts quickly. All our findings are supported by option trading behavior and are robust to different measures of IVOL, various stock characteristics and other competing explanations for the negative IVOL-return relation.

Keywords: Options, Stock Returns, Idiosyncratic Volatility, Volatility-of-Volatility

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* Finance Center Muenster, University of Muenster, Universitätsstr. 14-16, 48143 Münster, Germany. E-mail: nicole.branger@wiwi.uni-muenster.de, hendrik.huelsbusch@wiwi.uni-muenster.de, frederik.middelhoff@wiwi.uni-muenster.de.

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1 Introduction

The higher the exposure to systematic risk, the higher are an asset's expected returns. This fundamental relation between systematic risk and subsequent returns is one of the cornerstones in asset pricing theory. In contrast to systematic risk, the relation between idiosyncratic risk and subsequent returns offers a less clear picture until now. In classical asset pricing theory it has been common sense to assume that idiosyncratic risk is either positively priced (Merton (1987)) or has no pricing impact at all, which is in line with the CAPM (Sharpe (1964), Lintner (1965)). However, in the seminal work of Ang et al. (2006) both classical assumptions are challenged by the finding of a negative relation between the realized idiosyncratic return volatility (IVOL) and subsequent returns.¹ According to their findings investors are willing to accept lower returns if an asset is highly exposed to idiosyncratic risk. Since this finding seems irreconcilable with classical approaches, the negative relation between idiosyncratic risk and future returns has become known as the IVOL puzzle.

We exploit information from stock options to offer a resolution to the IVOL puzzle under the assumption that idiosyncratic risk is positively priced. Central in finding a negative IVOL-return relation so far is the measurement of IVOL, where the measure purposed by Ang et al. (2006) is purely historic (e.g. Fu (2009)). Using option prices enables us to overcome this pitfall by linking the pure historic measure of IVOL with forward looking expectations. In contrast to other studies, our method allows us not only to analyze the IVOL-return relation itself, but also to analyze why previous authors have found a negative relation between historic IVOL and realized returns. Therefore, we use option prices to estimate the volatility of idiosyncratic volatility (IVOLVOL) and show that it can be used as a proxy for the expected mean-reversion speed of IVOL. Our analysis indicates that for stocks with slow mean-reverting IVOL, the negative IVOL-return relation becomes insignificant.

¹In the following we use idiosyncratic risk and firm specific risk synonymously for IVOL.

In contrast, for stocks with fast mean-reverting idiosyncratic risk we find a strong amplification of the negative IVOL-return relation. Both findings are in line with a positive price of idiosyncratic risk. While for slow mean-reversion, the historically realized idiosyncratic risk level is expected to stay rather constant, low (high) historically realized IVOL will be subject to a strong increase (decrease) if it is fast mean-reverting and will thus result in higher (lower) realized returns. This explains the amplification of the negative IVOL-return relation for stocks with strong mean-reversion.

For our analysis, we use 19 years of daily stock and stock options data and focus on assets with the highest liquidity in stock and option trading only. Despite having a subset of the whole stock universe the negative IVOL-return relation is prevalent in this sample too. In particular, the median size and trading volume of the firms lies in the 90%-percentile of the universe taken in comparable studies. Still, a single sort on IVOL yields a monthly highly significant return and Fama-French 3-factor alpha ([Fama and French \(1993\)](#)) for a low-minus-high portfolio of 1.04% and 1.51%, respectively. This negative relation is not puzzling though, but can be explained by investors' expectations about future idiosyncratic risk which we extract from stock option prices. In doing so, we employ a largely model-free parametrization and link it intuitively to the baseline method of measuring idiosyncratic risk relatively to the Fama-French 3-factor model. Our method uses model-free techniques from [Bakshi et al. \(2003\)](#) to calculate moments of the individual return distribution and a simple linear model. This approach enables us to measure investors' expectation about individual variations in idiosyncratic risk over the next month on a single stock level by our measure of IVOLVOL.

Due to its nature, IVOLVOL is a good proxy for the expected mean-reversion speed in IVOL. We show that expectations are in line with realizations. It holds, the higher the current IVOLVOL the higher is the mean-reversion speed in idiosyncratic

risk. Therefore, we augment the purely historic measure of idiosyncratic risk with our measure of IVOLVOL. This allows to analyze not only the expected mean-reversion speed, but also to gain insights about the direction of expected IVOL innovations. The combination of both measures reveals that the IVOL-return relation heavily relies on the expected magnitude of mean-reversion in idiosyncratic risk, which we show by a double sort analysis. That is, if investors expect IVOL to change little over the next month, the return of the low-minus-high IVOL portfolio is statistically not distinguishable from zero. This observation is in line with rational investors' behavior. If realized stock IVOL is high (low) in the current period and it is expected to mean-revert only slowly, investors expect idiosyncratic risk to stay rather high (low). Consequently, if IVOL is positively priced, as implicated by [Merton \(1987\)](#), investors demand higher (lower) returns. This leads to a low return of the low-minus-high idiosyncratic risk portfolio, conditional on a low IVOLVOL. On the other hand, if IVOL is expected to be exposed to a fast mean-reversion, e.g., in the case of high IVOLVOL, investors adjust their expected returns. Thus, if idiosyncratic risk of a stock is high (low) investors demand lower (higher) returns, because they expect idiosyncratic risk to decrease (increase) by relatively large amount. This effect leads to highly significant monthly returns of the low-minus-high IVOL portfolio of 2.04% and an alpha of 2.61%.

All findings in the data support our line of reasoning. Especially, options trading data reveals that investors incorporate their expectations about the stickiness of idiosyncratic risk into their risk-return trade-off. Looking at trading volumes of puts relative to calls (put/call-ratios) indicates, if idiosyncratic risk of a stock is expected to be sticky and has a low (high) level, market participants expect lower (higher) future returns compared to other stocks. Supporting this finding, we document that conditional on low (high) IVOL the put/call-ratio decreases (increases) in the mean-reversion speed of idiosyncratic risk. This indicates that investors trade options in the direction of expected innovations of idiosyncratic risk, by assuming

corresponding innovations in the stocks price as a compensation for idiosyncratic risk. Market participants buy more calls if firm specific risk is likely to increase and more puts if it is likely to decrease next month.

In a comparative analysis we show that our measure for mean-reversion in idiosyncratic volatility captures indeed an important facet of the IVOL puzzle which was omitted by the literature up to now. Therefore, we use the techniques from [Hou and Loh \(2016\)](#) and document that our measure surpasses and dominates the explanatory power of competing explanations. A simple classification of stocks with respect to mean-reversion speed (proxied by IVOLVOL levels) helps to explain around 40% of the IVOL anomaly. In contrast, controlling for the mean-reversion speed, other explanatory variables make up roughly 20% in total only. Consequently, our measure combined with others helps to explain around 60% of the total IVOL-return relation.² For the test set of alternative explanations we follow closely [Hou and Loh \(2016\)](#). We include (co-)skew measures as well as the retail trading proportion (RTP) as proxies for lottery preferences. Further, we test for market frictions by including lagged returns, the liquidity measure of [Pastor and Stambaugh \(2003\)](#), the proportion of zero returns and the relative bid-ask spread. An additional robustness analysis confirms that our results are robust to various measures of IVOL as well as portfolio weighting schemes and cannot be explained by stock liquidity, short-sale constraints or size.

Our paper is related to different strands of the literature, focusing on the relation between idiosyncratic risk and subsequent returns as well as possible resolutions for the IVOL puzzle. [Ang et al. \(2006\)](#) are the first to document the negative IVOL-return relation. Stocks with low realized idiosyncratic risk exhibit higher subsequent returns than stocks with high realized idiosyncratic risk. They show in a sequential paper ([Ang et al. \(2009\)](#)) that the IVOL anomaly is prevalent in different markets

²[Hou and Loh \(2016\)](#) find values between 29-54% for the combined explanatory power of the most established measures.

and thus robust. However, the robustness of the negative relation is questioned by [Bali and Cakici \(2008\)](#) who argue that the effect is mainly driven by small stocks and the portfolio weighting scheme. In contrast, including the biggest stocks in our sample only, we provide strong support for the existence of a robust and generally negative IVOL-return relation.

Offering a possible resolution to the IVOL puzzle, some authors connect the anomaly to short-sale constraints. [Shleifer and Vishny \(1997\)](#) find that idiosyncratic risk dampens the willingness to short-sale. [Stambaugh et al. \(2015\)](#) and [Boehme et al. \(2009\)](#) argue that the underperformance of high IVOL stocks stems merely from short-sale constrained stocks, since they are too expensive. However, the authors still find the puzzle even when excluding the 60% smallest traded stocks. Our paper adds to this discussion, since we exclude stocks with illiquid option trading. Our study thus only focuses on assets with the weakest short-sale constraints and we find the IVOL anomaly still to be prevalent for these stocks.

Another strand of the literature argues that the incorporation of investors' expectations about future idiosyncratic risk is crucial to understand the IVOL-return relation. [Fu \(2009\)](#) and [Peterson and Smedema \(2011\)](#) use EGARCH models to proxy expectations about idiosyncratic risk innovations and show that the puzzle vanishes after accounting for those. They find the expected IVOL-return relation to be positive. However, [Fink et al. \(2012\)](#) question these methods by showing that the former studies are prone to a significant look-ahead bias. After controlling for the set of information they find the IVOL anomaly to be prevalent. In contrast to the former authors [Rachwalski and Wen \(2016\)](#) argue that investors only price perceived IVOL and thus incorporate idiosyncratic information only with a lag. Following them, the puzzle merely stems from mis-measurement of current IVOL, which can be proxied by perceived idiosyncratic risk, measured in terms of realization far in the past, and current IVOL. However, none of the former studies make use of implicit information

from the options market to extract investors' expectations but only use stock prices. Historic stock prices lag the forward-looking features options provide and thus the former authors omitted a large part of valuable information. Consequently, we add to the literature and, in contrast to [Fu \(2009\)](#) and [Peterson and Smedema \(2011\)](#), extract a measure for expected IVOL innovations in an almost model-free manner, using stock option prices.

We are not the first to use stock options in a joint analysis with idiosyncratic risk.³ [Aliouchkin \(2015\)](#) looks at the cross-section of S&P100 options and calibrates a flexible model for the dynamics of stock prices. Subsequently, he extracts moments of the expected return distributions. He finds that the absolute idiosyncratic skewness and co-skewness is negatively related to future returns. Different from his paper, we jointly use a considerably bigger cross-section of stocks and options together with almost model-free methods as well as an explicit focus on expected innovations of firm specific volatility.

Other authors extract information about expected idiosyncratic risk using only low parametrized models. [Dennis et al. \(2006\)](#), [Diavatopoulos et al. \(2008\)](#), [Moll and Huffman \(2016\)](#), amongst others, employ regression models to calculate implied idiosyncratic volatilities from option prices and aggregate implied volatility. They find that implied idiosyncratic risk is negatively priced and that investors care about its innovations. However, different from our approach, their measure for IVOL levels can get negative and the defined IVOL innovations are only loosely connected to IVOL estimated by factor models in the style of [Ang et al. \(2006\)](#). Further, these authors do not focus explicitly on big stocks with the most liquid options.

³ [Cao and Han \(2013\)](#) show that delta-hedged option returns are decreasing in IVOL. [Elkamhi et al. \(2011\)](#) use a measure for informed option trading and show that the more informed option traders the lower stock returns. [Bégin et al. \(2016\)](#) calibrate parametric models on single stock level for 260 stocks using options. They show that only idiosyncratic jump risk matters for the equity risk premium. Diffusive idiosyncratic risk is not priced.

Another strand of the literature finds that higher order risks on individual firm level such as skewness, co-skewness and volatility-of-volatility are closely related to the IVOL-return relation as well. [Boyer et al. \(2010\)](#) show that expected idiosyncratic skewness is negatively correlated with returns. In an extensive study [Conrad et al. \(2013\)](#) document a strong negative impact of individual risk neutral skewness on future returns. [Harvey and Siddique \(2000\)](#) find a significant risk-premium for market skewness. These findings are supported by [Dittmar \(2002\)](#) and [Schneider et al. \(2016\)](#) who provide further empirical evidence as well as theoretical explanations. [Baltussen et al. \(2014\)](#) document that realized volatility-of-volatility is negatively related to future returns.⁴ Our paper adds to this literature and shows that higher order idiosyncratic specific risk helps to explain the IVOL anomaly. Further, we show that the information contained in IVOLVOL with respect to the IVOL-return relation is different from the established (co-)skewness measures.

The remainder of the paper is structured as follows. Section 2 elaborates on the concept of measuring mean-reversion in idiosyncratic risk and its link to expected returns. In Section 3 we describe our data and methods to calculate IVOL and IVOLVOL. Section 4 contains our main results. There we show the existence of the IVOL-return anomaly in our dataset and afterwards reason it with expected IVOL innovations. The robustness analysis is conducted in Section 5 and, last, Section 6 concludes.

2 Idiosyncratic Risk and its Expectation

It is well recognized that volatility is mean-reverting and that the incorporation of this feature is essential for pricing risk.⁵ Idiosyncratic volatility is the risk of a stock in

⁴[Chen et al. \(2014\)](#) find the same using high-frequency data. [Bali et al. \(2009\)](#) relate large changes in IVOL to firm-level news.

⁵See for example [Merville and Pieptea \(1989\)](#) and [Heston \(1993\)](#).

excess of systematic risk and is naturally bounded from below as well as from above. In the extreme, a stock's volatility can either fully depend on the market or is subject to idiosyncratic volatility risk only. Consequently, IVOL should mean-revert too and the mean-reversion effect should affect prices.⁶ This is the cornerstone of our method to explain the negative IVOL-return relation. We incorporate implied expectations about mean-reversion in IVOL by using the cross-section of stock options. Our method isolates expectation about mean-reversion effects in IVOL from options data and puts them in relation to current levels of idiosyncratic risk. This relation then reveals if investors expect risk to increase, decrease or to remain rather unchanged. By doing so, we solely rely on time-series analysis techniques. Unlike other studies on expected idiosyncratic risk, we do not use a strong parametrization, but we provide a consistent framework which can handle expected IVOL innovations and IVOL levels at once. Further, we explicitly incorporate investors' views on future risk without introducing a look-ahead bias.

For the sake of simplicity, assume that IVOL is measured relatively to the market model, for now. In particular, assume temporarily that $IVOL_t$ at day t is estimated as follows:

$$R_{i,s} - r_{f,s} = \alpha_i + \beta_{i,M} (R_{M,s} - r_{f,s}) + \epsilon_{i,s}, \quad (1)$$

and $IVOL_t^i = \text{std}[\epsilon_{i,t-\tau:t}]$, with $\tau = 1$ month.⁷ If we calculate the risk-neutral expectation of the quadratic variation on both hand sides and assume the interest rate to be deterministic, we get:⁸

$$(\sigma_{i,s}^{\mathbb{Q}})^2 = \gamma_i + (\beta_{i,M}^{\mathbb{Q}})^2 (\sigma_{M,s}^{\mathbb{Q}})^2 + \mathbb{E}_s^{\mathbb{Q}} \left[\int_s^{s+\tau} (d\epsilon_{i,s})^2 ds \right] \quad (2)$$

$$= \gamma_i + \beta_{i,\sigma_M} (\sigma_{M,s}^{\mathbb{Q}})^2 + \eta_{i,s}^{\mathbb{Q}}, \quad (3)$$

⁶We validate the existence of mean-reversion in IVOL levels in the empirical section.

⁷In the empirical part of our paper we define IVOL relative to the 3-factor Fama-French model.

⁸The assumption of deterministic interest rate is quite common when calculating expected variations. See for example [Bakshi et al. \(2003\)](#) or [Jiang and Tian \(2005\)](#).

where $(\sigma_{i,s}^{\mathbb{Q}})^2$ describes the expected variation in the individual stock returns and $(\sigma_{M,s}^{\mathbb{Q}})^2$ the expected variation in returns of the market portfolio over the future period. The process $\eta_{i,s}^{\mathbb{Q}}$ describes the risk-neutral expectation of the variation in residuals $\epsilon_{i,s}$. Thus, $\eta_{i,s}^{\mathbb{Q}}$ defines the expected idiosyncratic variance over the next month. Consequently, it proxies for the expected next month IVOL on day s , too. Considering a time series of $\eta_{i,s}^{\mathbb{Q}}$ enables us to extract the variation in these expectations on IVOL.⁹ We define the volatility of idiosyncratic volatility on day t as

$$\text{IVOLVOL}_t^i = \text{std} [\eta_{i,t-\tau:t}^{\mathbb{Q}}]. \quad (4)$$

By definition, IVOLVOL expresses the second moment of the expected idiosyncratic variance. Empirically, we construct $(\sigma_{\bullet,s}^{\mathbb{Q}})^2$ in a model-free fashion from portfolios of options on individual stocks and the market, respectively. By using options we explicitly incorporate investors' view on future returns and, by construction, their expectation about changes in IVOL.

If we assume IVOL to follow some mean-reversion process, as we confirm in Section 4.2, we can infer the mean-reversion speed from its IVOLVOL. For the sake of illustration, assume IVOL follows a simple Ornstein-Uhlenbeck process:

$$d\text{IVOL}_t^i = \kappa^i \left(\overline{\text{IVOL}}^i - \text{IVOL}_t^i \right) dt + \sigma_{\text{IVOL}}^i dW_t^i. \quad (5)$$

where κ^i denotes the mean-reversion speed, $\overline{\text{IVOL}}^i$ the long-run mean of IVOL and dW_t^i describes a Wiener process, scaled by σ_{IVOL}^i .¹⁰ Core in analyzing the relation between IVOL and returns with portfolio sorts is the concentration on extreme

⁹In the empirical analysis, we are not able to compute point estimates of expected idiosyncratic variance. In order to estimate $\eta_{i,s}^{\mathbb{Q}}$ from equation (3) we use linear regressions and define $\eta_{i,s}^{\mathbb{Q}}$ as its residuals. Thus, these will proxy for the expected variation in IVOL only.

¹⁰For simplicity we assume the parameters to be the same under \mathbb{P} and \mathbb{Q} . This assumption will have no qualitative impact on our results, as long as the parameters for the different measures are positively related.

realizations of IVOL and the return difference in them. Thus, for a portfolio of high (low) IVOL stocks it is quite reasonable that these realized IVOL levels lie not only above (below) the long run mean, but distort from it by a large amount.¹¹ IVOLVOL proxies for the realized variation in the expectation on IVOL over a certain period, as we argue above. Figure 1 plots IVOLVOL in relation to κ^i for different levels of distortion from the long run mean.¹² The figure shows that the higher IVOLVOL the larger the mean-reversion coefficient κ^i . Further, the increase of IVOLVOL in κ^i is stronger, the higher the distortion from the long run mean. The intuition underlying this mechanism can be understood best, when considering a simple example.

Assume a stock shows a high realized IVOL at time $t_0 + \tau$ which is well above its long-run mean. In such a case the drift component in equation (5) gets highly negative and thus has a major impact on future IVOL innovations. A higher κ^i even strengthens this impact and therefore the realized IVOL in $t_0 + (\tau + 1)$ will decrease towards its long-run mean by larger amounts, the larger κ^i . This in turn will give rise to a higher difference in the expected IVOLs for time $t_1 + \tau$ and $t_1 + (\tau + 1)$. Since the calculation of volatility involves a sum of squared differences, volatility of expectations on IVOL rise in κ^i conditional on a distortion from its long-run mean.¹³ Thus, IVOLVOL can be interpreted as a measure to proxy for the mean-reversion speed of idiosyncratic volatility, especially if the current IVOL is either high or low. This interpretation connects IVOLVOL closely to the idiosyncratic risk of an investment that an investor is expected to bear in the future and offers

¹¹In the following we assume the mean-reversion speed and the long run mean to be time invariant. This is no harsh restriction, since our later analysis concentrates on rather short holding periods of one month.

¹²We simulate 200,000 paths of IVOL, using equation (5) over one month. In every point in time we compute the expected IVOL^{*i*} next month, conditional on the current realization. IVOLVOL is measured as the mean of the standard deviations of the expectations on IVOL.

¹³In the later analysis we control explicitly for the level of IVOL. It follows that any differences in IVOLVOL should be mainly driven by κ^i .

a convenient method to rationalize their behavior. Therefore, IVOLVOL allows to draw conclusions on the drivers of the classical IVOL puzzle.

In general, if a stock is currently in a regime of low (high) IVOL, its idiosyncratic risk is likely to increase (decrease) over the next period due to the mean-reversion effect in IVOL. The expected mean-reversion may be quite distinct for different stocks. These different expectations can be reconciled from the expected variation in idiosyncratic volatility. For example, a stock's idiosyncratic risk being in a regime of low IVOL and high IVOLVOL (low/high) is strongly expected to increase. It is currently quite low and is subject to a large mean-reversion effect. In comparison, for a stock exposed to the same low level of IVOL, but low IVOLVOL (low/low) the mean-reversion effect in idiosyncratic volatility is expected to be weaker. The reason is that a lower IVOLVOL level signals a smaller κ^i . Overall, the low/high stocks are more risky than the low/low stocks and investors demand higher returns for the former compared to the latter. The same reasoning applies to stocks with currently high IVOL. If a stock is in a regime of high IVOL and high IVOLVOL (high/high), investors strongly expect IVOL to change over the next period. In contrast, stocks with low IVOLVOL have an IVOL of higher expected persistence. For these stocks the magnitude of changes in idiosyncratic risk is expected to be less pronounced than for high IVOLVOL stocks. Consequently, if the IVOL of a stock is currently high and IVOLVOL is low (high/low), IVOL is expected to be more sticky and therefore less likely to decrease by a great amount. As a result, investors demand higher returns for high/low than for high/high stocks.

The above described relation between IVOL and IVOLVOL leads to two hypotheses regarding the risk-return relation of stocks with respect to idiosyncratic risk. These hypotheses should hold, as long as investors demand compensation for bearing idiosyncratic risk (Merton (1987)) and given that expected variation in idiosyncratic risk is really a proxy for the mean-reversion effect.

Hypothesis 1: The negative IVOL-return relation should vanish for low IVOLVOL stocks, because these stocks are exposed to idiosyncratic risk with low mean-reversion speed. In this case the realized IVOL level is a rather good proxy for the expected future IVOL level. Hence, realized IVOL signals low (high) future expected idiosyncratic risk when it is currently low (high). Therefore, conditional on low IVOLVOL, investors demand lower returns for low IVOL stocks and higher returns for high IVOL stocks.

Hypothesis 2: For high IVOLVOL stocks, the difference in returns between low and high IVOL stock should increase, because high IVOLVOL stocks have idiosyncratic risk with a high mean-reversion speed. In this case the realized IVOL is a poor proxy for expected future IVOL levels, because currently high (low) IVOL signals lower (higher) expected future IVOL.

3 Data and Methodology

This section describes the data used in the empirical part later on and explains the construction of key measures of our analysis, IVOL and IVOLVOL.

3.1 Data

We merge three different databases for 1996/01–2014/12 sample period, and thereby analyse 19 years of data. We use daily bid/ask prices, implied volatilities, trade volumes, and open interests of American stock-options as well of SPX options and the zero yield curve from Ivy DB US provided by OptionMetrics. From CRSP we obtain daily and monthly stock data, such as split-adjusted returns, prices, dividend amounts, dividend frequency and trade volume. Further, to calculate the book-to-market ratio we include the book-value on annual basis from Compustat in our

analysis. Last, we obtain daily Fama-French factors from Kenneth French’s data library.

Overall, our raw sample consists of 8290 firms for which options are traded at some point in time and 170 million daily data points of options with non-zero prices, where we calculate the option price as the mid-point of bid/ask prices. To provide a reliable data basis for our analysis we employ several filters, which are quite similar to [Goyal and Saretto \(2009\)](#). First, we exclude all options with zero open interest, zero volume, no implied volatilities and which violate standard no-arbitrage bounds or where the bid price is lower than the ask price. Second, we follow OptionMetrics’ pricing approach for American options to calculate synthetic prices of corresponding European stock options. Given an implied volatility, we re-price all quoted American options using a [Cox et al. \(1979\)](#) (CRR) tree with 1000 steps and incorporate discrete dividends. For dividend amounts and frequencies we use OptionMetrics quotes. For precision, we discard all options with a relative pricing error of the calculated CRR American option price to the quoted mid bid/ask price, being larger than 1%. Next, we use the same CRR trees to calculate European Option prices and thereby account explicitly for the early exercise premium of American options. This is essential for the later construction of the $(VIX)^2$ on single stock level $(VIX^i)^2$. This approach is quite accurate as shown for example in [Tian \(2011\)](#) and [Ju and Zhong \(1999\)](#) and comparable to [Broadie et al. \(2007\)](#), since the CRR pricing model converges to the Black-Scholes model if the step size goes to zero. After these filtering methods and after controlling for the number of data points in our later regressions, our whole sample spans 3087 firms and over 11 million options. Each month, our cross-section consists of 383 firms on average and of more than 46000 options in total.

3.2 Measurement of IVOL and IVOLVOL

To estimate the idiosyncratic volatility on individual stock level we follow the established approach by measuring it relative to the Fama-French 3-factor model. We first regress contemporaneously daily excess returns over one month on the three factors, excess return of the market portfolio, high-minus-low book-to-market ratio and size. Afterwards, we define IVOL as the standard deviation of the model's pricing errors. This leads to the following measurement of IVOL for month t

$$R_s^i - r_{f,s} = \alpha^i + \beta_{MKT}^i MKT_s + \beta_{HML}^i HML_s + \beta_{SMB}^i SMB_s + \epsilon_s^i, \quad (6)$$

$$IVOL_t^i \equiv \text{std} [\epsilon_{t-\tau:t}^i], \quad (7)$$

where τ equals one month, $r_{f,s}$ is the risk-free rate and R_s^i are daily stock returns.

Measuring volatility of idiosyncratic volatility follows a quite similar pattern, since we define IVOLVOL as the standard deviation of a contemporaneously regression again. However, equation (3) requires to measure the risk-neutral expectations of the variation in market and stock returns. A natural and model-free measure of expected market volatility under the risk-neutral measure is the VIX, provided by the CBOE. Thus, we rely on it and compute a VIX^i on a single stock level. This allows for computing IVOLVOL from equation (3). More accurately, for month t we regress daily VIX^i levels on the market VIX^M

$$(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i, \quad (8)$$

$$IVOLVOL_t^i \equiv \text{std} [\eta_{t-\tau:t}^i]. \quad (9)$$

To ensure reliable results of our estimation, we consider only stocks where we have at least 15 daily returns R_s^i and at least 15 daily VIX_s^i observations within a month. We therefore isolate the stocks which have the most liquid options. Nevertheless, our IVOLVOL estimate proxies for the exact IVOLVOL only. The calculation of our IVOL relies on the Fama-French 3-factor model, while in equation (8) we assume

the market model. However, only including the VIX leads to a more noisy measure of IVOLVOL and therefore tends to weaken our findings. In addition, all our results hold if we estimate idiosyncratic risk relative to the CAPM or the Fama-French 5-factor model (Fama and French (2015)), as we show in the robustness part. Including measures for systematic volatility and jump risks, like the realized variance or bipower variation from Corsi et al. (2010) for the S&P500 or VIX, increases the explanatory power of the model to estimate IVOLVOL only, but has little impact on the results of the later sorting exercise. Therefore, we stick to the most straightforward model to estimate IVOLVOL. We calculate VIX_t^i of the individual stock on day t as

$$(VIX_t^i)^2 = \frac{2e^{r_{f,t}\tau}}{\tau} \left[\int_0^{S_t^i} \frac{P_t^i(K)}{K^2} dK + \int_{S_t^i}^{\infty} \frac{C_t^i(K)}{K^2} dK \right], \quad (10)$$

where S_t^i is the stock price, $P_t^i(K)$ are put prices and $C_t^i(K)$ are call prices with strike K and maturity $\tau = 1$ month. We use the set of options with maturity of exactly one month as long as they are available. Otherwise, we use two sets, one with maturity τ_1 below one month and a second with maturity τ_2 above one month. In each case we follow Jiang and Tian (2005). We interpolate implied volatilities across strikes using spline interpolation and extrapolate using the quoted implied volatility of the highest or lowest strike, respectively. If necessary, we interpolate the implied volatilities linear across maturity to get prices of options with one month to maturity and employ the above formula. On average we use a quoted subset of 8 options per day, which is extended to roughly 10 options per day using the put-call parity, to build the $(VIX^i)^2$.

4 Results

In this section we present our results, by analyzing the relation between realized IVOL and subsequent returns first and then by testing our hypothesis with respect

to the IVOL anomaly and the IVOLVOL.

4.1 Realized Idiosyncratic Risk and Expected Returns

[Ang et al. \(2006\)](#) and most followup studies analyze the IVOL puzzle by looking at the full stock universe quoted at NYSE, AMEX and NASDAQ. In contrast, we use stocks for which options are traded only and in addition filter for options liquidity. Thus, we analyze a subsample compared to previous work. [Table 1](#) highlights some key differences of our sample compared to the usual NYSE, AMEX and NASDAQ sample for the same sample periods. The table reports the mean, median and percentiles of firm size and trading volume in stocks. With regard to both aspects our sample consist of the largest and most liquid stocks, compared to the total universe. The median size of \$ 2,325 MM in our sample is higher for more than 90% of firms in the total universe. The same holds true for the median trading volume of \$ 956 MM. In presence of this quite different sample we run an analysis of the relation between realized idiosyncratic risk and subsequent returns first, before turning to the pricing effect of expected idiosyncratic risk. Finding the negative relation between realized IVOL and subsequent returns to be prevalent enables us to analyze the drivers of this interplay in a next step. In addition, finding evidence for a negative relation should be challenging for explanations based on limits of arbitrage.¹⁴ Those explanation rely their reasoning on the argument that investors might be unable to exploit an arbitrage opportunity, since they might face short selling restrictions or short selling might be too expensive. However, our sample consists of very large stocks with high liquidity in option and stock trading only. With these stocks short selling is considered to be comparatively easy and less expansive. [Table 2](#) reports results of

¹⁴Using the whole stock universe [Stambaugh et al. \(2015\)](#) argue that the IVOL puzzle can be partially explained by limits of arbitrage. Nevertheless, [Table 7](#) in their paper documents the puzzle's existence for the 40% biggest stocks.

single IVOL sorts. Following [Ang et al. \(2006\)](#) we sort stocks into quintile portfolios each month, such that their realized one month IVOL is increasing in portfolio rank. Next to mean excess returns of equally weighted and value weighted portfolios, we report Fama-French 3-factor alpha for both.

Table 2 clearly indicates, that the puzzle is prevalent on an alpha level. The highest IVOL portfolio yields significantly lower alpha than the low IVOL portfolio, for both, value and equally weighted portfolios. For value weighted portfolios, there is a significant difference of 1.22% between the low and high IVOL portfolio, while for equally weighted portfolios the difference is 1.64% and highly significant, too. The puzzle is mainly driven by the highest IVOL portfolio, which has a highly significant negative alpha, while the alpha for the low IVOL portfolio is not significantly different from zero for equally weighted portfolios. Even though [Bali and Cakici \(2008\)](#) find that the negative relation between realized idiosyncratic risk and subsequent returns is quite sensitive to the portfolio weighting scheme and that it is only prevalent for value weighted portfolios, we find the exact opposite result in our sample with respect to excess returns. The return difference is highly significant for equally weighted portfolios on the 1% level, but not for value weighted portfolios. This suggests that the puzzle might be driven by stocks which are among the smallest in our sample. Nevertheless, these stocks are still quite large compared to the total stock universe, as indicated by Table 1, so that this finding is not in contradiction to previous work. Interestingly, the mean monthly return difference between the highest and lowest IVOL portfolio is 1.13% for equally weighted returns, which is close to [Ang et al. \(2006\)](#) who report a mean monthly return difference of 1.06%. All in all, we conclude that the negative relation between realized idiosyncratic risk and subsequent returns is evident in our sample, too.

4.2 Variation in Expected Idiosyncratic Risk and its Mean-Reversion Speed

In this section we analyze whether our basic assumptions for our hypothesis hold. As pointed out, our hypothesis is based on two key aspects. First, we expect IVOL to be mean-reverting as has been shown often for total volatility. Next, we argue that the expected variation in future IVOL levels, expressed by our measure of IVOLVOL, is a proxy for the mean-reversion speed and together with the current level of IVOL a measure for the direction of future IVOL movement.

Table 3 reports results of an augmented Dicky-Fuller test. There we test the stationarity of the IVOL time series of every stock in our sample. The values in the first three rows report the percentage of rejected null hypothesis in favor for the alternative hypothesis for different significance levels. The null states that the time series has a unit root, while the alternative hypothesis assumes stationarity of the time series without a drift and trend. We report results for different required minimum lengths of each time series (12 to 120 month).¹⁵ Taking a look at the results indicates that a large majority of the IVOL time series is stationary. On the 5% significance level, the null is rejected for 78.89% (minimum of 12 observations) up to 98.15% (minimum of 120 observations) of all IVOL time series included. Therefore, we conclude that IVOL is in general stationary and thus has to show a mean-reverting behavior.

Next, we turn to the mean-reversion speed and its direction. As mentioned before, we expect that current low (high) IVOL will tend to increase (decrease) on average. This increase (decrease) should be stronger the higher the IVOLVOL. Therefore, Table 4 reports the mean-reversion effect for different IVOL/IVOLVOL portfolios. With this table, we follow our later analysis and form dependent double

¹⁵Note, the time series of IVOL is only monthly and thus no more than 227 observations long. However, testing for stationarity longer time series are favorable.

sorts first. For these we sort our stock universe into quintile portfolios basing on their IVOL in a first step and then split each portfolio into three sub-portfolios basing on their IVOLVOL. For these portfolios, we look at the average change in IVOL and run the following regression:

$$\text{IVOL}_{t+1}^{PF} - \text{IVOL}_t^{PF} = \alpha^{PF} + \kappa^{PF} \text{IVOL}_t + \epsilon_{t+1}^{PF}, \quad (11)$$

Here, $\text{IVOL}_{t+1}^{PF} - \text{IVOL}_t^{PF}$ is the change of the average IVOL of a portfolio over the next month and α^{PF} is a constant. In this regression κ^{PF} is the mean-reversion effect. It states the direction of the mean-reversion and its absolute value is the mean-reversion speed.¹⁶ Looking at Table 4 reveals that the direction of IVOL movement is as expected. Regardless of the level of IVOLVOL, there is an average increase in IVOL if the current IVOL level is very low and a decrease if it is very high. However, there are differences in how fast the direction changes. While the effect for low IVOLVOL is only positive for the lowest IVOL portfolio, for high IVOLVOL it changes sign only for the fourth and fifth portfolio. More important however, the mean-reversion speed clearly depends on our measure of IVOLVOL. Conditional on low IVOL, we find an insignificant speed of 0.03 for the low and medium IVOLVOL portfolios. In contrast, for high IVOLVOL it gains economically and statistically much more power and equals 0.16. The same holds true for high IVOL. There the speed for all IVOLVOL portfolios is statistically highly significant. While it is 0.23 for lowest IVOLVOL, and is 0.32 for the high IVOLVOL portfolio. Therefore, we conclude that IVOLVOL indeed indicates the mean-reversion speed and together with IVOL the direction of the future IVOL movement.

In the analysis so far, we look at the average IVOL of a portfolio only. To enhance our analysis, we confirm the results using a cross-sectional dummy regression to estimate the direct impact of IVOLVOL on changes in IVOL levels. We look at

¹⁶Note, α^{PF} contains the long run means of each IVOL portfolio, which we assume to be constant, times the mean-reversion speed ($|\kappa|^{PF} \times \overline{\text{IVOL}}^{PF}$). Therefore, $|\kappa|^{PF}$ is the mean-reversion speed.

IVOL innovations, $\Delta\text{IVOL}_{t+1}^i \equiv \text{IVOL}_{t+1}^i - \text{IVOL}_t^i$, on single stock level and distinguish between different regimes with our dummies. Thus, we analyze the time series of every stock separately now. The base case of our regression is the regime of low IVOL and low IVOLVOL (low/low). We use two dummies to control for times of high IVOL ($D1_t^i$) and times of high IVOLVOL ($D2_t^i$). The product of our dummies ($D1_t^i \times D2_t^i$) indicates times of simultaneously high IVOL and high IVOLVOL. Thus, the regression corresponds to a independent double sort and a stock will belong to the high IVOL (IVOLVOL) bucket only in relation to the remaining IVOLs (IVOLVOLs) of the whole cross section. We use the following regression model:

$$\begin{aligned} \Delta\text{IVOL}_{t+1}^i &= \alpha^i + \beta_1^i \times \text{IVOLVOL}_t^i & (12) \\ &+ \beta_2^i \times D1_t^i \times \text{IVOLVOL}_t^i \\ &+ \beta_3^i \times D2_t^i \times \text{IVOLVOL}_t^i \\ &+ \beta_4^i \times (D1_t^i \times D2_t^i) \times \text{IVOLVOL}_t^i + \epsilon_{t+1}^i. \end{aligned}$$

Table 5 reports the mean effect of IVOLVOL on subsequent realized levels of idiosyncratic risk and t-statistics. For robustness we only consider β_j^i if at least five observations of firm i being in a certain regime j are available, e.g., we include a stocks β_j^i to one of the four regimes if we have at least five month of observations for it in that specific regime. To show the overall effect on changes in idiosyncratic risk, we report the absolute change due to IVOLVOL, which we calculate as follows. First, for each bucket and for each stock we sum the betas according to the regime and afterwards we multiply by the mean IVOLVOL level of the firm conditional on being in the particular regime. This gives us the average impact of expected future variation in idiosyncratic risk on realized changes in idiosyncratic risk (ΔIVOL^i) for each firm in each regime. Afterwards we average over each bucket.

Once more, we find a highly significant impact of the expected variation in future idiosyncratic risk on subsequent realized IVOL levels. Due to the mean-reversion in IVOL, IVOLVOL has a positive effect if the currently realized idiosyncratic risk

is low (thus it will increase), while the effect is negative if currently realized idiosyncratic risk is high. More importantly, the effect for high IVOLVOL stocks is significantly higher than for stocks with low IVOLVOL. More precisely, in the low/low case the effect is 0.11%, but in contrast in the low/high case it is much stronger with 0.26%. For currently high realized IVOL we find in the high/low case an impact of -0.37% and in the high/high case an impact of -0.60%, which is larger in magnitude. The differences conditional on IVOL levels of -0.05% for low IVOL and 0.28% for high IVOL are statistically highly significant as well.¹⁷ As both analyses point in the very same direction, we conclude that IVOLVOL indeed indicates the mean-reversion speed and together with IVOL the direction of the future IVOL movement.

4.3 Expected Idiosyncratic Risk and Expected Returns

In this section we analyze the pricing of expected future idiosyncratic risk. Therefore, we examine the effect of lagged IVOL levels on realized returns. We do this conditional on the expected variation in the IVOL levels, that is for different levels in IVOLVOL. Therefore, Tables 6 – 9 report results for dependent 5×3 portfolio double sorts. For these, we sort our stock universe into quintile portfolios each month based on their realized IVOL level first. Then each IVOL portfolio is split into three independent portfolios according to the stocks measures of IVOLVOL. Conditional on the level of IVOLVOL, Figure 2 shows the cumulative return for the low - high IVOL strategy. As can be seen, accounting for the mean-reversion direction of IVOL and its speed has a strong impact on the strategy’s performance. It holds the higher the IVOLVOL the larger the return of the difference portfolio. The strategy of in-

¹⁷Note, the reported differences -0.05% in the case of low IVOL and 0.28% in the case of high IVOL are not simply the differences of the mean effects of the single buckets. We can only calculate the difference if a firm was in both buckets at least once with five observations each.

vesting in the difference portfolio for high IVOLVOL stocks only yields a cumulative log-return of 3.5 over the whole sample period. Further, for low expected variation in idiosyncratic risk levels the cumulative return of a low-minus-high IVOL portfolio is slightly negative and almost zero.

For a deeper analysis, Table 6 reports Fama-French alphas of equally weighted portfolios for the conditional sort. With these results the extreme IVOL/IVOLVOL portfolios are of most interest, that is the low/low, low/high, high/low and high/high portfolios and the differences in alpha between those. As for the single sort, the alpha is positive for low IVOL portfolios and changes sign for higher IVOL portfolios. Here, the alpha decreases faster with the IVOL for stocks with higher IVOLVOL. The last column of the table reports the alpha for the difference between the low minus high IVOL portfolio for every IVOLVOL bucket. All results speak strongly in favor of our hypothesis. Precisely, there is only a weak statistical significance of the alpha for the difference portfolio of 0.70% if the IVOLVOL is low. As pointed out the mean-reversion effect is much slower for these stocks and high (low) IVOL will stay rather high (low). Thus, investors seem to demand less compensation. In contrast, the significance is economically and statistically much stronger if we look at the alpha of the difference portfolio between low/high and high/high, which is 2.61%. Again, these stocks constitute of an IVOL that is much faster mean-reverting. Therefore, it is very likely that a current low (high) IVOL will increase (decrease) by a greater amount and thus investors might demand higher compensation for this increase. The last row of Table 6 reports the difference in alpha between the lowest and highest IVOLVOL portfolio for every realized IVOL bucket. Looking at these results, there is no significant difference in alpha between the low/low and low/high portfolios. However, there is a highly significant difference of 1.69% between the alpha of high/low and high/high portfolios. Putting together, this indicates that the negative relation between lagged idiosyncratic risk and subsequent returns as found in the single sort is driven by high IVOLVOL stocks, which have a faster mean-

reverting IVOL. In addition, among the high IVOLVOL stocks, the negative relation between lagged IVOL and realized returns is driven by the high/high portfolio, since there is only a significant difference between high/low and high/high.

Table 7 reports mean excess returns for the very same equally weighted portfolios. The overall results are as in the case of the Fama-French alpha. Returns tend to be positive for low IVOL portfolios and decrease in the IVOL rank. However, they get only negative for the highest IVOL portfolios which have either a medium or high IVOLVOL. Looking at the differences in returns between low and high realized idiosyncratic risk portfolios in the last column reveals that the difference of 0.28% between low/low and high/low is not significantly different from zero. In contrast, the difference between low/high and high/high of 2.04% is highly significant in a statistical and economical sense. The last row in Table 7 shows that only the 1.34% difference between low/high and high/high is significant. Once again, this highlights that returns of the high/high portfolio are the reason for the existence of a negative relation between realized idiosyncratic risk levels and subsequent returns, as documented by the single sort. Table 8 and 9 report alpha and excess returns for conditional sorts of value weighted portfolios. The results on alpha level are exactly the same as for equally weighted portfolios. If the IVOLVOL is low, the alpha of the difference portfolio is not significant and 0.41%. If the IVOLVOL is high, it gets highly significant and equals 1.84%. Again given a certain IVOL level, only the difference between high/low and high/high has an alpha of 1.38% and is highly significant, highlighting the importance of the high/high portfolio in explaining the negative relation between lagged IVOL and realized returns. In contrast, the results for excess returns are much weaker. Nevertheless, this fact is not surprising, since there is no relation between realized idiosyncratic risk and subsequent excess returns found for single sorts. Apart from this, the results point still in the same direction. There is a weak significance for the difference between low/high and high/high portfolio returns (1.30%), but no significance between low/low and high/low portfolio

returns (0.06%).

All in all, the findings speak strongly in favour for our hypothesis. The negative relation between lagged IVOL and realized returns can be explained if accounting for the mean-reversion effect of IVOL and the differences in its mean-reversion speed. However, in order to verify whether this pattern is indeed in line with the behavior of rational investors, we have to analyze whether investors do anticipate this pattern.

4.4 Evidence from Option Trading

In the previous sections we argue that idiosyncratic risk innovations are related to variations of future expected idiosyncratic risk levels and thus IVOLVOL is a proxy for the mean-reversion speed of IVOL. In this section we provide evidence that investors indeed incorporate this relationship into their trading decisions, especially when trading options.

Table 10 reports average ratios of traded put volumes to call volumes in the portfolio formation month. This trading behavior should be related to the expected drift in the stock price and thus indicate whether investors expect a certain compensation given a specific level of idiosyncratic risk. In line with the literature our data indicates that on general more calls are traded than puts.¹⁸ More importantly, the put/call-ratio greatly depends on idiosyncratic risk and our measure of IVOLVOL. For stocks in the low/low portfolio, the ratio is overall the largest with a value of 0.89. This indicates the highest demand for put options relative to calls. Therefore, the ratios document that investors expect future returns for assets with current low and slowly mean-reverting IVOL to be lower compared to the stocks in other portfolios. In comparison, for stocks with currently high IVOL, which is slowly mean-reverting, the average put/call-ratio is 0.62 – the lowest value across all portfolios. These stocks

¹⁸See for example, [Dennis and Mayhew \(2002\)](#) who report a median of 0.2950 or [Bali and Murray \(2013\)](#) who report a larger open interest for calls than for puts.

are currently exposed to high firm specific risk, which is expected to be quite sticky. Therefore, these stocks are expected to be risky next period, too. Market participants demand future high returns for bearing this risk and in turn incorporate the expected returns into their option trading decisions. Therefore, investors buy more call options.

Further, conditional on the IVOL regime we find highly significant differences in the put/call-ratios for low and high IVOLVOL buckets. If idiosyncratic risk is low, the ratio for high IVOLVOL stocks is with 0.78 significantly smaller than for low IVOL stocks for which we report a value of 0.89. Investors are more willingly to trade calls on low/high stocks than on low/low stocks. They take into account a likely increase in individual risk and thus higher stock returns, for stocks in the low/high portfolio.¹⁹ In a similar spirit, conditional on high IVOL the put/call-ratio is significantly larger for high IVOLVOL stocks than for low IVOLVOL assets. The difference is with 0.06 highly significant. These findings indicate a higher demand for puts on assets in the high/high regime. Rational investors trade more put option for these assets because they expect returns to decrease due to a large deterioration in idiosyncratic risk. Also the differences in put/call-ratios between low and high IVOL for given IVOLVOL buckets speak in favor for our hypothesis. The difference of 0.27 is the largest for low IVOLVOL stocks. For these investors expect low mean-reversion in idiosyncratic risk and thus, the risk levels to be good distinguishable from each other. Here, the difference in option trading - more puts for low IVOL and more calls for high IVOL - stays the most pronounced. This difference shrinks in the IVOLVOL buckets. For high IVOLVOL we find the lowest difference of 0.10. Here, investors expect an increase in current low idiosyncratic risk and a decrease in high idiosyncratic risk, so that risk levels move from the extreme in the opposite

¹⁹In unreported results, we find that low/low stocks are significantly less liquid than low/high stocks measured in terms of the average liquidity factor of [Pastor and Stambaugh \(2003\)](#). So investors' interest in low/high stocks seems to be higher than in low/low stocks.

direction. Therefore, investors trade options according to that behavior and decrease the demand for puts (calls) of the low/high (high/high) portfolio. This results in a shrinking difference in the put/call-ratio. All in all, we conclude that the observed patterns in option trading speak in favor for our hypothesis and that investors seem to account for the mean-reversion in IVOL.

4.5 Mean-Reversion Speed vs. Competing Explanations

All evidence thus far speak strongly in favor of that controlling for the mean-reversion speed in IVOL disentangles the IVOL puzzle. However, there exists a battery of other attempts to solve the puzzle, as pointed out before. If the mean-reversion speed in IVOL is the central driver in observing a negative relation between past realized IVOL levels and subsequent returns, it should be able to beat these competing explanations in a direct horse race. Therefore, we make use of the method in [Hou and Loh \(2016\)](#), which allows to decompose the IVOL coefficient of a cross-sectional regression of IVOL on returns, with respect to various competing explanations. Table 16 reports results of the decomposition. Panel A displays the intercepts and coefficients of cross-sectional regression, where we regress the lagged $IVOL^i$ on returns at every point in time and then average over time. Next to raw returns, we use Fama-French 3-factor and 5-factor alpha and stock characteristic-adjusted returns, following [Daniel et al. \(1997\)](#) (DGTW), as independent variable. The coefficients are all negative and range from -0.2801 for raw returns to -0.0144 for 3-factor alpha. This fact highlights once more the existence of the IVOL puzzle in our data sample, since all coefficients are highly significant except for the Fama-French 5-factor alpha.²⁰

In Panel B we report results of an instantaneously cross-sectional regression of

²⁰In the robustness part, we show that the negative relation between IVOL and 5-factor alpha is prevalent for our highest IVOLVOL bucket.

control variables, which are propagated to explain the IVOL puzzle, on the realized IVOL levels. This regression is the same for all independent variable sets in Panel A and thus yields the same coefficients in all regressions. To test our hypothesis we include three dummy variables indicating if a stock's IVOLVOL lies in the lowest, middle or highest tertile. Motivated by our previous findings, we expect the negative influence of IVOL on returns to be strongest in our highest tertile. However, by conditioning on the IVOLVOL level only, this method imposes a larger hurdle in explaining the IVOL return relation compared to our double sort analysis. As pointed out before, we expect the negative relation to be prevalent for stocks which show a large distortion from its long-run mean and a fast mean-reversion at the same time. However, we are not able to control for the IVOL level in this method by design. Nevertheless, if we find an effect of IVOLVOL alone, it indicates a lower bound for the potential of the mean-reversion to explain the IVOL puzzle. We follow [Hou and Loh \(2016\)](#) and include next to our IVOLVOL measure the most promising alternative explanations. Precisely, we add realized skewness, risk-neutral skewness, co-skewness, the retail trading proportion (RTP), lagged returns, the proportion of zero returns, the liquidity measure of [Pastor and Stambaugh \(2003\)](#) and the relative bid-ask spread. The results in Panel B show that all coefficients of the IVOLVOL dummies are highly significant. While the coefficient for the lowest IVOLVOL tertile is -0.0028, it increases to 0.0111 for the highest tertile, capturing the positive correlation of both measures. Next to these, the risk-neutral skew, lagged returns, the proportion of zero returns and the bid-ask spread are significant only.

Panel C displays the main results of the analysis. Following [Hou and Loh \(2016\)](#) we use the coefficients from the regression in Panel B to disentangle the IVOL coefficient from Panel A. Therefore, we compute sensitivities for these explanatory measures, which are embedded in the IVOL coefficient. We do so by computing the covariance between returns and the fraction of the explanatory variable, which explains the variation in IVOL e.g., the coefficient times the beta of the variable

from the regression in Panel B. These are then in turn normalized by the variance of IVOL, to ensure that all computed sensitivities plus the residual part, which is not explained by any of these variables, add up to the coefficient in Panel A. The disentangled coefficients are then gained by averaging over time. Panel C of Table 16 clearly indicates that the proposed relation of our measures holds. The high IVOLVOL dummy has the highest impact, compared with the other two dummies. It is able to explain the IVOL coefficient by 27.56% up to 86.25%, while the low IVOLVOL dummy has an explanatory power of 9.01% to -26.14%. The results for the 5-factor alpha show the strongest divergence and, thus, support our reasoning. While a simple dummy indicating whether mean-reversion is expected to be high explains nearly the whole negative coefficient of IVOL, low mean-reversion turns the relation around and signals a positive or only weak negative dependence of IVOL and returns. The findings are a strong support for the IVOL puzzle to be driven by the mean-reversion. This fact is even bolstered when looking at the other controls. In line with Hou and Loh (2016), we find that many potential explanations have little power in explaining the puzzle. RTP and ZeroRet are among the best performing competing explanations, showing an explanatory power of 3.08% – 12.66% and 8.27% – 12.25%, respectively. However, these lie far below the explanatory power of IVOLVOL_{high}. Moreover, all competing explanations taken together are in no case able to beat the explanatory power of the high IVOLVOL dummy. In addition, the three dummies together explain by far the largest fraction of the IVOL puzzle. While all explanations together are able to explain 63.14%, 56.54% and 61.54%, the dummies alone are responsible for 41.17%, 38.41% and 38.15%, respectively. Thus, due to IVOLVOL we are able to boost the explanatory power way above the reported 29%-54% in Hou and Loh (2016). All in all, these results fully support our hypothesis and show that simple dummies to control for mean-reversion effects are superior to many other famous explanations for the IVOL puzzle.

5 Robustness

In this section we undertake a robustness analysis with respect to the model which is used in the estimation of idiosyncratic risk. Further we provide evidence that the underperformance cannot be explained by other stock characteristics.

5.1 Controlling for Different Measurements of IVOL

Throughout the paper we estimate idiosyncratic risk relative to the Fama-French 3-factor model. However, our IVOLVOL measure is calculated relative to the market VIX and thus measures mean-reversion effects of idiosyncratic volatility relative to the market model. To check if our main findings still hold in a more consistent setting we calculate IVOL relative to the CAPM and report excess returns the same as the mean-reversion effect in Table 11 and Table 12. Table 11 shows that our main findings remain unchanged overall. When we calculate idiosyncratic volatility relative to the CAPM the IVOL-return relation still vanishes completely for low IVOLVOL stocks and is strengthened for high IVOLVOL stocks. As before the IVOL-return relation seems to be driven by high/high stocks. In addition, Table 12 documents that our explanation for the existence of the negative IVOL-return relation is still valid. The mean-reversion effect is still larger for high IVOLVOL stocks with signs in the right direction. Interestingly, the absolute effect remains on the same level overall, but the significance for low/high stocks is dampened to a 10% significance level.

[Fama and French \(2015\)](#) extend their three-factor model and show that investment as well as profitability are crucial to price the cross-section of stocks. These factors are different from the classical ones and may proxy risks factors which are important for the IVOL-return relation. Thus, we calculate idiosyncratic risk relative to the five-factor model. Table 13 and Table 14 show excess returns and the

mean-reversion effect for portfolios sorted on IVOL and IVOLVOL. The tables indicate that our findings and our explanation are robust to the 5-factor model of Fama and French. The IVOL-returns of the difference portfolios vanish for low IVOLVOL stocks and amplify for high IVOLVOL stocks. Further, high/high stocks are still the main driver and the mean-reversion effect increases in IVOLVOL levels.

Finally, we test a different sorting scheme. Section 4.3 focuses on analyzing possible explanations of the negative IVOL-return relation, documented by earlier studies. Therefore we run dependent double sorts. Table 15 reports equally weighted results of a simple independent 2×2 double sort.²¹ Both, for alpha and excess returns the results point in the very same direction as before. For low IVOL, we find no significant difference in return or alpha for the high minus low IVOL portfolio. In contrast, the difference portfolio shows a highly significant excess return (alpha) of 0.66% (0.88%) on average if the IVOLVOL is high. Again, high/high stocks seem to drive the effect. Therefore we conclude that our results are neither driven by a biased estimator of idiosyncratic risk, nor by a specific sorting technique.

5.2 Controlling for Stock Characteristics

This part of our robustness analysis concentrates only on portfolios of high IVOLVOL stocks since we find significant differences in returns for those only. However, these results might be driven by some special stock characteristics. In doing so, we first perform the usual conditional 5×3 sort on idiosyncratic risk and IVOLVOL. Afterwards we only concentrate on the stocks in the highest IVOLVOL tertiel and use the same method as in [Ang et al. \(2006\)](#). That is, we first sort conditionally for the robustness variable in a high/low fashion and then for IVOL. Thereupon, we average the portfolios along the robustness variable and report excess returns as well.

²¹Since the negative IVOL-return relation is generally weaker for value weighted portfolios in the sample, we find no significant differences for a rather sparse 2×2 sorting scheme.

Table 17 reports the resulting excess returns after controlling for various characteristics. Our analysis reveals that none of the included variables are capable to explain the excess return of the low-minus-high portfolio since both stay highly economically and statistically significant. Further, for almost all control variables the magnitudes of returns are in the same range as before. In the following we will explain the economic reasons for including the control variables.

Size – We control for size since [Bali and Cakici \(2008\)](#) suggest that the IVOL puzzle might be driven by small stocks mainly. Controlling for size has a small marginal effect on return level. It only leads to slightly deteriorated returns of 1.72% for the difference portfolio.²²

Book-to-Market Ratio – As [Fama and French \(1992\)](#) and others show, the book-to-market (B/M) ratio is a strong driver of returns. The higher the ratio the higher the future realized return. It might be possible that low IVOL stocks are mainly value stocks and that high IVOL stocks are more likely to be growth stocks. This relation might explain the large returns of the low-minus-high IVOL portfolio. However, after controlling for B/M the return for the low-minus-high portfolio is still highly significant positive.

Liquidity – Several authors argue that liquidity positively influences returns and that high IVOL stocks might be less liquid.²³ We measure liquidity risk using the liquidity beta introduced by [Pastor and Stambaugh \(2003\)](#) estimated over the formation period of one month. We find, even after controlling for liquidity, the excess return of the low-minus-high portfolio remains significant.

Bid-Ask Spread of Stocks – Another measure for liquidity is the bid-ask spread, which we measure as the daily average over the formation month.²⁴ Similar to the findings for the liquidity beta the significance of the excess return of low IVOL stocks

²²Table 7 reports a return of 2.04% in the case of no controls.

²³See for example [Amihud \(2002\)](#) and [Liang and Wei \(2012\)](#) amongst others.

²⁴[Brennan and Subrahmanyam \(1996\)](#) argue that the spread is a measure for liquidity, although a noisy one.

remains untouched.

Bid-Ask Spread of Puts – The underperformance of the high IVOL portfolio could be driven by too high prices of stocks for which investors face short sale constraints. [Lin and Lu \(2015\)](#) show that, since the replicating portfolio for puts includes a short position in the underlying stock, the bid-ask spread of put options is positively related to lending fees. Those fees can be interpreted as a level of short-sale constraints, because the higher the fee the more costly to short the stock. However, after controlling for the spread we still find highly significant abnormal returns for the difference portfolio.

Return Reversal – [Huang et al. \(2010\)](#) show that, if the whole stock universe is considered, the IVOL puzzle can on average be explained by return reversal. After controlling for return reversal the return of the difference portfolio is still highly significant in statistical and economic terms, so we do not find evidence for this explanation in our subsample.

6 Conclusion

Our paper analyzes the widely documented negative relation between historically realized IVOL and subsequent realized returns. Thereby, we highlight the importance of measuring expected IVOL to infer the real relation between expected idiosyncratic risk and expected returns. If IVOL obeys a mean-reversion process, a high (low) IVOL might indicate a rather large distortion from its long-run mean. In such a case it is obvious that the higher the mean-reversion speed, the larger the expected decrease (increase) in IVOL. Further, we demonstrate theoretically and empirically that the variation in expected future idiosyncratic risk will increase in the mean-reversion speed. Therefore, we propose the measure of IVOLVOL as a natural proxy for the expected mean-reversion speed in IVOL. Then the magnitude of the negative IVOL-return relation should increase in IVOLVOL.

The empirical assessment of our paper speaks strongly in favor for these hypotheses. First, we document the existence of the classical IVOL anomaly in a subsample of highly liquid stocks and options if a historic measure of IVOL is used. Stocks with high idiosyncratic volatility underperform compared to stocks with low idiosyncratic volatility. Second, we verify that the time series of IVOL is stationary. Therefore, it is reasonable to assume a mean-reversion behavior of IVOL. In addition, we show that our measure of IVOLVOL indeed proxies for the mean-reversion speed in IVOL. In essence, we can expect a higher speed of mean-reversion if the IVOLVOL is large. Thereby, we construct our measure of IVOLVOL in an almost model-free fashion by using option implied information which we extract from the cross-section of options. Thus, our measure of IVOLVOL is entirely forward looking and can be used together with the current level of IVOL to infer the expected innovation in IVOL. Third, this interplay resolves the previously documented negative IVOL-return relation. More precisely, the negative IVOL-return relation vanishes for low IVOLVOL stocks. Here, idiosyncratic risk levels are expected to be rather constant. On the other hand, the negative IVOL-return relation is economically and statistically bolstered, looking at high IVOLVOL stocks. Again, the idiosyncratic risk of these stocks is expected to move in opposite direction and thus investors demand a higher (lower) compensation for current low (high) IVOL stocks, which are expected to suffer a larger increase (decrease) in idiosyncratic risk.

Our findings are backed up by options data. Options trading behavior indicates that investors buy more call (put) options if IVOL is more likely to increase (decrease), because they have the incentive to participate on future high (low) returns. In a comparative analysis in the style of [Hou and Loh \(2016\)](#) we show that IVOLVOL is distinct from other explanations for the IVOL anomaly. Our proxy for mean-reversion can explain more than 40% of the total IVOL anomaly on its own. Whereby other explanatory variables, which proxy market frictions or the lottery preference of investors, only manage to explain around 20%. In addition, our

results are not driven by stock characteristics other than expectations about idiosyncratic risk. In a robustness analysis we rule out stock liquidity and short-sale constraints. Further, we show that our results are insensitive to different measures of idiosyncratic risk and different sorting techniques. Our paper gives a new valuable perspective on the existence of the negative IVOL-return relation for the biggest, most liquid stocks if a historic measure of IVOL is used. In addition, it demonstrates that this observation can be explained by a mean-reversion in IVOL alongside with a rational behavior of investors.

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Stock Fundamentals

		Mean	Median	Lowest 20%	Next 40%	Next 60%	Next 80%	Highest 90%
Sample	Size	7719	2325	764	1582	3304	8541	17682
	Dollar Vol.	2095	956	410	738	1259	2435	3907
Whole Universe	Size	1364	158	34	97	254	836	2069
	Dollar Vol.	289	18	2	9	34	144	411

Table 1: The table shows descriptives of size and Dollar Vol, the average monthly trading Dollar volume, in million \$ each. Whole Universe covers all stocks from the three exchanges AMEX, NYSE and NASDAQ. Sample is the subsample of the universe, which we use throughout the paper. The sample only contains stocks for which we have at least 15 days of return and VIXⁱ observations for some month. We only use stock characteristics for the months the stock is included. Details can be found in the data section. The sample period is 1996/01–2014/12.

Single Sort on Idiosyncratic Volatility					
Excess Return – Equally Weighted					
1 LOW	2	3	4	5 HIGH	1 - 5
0.71*** (0.33)	0.79* (0.42)	0.68 (0.54)	0.54 (0.65)	-0.43 (0.73)	1.13*** (0.54)
Excess Return – Value Weighted					
1 LOW	2	3	4	5 HIGH	1 - 5
0.67*** (0.31)	0.69* (0.40)	0.37 (0.49)	0.55 (0.65)	-0.12 (0.75)	0.79 (0.57)
FF-Alpha – Equally Weighted					
1 LOW	2	3	4	5 HIGH	1 - 5
0.18 (0.11)	0.11 (0.15)	-0.16 (0.18)	-0.35 (0.24)	-1.47*** (0.27)	1.64*** (0.31)
FF-Alpha – Value Weighted					
1 LOW	2	3	4	5 HIGH	1 - 5
0.21*** (0.08)	0.09 (0.14)	-0.34 (0.18)	-0.26 (0.24)	-1.01*** (0.35)	1.22*** (0.39)

Table 2: The table shows monthly excess returns, averaged over the sample period 1996/01–2014/12, and 3-factor Fama-French alphas for different portfolios. We sort stocks into five equally/value weighted portfolios basing on realized IVOL in formation month. Then, we calculate excess returns for the next month. We calculate IVOL relative to the Fama-French 3-factor model. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Stationarity of IVOL				
	Required Observations			
	12	24	60	120
10% Level	85.83	87.81	95.41	99.27
5% Level	78.89	80.99	91.27	98.15
1% Level	62.23	64.21	75.57	89.36
Number of Stocks	3,056	2,962	2,464	1,785

Table 3: The table shows results of an augmented Dicky-Fuller test for an unit root against a stationary time series. Values report the percentage of rejected null hypothesis against the alternative of a stationary process without drift and trend for the 10, 5 and 1% significance level. To include a stock’s time series at least 12, 24, 60 or 120 observations of the monthly IVOL are required. The last row reports the number of time series included to the test.

Mean Reversion Effect in IVOL

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
IVOLVOL	1 LOW	0.03 (0.03)	-0.14*** (0.04)	-0.11*** (0.03)	-0.13*** (0.04)	-0.23*** (0.03)
	2	0.03 (0.05)	-0.04 (0.05)	-0.01 (0.03)	-0.08*** (0.03)	-0.28*** (0.03)
	3 HIGH	0.16*** (0.07)	0.12*** (0.05)	0.06* (0.03)	-0.01 (0.03)	-0.32*** (0.04)

Table 4: The table reports the mean reversion effect κ^{PF} in IVOL for different IVOL/IVOLVOL portfolios. For each portfolio we measure the mean reversion effect κ^{PF} by running the regression $IVOL_{t+1}^{PF} - IVOL_t^{PF} = \alpha^{PF} + \kappa^{PF}IVOL_t^{PF} + \epsilon_{t+1}^{PF}$, where $IVOL_t^{PF}$ is the average portfolio IVOL in month t . The portfolio formation date is month t . *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean Effect of IVOLVOL on IVOL Movements

		Ranking on Idiosyncratic Volatility	
		1 LOW	2 HIGH
IVOLVOL	1 LOW	0.11*** (0.03)	-0.37*** (0.02)
	2 HIGH	0.26*** (0.03)	-0.60*** (0.09)
1 - 2		-0.05*** (0.02)	0.28*** (0.09)

Table 5: The table shows the average impact of current IVOLVOL on idiosyncratic risk innovations over the next month in percent. We estimate the impact by a cross-sectional dummy regression $\Delta IVOL_{t+1}^i = \alpha^i + \beta_1^i \times IVOLVOL_t^i + \beta_2^i \times D1_t^i \times IVOLVOL_t^i + \beta_3^i \times D2_t^i \times IVOLVOL_t^i + \beta_4^i \times (D1_t^i \times D2_t^i) \times IVOLVOL_t^i + \epsilon_{t+1}^i$, where $D1$ indicates times of high IVOL and $D2$ times of high IVOLVOL. The table reports average $\beta_j^i \times \text{mean}[IVOLVOL_t^i | \text{Stock is in regime } j]$. The sample period is 1996/01–2014/12.

FF-Alphas of Equally Weighted Portfolios

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.04 (0.16)	0.16 (0.16)	0.03 (0.26)	-0.07 (0.31)	-0.65* (0.33)	0.69* (0.40)
	2	0.20 (0.14)	0.03 (0.18)	-0.18 (0.19)	-0.16 (0.30)	-1.45*** (0.37)	1.64*** (0.41)
	3 HIGH	0.27 (0.16)	0.15 (0.27)	-0.34 (0.28)	-0.85*** (0.36)	-2.35*** (0.37)	2.61*** (0.42)
1 - 3		-0.23 (0.22)	0.02 (0.27)	0.37 (0.29)	0.77* (0.45)	1.69*** (0.44)	

Table 6: The table shows 3-factor Fama-French alphas for portfolios from a conditional 5×3 double sort on IVOL and IVOLVOL over the sample period 1996/01–2014/12. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we weight the constituents equally and calculate excess returns for the next month. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std}[\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.46 (0.30)	0.71*** (0.32)	0.74 (0.48)	0.73 (0.62)	0.18 (0.70)	0.28 (0.60)
	2	0.77*** (0.35)	0.72* (0.41)	0.68 (0.55)	0.71 (0.67)	-0.32 (0.77)	1.09* (0.60)
	3 HIGH	0.88*** (0.41)	0.96* (0.57)	0.63 (0.67)	0.18 (0.77)	-1.16 (0.84)	2.04*** (0.64)
1 - 3		-0.43 (0.27)	-0.25 (0.34)	0.10 (0.37)	0.55 (0.43)	1.34*** (0.53)	

Table 7: The table shows monthly excess returns, averaged over the sample period 1996/01–2014/12, from a conditional 5×3 double sort on IVOL and IVOLVOL. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we weight the constituents equally and calculate excess returns for the next month. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std}[\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

FF-Alphas of Value Weighted Portfolios

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.24* (0.14)	0.25** (0.13)	-0.18 (0.22)	-0.15 (0.30)	-0.17 (0.40)	0.41 (0.45)
	2	0.15 (0.14)	-0.04 (0.19)	-0.21 (0.24)	-0.01 (0.27)	-1.56*** (0.47)	1.70*** (0.49)
	3 HIGH	0.30 (0.19)	0.03 (0.29)	-0.39 (0.37)	-0.52 (0.47)	-1.55*** (0.50)	1.84*** (0.56)
1 - 3		-0.06 (0.27)	0.22 (0.29)	0.21 (0.41)	0.36 (0.55)	1.38*** (0.59)	

Table 8: The table shows 3-factor Fama-French alphas for portfolios from a conditional 5×3 double sort on IVOL and IVOLVOL, over the sample period 1996/01–2014/12. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we value-weight the constituents and calculate excess returns for the next month. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std}[\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Value Weighted Portfolios

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.61*** (0.28)	0.72*** (0.31)	0.44 (0.42)	0.55 (0.59)	0.55 (0.72)	0.06 (0.62)
	2	0.66* (0.34)	0.59 (0.41)	0.51 (0.53)	0.80 (0.65)	-0.54 (0.79)	1.20* (0.63)
	3 HIGH	0.84** (0.43)	0.82 (0.59)	0.51 (0.71)	0.47 (0.90)	-0.45 (1.01)	1.30* (0.78)
1 - 3		-0.24 (0.31)	-0.10 (0.38)	-0.07 (0.49)	0.08 (0.56)	1.00 (0.67)	

Table 9: The table shows monthly excess returns, averaged over the sample period 1996/01–2014/12, from a conditional 5×3 double sort on IVOL and IVOLVOL. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we value-weight the constituents and calculate excess returns for the next month. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std}[\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Put/Call-Ratio

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.89 (0.14)	0.68*** (0.10)	0.65*** (0.10)	0.63*** (0.10)	0.62*** (0.09)	0.27*** (0.05)
	2	0.81 (0.13)	0.70*** (0.11)	0.67*** (0.10)	0.65*** (0.10)	0.64*** (0.10)	0.18*** (0.04)
	3 HIGH	0.78* (0.12)	0.69*** (0.11)	0.67*** (0.10)	0.67*** (0.10)	0.68*** (0.10)	0.10*** (0.02)
	1 - 3	0.11*** (0.04)	-0.01 (0.01)	-0.01 (0.01)	-0.04*** (0.01)	-0.06*** (0.02)	

Table 10: The table shows monthly put/call-ratio, averaged over the sample period 1996/01–2014/12, for a conditional 5×3 double sort on IVOL and IVOLVOL. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, calculate the put/call-ratio as the one-month average of put trading volume divided by call trading volume. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std} [\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. The test hypothesis for the ratios is that they equal one, whereas the hypothesis for the differences is that they equal zero. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios - IVOL relative to CAPM

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.43 (0.29)	0.75*** (0.34)	0.78 (0.48)	0.81 (0.62)	0.04 (0.71)	0.39 (0.6)
	2	0.80*** (0.36)	0.71* (0.43)	0.73 (0.57)	0.80 (0.71)	-0.21 (0.79)	1.00* (0.6)
	3 HIGH	0.92*** (0.40)	0.74 (0.54)	0.56 (0.64)	0.07 (0.75)	-1.05 (0.83)	1.97*** (0.64)
1 - 3		-0.48* (0.25)	0.01 (0.31)	0.22 (0.37)	0.74 (0.47)	1.10*** (0.48)	

Table 11: The table shows a robustness analysis for the existence of the IVOL-return relation. We report monthly excess returns, averaged over the sample period 1996/01–2014/12, from a conditional 5×3 double sort on IVOL and IVOLVOL. We calculate IVOL relative to the CAPM and estimate IVOLVOL as before. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we weight the constituents equally and calculate excess returns for the next month. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean Reversion Effect in IVOL - IVOL relative to CAPM

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
IVOLVOL	1 LOW	0.03 (0.04)	-0.15*** (0.04)	-0.11*** (0.03)	-0.13*** (0.03)	-0.23*** (0.03)
	2	0.01 (0.05)	-0.05 (0.05)	-0.03 (0.03)	-0.08*** (0.04)	-0.27*** (0.03)
	3 HIGH	0.11* (0.06)	0.10** (0.05)	0.05* (0.03)	-0.04 (0.04)	-0.30*** (0.04)

Table 12: The table shows robustness analysis for the mean reversion effect. We report the mean reversion effect κ^{PF} in IVOL for different IVOL/IVOLVOL portfolios, where we calculate IVOL relative to the CAPM and estimate IVOLVOL as before. For each portfolio we measure the mean reversion effect κ^{PF} by running the regression $IVOL_{t+1}^{PF} - IVOL_t^{PF} = \alpha^{PF} + \kappa^{PF} IVOL_t^{PF} + \epsilon_{t+1}^{PF}$, where $IVOL_t^{PF}$ is the average portfolio IVOL in month t . The portfolio formation date is month t . *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios - IVOL relative to FF5

		Ranking on Idiosyncratic Volatility					
		1 LOW	2	3	4	5 HIGH	1 - 5
IVOLVOL	1 LOW	0.47* (0.28)	0.60* (0.32)	0.87* (0.48)	0.76 (0.58)	0.14 (0.67)	0.34 (0.57)
	2	0.80*** (0.37)	0.72* (0.42)	0.79 (0.57)	0.60 (0.7)	-0.32 (0.77)	1.12* (0.60)
	3 HIGH	0.79* (0.41)	0.91 (0.60)	0.72 (0.69)	0.00 (0.76)	-1.00 (0.82)	1.79*** (0.61)
	1 - 3	-0.32 (0.26)	-0.31 (0.37)	0.15 (0.40)	0.75* (0.45)	1.13*** (0.50)	

Table 13: The table shows a robustness analysis for the existence of the IVOL-return relation. We report monthly excess returns, averaged over the sample period 1996/01–2014/12, from a conditional 5×3 double sort on IVOL and IVOLVOL. We calculate IVOL relative to the five-factor Fama-French model (Fama and French (2015)) and estimate IVOLVOL as before. First, we sort stocks on IVOL and subsequently on IVOLVOL basing on realizations in formation month. Then, we weight the constituents equally and calculate excess returns for the next month. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Mean Reversion Effect in IVOL - IVOL relative to FF5

		Ranking on Idiosyncratic Volatility				
		1 LOW	2	3	4	5 HIGH
IVOLVOL	1 LOW	0.02 (0.03)	-0.15*** (0.04)	-0.11*** (0.03)	-0.15*** (0.03)	-0.23*** (0.03)
	2	0.02 (0.05)	0.00 (0.06)	-0.01 (0.03)	-0.05 (0.03)	-0.29*** (0.03)
	3 HIGH	0.15*** (0.07)	0.10*** (0.05)	0.04 (0.03)	-0.06* (0.03)	-0.31*** (0.04)

Table 14: The table shows robustness analysis for the mean reversion effect. We report the mean reversion effect κ^{PF} in IVOL for different IVOL/IVOLVOL portfolios, where we calculate IVOL relative to the five-factor Fama-French model (Fama and French (2015)) and estimate IVOLVOL as before. For each portfolio we measure the mean reversion effect κ^{PF} by running the regression $IVOL_{t+1}^{PF} - IVOL_t^{PF} = \alpha^{PF} + \kappa^{PF} IVOL_t^{PF} + \epsilon_{t+1}^{PF}$, where $IVOL_t^{PF}$ is the average portfolio IVOL in month t . The portfolio formation date is month t . *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Independent Double-Sort

		Return – Equally Weighted		
Ranking on Idiosyncratic Volatility				
		1 LOW	2 HIGH	1 - 2
IVOLVOL	1 LOW	0.71** (0.35)	0.78 (0.53)	-0.07 (0.27)
	2 HIGH	0.71 (0.58)	0.05 (0.70)	0.66*** (0.31)
1 - 2		0.00 (0.30)	0.74*** (0.30)	
		FF-Alpha – Equally Weighted		
		1 LOW	2 HIGH	1 - 2
IVOLVOL	1 LOW	0.12 (0.11)	0.04 (0.22)	0.09 (0.19)
	2 HIGH	-0.10 (0.25)	-0.98*** (0.22)	0.88*** (0.24)
1 - 2		0.22 (0.26)	1.01*** (0.21)	

Table 15: The table shows monthly excess returns, averaged over the sample period 1996/01–2014/12, and 3-factor Fama-French alphas for portfolios from unconditional 2×2 double sorts. We sort stocks into four equally weighted portfolios basing on realized IVOL/IVOLVOL in formation month. Then, we calculate excess returns for the next month. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std}[\eta_{t-30D:t}^i]$. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Decomposition of IVOL Puzzle

	Raw Returns		Alpha - FF3		Alpha - FF5		Returns - DGTW adj.	
	Coeff.	Expl.	Coeff.	Expl.	Coeff.	Expl.	Coeff.	Expl.
<i>Panel A: IVOL on Return</i>								
Intercept	0.0120*** (0.0037)		0.0004*** (0.0001)		0.0003*** (0.0001)		0.0044 (0.0027)	
IVOL	-0.2801*** (0.1329)		-0.0144*** (0.0066)		-0.052 (0.0051)		-0.2708*** (0.1221)	
<i>Panel B: Controls on IVOL</i>								
Intercept	0.0122*** (0.0026)		0.0122*** (0.0026)		0.0122*** (0.0026)		0.0122*** (0.0026)	
IVOLVOL _{low}	-0.0028*** (0.0010)		-0.0028*** (0.0010)		-0.0028*** (0.0010)		-0.0028*** (0.0010)	
IVOLVOL _{mid}	0.0029*** (0.0010)		0.0029*** (0.0010)		0.0029*** (0.0010)		0.0029*** (0.0010)	
IVOLVOL _{high}	0.0111*** (0.0019)		0.0111*** (0.0019)		0.0111*** (0.0019)		0.0111*** (0.0019)	
Skew _{real.}	-0.0003 (0.0002)		-0.0003 (0.0002)		-0.0003 (0.0002)		-0.0003 (0.0002)	
Skew _{RN}	0.0036*** (0.0008)		0.0036*** (0.0008)		0.0036*** (0.0008)		0.0036*** (0.0008)	
CoSkew	-0.0025 (0.0021)		-0.0025 (0.0021)		-0.0025 (0.0021)		-0.0025 (0.0021)	
RTP	0.0000 (0.0001)		-0.0000 (0.0000)		-0.0000 (0.0000)		-0.0000** (0.0001)	
LagRet	0.0034*** (0.0013)		0.0034*** (0.0013)		0.0034*** (0.0013)		0.0034*** (0.0013)	
ZeroRet	0.0004*** (0.0001)		0.0004*** (0.0001)		0.0004*** (0.0001)		0.0004*** (0.0001)	
Liquidity	202.50 (348.61)		202.50 (348.61)		202.50 (348.61)		202.50 (348.61)	
Spread	129.31*** (41.50)		129.31*** (41.50)		129.31*** (41.50)		129.31*** (41.50)	
<i>Panel C: Decomposition of IVOL Coefficient</i>								
IVOLVOL _{low}	-0.0171 (0.0692)	6.14%	-0.0012 (0.0023)	9.01%	0.0011 (0.0019)	-26.14%	-0.0241 (0.0565)	8.99%
IVOLVOL _{mid}	-0.0085 (0.0203)	3.06%	0.0011 (0.0015)	-8.15%	0.0022 (0.0015)	-5.67%	-0.0043 (0.0225)	1.60%
IVOLVOL _{high}	-0.0889 (0.0939)	31.97%	-0.0051 (0.0046)	37.55%	-0.0036 (0.0032)	86.25%	-0.0739 (0.0834)	27.56%
Skew _{real.}	0.0049 (0.0066)	-1.77%	0.0001 (0.0004)	-1.03%	0.0002 (0.0004)	-4.34%	0.0080 (0.0065)	-2.98%
Skew _{RN}	-0.0126 (0.0096)	4.53%	-0.0009 (0.0006)	6.75%	-0.0004 (0.0005)	8.37%	-0.0144 (0.0089)	5.35%
CoSkew	0.0045 (0.0031)	-1.64%	0.0002 (0.0001)	-1.68%	0.0002 (0.0001)	-4.71%	0.0036 (0.0030)	-1.34%
RTP	-0.0336 (0.0183)	12.10%	-0.0004 (0.0010)	3.08%	-0.0005 (0.0008)	11.54%	-0.0339 (0.0184)	12.66%
LagRet	-0.0031 (0.0191)	1.10%	0.0001 (0.0010)	-0.51%	0.0005 (0.0008)	-10.77%	-0.0035 (0.0180)	1.31%
ZeroRet	-0.00230 (0.0261)	8.27%	-0.0017 (0.0013)	12.25%	-0.011 (0.0012)	26.83%	-0.0256 (0.0262)	9.53%
Liquidity	-0.0060 (0.0068)	2.15%	-0.0001 (0.0003)	0.53%	0.0001 (0.0003)	-1.44%	-0.0041 (0.0067)	1.53%
Spread	0.0077 (0.0058)	-2.79%	0.0002 (0.0002)	-1.26%	0.0000 (0.0003)	0.00%	0.0072 (0.0055)	-2.68%
Controls _{total}	-0.1755** (0.1002)	63.14%	-0.0077** (0.0039)	56.54%	-0.0014 (0.0031)	33.84%	-0.1650** (0.0877)	61.54%
Residual	-0.1025** (0.0594)	36.86%	-0.0059** (0.0032)	43.46%	-0.0028 (0.0030)	66.16%	-0.1031** (0.0536)	38.46%

Table 16: The table shows a multivariate analysis for the decomposition of the idiosyncratic volatility puzzle following [Hou and Loh \(2016\)](#). Panel A shows the average coefficients of cross-sectional regressions of monthly (risk-adjusted) returns on IVOL. DGTW refers to risk adjusted returns from [Daniel et al. \(1997\)](#). Panel B shows results from a multivariate regression of IVOL on a set of explanatory variables. Panel C shows the normalized covariation of the explanatory variables as well as their overall explanatory power for the IVOL anomaly. We calculate IVOL relative to the 3-factor Fama-French model and IVOLVOL relative to the market VIX as $(VIX_s^i)^2 = \gamma^i + \beta_{VIX}^i (VIX_s^M)^2 + \eta_s^i$, $IVOLVOL_t^i \equiv \text{std} \left[\eta_{t-30D:t}^i \right]$. Liquidity is the liquidity beta of [Pastor and Stambaugh \(2003\)](#) and $Skew_{real.}$ is the realized skewness of raw daily in formation month. $Skew_{RN}$ is the risk neutral skew from [Bakshi et al. \(2003\)](#) at the end of formation month. CoSkew is the coskewness measure in [Chabi-Yo and Yang \(2010\)](#). RTP is the retail trading proportion. LagRet is the one month lagged return, ZeroRet is the proportion of zero returns and Spread is the average relative bid-ask-spread in formation month. Controls_{total} is the sum of the variables explanatory power and residual captures the unexplained part of the IVOL-return relation. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

Returns of Equally Weighted Portfolios

Ranking on Idiosyncratic Volatility						
Controlling for	1 LOW	2	3	4	5 HIGH	1 - 5
Size	0.97*** (0.46)	0.60 (0.56)	0.63 (0.64)	0.07 (0.68)	-0.75 (0.80)	1.72*** (0.57)
B/M	0.88** (0.43)	1.06* (0.6)	0.48 (0.66)	0.26 (0.75)	-1.14 (0.81)	2.03*** (0.59)
Liquidity	0.97*** (0.41)	0.80 (0.54)	0.69 (0.66)	0.18 (0.75)	-1.10 (0.82)	2.07*** (0.64)
Volume	0.85*** (0.40)	0.84 (0.55)	0.63 (0.67)	0.36 (0.74)	-1.25 (0.82)	2.10*** (0.63)
Bid-Ask Spread: Stock	0.79* (0.42)	0.96* (0.56)	0.65 (0.65)	0.37 (0.74)	-1.26 (0.81)	2.05*** (0.61)
Bid-Ask Spread: Put	0.89*** (0.40)	0.81 (0.54)	0.62 (0.65)	0.39 (0.75)	-1.16 (0.83)	2.05*** (0.65)
Return Reversal	0.73* (0.40)	0.99* (0.57)	0.75 (0.63)	0.11 (0.76)	-1.13 (0.83)	1.86*** (0.62)

Table 17: The table shows a robustness analysis for the IVOL phenomenon conditional on the high IVOLVOL regime, for the sample period 1996/01–2014/12. First, we sort stocks on stock characteristics in a high/low fashion and subsequently on IVOL basing on realizations in formation month. Then, we calculate next month returns and afterwards we average again along characteristics. Size is the market capitalization. We measure Liquidity by the liquidity beta of [Pastor and Stambaugh \(2003\)](#) and Volume by the average stock trading volume of one month. The bid-ask spreads for stocks and puts are calculated as the difference of bid- and ask-prices divided by the mid-prices. We measure return reversal as in [Huang et al. \(2010\)](#) by returns in formation month. *, ** and *** indicate statistical significance at the 90, 95, and 99% confidence level. Newey-West adjusted standard errors are stated in parentheses.

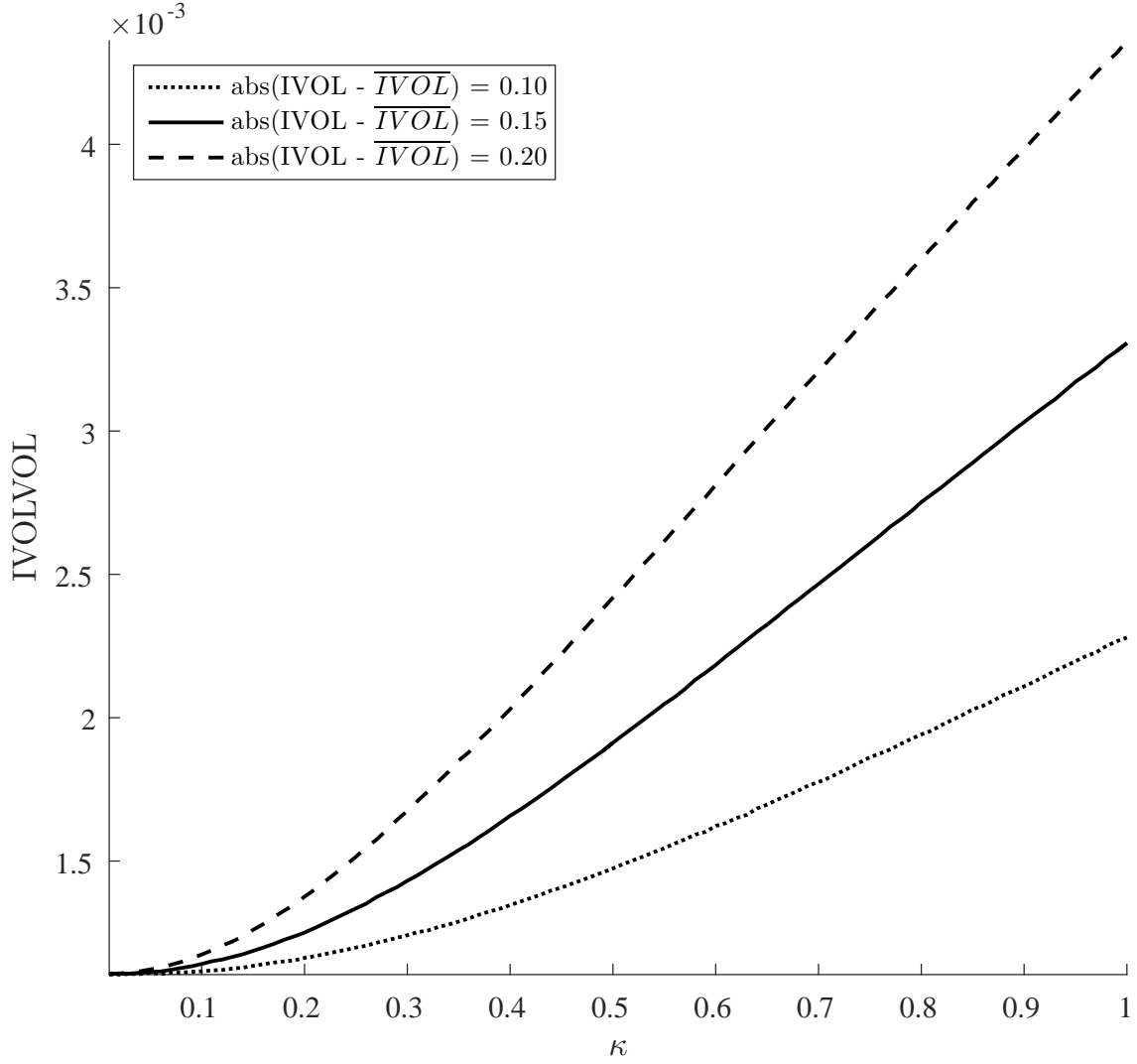


Figure 1: The figure displays the dependence of IVOLVOL on the mean-reversion coefficient κ^i in our model from section 2 for different IVOL levels. The model's coefficients are $\overline{\text{IVOL}}^i = 0.15$, $\sigma_{\text{IVOL}}^i = 0.01$. We simulate 200,000 paths of IVOL, using equation (5) over one month to generate the data.

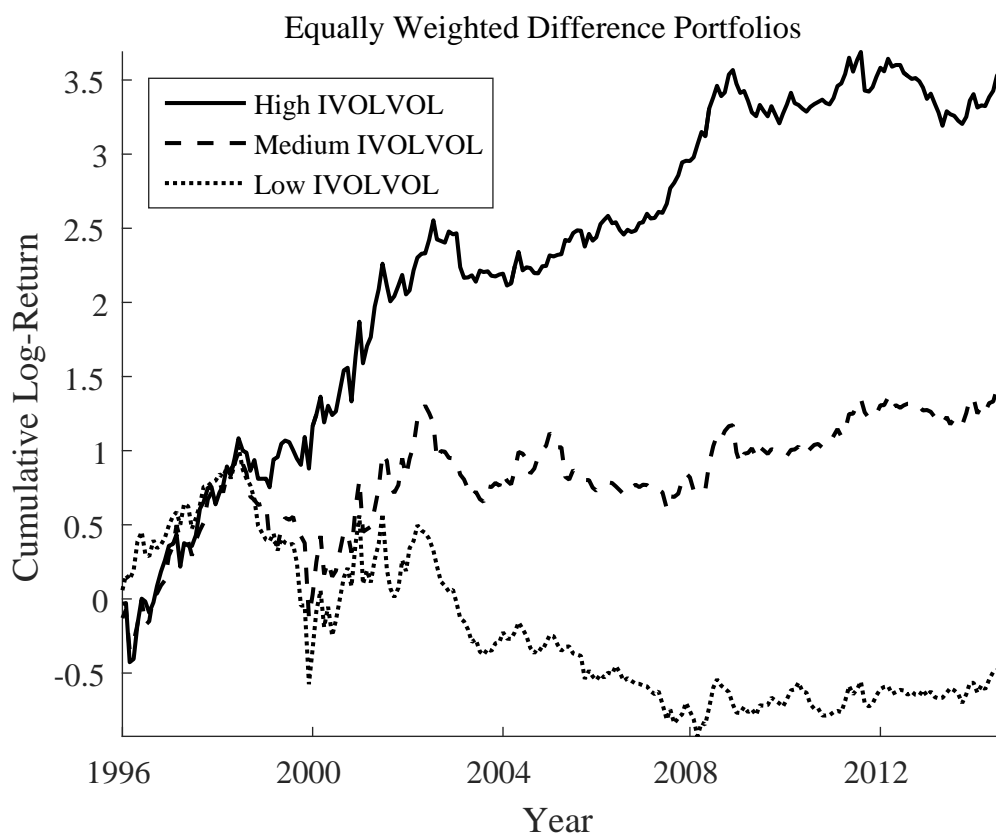


Figure 2: The figure displays cumulative log-returns of low-minus-high IVOL difference portfolios across different IVOLVOL regimes. Each month we conditional sort stocks first into five idiosyncratic risk quintiles and subsequent into three IVOLVOL terziles. Afterwards, for each IVOLVOL regime we calculate returns of equally weighted low-minus-high IVOL portfolios over the next month and accumulate.