The Best in Town:

A Comparative Analysis of Low-Frequency Liquidity Estimators

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This Draft§

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Abstract

In this paper we conduct the most comprehensive comparative analysis of low-frequency liquidity measures so far. We review a large number of estimators and use a broad range of procedures to evaluate them. We find that the performance of the estimators is highly dependent on the particular application, and that no single best estimator exists. Against this background, we further analyze which firm characteristics determine the accuracy of the low-frequency estimators, we analyze whether a composite low-frequency estimator can outperform the best individual measures, and we analyze whether changes in the trading protocol (such as a reduction of the minimum tick size or the introduction of NYSE Open Book and NYSE Hybrid) affect the performance of the low-frequency estimators. Our ultimate objective is to guide researchers in their search for the right measure for a particular application.

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1 Introduction

The availability of accurate measures of liquidity is of utmost importance for empirical research in finance. This is obviously true for research in market microstructure where liquidity is recognized to be one of the most important, if not the most important measure of market quality. The empirical asset pricing literature has accumulated convincing evidence that the liquidity of an asset affects its expected rate of return and, in turn, the cost of capital of the issuer (see e.g. Amihud and Mendelson (1986), Pastor and Stambaugh (2003), Acharya and Pedersen (2005)). More recently, research in corporate finance has uncovered several channels through which liquidity and corporate financing decisions are interrelated.[1]

The most widely used measures of liquidity are quoted and effective bid-ask spreads. Direct estimation of the spread requires intraday data on bid and ask prices and (for the effective spread) transaction prices. This data is often unavailable. Even if the data is available direct estimation of the spread may be burdensome because of the tremendous increase in trading and quotation activity we have witnessed in the last two decades. Therefore, researchers have developed various methods to estimate the spread from low-frequency (usually daily) data. This immediately raises the questions (1) of the general accuracy of these low-frequency measures and (2) of their relative performance.

In the present paper we address both questions. We use the effective spread and the price impact calculated from high-frequency data as benchmark measures and then evaluate the low-frequency estimators against these high-frequency benchmarks. The main metrics to assess the performance of the low-frequency estimators are their cross-sectional and time-series correlations with the high-frequency benchmarks and the mean absolute and root mean squared error (RMSE).

We are not the first to evaluate the relative performance of alternative liquidity proxies. Several papers that propose a new low frequency estimator compare its performance

[1] Recent examples include research on share repurchases (Hillert et al. (2016)), on corporate governance (Chung et al. (2010)) and on shareholder activism (Norli et al. (2015)). See also the survey by Amihud and Mendelson (2008).

to that of existing measures in order to demonstrate the superiority of the measure that is advocated in the paper. These horse races yield ambiguous results (see Goyenko et al. (2009), Hasbrouck (2009), Holden (2009), Fong et al. (2011), Corwin and Schultz (2012), Abdi and Ranaldo (2016), Tobek (2016)).

Some papers have extended the evaluation approach to asset classes other than equities. Marshall et al. (2012) evaluate liquidity proxies in commodities markets and conclude that the Amihud (2002) illiquidity ratio, the Amivest measure and the effective tick measure (Holden (2009), Goyenko et al. (2009)) perform well. In contrast, Karnaukh et al. (2015) find that the Corwin and Schultz (2012) high-low estimator performs well in FX markets. This result is confirmed for bond markets by Schestag et al. (2016). These authors conclude that the Roll (1984) serial covariance estimator and the Hasbrouck (2009) Gibbs sampling approach also perform well in bond markets.

The contribution of our paper to the literature is threefold. First, ours is the most comprehensive study so far. We evaluate a large number of low-frequency estimators. We estimate both cross-sectional and time-series correlations as well as mean absolute errors and root mean squared errors, we employ different weighting schemes (equal-weighted, value-weighted and observation-weighted), apply the liquidity proxies to individual stocks as well as to portfolios, and use data from different markets (NYSE, Nasdaq and Amex). Further, we use both the effective spread and the price impact as high-frequency benchmarks. This is potentially important because some of the low-frequency estimators we evaluate (most notably the Amihud (2002) illiquidity ratio) do not try to estimate the bid-ask spread but are rather measures of the price impact. We also follow Chung and Zhang (2014) and include the daily closing bid-ask spreads contained in the CRSP data base in our evaluation. Second, inspired by Baker and Wurgler (2006) we construct two composite liquidity measures. The first is based on the first principal component of a set of low frequency estimators while the second is based on an approach that maximizes the correlation between a linear combination of low-frequency estimators and the effective bid-ask spread. We then perform out-of-sample tests to assess the extent to which these composite measures improve upon the performance of the best individual low-frequency estimators. Third, we shed light on the variables that determine the performance of the estimators. In this context we show that the time-series correlation of the low-frequency estimators with the effective spread benchmark depends, in predictable ways, on the liquidity, market capitalization, turnover, age and listing location of a stock. It is further conceivable that the performance of some or all of the estimators we analyze depends on the regulatory regime. We therefore analyze how the accuracy of the liquidity proxies is affected by changes in the minimum tick size and other regulatory changes on the NYSE (NYSE Open Book and NYSE Hybrid) and Nasdaq (Nasdaq Order Handling Rules).

Our results can be summarized as follows. The low-frequency estimators are generally better able to track the (cross-sectional and time-series) variation in the effective spread than variation in the price impact. They are further better at tracking levels than first differences. The performance of some of the low-frequency estimators is extremely sensitive to minor changes in methodology. Some estimators (e.g. the CRSP closing spread and the measures recently proposed by Tobek (2016)) generally perform well while other estimators display good performance only in specific settings or fail completely. Composite estimators do not improve upon the performance of the best individual estimators. The introduction of the Nasdaq order handling rules in 1997 tended to increase the accuracy of the low-frequency estimators while the reduction of the tick size on the NYSE from sixteenths to decimals in 2001 had the opposite effect. Other regime changes (most notably the introduction of NYSE Open Book and NYSE hybrid) did not have a first-order effect on the performance of the low-frequency proxies. The estimators we evaluate differ with respect to their data requirements. While some only require daily prices or returns, others also require data on trading volume and/or daily high and low prices. Data availability is thus also a decisive factor in the choice of the best estimator. Our results, summarized in Figure 1, allow researchers to choose the best low-frequency estimator in a specific research setting.

The paper is structured as follows. Section 2 introduces the liquidity measures that we analyze. Section 3 describes our data and the methodology. The results of our empirical analysis are presented in section 4. Section 5 concludes.

[Insert Figure 1 about here]

2 Liquidity Measures

In this section we describe the liquidity measures analyzed in this paper. We start by briefly introducing the high-frequency benchmark measures (based on intraday data), the effective bid-ask spread and the price impact. We then introduce the low-frequency measures (based on daily data). We sort these into two categories, low-frequency spread estimators and low-frequency price impact estimators.

2.1 Benchmark Measures and CRSP Closing Spread

The low frequency measures we evaluate are based on transaction prices. Therefore the appropriate benchmark measure is the effective spread because (1) it accounts for possible price improvement, and (2) it implicitly accounts for the fact that transactions tend to occur when the quoted bid-ask spread is low. The effective spread and the relative effective spread are calculated as

$$s_t^e = 2|p_t - m_t| \quad ; \quad s_t^{e,rel} = \frac{2|p_t - m_t|}{m_t} \tag{1}$$

where p_t denotes the transaction price and m_t the quote midpoint in effect immediately prior to the transaction.

In the presence of informed traders order flow is informative. Consequently, transactions will have a (permanent) impact on prices. This price impact can be measured by the change in the quote midpoint in an interval of length Δt after a trade,

$$s_t^{pi} = Q_t\left(m_{t+\Delta t} - m_t\right) \quad ; \quad s_t^{pi,rel} = Q_t \frac{(m_{t+\Delta t} - m_t)}{m_t} \tag{2}$$

where the trade indicator Q_t is 1 when the trade is buyer-initiated and (-1) when it is seller-initiated. Trade classification is based on the Lee and Ready (1991) algorithm. A common choice for Δt is 5 minutes. We follow this convention.[2]

The CRSP database provides data on closing bid and ask prices for our entire sample period.[3] Obviously, this data can be used to construct an estimate of the quoted bid-ask spread. Chung and Zhang (2014) provide evidence that the CRSP spread estimate is highly correlated with the spread estimated from intraday (TAQ) data in cross-section. The time-series correlation is high only for Nasdaq stocks. Against the backdrop of this favorable evidence we include the CRSP closing spread among the low-frequency estimators we evaluate in our empirical analysis.

2.2 Low-Frequency Estimators of the Bid-Ask Spread

2.2.1 The Roll measure

Roll (1984) has proposed a simple procedure to estimate the spread from transaction prices. Under a set of assumptions that effectively assumes away traders with private information he shows that the effective bid-ask spread is related to the serial covariance of successive price changes. Similarly, the relative effective bid-ask spread is related to the serial covariance of successive returns:

$$s^{Roll,level} = 2\sqrt{-Cov\left(\Delta p_t, \Delta p_{t-1}\right)} \quad ; \quad s^{Roll,ret.} = 2\sqrt{-Cov\left(\Delta r_t, \Delta r_{t-1}\right)} \qquad (3)$$

The logic of the Roll (1984) spread estimator applies to price changes at any frequency. Therefore, the Roll estimator can be applied to intradaily prices as well as to daily prices.[4]

[2]A five-minute interval to estimate the price impact is excessively long in the presence of high-frequency trading. However, given that (1) our sample period starts in 1993, long before high-frequency traders appeared in the markets, and (2) our sample is dominated by small firms for which the amount of high frequency trading is likely to be low, we decided to use five-minute intervals in our analysis. We also note that the choice of the interval length does not have a first-order effect on the results. This has been shown at the "short end" (1 to 20 seconds) by Conrad et al. (2015) and at the "long end" (5 to 30 minutes) by Huang and Stoll (1996).

[3]For a detailed account of the availability of closing bid and ask price data see Chung and Zhang (2014).

[4]Roll (1984) applied his estimator to daily and weekly prices.

Empirically it is often the case that the serial covariance of successive price changes is positive.[5] This is particularly true for stocks with low spreads. In these cases the Roll measure is not defined. Three procedures that are commonly applied in these cases are (1) to set the spread estimate to zero in these cases or (2) to drop the corresponding observations or (3) to calculate the Roll estimator as $s^{Roll} = -2\sqrt{Cov\left(\Delta p_t, \Delta p_{t-1}\right)}$ in those cases (which will result in negative spread estimates). We implemented all of those procedures. However, we only present results for the first version (i.e. we set the spread to zero if the covariance is positive) because this specification resulted in the most accurate estimates.[6] In the following we refer to the version of the measure based on price changes as *Roll 0* and to the version based on returns as *Roll 0 (ret)*.

Hasbrouck (2009) builds on the Roll (1984) measure and proposes a Bayesian estimation approach. The spread estimates are constructed using a Gibbs sampling procedure. The programs to calculate this measure are available on Joel Hasbrouck's homepage.[7] We also include Hasbrouck's Gibbs measure (denozed as *Gibbs*) in our analysis.

2.2.2 Zero-return based estimators

Lesmond et al. (1999) develop an estimator of total transaction costs denoted LOT. Total transaction costs include brokerage commissions and exchange fees besides the spread. Consequently, the LOT estimator should be larger than direct estimates of the effective bid-ask spread. The LOT estimator is based on a simple intuition. Absent transaction costs a trader with private information on the value of a security will trade on her information up to the point where the marginal price is equal to her estimate of the asset value. The price will thus eventually reflect her private information. If, however, the total transaction cost exceeds the expected gain from trading the trader will refrain from trading. Her information will then not be impounded into prices. If

[5] In our sample this is the case for 33% of the stock-month observations. See also Fama (1970), Ohlson and Penman (1985) and Fama and French (1988). For a detailed discussion of the statistical properties of the Roll estimator see Harris (1990). He puts special emphasis on the small sample properties of the estimator.

[6] The results for the other specifications are available upon request.

[7] http://people.stern.nyu.edu/jhasbrou/Research/GibbsCurrent/gibbsCurrentIndex.html.

transaction costs even for the trader with the highest expected gain from trading exceed those expected gains, a zero return will be recorded. By this argument a zero return observation is indicative of high transaction costs. Therefore, the fraction of zero return observations in a period can be used as a very simple proxy for transaction costs.

$$Zero = \frac{\#\ of\ zero\ return\ days\ in\ period}{\#\ of\ trading\ days\ in\ period} \tag{4}$$

We calculate two versions of the Zero estimator. In the first we use all trading days and in the second we only include days with positive trading volume. The results of both approaches are very similar. Therefore, we only report the results for the first version in the paper.[8]

Lesmond et al. (1999) then develop an extended model that also uses the information provided by non-zero returns. Assume that the unobservable "true" returns are generated by a market model

$$r^*_{i,t} = \beta_i r_{m,t} + \epsilon_{i,t} \tag{5}$$

If transaction costs were zero, observable returns would also be generated by that market model. With positive transaction costs, however, observed returns will be

$$r_{i,t} = \begin{cases} r^*_{i,t} - \alpha_{1,i} & \text{if } r^*_{i,t} \leq \alpha_{1,i} \\ 0 & \text{if } \alpha_{1,i} < r^*_{i,t} \leq \alpha_{2,i} \\ r^*_{i,t} - \alpha_{2,i} & \text{if } r^*_{i,t} > \alpha_{2,i} \end{cases} \tag{6}$$

where $\alpha_{1,i} < 0; \alpha_{2,i} > 0$ denote the transaction costs for a sale and a purchase, respectively. The intuition is similar to the one presented above. The marginal trader (the trader with the highest expected benefit from trading) will only trade if the true expected return exceeds the transaction costs. Otherwise, a zero return is observed. The model allows for different transaction costs for buying and selling as the marginal

[8]Results for the second specification are available upon request.

seller might be a short seller, and short sales may cause higher transaction costs than regular trades. Lesmond et al. (1999) derive the likelihood function which can be used to obtain maximum likelihood estimates of the parameters $\alpha_{1,i}$ and $\alpha_{2,i}$. The measure of the proportional roundtrip transaction costs is then

$$LOT = \alpha_{2,i} - \alpha_{1,i} \tag{7}$$

To estimate their model Lesmond et al. (1999) categorize the trading days in their sample into three groups, namely days with zero return of the stock under consideration, days with non-zero stock returns and negative market returns, and days with non-zero stock returns and positive market returns. Goyenko et al. (2009) propose an alternative categorization. They sort by the stock return only and thus categorize the observations into zero return days, positive return days and negative return days. They denote the resulting modified estimator LOT Y-split.

Fong et al. (2011) simplify the LOT measure. They assume that transaction costs for buying and selling are symmetrical ($-\alpha_{1,i} = \alpha_{2,i}$). Additionally, they replace the market model assumption by the assumption that true returns are normally distributed. Thus, they obtain the estimator

$$FHT = 2\sigma\Phi^{-1}\left(\frac{1 + Zero}{2}\right) \tag{8}$$

where Φ denotes the cumulative density of the standard normal distribution, σ is the standard deviation of daily returns, and $Zero$ is the proportion of zero return days as defined above.

Tobek (2016) proposes a modification of the LOT and FHT estimators. He does not differentiate between zero return and non-zero return days but rather between zero volume and positive volume days.

In our empirical analysis we include five of the estimators discussed in this section, namely, the number of zero returns (denoted *Zero*), the original LOT estimator (LOT), the LOT Y-split estimator (*LOT y-split*), the FHT estimator (*FHT*) and the modification of the FHT estimators proposed by Tobek (2016) (*Tobek FHT*).

2.2.3 The effective tick estimator

The minimum tick size set by the exchange determines the set of admissible prices. If the minimum tick size is one cent, all prices ending on full cents are admissible while prices ending on a fraction of a cent (sub-penny prices) are not. However, observed prices are not uniformly distributed over the full set of admissible prices. Rather, traders have a preference for particular (e.g. round) numbers. This phenomenon is referred to as price clustering (see Harris (1991)). The observed price clustering can be used to draw inferences on the spread (Holden (2009), Goyenko et al. (2009)). Assume the minimum tick size is one cent and the spread is five cent. It is assumed that a five cent spread is implemented on a five cent price grid. That is, even so the minimum tick size is one cent, traders behave as if it was five cents. By that assumption, we will not observe bid and ask prices of 40.41 and 40.46, respectively. Rather, we would observe 40.40 and 40.45.

Now assume a transaction price of 40.41 is observed. This price will only be observed when the spread (and thus the price grid that traders use) is one cent. Thus, we can attach a 100% probability to a one cent price grid to the observation. Assume next we observe a price of 40.45. This price can result from a one cent grid or from a five cent grid. The probability of observing a price ending on x5 cent when a one cent grid is used is 10% (10 prices out of a total of 100). The probability of observing a price ending on x5 cents when a five cent grid is used is 50% (10 prices out of a total of 20 because it is assumed that a 5 cent spread is implemented on a price grid that only comprises prices ending on x0 and x5). Thus, the price of 40.45 comes from a one cent grid with probability 0.167 ($= \frac{0.1}{0.6}$) and from a five cent grid with probability 0.833 ($= \frac{0.5}{0.6}$). Combining these numbers results in an expected spread equal to 4.33 cent ($= 0.167 * 0.01 + 0.833 * 0.05$). By this logic each price implies an expected distribution of price grids from which it is drawn. We can then calculate the expected spread that is implied by the observed price.

Averaging this over a sample of closing prices yields an estimate of the effective bid-ask spread(Holden (2009)[9], Goyenko et al. (2009)).

The resulting estimator, known as the effective tick estimator, can be calculated with or without observations from zero-volume days. If these observations are included, the quote midpoint is used to infer the bid-ask spread. We have implemented both versions. Because the results were almost identical we only report those for the version that excludes zero-volume days. We denote the estimator *Effective Tick*.

Obviously the effective tick estimator has to be adjusted to the prevailing minimum tick size.[10] Our sample period covers three minimum tick size regimes, eighths, sixteenths, and decimals. We derive an appropriate version of the effective tick estimator for each of these regimes[11] and apply it to our data during the period in which the respective regime was in effect.

2.2.4 High-low spread estimators

Corwin and Schultz (2012) propose an estimator that is based on the following intuition: The highest [lowest] price observed on a trading day will typically result from a transaction at the ask [bid] price. The difference between the daily high and low price thus contains one component which is related to the spread and one component which is related to the volatility of asset returns. The problem is to disentangle these components. Corwin and Schultz (2012) assume that (a) true asset prices follow a diffusion process and (b) the bid-ask spread is constant over time. Consequently, the variance of changes in the true asset value increases proportionally with time while the contribution of the spread to the high-low difference does not. Under these assumptions the difference between the daily high and low price contains once the component related to the variance of price changes and once the component related to the spread. The difference between the highest and the lowest price measured over a two-day interval contains twice the component related

[9]Holden (2009) also constructs combined estimators which are a linear combination of the effective tick and the Roll estimators.

[10]See appendix A in Holden (2009).

[11]Details are available upon request.

to the variance of price changes but still only once the component related to the spread.[12] We thus essentially have two equations and two unknowns and can solve for the spread estimator

$$CS = \frac{2\left(e^{\alpha} - 1\right)}{1 + e^{\alpha}} \tag{9}$$

with $\alpha = \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}}$, where β and γ are the sample estimates of $E\left\{\sum_{j=0}^{1}\left[ln\left(\frac{H_{t+j}}{L_{t+j}}\right)\right]^2\right\}$ and $E\left\{\left[ln\left(\frac{H_{t,t+1}}{L_{t,t+1}}\right)\right]^2\right\}$, respectively. H and L denote the observed high and low prices. The parameter β contains the sum of the high-low price ratio for two individual days t and (t+1) while the parameter γ contains the high-low price ratio calculated from the high and low prices observed over the two-day interval from day t to day (t+1). One advantage of the CS estimator is that it does not require a long time series. Observations from any two trading days are sufficient to derive a spread estimate. The CS estimator can become negative. As with the Roll estimator, this is more likely to happen when the spread is small.

The derivation of the CS estimator as presented above is based on a simplifying assumption that essentially treats Jensen's inequality as an equality. We also implement a version of the CS estimator that does not require this assumption. The modified estimator has the drawback that it can only be obtained numerically. The results were similar to (but slightly worse than) those obtained when using the simple CS estimator. We therefore only include the simple CS estimator in our analysis.[13]

Tobek (2016) proposes a modified version of the CS estimator. The main difference is that Tobek (2016) uses the arithmetic mean of the log price range over a two-day interval, $\frac{1}{2}\left[ln\frac{H_t}{L_t} + ln\frac{H_{t+1}}{L_{t+1}}\right]$, while Corwin and Schultz (2012) use the square root of the

[12]This idea is reminiscent of the market efficiency coefficient (MEC) proposed by Hasbrouck and Schwartz (1988). The MEC is simply the ratio of a stock's return variance measured over a long interval divided by T times the return variance over a short interval. T is the length of the long interval divided by the length of the short interval. The MEC is expected to be smaller than one, and to decrease in the illiquidity of a stock.

[13]The results for the modified estimator are available upon request.

sum of squared price ranges, $\left\{\left[ln\frac{H_t}{L_t}\right]^2 + \left[ln\frac{H_{t+1}}{L_{t+1}}\right]^2\right\}^{0.5}$. Tobek (2016) argues that the arithmetic mean is more robust and that, therefore, his estimator will be less affected by variations in volatility.

We implement two versions of the CS estimator, the original version (denoted *Corwin 0*) and the modified version developed by Tobek (2016) (denoted *Tobek Corwin 0*). In both cases negative spread estimates are set to zero.[14]

Abdi and Ranaldo (2016) propose an alternative estimator based on daily high and low prices. They argue that the average of the mid-range between the daily high and low prices on day t and the midrange on day $(t+1)$ is a natural proxy for the quote midpoint (or, for that matter, the efficient price) at the close of day t. The squared difference between the actual closing price and this estimator of the efficient price can then be interpreted as an estimate of the sum of the squared effective half-spread and a term that captures the transitory volatility of the efficient price. The squared differences between the midrange of the daily high and low prices on day $(t+1)$ and the midrange on day t delivers an estimate of the transitory volatility. Combining both expressions yields a spread estimator of the form

$$s^2_{Abdi} = 4E\left[\left(c_t - \frac{\eta_t + \eta_{t+1}}{2}\right)^2\right] - E\left[(\eta_{t+1} - \eta_t)^2\right] \tag{10}$$

with c_t being the log closing price on day t and $\eta_t = \frac{ln(H_t)}{ln(L_t)}$. Obviously, the expectations have to be replaced by appropriate estimates. Abdi and Ranaldo (2016) propose two approaches. In the first approach (which we denote *Abdi monthly*) expectations are replaced by monthly averages to obtain a monthly spread estimator. If the resulting estimator of the squared spread is negative it is replaced by zero. In the second approach an estimator of the squared spread is obtained for consecutive two-day periods. Negative estimates are again replaced by zero. The square roots of these estimates are then

[14] We also implemented versions of both estimators which include negative values in the calculation of the monthly or yearly averages. We found, however, that the performance of the estimators improves when negative values are set to zero. Therefore, only results for this latter version are reported in the paper.

averaged over the days of the month to obtain a monthly spread estimate. We refer to this version as *Abdi 2-day*. We include both versions of the Abdi and Ranaldo (2016) estimator in our empirical analysis.

2.2.5 The Tobek measure

As mentioned above, Tobek (2016) develops modified versions of several low frequency estimators. However, the main contribution of his paper is to show empirically that the bid-ask spread is closely related to a function of volume and volatility.[15] Specifically, he finds that the ratio

$$VoV_i = 2.5 \frac{\sigma_i^{0.6}}{V_i^{0.25}} \tag{11}$$

has very high cross-sectional correlation with the bid-ask spread. σ is estimated either by the sum of squared daily returns or by the Parkinson (1980) high-low volatility estimator, and V_i is the average of the daily trading volume. The factor 2.5 is simply a scaling factor that aligns the mean of the volatility-to-volume ratio with the average spread in the US during Tobek's sample period 1926-2015. We include the Tobek estimator in our horse race and denote it VoV (for "volatility over volume"). We follow the recommendation by Tobek (2016) and estimate the standard deviation using the high-low variance estimator proposed by Parkinson (1980). However, since there may be occasions where data on high and low prices is unavailable we also include the version of the estimator that uses the sum of squared daily returns to estimate sigma. The two estimators are denoted *VoV High-Low* and *VoV Sigma*, respectively.

[15]Kyle and Obizhaeva (2014) develop, on theoretical grounds, a liquidity measure which is closely related to Tobek's measure. It is defined as the dollar trading volume to the power of $\frac{1}{3}$ divided by the standard deviation of returns to the power of $\frac{2}{3}$.

2.3 Low-Frequency Estimators of the Price Impact

2.3.1 The Amihud illiquidity ratio

In a liquid market the price change in response to a given trading volume will be small; in an illiquid market it will be large. This intuition suggests relating price changes to trading activity. The first measure we are aware of that builds on this intuition is the Amivest ratio.[16] It is defined as the sum of daily volume divided by the sum of absolute daily returns.

$$Amivest_{it} = \frac{\sum V_{i,t}}{\sum |r_{it}|}, \tag{12}$$

Amihud (2002) has proposed the illiquidity ratio

$$Illiq_i = \frac{1}{D_i} \sum_{t=1}^{D_i} \frac{|r_{it}|}{V_{it}}, \tag{13}$$

$r_{i,t}$ and $V_{i,t}$ are the return and the dollar trading volume of stock i on day t, respectively, and D_i is the number of days in the evaluation period (often a month or a year). Only days with non-zero volume are included. The illiquidity ratio has several advantages. It has low data requirements, it is easy to calculate, and it has a theoretical foundation based on Kyle (1985). Therefore it has become very popular and is widely used. However, the measure also has its drawbacks.[17] Most importantly, it is unable to differentiate between price changes that are related to new information and those that are not. Every event that causes a large price change (such as a merger announcement) is taken as evidence of illiquidity.

The Amihud illiquidity ratio measures by how much one dollar of trading volume

[16] The Amivest ratio has been applied in academic research by Cooper et al. (1985).

[17] Grossman and Miller (1988) discuss the suitability of the Amivest ratio. Their arguments also apply to the illiquidity ratio. Acharya and Pedersen (2005) contend that the illiquidity ratio is not stationary. Its unit of measurement is percent return per dollar of trading volume. Thus, the measure ignores inflation. This is an important issue in asset pricing studies which typically cover very long sample periods. Acharya and Pedersen (2005, p. 386) propose to solve this problem by scaling the illiquidity ratio. Brennan et al. (2013) analyze the asset pricing implications of the illiquidity ratio in detail. They find that it is reliably priced, but that the pricing is caused by those components of the illiquidity ratio that are related to negative return days.

moves the price of an asset. An alternative question is 'how much volume does it take to move the price of an asset by one dollar?'. This is the question the LIX measure, proposed by Danyliv et al. (2014), tries to answer. The measure is defined as

$$LIX_{it} = log_{10}\left(\frac{V_{i,t}P_{close}}{H_{i,t} - L_{i,t}}\right) \tag{14}$$

The authors propose the log specification in order to restrict the range of values their measure can assume. They argue that a log with the base 10 would result in values between 5 and 10.

We include in our empirical analysis all three measures, the Amivest ratio (denoted *Amivest*), the Amihud (2002) illiquidity ratio (*Amihud*) and the LIX measure (*LIX*). Note that the Amivest ratio and the LIX measure are measures of liquidity because larger numerical values indicate higher levels of liquidity. We therefore multiply both measures by (-1) before including them in our horse race.

Tobek (2016) also proposed a modified version of the Amihud (2002) illiquidity ratio based on the volatility-to-volume ratio. The daily version is defined as

$$VoV_{daily} = \frac{log\left[\frac{H_t}{L_t}\right]^{0.6}}{V_{i,t}^{0.25}} \tag{15}$$

Monthly and yearly estimates are obtained by averaging over the daily values. We include the modified illiquidity ratio in our horse race and denote it *VoV daily*.

2.3.2 The Pastor/Stambaugh measure

Pastor and Stambaugh (2003) propose to run the following regression

$$\left(r_{i,(t+1)} - r_{m,(t+1)}\right) = \alpha_i + \phi_i r_{i,t} + \gamma_i * \left(sign\left(r_{i,t} - r_{m,t}\right) * V_{i,t}\right) + \epsilon_{i,t}, \tag{16}$$

where $r_{i,t}$ is the return of stock i on day t, $r_{m,t}$ is the return on a stock index on day t and $V_{i,t}$ is the dollar trading volume of stock i on day t. The coefficient γ_i measures the sensitivity of a stock's excess return over the index with respect to lagged signed volume.

The intuition is as follows: Volume moves prices. However, some of the price change is transitory and will be reversed on the next trading day.[18] The coefficient γ_i measures this reversal and is thus expected to be negative. The less liquid a stock, the higher the temporary price change and the reversal should be. Thus, less liquid stocks should have higher absolute (i.e. more negative) γ_i. We multiply γ_i by (-1) in order to obtain larger values for less liquid stocks.

Estimation of γ_i requires a market proxy $r_{m,t}$. We use the CRSP value weighted index, the CRSP equally weighted index and the S&P500. The results are very similar. We therefore only report the results for the CRSP value weighted index. The resulting estimator is denoted *Gamma*.

2.4 Summary of Estimators

Table 1 lists all estimators that we include in our empirical analysis and the data that is necessary to apply them. While some of the estimators only require data on closing prices or daily returns, others also require volume data or a time series of daily high and low prices. Three estimators (the two versions of LOT and the Pastor and Stambaugh (2003) γ) require a time series of market returns. Consequently, there may be situations in which only a subset of the low-frequency estimators can be applied because of unavailability of data. The results of our empirical analysis may inform researchers about which of the feasible estimators (i.e. those for which the required data is available) is expected to perform best in a specific application.

[Insert Table 1 about here]

[18]Pastor and Stambaugh (2003) implicitly assume that aggregate order flow has a transitory price impact which shows up in daily returns and is reversed on the next day. In contrast, other price impact measures (e.g. the 5-minute price impact introduced earlier or the trade indicator models proposed by Glosten and Harris (1988), Huang and Stoll (1997) and Madhavan et al. (1997)) implicitly assume that the transitory price impact is very short-lived.

3 Data

In order to calculate the different estimators daily information on the entire US equity market is collected from CRSP. As mentioned in the preceding section, estimators differ in terms of data requirements and computation time. For some estimators only daily closing prices or daily returns are needed.[19] Other measures, however, require additional information. Calculation of the Amihud illiquidity ratio, for example, requires information on daily trading volume. As a consequency of the differing data requirements there are cases in which some of the low-frequency estimators can be calculated for a given stock-month while others (because of data availability) cannot. The number of stock-month observation is thus slightly different for different estimators.[20] Most estimators are easy to compute, however, some estimators (e.g. the LOT and the Gibbs estimators) are computation-intensive.

To assess the quality of the different estimators, we use (as described in section 2) effective spreads and price impacts calculated from the TAQ data base as benchmark measures. We obtained daily averages of these variables from the Market Microstructure Database maintained by the Vanderbilt University.[21] These daily averages are later aggregated to stock-month and stock-year averages in order to compare them to the Low-frequency estimators which we also calculate at the stock-month and stock-year level.

3.1 Sample Selection

Requirements on our dataset are nor very restrictive. We include all (common) stocks (sharecode 10 and 11) that were listed on one of the three exchanges Nyse, Amex, Nasdaq during the period from January 1^{st} 1993 until December 31^{st} 2012. We then eliminate

[19]We use returns corrected for dividends and stock splits.

[20]As a robustness check we repeat our entire analysis on a subsample for which all estimators are available in each month. The results are qualitatively similar to those presented in the paper and are available upon request.

[21]We thank the Vanderbilt University for providing the data. The daily averages for NYSE [Amex] stocks are based on NYSE [Amex] quotes only. Daily averages for Nasdaq stocks are based on the NBBO. We checked the quality of the data by directly calculating effective spreads from TAQ data for one year. The daily average spreads were identical for 99% of the sample.

months with stock splits, exchange, ticker or cusip changes and months with special trading or security status. We thus end up with a sample of about 27 million firm-day observations. This includes both firms that were delisted as well as firms that were newly listed during our sample period. In contrast to previous literature, we use the whole universe of stocks listed on one of the three exchanges and not just a random subsample.[22]

[Insert Table 2 about here]

Based on this sample we calculate the different estimators for each firm-month and each firm-year. We implement both versions because in some applications (e.g. in asset pricing) researchers typically use monthly data while in other application yearly data is preferred.

Table 2 shows that we end up with about 1.3 million firm-month observations after calculating all estimators. We match this data set with the intraday data (aggregated to the monthly/yearly level) based on 8-digit CUSIPs. The matched data set contains 1,083,680 observations. In some cases the stock-month liquidity estimates are based on a small number of daily observations. To reduce estimation error we therefore include only stock-month observations that are based on at least 12 daily observations. The final data set contains 1,079,509 stock-month observations.

Table 3 shows the number of firms in our sample, both in total and in each year of the sample period. The number of stocks peaks in the late 1990s while it reaches its minimum towards the end of our sample period after the financial crisis. Market shares of the three different exchanges in terms of listed firms are actually very stable over time. 60% and 30% of all firms are listed on NASDAQ and NYSE, respectively.

[Insert Table 3 about here]

[22]See Goyenko et al. (2009, p.161).

3.2 Summary Statistics

Table 4 provides descriptive statistics for our sample. We only include observations for which our benchmark measure, the effective spread, is available. The market capitalization, averaged over more than 1 million stock-month observations, is $ 1,840 million. The median value is only $ 248 million, implying that the size distribution is heavily skewed. The same applies to the distributions of the daily turnover ratio (defined as the ratio of dollar trading volume and market capitalization) and the number of trades. The average percentage quoted and effective spreads amount to 1.88% and 1.47%, respectively. The average CRSP closing spread is larger, at 2.16%.

[Insert Table 4 about here]

The summary statistics shown in Table 4 mask the significant changes that occurred during the sample period. Figure 2 reveals that the daily dollar trading volume increased almost tenfold between 1993 and 2013 while the effective spread decreased from slightly below 2% to approximately 0.5% (with a temporary increase during the financial crisis).

[Insert Figure 2 about here]

Table 4 also shows summary statistics for all the spread estimators that we include in our analysis. When comparing the mean values to the average quoted and effective spreads it should be kept in mind that not all estimators attempt to estimate the spread level. This holds for the Amivest ratio, the Amihud illiquidity ratio and the LIX measure, for the percentage of zero returns, and for the Pastor and Stambaugh (2003) gamma. The LOT and FHT measures estimate the total transaction costs and should therefore be larger than the effective spread. The Roll 0 estimator and the Hasbrouck (2009) Gibbs sampler estimate the dollar spread while the remaining estimators estimate the percentage spread. Figure 3 visualizes the mean values. Only the LOT y-split estimator yields a mean value that is within 10% of the mean effective spread. An additional two low-frequency estimators (Abdi 2-day and Corwin 0) yield a mean within a 20% range around the mean effective spread. Of course a mean value close to the benchmark value does not guarantee that a low-frequency estimator is an accurate liquidity proxy. In the main analysis of the paper we will therefore analyze the cross-sectional and time series correlation between the proxies and the benchmark, and we will evaluate the mean absolute errors and the root mean squared errors of the low-frequency estimators.

[Insert Figure 3 about here]

3.3 Methodology

Our main analysis proceeds as follows: As described above, we first calculate the benchmark measures and all low-frequency estimators (including the CRSP closing spread) for each stock and each month. Based on this data we then estimate correlations between each of our low-frequency estimators and (a) the percentage effective spread and (b) the 5-minute price impact. We repeat the procedure using stock-year observations instead of stock-month observations.

The correlations are estimated in three different ways. First, we calculate cross-sectional correlations (both in levels and in first differences) for each month of the sample period, resulting in a time-series of 240 (239 for the first-differenced data) monthly correlations. Second, we calculate time-series correlations at the portfolio level. To this end, we first calculate the (equally-weighted and value-weighted) average liquidity for all sample stocks in a given months. This procedure is implemented both for the benchmark measures and the low-frequency estimators. We thus obtain one time series of portfolio-level liquidity for each measure. Based on these time series we then calculate time-series correlations in levels and first differences. We refer to this procedure as "time-series portfolio". Third, we calculate time-series correlations between the benchmark measures and the low-frequency estimators at the individual stock level, again both in levels and in first differences. We then calculate (equally-weighted and value-weighted) cross-sectional averages of these time-series correlations. We refer to this procedure as "time-series stock-by-stock". Finally we calculate mean absolute errors (MAE) and root mean squared errors (RMSE) for those low-frequency estimators that attempt to estimate the percentage effective spread.

To put our approach into perspective, Table 5 lists which previous papers have used what methodology to evaluate low-frequency spread estimators. The table reveals that our paper is the most comprehensive study so far. Ours is the only paper besides Goyenko et al. (2009) that uses both monthly and yearly liquidity estimates as the basic unit of investigation. It further is the only paper that implements both a stock-by-stock and a portfolio approach to evaluate the time-series correlation, and it is the only one that implements several weighting schemes for the correlation analysis (equal-/value- and observation-weighting).

[Insert Table 5 about here]

4 Empirical Findings

4.1 Cross-Sectional Analysis

Table 6 shows the results for the cross-sectional correlations in levels. It reports the average number of stocks included in the monthly cross-sections, the time-series average of the monthly cross-sectional correlations (including the result of a t-test of the time-series average against zero), the percentage of months with a positive cross-sectional correlation and the percentage of month with a cross-sectional correlation that is significantly larger than zero at the 5% level. This information is provided for both benchmark measures, the percentage effective spread and the price impact.

[Insert Table 6 about here]

We first consider the effective spread as benchmark measure. There are huge differences in the performance of the various low-frequency estimators. The four best estimators exhibit average cross-sectional correlations above 86%. The highest correlation

(87.6%) is achieved by the version of the Tobek (2016) estimator that relies on daily high and low prices to estimate volatility (denoted VoV High-Low), closely followed by the CRSP closing spread (87.4%) and the two other versions of the Tobek (2016) estimator, the VoV daily estimator(86.9%) and the version that uses squared daily returns to estimate volatility (VoV sigma, 86.4%). All other estimators achieve markedly lower correlations. The Tobek (2016) version of the FHT estimator, the Abdi monthly and Abdi 2-day estimator, the LIX estimator and the Amihud (2002) illiquidity ratio are the "best of the rest", with average cross-sectional correlations ranging from 73.5% to 64.8%. At the other end of the spectrum, three estimators (the Roll (1984) estimator based on price changes, the Hasbrouck (2009) Gibbs sampler and the Pastor and Stambaugh (2003) gamma) achieve average correlations below 10%. In addition, there are months in which the cross-sectional correlation between these estimators and the effective spread is actually negative.

Figure 4 plots the cross-sectional correlation between ten of the low-frequency estimators and the effective spread for each month of the sample period. The VoV High-Low measure appears to be the most consistent estimator. It achieves correlations above 80% in every single month. The CRSP closing spread achieves higher correlation than the VoV High-Low measure in the beginning of the sample period (until 1997) and from 2003 onwards. Between 1998 and 2002 the performance of the CRSP closing spread deteriorates. The performance of some of the other low-frequency measures declines over time. This is particularly true for the Roll measure, the Abdi 2-day estimator and for the Corwin and Schultz high-low estimator. A potential explanation for this finding is that effective spreads have generally decreased over time (partly because of decimalization)[23], and that these estimators may perform worse in a low-spread environment. We return to this issue in section 4.5. In this context it is also interesting to note that the performance of some estimators appears to improve during the financial crisis. This is particularly true for the Amihud illiquidity ratio which achieves its best performance between November 2008

[23]see Figure 2.

and January 2011.

[Insert Figure 4 about here]

The cross-sectional correlations based on first differences are much lower than those based on levels. However, as is documented in Table 7, the same four measures as before perform best, with the CRSP closing spread being the top performer (correlation 58.3%) followed by the three versions of Tobek's measure (correlations between 46.8% and 41.4%).

[Insert Table 7 about here]

Columns 6-9 in Table 6 reveal that the cross-sectional correlations between the low frequency estimators and the price impact are much lower than those with the effective spread. This even holds for those estimators that are constructed as measures of price impact (the Amihud illiquidity ratio and the Pastor and Stambaugh gamma). The four best performing measures are again the CRSP closing spread and the three version of Tobek's measure, with the VoV daily measure being the top performer (average correlation 40.3%).

None of the low-frequency estimators performs well when first differences are benchmarked against the price impact. Even the best performing proxy, the Tobek daily measure, has an average cross-sectional correlation below 10%.

To summarize, there are remarkable differences in the performance of the low-frequency liquidity estimators. The best proxies capture the cross-sectional pattern of the effective spread levels very well (with average correlations above 86%), while the worst-performing proxies achieve values below 10%. The proxies do much worse when benchmarked against the price impact rather than the effective spread. Further, we find that the low-frequency liquidity estimators are much better at tracking levels than at tracking first differences. The best performing measures are the three version of Tobek's measure and the CRSP closing spread.

4.2 Timeseries Analysis

4.2.1 Portfolio-Level Correlations

Table 8 shows the portfolio-level time series correlations. Two striking findings emerge immediately. First, the correlations are much higher than the cross-sectional correlations discussed in the previous section. Second, the results for some of the low-frequency proxies are extremely sensitive to the weighting scheme (i.e. equally-weighted versus value-weighted portfolios). This is particularly true for the Roll estimator, the two versions of the high-low estimator, the two versions of the Abdi and Ranaldo estimator and the Gamma estimator. These are precisely the measures that exhibit strongly decreasing cross-sectional correlation over time (see Figure 4). We have conjectured in the preceding section that these measures perform poorly in a low-spread environment. Because larger firms have lower spreads, these measures are likely to perform poorly when the liquidity of a value-weighted portfolio is considered.

[Insert Table 8 about here]

When the effective spread is used as benchmark, the FHT estimator and the effective tick estimator perform very well both for equally-weighted and for value-weighted portfolios (correlations range from 97.7% to 98.7%). The CRSP closing spread, the zero estimator and the two versions of the LOT measure perform well for the equally-weighted portfolio but slightly less well for the value-weighted portfolio. The reverse is true for the LIX ratio.

The ranking of the measures changes considerably when we consider first differences instead of levels (see Table 9). Now the three versions of the Tobek measure perform best both for the equally-weighted and for the value-weighted portfolio. Correlations range from 75.2% to 87.8%. The CRSP closing spread performs reasonably well for the equally-weighted portfolio (correlation 78.9%) but does much less well for the value-weighted portfolio (37.4%).

[Insert Table 9 about here]

The ranking of the low-frequency proxies changes yet again when we consider the price impact as benchmark measure. Now the return-based Roll measure performs best for the equally-weighted portfolio (correlation 80.5%) while the Abdi two-day measure comes out first for the value-weighted portfolio. As before, correlations drop when we consider first differences rather than levels. The Abdi two-day measure is still the best performing measure for the equally-weighted portfolio (59.0%). However, several other measures (most notably VoV daily, VoV sigma and the return-based version of the Roll measure, with correlations ranging from 40.3% to 38.8%) come out ahead for the value-weighted portfolio.

In summary, the results for the portfolio-based time-series approach are heterogeneous. The relative performance of the low-frequency proxies depends on the weighting scheme (equally versus value-weighted portfolio), on the benchmark measure (effective spread or price impact), and on whether levels or first differences are considered. None of the measures we analyze performs well under all conditions.

In our opinion the portfolio-level time-series correlation, even so it has been applied in several previous studies (see Table 5) is not a very good metric to assess the performance of the low-frequency liquidity estimators. The portfolio-level time-series correlation identifies low-frequency liquidity measures that capture the time-series variation of liquidity at the level of the entire market or at the level of a broadly diversified portfolio. However, in most applications we can think of, a researcher will either be interested in a measure that captures the cross-sectional variation of liquidity or in a measure that captures the time-series variation at the stock level. In the first case the choice of a measure should be based on cross-sectional correlations (discussed in the previous section) while in the second case it should be based on stock-level time-series correlations (to be discussed in the next section).

4.2.2 Stock-Level Correlations

The results presented in the previous section were obtained by estimating the correlation between (weighted) averages of liquidity measures. In this section we report (weighted) averages of correlations estimated at the stock level. We apply three different weighting schemes, an equally-weighted average, a value-weighted average and an average weighted by the number of stock-month observations available for a given stock. The latter weighting scheme puts less weight on stocks with missing data, but also on stocks which left the sample (e.g. because of a merger or because of bankruptcy) and on stocks which went public after the beginning of our sample period.

Table 10 shows the results when the correlations are estimated for the levels of the liquidity measures. The correlations are generally lower than the portfolio-level correlations reported in the previous section. Five estimators (the CRSP closing spread, all three version of Tobek's estimator and the LIX estimator) achieve average correlations above 60% irrespective of the weighting scheme that is applied. The CRSP closing spread performs best when equally-weighted and observation-weighted averages are considered (correlations 79.7% and 81.3%, respectively) while the VoV daily estimator performs best (72.0%) when value-weighted averages are considered. Of the remaining estimators, the effective tick estimator and the Amihud illiquidity ratio show a reasonably good and consistent performance.

[Insert Table 10 about here]

Correlations obtained from first-differenced liquidity measures are considerably lower (Table 11). The same five measures as before perform best (with correlations ranging from 40.7% to 57.6%) when we consider equally-weighted or observation-weighted averages of stock-by-stock correlations. The Tobek measures and the CRSP closing spread continue to perform well (correlations between 33.5% and 52.8%) when value-weighted averages are considered while the LIX ratio (28.2%) does considerably worse in this case.

[Insert Table 11 about here]

As in the preceding sections correlations are much lower when the price impact rather than the effective spread is used as benchmark. The largest value drops to 37.0%. The five measures listed above continue to perform relatively well. When first differences are used instead of levels the correlations drop even further, with the largest value now being 15.9%. The VoV high-low estimator and the VoV daily estimator are still among the top four measures.

All in all, the results are rather similar to those obtained for the cross-sectional correlations. We observe huge performance differences across the low-frequency measures, we find that correlations are higher in levels than in first differences, and we find that the low-frequency proxies are better able to track the effective spread than the price impact. The best performing measures are the two version of Tobek's measure (VoV high-low and VoV daily), the CRSP closing spread and the LIX ratio.

4.3 RMSE/MAE

The correlations discussed in the preceding sections indicate how well the various low-frequency proxies capture the cross-sectional and time-series dispersion of the benchmark measures, but they do not provide information on how close the low-frequency estimators are to the benchmark measures. Therefore we also calculate mean absolute errors (MAE) and root mean squared errors (RMSE). This analysis is confined to those low-frequency proxies that estimate the percentage effective spread. We proceed as follows. We calculate, for each of the low-frequency estimators included, the absolute difference and the squared difference between the proxy and the effective spread for each stock-month observation. These deviations are then aggregated across stocks and months to obtain the MAE and RMSE.

The results are presented in columns 2 and 3 of Table 12. The RMSEs are generally larger than the MAEs (mean 1.97% as compared to 1.57%). There are four measures with a mean absolute error below 1%, the Abdi 2-day estimator (0.86%), the VoV high-low estimator (0.91%), the VoV sigma estimator (0.97%) and the effective tick estimator

(0.995%). Most of the other estimators have MAEs between 1.1% and 1.21%. Three estimators perform worse, with MAEs ranging from 1.56% (LOT y-split) to 5.86% (Roll). The same four estimators that have the lowest MAEs also have the lowest RMSEs, and the same three measures that have the highest MAEs also have the highest RMSEs.

It is noteworthy that the CRSP closing spread, which performed very well in the correlation analysis, is not among the top estimators in terms of MAEs and RMSEs. This may be due to the relatively poor performance, documented in Figure 4, of the CRSP closing spread between 1998 and 2002. It should also be noted that the CRSP closing spread is a quoted spread, and quoted spreads are larger, on average, than effective spreads.

[Insert Table 12 about here]

4.4 Yearly versus Monthly Aggregation

So far we have used liquidity measures at the stock-month level as our basic unit of observation. In many applications (e.g. panel studies with annual data) only a yearly measure of liquidity is needed. We therefore repeat the analysis using liquidity measures at the stock-year level as the basic unit of observation. Those yearly estimators are based on more (daily) observations and thus might be more precise. The main question we wish to address is whether the low-frequency estimators that perform best on monthly data also perform best when yearly data is used. We only present the results that we obtain when using the levels of the effective spread as benchmark measures.[24]

The results for the correlation analysis are shown in Table 13. For ease of comparison the correlations obtained from stock-month level data are repeated in the table. Columns 2 and 3 report the results for the cross-sectional correlation, columns 4 and 5 those for the time-series analysis at the (equally-weighted) portfolio level and columns 6 and 7 those for the time-series analysis at the stock-by-stock level (equally-weighted). Two

[24]Results with first differences of the effective spread and with the price impact as benchmark measures are available upon request.

main findings emerge. First, in most cases yearly liquidity measures result in higher correlations between the low-frequency proxies and the effective spread benchmark than monthly liquidity measures. Second, the ranking of the proxies is very similar for yearly and monthly data. For the cross-sectional correlation and the time-series correlations at the stock-by-stock level, the top-performing low frequency proxy is the same in both cases, and the measures on ranks 2 to 4 are also identical (although the ordering is slightly changed). For the portfolio-level time-series correlations, the top-three performers in the monthly analysis are also the top-three performers in the yearly analysis (although not in the same order).

[Insert Table 13 about here]

The mean absolute errors (MAE) and root mean squared errors (RMSE) are shown in columns 4 and 5 of Table 12. They are very similar to the results obtained from monthly data shown in columns 2 and 3 of the same table.

To summarize, the results presented in Table 12 and Table 13 allow a simple conclusion. The choice of a low-frequency liquidity proxy can be made independent of the data frequency (i.e. monthly or yearly) at which the proxy is used.

4.5 Sample splits

The results presented previously suggest that the performance of some of the low-frequency estimators depends on the level of the bid-ask spread. To explore this issue further we subdivide the sample into five quintiles according to the size of the effective bid-ask spread. We resort the stocks every month. A stock can thus be sorted into different quintiles over time. Based on these quintile sorts we then estimate the cross-sectional and time-series correlations as before. Specifically, we calculate the cross-sectional correlation between the low-frequency estimators and the effective spread benchmark for each quintile and each month and then calculate, for each quintile, the time-series average of the cross-sectional correlations. Next, we calculate the (equally-weighted) average liquidity of all stocks in a given quintile for each month, and for both the benchmark measure and the low-frequency estimators, resulting in one portfolio-level time series for each measure and each quintile. Based on these time series we then calculate portfolio-level time-series correlations. Finally, we calculate the time-series correlation between each low-frequency proxy and the benchmark measure for each stock within a quintile and then average the correlations across the stocks in a quintile. In the following we show results for quintiles one, three and five.

The results for the cross-sectional correlations are presented in Table 14. Three main findings emerge. First, there appears to be a pronounced u-shaped pattern across the quintiles. For most estimators the correlations are high in the quintile of stocks with the smallest spreads, then decrease and increase again in the quintile of stocks with the largest spreads. Second, when tracking the performance of the same estimator across the quintiles, we find that most estimators perform best in the largest spread quintile. However, some estimators (for example the effective tick estimator and the Corwin and Schultz (2012) high-low estimator) perform best in the smallest spread quintile. Remarkably, none of the estimators we analyze performs best in the intermediate quintile. Third, some low-frequency estimators perform well in all quintiles. The three versions of Tobek's estimator (VoV high-low, VoV daily and VoV sigma) are among the top four estimators in all three quintiles shown in Table 14. The CRSP closing spread performs well in the large-spread quintile but does much less well in the small spread quintile. The LIX estimator, on the other hand, does well in the small-spread quintile but is not among the top estimators in the larger spread quintiles.

[Insert Table 14 about here]

Table 15 shows the portfolio-level time-series correlations. The most consistent estimators, with correlations above 93% in all three quintiles, are the LOT and LOT y-split estimators, the FHT estimator and the effective tick estimator. While these estimators perform very well in the small spread quintile (LOT, FHT and Effective Tick) and in the medium spread quintile (LOT y-split, FHT and Effective Tick), they are not among the best estimators in the large spread quintile. Here, the CRSP closing spread performs best (correlation 99.3%), followed by the two versions of the Abdi and Ranaldo (2016) measure (99.2% and 99.1%, respectively). The different versions of Tobek's estimator, which performed very well in the cross-sectional analysis, achieve correlations between 87.3% and 95.6% but are not among the top four estimators in any of the three quintiles. Comparing the results across the quintiles reveals that most estimators yield reasonably high correlations in the large spread quintile. However, the performance of some of the estimators deteriorates when the medium and low spread quintiles are considered. This is particularly true for the Roll estimator, the Corwin and Schultz (2012) high-low estimator and the two versions of the Abdi and Ranaldo (2016) estimator. The popular Amihud (2002) illiquidity ratio, on the other hand, performs well in the small and medium spread quintiles (correlations 91.3% and 96.7%, respectively) but does not very well in the large spread quintile (48.2%).

[Insert Table 15 about here]

The results for the stock-by-stock time series correlations are shown in Table 16. As was the case in the main analysis (see Tables 8 and 10) the stock-by-stock correlations are much lower than the portfolio-level correlations. On the other hand, the results for the stock-by-stock correlations are much more homogeneous than those for the portfolio-level correlations. This is true both when we consider the performance of different estimators in the same quintile and when we consider the performance of the same estimator across quintiles. Three estimators stand out. The VoV high-low estimator, the VoV daily estimator and the CRSP closing spread are the top-three estimators in all three quintiles. They achieve correlations between 51.8% and 72.5%. While VoV daily is the best estimator in the small spread quintile, the CRSP closing spread achieves the highest correlation in the medium and large spread quintiles. No other estimator achieves correlations above 50% in all quintiles.

[Insert Table 16 about here]

The results can be summarized as follows. Some estimators perform very differently in different spread quintiles. It may therefore be the case that an estimator produces reasonably good result in a sample of low-liquidity stocks while the same estimator may fail completely when applied to a sample of high-liquidity stocks (or vice versa). Tobek's VoV high-low and VoV daily estimators appear to be a very good choice overall. Both are among the top performers in all spread quintiles when either cross-sectional or stock-level time series correlations are considered.

4.6 NYSE versus Nasdaq

The performance of the low-frequency estimators may be affected by the trading protocol of the market in which a stock is traded. Consequently, both the absolute and the relative performance of the estimators may be different for NYSE and Nasdaq listed stocks. To explore this issue we perform a matched-sample analysis. We select, without replacement, a Nasdaq control stock for each NYSE stock.[25] We follow Hendershott and Moulton (2011) and select the control stocks such that the score

$$score = \frac{\left|\frac{Size_{NYSE}}{Size_{NASDAQ}} - 1\right| + \left|\frac{Spread_{NYSE}}{Spread_{NASDAQ}} - 1\right|}{2} \tag{17}$$

is minimized.[26] The matching is performed for the first month of the sample period. For NYSE stocks which are added to the CRSP database later, the matching is performed for the first month for which data is available. When a matched Nasdaq stock leaves the sample (e.g. because of a merger) we select a new Nasdaq match for the corresponding NYSE stock.[27] All other procedures are as described previously. We analyze

[25] We select a NYSE stock randomly and then identify the best match from the sample of Nasdaq stocks. We then randomly select the next NYSE stock and identify the best match from the sample of remaining Nasdaq stocks. We proceed in this way until we have found a match for all NYSE stocks.

[26] We also implemented a version of the procedure where we required that the score is below 1. The results were very similar to those presented below and are therefore omitted.

[27] When a NYSE stock has two (or more) Nasdaq matches we calculate two time-series correlations between the effective spread and the low-frequency estimators, one for the original match and one for

cross-sectional correlations and time-series correlations at the individual stock level.[28] The results are presented in Table 17. Columns 1-4 show the average cross-sectional correlation between the low-frequency proxies and the effective stock for NYSE stocks (column 1), the average correlation for the matched Nasdaq stocks (column 2), the percentage of months in which the correlation is higher for the Nasdaq stocks (column 3), and the result of a t-test of the null hypothesis that the cross-sectional correlation is the same for NYSE and Nasdaq stocks. Columns 5-8 provide similar information on the stock-level time-series correlations.

The results for the cross-sectional and time-series correlations are very similar. While some of the low-frequency estimators (e.g. the Amihud illiquidity ratio and the effective tick estimator) perform better for NYSE stocks, the majority of the estimators perform better for Nasdaq stocks. The relative performance of the estimators is very similar in the two markets. The three measures that achieve the highest cross-sectional and time-series correlation for NYSE stocks (the CRSP closing spread, the VoV high-low estimator and the VoV daily estimator) also achieve the highest correlation for Nasdaq stocks. These three estimators are among those estimators that perform better for Nasdaq stocks.

[Insert Table 17 about here]

To summarize, the results in this section indicate that the ranking of the estimators is similar for NYSE and Nasdaq stocks. Therefore, researchers can choose the same estimator for NYSE and Nasdaq samples.

4.7 Regression Analysis

The results in sections 4.5 and 4.6 indicate that the performance of some of the low-frequency estimators depends on the size of the bid-ask spread. Other firm characteris-

the new match. We then calculate an observation-weighted average of these two correlations. This average is then compared to the time-series correlation between the effective spread and the low-frequency estimators for the NYSE stock.

[28]In both cases we present results for the equal-weighted correlations. Other weighting schemes lead to very similar results.

tics besides the spread may also affect the accuracy of the estimators. We explore this possibility by estimating cross-sectional regressions. We again start from the full set of stock-month observations. We calculate, for each stock, the time-series correlations between the effective spread benchmark and each of the low-frequency proxies. These correlations are the dependent variable in our regression analysis.[29] We include as independent variables the effective spread, the squared effective spread (to account for the non-linear pattern documented above), firm size (measured by the market value of equity) and the turnover ratio (defined as the ratio of daily dollar trading volume and market capitalization). These variables are calculated as time-series averages over the sample period. We further include the standard deviation of daily returns and the log of the age of the firm (measured from the first availability of data for the firm in the CRSP database). Finally, to account for the result reported in section 4.6 that the listing venue may also affect the performance of the liquidity estimators, we include dummy variables indicating whether the firm is listed on Nasdaq, the NYSE or AMEX.[30]

We estimate one cross-sectional regression for each low-frequency estimator. Because the number of time-series observations included in the correlation estimates and in the time-series averages differs across stocks we use weighted least squares with weights that reflect the number of time-series observations for each stock. The results are shown in Table 18. To enhance the readability of the Table we do not show results for all low-frequency estimators. Rather, for each group of conceptually similar estimators (e.g. the three versions of the VoV estimator) we report results for the estimator that performs best. The explanatory power of the regressions, measured by the adjusted regression R^2, ranges from 0.03 for the Pastor and Stambaugh (2003) gamma to 0.96 for the CRSP closing spread. The coefficient estimates indicate that the determinants of the time-series correlations between the effective spread and the low-frequency proxies differ widely be-

[29]The dependent variable can obviously only take on values between -1 and 1. Approximately 1% of the predicted values from the OLS regressions are outside of this range.

[30]Firms that changed their listing during our sample period are excluded. We calculated the variance inflation factors for all explanatory variables. They were all below the critical value of 10, indicating that multicollinearity is not an issue.

tween the different proxies. The most common pattern for the coefficients on the effective spread is a positive coefficient for the spread and a negative coefficient for the squared spread. This pattern is consistent with the u-shaped relation documented above. For three estimators (Zero, Effective Tick and LIX) both coefficients are negative, implying that these measures achieve higher time-series correlations for lower-spread stocks. No clear pattern emerges for the coefficients on firm size. Six coefficients are significantly negative while five coefficients are significantly positive. For most (but not all) low-frequency estimators, the time-series correlations are increasing in turnover, firm age, and volatility. The coefficients on the listing dummies indicate that all low-frequency estimators (with the exception of the Pastor and Stambaugh (2003) gamma) achieve higher correlations for Nasdaq stocks. This is a stronger result than the one we obtained from the matched-sample analysis in the previous section.

[Insert Table 18 about here]

The results shown in Table 18 indicate that the time-series correlations between the effective spread and the low-frequency proxies can indeed be explained by firm characteristics. In principle, then, the predicted values from the regression can be used to predict the accuracy of a low-frequency estimator for a particular stock. However, when used in this way the regression should not include the effective spread on the right-hand side because researchers use the low-frequency estimators precisely because data on effective spreads is unavailable. We therefore now take the perspective of a researcher who has access (only) to the CRSP data base. All variables used on the right-hand side of our regression model except the effective spread are available in the CRSP data base. We therefore replace the effective spread by the CRSP closing spread and re-estimate our model. The results are shown in Table 19. The explanatory power, as measured by the adjusted R^2, is essentially unchanged. The coefficient estimates in Table 19 can thus be used to forecast the accuracy of the low-frequency estimators. Consider, for example, a 5-year old NYSE-listed firm with a CRSP closing spread (averaged over the sample period) of 0.02 (corresponding to 2%), a market capitalization of 1 billion dollars, an average daily turnover ratio of 0.006 and a standard deviation of daily returns of 0.9. Our regression results predict that the time-series correlation between the effective spread and the CRSP closing spread is 69.8%. The prediction for the VoV high-low estimator is a correlation of 68.1%. The high regression R^2 indicates that these predictions are precise.

[Insert Table 19 about here]

One plausible way to make use of the results of the predictive regressions is to select, for each stock, the estimator which is expected to perform best. We therefore predicted the best estimator for each stock in our sample. The results are shown in Table 20. The CRSP closing spread is the clear winner. It is predicted to be the best estimator for 78.4% of the sample stocks, followed by the VoV high-low estimator (8.0%). All other estimators are rarely predicted to perform best. In the next step we analyze how the accuracy of low-frequency spread estimation can be improved when we use the predicted best estimator for each stock rather than using the same estimator for all stocks. We use the CRSP closing spread as the benchmark measure. It achieves average stock-level time-series-correlations between 68.1% (value-weighted average) and 81.3% (observation-weighted average). Using the best estimator for each stock improves upon this performance. The average value-weighted correlation increases from 68.1% to 79.6%. The improvement is more modest if one of the two other weighting schemes is applied (from 79.7% to 80.8% for the equally-weighted average and from 81.3% to 84.6% for the observation-weighted average).

[Insert Table 20 about here]

In summary, the results in this section indicate that the accuracy of the low-frequency estimators depends on firm characteristics in a predictable way. This predictability can be exploited to forecast the accuracy of the low-frequency estimators. A potentially useful application is to determine which of the various low-frequency estimators is expected to perform best for a particular stock. Our results indicate that the accuracy of low-frequency spread estimation can indeed be improved when the predicted best estimator is used for each stock. The improvement is substantial when value-weighted averages of time-series correlations are considered while it is modest when equally-weighted or observation-weighted averages are considered. It should also be noted that the procedure proposed above is only feasible when one is interested in the time-series patterns of liquidity. The procedure is not applicable in a cross-sectional context.

4.8 Construction of a Composite Spread Estimator

The various low-frequency estimators that we analyze are conceptually very different. It is therefore conceivable that some combination of estimators may result in a liquidity measure that is superior to any individual estimator. In this section we explore this possibility. However, rather than including all low-frequency estimators in the investigation, we confine the analysis to the following estimators: the CRSP closing spread, the return-based Roll estimator, the Zero estimator, the Corwin and Schultz (2012) high-low estimator, the effective tick estimator, the FHT estimator, the Amihud illiquidity ratio, the Pastor and Stambaugh (2003) gamma, the LIX estimator, the Abdi 2-day estimator, the Tobek VoV high-low estimator and the LOT y-split estimator. These estimators were chosen such that the whole spectrum of estimation concepts is represented. Whenever there are several conceptually similar estimators (e.g. the different versions of the Tobek estimator) we select the one that performed best in the correlation analysis presented above.[31]

We implement two different approaches to construct a composite low-frequency liquidity measure. The first approach starts from the full set of stock-month observations and determines the first principal component of the low-frequency liquidity estimators

[31] This implies that the estimators were selected with hindsight. As is shown below, however, even though we selected the components of our composite estimators with hindsight, the composite estimators do not perform better than the best of the individual estimators. Our conclusions are thus not affected by a hindsight bias.

listed above. We then construct a combined measure as a weighted average of the individual low-frequency estimators, using their loadings in the first principal component as weights.[32] We refer to this estimator as the principal component (PC) estimator.

The second approach proceeds as follows. We again start from the full sample of stock-month observations and then calculate, for each month, the equally-weighted average effective spread and the equally-weighted average of the low-frequency estimators, resulting in one time series for the effective spread benchmark and one time series for each of the low-frequency estimators. We then construct a linear combination of the time series of the low frequency estimators and determine the weights such that the resulting time series has maximum correlation with the time series of effective spreads. We then use these weights to construct a composite liquidity estimator which we denote the equally-weighted maximum correlation (MC_{ew}) estimator. We repeat the procedure using a value-weighted instead of the equally-weighted average. The resulting estimator is denoted value-weighted maximum correlation (MC_{vw}) estimator.

We calculate the PC and MC estimators for each stock-month and then evaluate their performance based on the correlations with the effective spread benchmark. We proceed as follows. We first calculate the correlation between the composite estimator and the effective spread benchmark for each stock. We then calculate the average correlation across stocks using three weighting schemes, an equally-weighted average, a value-weighted average, and an observation-weighted average as described above.

The procedure described above is applied to the full sample and thus results in an in-sample evaluation of the composite estimators because we perform the evaluation on the same data set that was used to obtain the weights for the PC/MC estimators. We additionally perform two out-of-sample evaluations. For the first out-of-sample evaluation we split the sample in the middle of the sample period. We then use the first half (1993 to 2002) of the sample to obtain the weights for the composite estimators and use the second half (2003 to 2012) of the sample for the evaluation. For the second out-of-sample

[32]The approach is inspired by Baker and Wurgler (2006) who use a similar approach to construct a composite sentiment measure.

evaluation we randomly select 50% of the sample stocks. We then obtain the weights for the PC/MC estimators from the resulting sub-sample and use the other sub-sample to evaluate the estimators.

In order to put the correlations between the composite estimators and the effective spread into perspective, we compare them to the correlations between the CRSP closing spread and the effective spread benchmark. The CRSP closing spread is easily available and easily applicable.[33] If it results in correlations that are equal to, or higher than, those obtained for the composite estimators, then the latter are obviously not a recommendable choice.

The results for the PC estimator are shown in Panel A of Table 21. The equally-weighted average correlation between the PC estimator and the effective spread is 70.3%. The value- and observation-weighted averages are larger, at 77.6% and 71.1%. The corresponding correlations for the CRSP closing spread are 79.7%, 81.3% and 68.1%, respectively. We therefore conclude that the PC estimator is inferior to the CRSP closing spread when equally-weighted and observation-weighted averages are considered and slightly improves upon the CRSP closing spread when value-weighted average correlations are considered. However, as can be seen from Table 10, the CRSP closing spread is not the best of the individual low-frequency estimators when value-weighted average correlations are considered. The best estimator, the Tobek VoV daily estimator, achieves an average correlation of 72.0% and is thus better than the PC estimator. The out-of-sample performance of the PC estimator is not inferior to the in-sample performance. This is true irrespective of whether we split the sample by time or by randomly sorting the sample stocks into two groups. The out-of-sample average correlations range between 69.9% and 77.6%. However, it is again true that the PC estimator is inferior to the CRSP closing spread when equally-weighted or observation-weighted averages are considered.

Panel B of Table 21 shows the results for the MC estimator. They are unambiguous.

[33] We could, of course, use any other individual low-frequency estimator for comparison. Equally-weighted, value-weighted and observation-weighted average correlations for all estimators for the full sample are displayed in Table 10.

No matter which weighting scheme is considered, and no matter whether the in-sample evaluation or the out-of-sample evaluation is considered, the MC estimator is inferior to the CRSP closing spread.[34]

[Insert Table 21 about here]

The results of this section can be briefly summarized as follows. Both composite estimators that we analyze, the principal component estimator and the two versions of the maximum correlation estimator, are not better than the best of the individual low-frequency estimators. We therefore recommend against practical application of the composite estimators.

4.9 Estimator Performance in Different Regimes

During our sample period the trading protocols of the NYSE and Nasdaq underwent considerable change. Major changes include the adoption of the Nasdaq Order Handling Rules in 1997 (see McInish et al. (1998), Chung and Van Ness (2001)), the introduction of NYSE Open Book in 2002 (see Boehmer et al. (2005)) and the introduction of NYSE Hybrid in 2006/2007 (see Hendershott and Moulton (2011)). Further, the minimum tick size was reduced from eights to sixteenths and later to decimals. In this section we analyze whether the performance of the low-frequency estimators is affected by these changes. For each event we perform a difference-in-differences analysis using a matched control sample of firms that were not affected by the regime change under consideration.

For the two major changes on the NYSE (NYSE Open Book and NYSE Hybrid) we proceed as follows. We define a 6-months pre-event period (July-December 2001 for NYSE Open Book, April-September 2006 for NYSE Hybrid) and a 6-months post-event period

[34] This result may seem surprising given that (a) the MC estimator was constructed such that it has maximum correlation with the effective spread and that (b) the CRSP closing spread is included in the set of low-frequency estimators that enter the MC composite estimator. However, it should be kept in mind that the MC estimator maximizes the portfolio-level correlation between the effective spread and a linear combination of the low-frequency estimators. The evaluation, on the other hand, is based on the average of the stock-level correlations. In theory one could identify a maximum correlation estimator for each stock individually.

(February-July 2002 for NYSE Open Book, February-July 2007 for NYSE Hybrid). The period during which the change took place (January 2002 for NYSE Open Book, October 2006-January 2007 for NYSE Hybrid) is discarded. We then select a Nasdaq match for each NYSE stock. The matching procedure is as described in section 4.6. The matching is performed for the first month of the pre-event period.

The reduction of the minimum tick size from sixteenths to decimals occurred in January 2001 for most NYSE stocks and in April 2001 for most Nasdaq stocks.[35] In order to avoid event contamination we define a six-months pre-event period and a two-months post-event period (July-December 2000 and February-March 2001, respectively) for the NYSE and a two-months pre-event period and a six-months post-event period (February-March 2001 and May-October 2001, respectively) for Nasdaq. The matching procedure is as described above.

The Nasdaq Order Handling Rules were introduced between January and October 1997. During this period both the NYSE and Nasdaq also reduced the minimum tick size from eights to sixteenths. The difference-in-differences analysis we perform thus compares the joint effect of the order handling rules and the tick size reduction on Nasdaq to the effect of a tick size reduction on the NYSE. The pre-event period extends from July-December 1996 and the post-event period from November 1997-April 1998. The matching procedure is as described above.

For all five events we perform a difference-in-differences analysis based on cross-sectional correlations. We calculate, for each month of the pre-event and the post-event period, and separately for the treatment and the matched control sample, the cross-sectional correlation between the low-frequency liquidity estimators and the effective spread benchmark. We then regress these correlations on a constant, a dummy variable that identifies the observations in the post-event period, a dummy that identifies observations from the treatment group, and the interaction between the post-event dummy and

[35]In both markets a small number of stocks was transferred to the new regime earlier. These stocks are completely discarded from our analysis. They are thus included neither in the treatment group nor in the control group.

the treatment dummy. We confine the analysis to the subset of low-frequency estimators described in section 4.8. The results are displayed in Table 22.

The introduction of the Nasdaq Order Handling Rules has generally increased the ability of the low-frequency proxies to capture cross-sectional variation in spreads. Nine out of twelve coefficients are positive, and eight of them significantly so. A notable exception is the CRSP closing spread which displays significantly lower cross-sectional correlation after the introduction of the order handling rules.

The tick size reduction from sixteenths to decimals on the NYSE had an overall negative impact on the accuracy of the estimators. Only the performance of the effective tick estimator improves significantly. On the other hand, six estimators (the Roll measure, the high-low spread estimator, the CRSP closing spread, the FHT estimator, the Abdi 2-day estimator and the LOT y-split estimator) perform significantly worse after the tick size reduction. The effective tick estimator appears to generally perform better under a low tick-size regime. It also displays significantly improved performance after the tick size reduction in Nasdaq. The difference-in-differences coefficients for the other estimators are predominantly positive, but much smaller in magnitude than the coefficient for the effective tick estimator. Only two of them (the coefficients for the LIX and the VoV high-low estimators) are significant.

The introduction of NYSE Open Book and NYSE Hybrid did not have much impact on the accuracy of the low-frequency estimators. The majority of the coefficients for the introduction of NYSE Hybrid is negative, but only the coefficient for the effective tick estimator is significant. The results for the introduction of NYSE Hybrid are ambiguous. Six coefficients are positive (but only the coefficient for the LIX estimator is significant) and six are negative (but only the coefficient for the Amihud illiquidity ratio is significant).

[Insert Table 22 about here]

Summarizing the results of this section, changes in the trading protocol can affect the performance of the low-frequency estimators. While the introduction of the Nasdaq

Order Handling Rules has improved the performance of a majority of the estimators, the tick size reduction on the NYSE has predominantly resulted in lower performance.

5 Conclusion

In this paper we perform a comprehensive comparative analysis of low-frequency measures of liquidity. Our main objective is to provide researchers with clear guidelines for the selection of the best liquidity estimator in a specific research application. The cornerstone of our analysis is a horse race between a comprehensive set of low-frequency liquidity estimators (including the CRSP closing spread) proposed in the literature.

Several variables characterize a specific research application. Among the most important are data availability (e.g. only transaction prices, prices and volume, or prices, volume and high-low prices) and the sample at hand (e.g. large-caps or small-caps). Further, researchers may be primarily interested in the bid-ask spread or in the price impact, and they may be interested in levels or in first differences of the variable of interest. Finally, in some applications the cross-sectional differences in liquidity are of prime importance while in other applications the time-series properties of liquidity are most important.

In order to capture all these aspects we implement several approaches aiming to compare the low-frequency liquidity measures. Specifically, we consider both time-series correlations and cross-sectional correlations, we apply different weighting schemes, we calculate mean absolute and root mean squared errors, and we use both the effective spread and the price impact (both in levels and in first differences) as benchmark measures. We further analyze how stock characteristics such as firm size and market characteristics such as the minimum tick size regime or the level of transparency affect the performance of the low-frequency liquidity proxies. Finally, we develop two composite low-frequency estimators and test whether they perform better than the best of the individual estimators.

We implement our analysis on a broad sample of more than 10,000 US stocks listed on the NYSE, AMEX and Nasdaq and covering 1993-2012. A central finding is that both the absolute and the relative performance of many of the low-frequency estimators is highly dependent on the specific setting and on the criterion used to evaluate the performance of the estimators.

In spite of these differences several general patterns emerge. First, the estimators are generally better at explaining levels than at explaining first differences, and they are better at explaining the effective spread than the price impact. On the other hand, the data frequency (i.e. the question whether the low-frequency estimators are calculated at the stock-month level or the stock-year level) does not materially affect the relative performance of the estimators. The composite estimators that we develop do not improve upon the performance of the best individual estimators. The introduction of the Nasdaq Order Handling Rules has improved the performance of a majority of the estimators while the tick size reduction on the NYSE has predominantly resulted in lower performance. Other changes in the trading protocol, namely the introduction of NYSE Open Book or NYSE Hybrid, did not have first-order effects on the performance of the low-frequency liquidity proxies.

The estimators that display the highest cross-sectional and stock-level time-series correlation with the benchmark measures are the estimators recently proposed by Tobek (2016) and the CRSP closing spread. The estimator that results in the smallest mean absolute and root mean squared error, on the other hand, is the Abdi 2-day estimator proposed by Abdi and Ranaldo (2016). The CRSP data set contains all data that is required to calculate these estimators from 1992 onwards.[36] Therefore, researchers using post-1992 US data can indeed select the best-performing low-frequency estimators. This may be different when data for other countries or pre-1992 US data is used for which reliable closing bid-ask spreads, daily high and low prices, or trading volume data may be

[36]The CRSP database contains data on closing bid and ask prices for NYSE and Amex stocks from December 28, 1992 onwards and for Nasdaq stocks from November 1, 1982 onwards. For details see Chung and Zhang (2014).

unavailable. In these cases data availability may be a limiting factor in the choice of an estimator. Figure 1 provides a brief summary of our results and may guide researchers' selection of an appropriate estimator in a specific research setting. It should be noted, though, that the recommendations given in Figure 1 are based on evidence from the US. The extent to which they are valid for other countries is an open issue that may be explored in future research.

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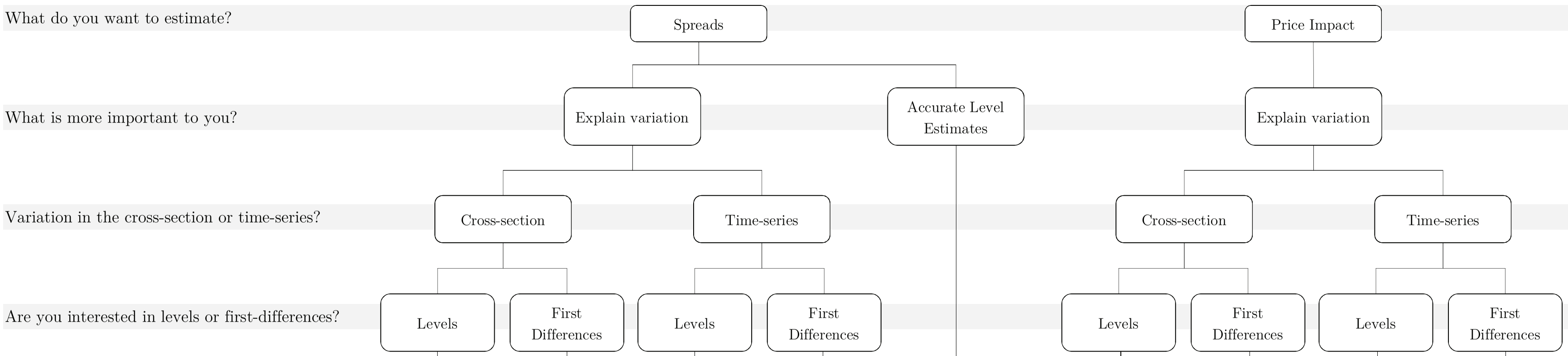
Figure 1: Summary of Results
This flowcharts summarizes the main findings of this paper described in Section 4. Every column of the table at the end of the flowchart refers to one specific analysis conducted in this paper. The abbreviations in the lower left of the chart are:
P/r for daily Closing Price/return; V for daily dollar trading volume; H/L for daily high/low prices; and B/A for daily clsoing bid/ask prices.
We decided to recommend *none* estimator if the correlation of all eligible estimators with the respective variable of interest (effective spread or 5-minute price impact) was below 10%.



If you have data on ..., use...

P/r	V	H/L	B/A	Spreads – Cross-section – Levels	Spreads – Cross-section – First Differences	Spreads – Time-series – Levels	Spreads – Time-series – First Differences	Spreads – Accurate Level Estimates	Price Impact – Cross-section – Levels	Price Impact – Cross-section – First Differences	Price Impact – Time-series – Levels	Price Impact – Time-series – First Differences
✓	✓	✓	✓	VoV High-Low	Closing Spread	Closing Spread	Closing Spread	Abdi 2-day	VoV Daily	none	VoV daily	VoV daily
✓	×	✓	✓	Closing Spread	Closing Spread	Closing Spread	Closing Spread	Abdi 2-day	Closing Spread	none	Closing Spread	Closing Spread
✓	✓	×	✓	Closing Spread	Closing Spread	Closing Spread	Closing Spread	Effective Tick	Closing Spread	none	VoV Sigma	VoV Sigma
✓	✓	✓	×	VoV High-Low	VoV daily	VoV daily	VoV daily	Abdi 2-day	VoV Daily	none	VoV daily	VoV daily
✓	×	×	×	Roll (0) ret	Roll (0) ret	Roll (0) ret	Roll (0) ret	Effective Tick	Lesmond	none	Roll (0) ret	none

Figure 2: Dollar-Volume / Effective Spreads over Time
This table shows the equal-weighted average daily dollar-volume (in millions) and effective spread per stock in our sample. Both variables are winsorized at the 1% level.

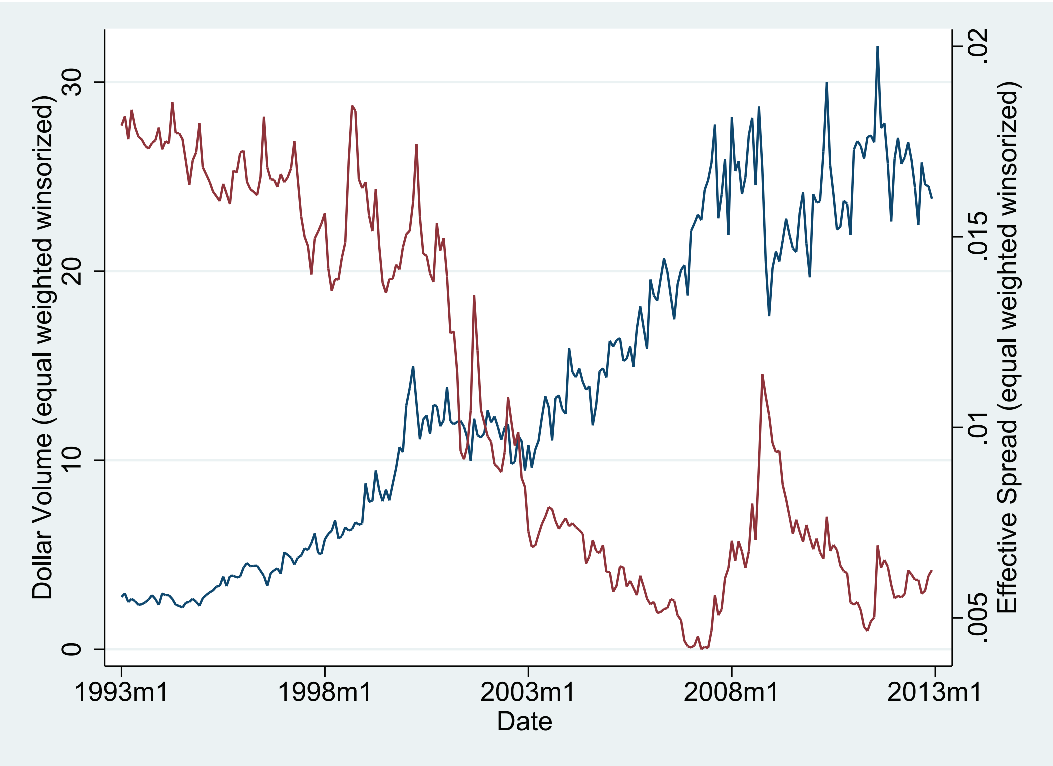


Figure 3: Mean Spread Levels with non-missing spreads
This figure shows the equal-weighted mean relative spread level across those estimators that actually aim to predict the relative spread level. In addition to the estimators, the actual effective spread calculated from TAQ data is depicted as a red line in the graph.

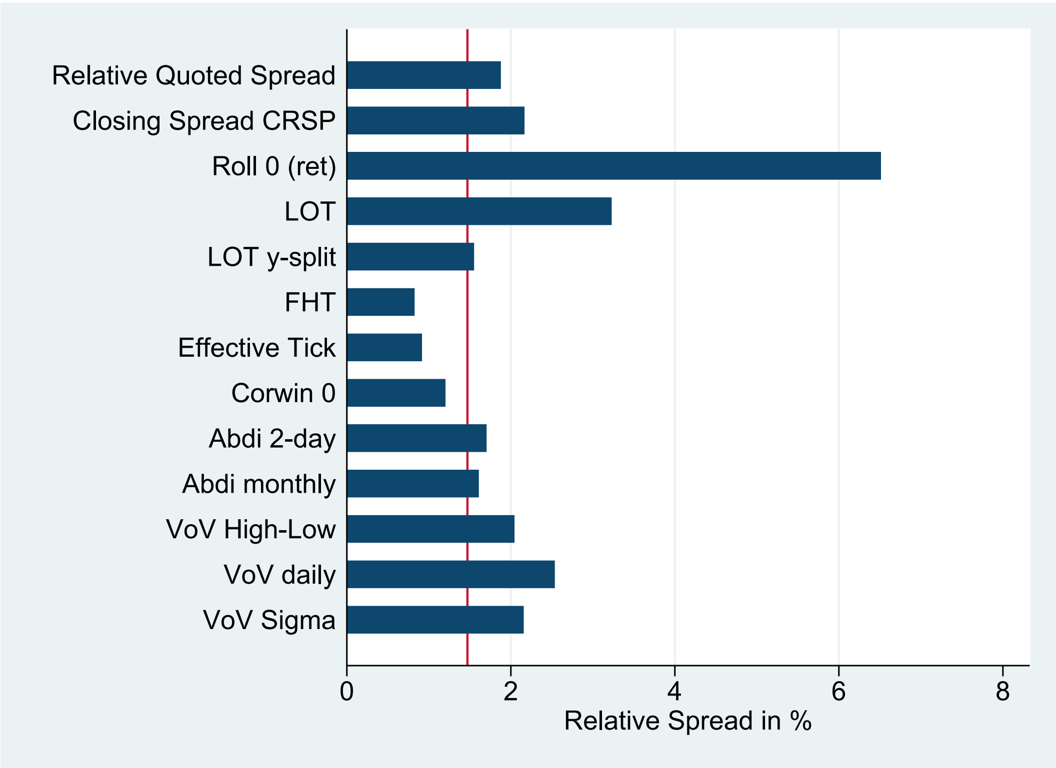


Figure 4: **Crosssectional Correlation**
This table shows the development of the 12 months rolling average of monthly crosssectional correlations over time.

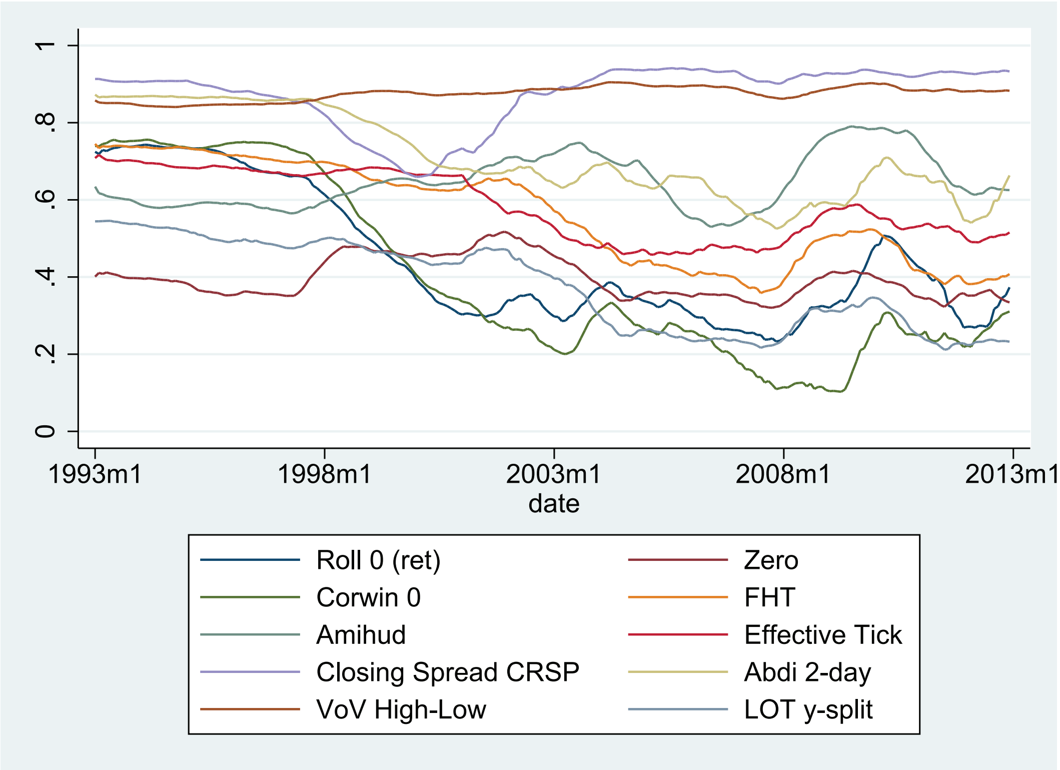


Table 1: Overview of Spread Measures and Variable requirements
This table provides an overview over the different Spread Measures regarded in this paper and required input variables. Columns are set equal to 1 if the respective input variable is needed to calculate the estimator. We document all our results for estimators in **bold**. Results for all other estimators are available upon request.

Estimator	Required Input					
	Close	**Return**	**Volume**	**High/Low**	**Bid/Ask**	**Market Return**
Roll	1					
Roll 0	1					
Roll Inverted	1					
Roll (ret)		1				
Roll 0 (ret)		1				
Roll Inverted (ret)		1				
LOT		1				1
LOT y-split		1				1
Zero		1				
Zero 2		1	1			
Corwin				1		
Corwin 0				1		
Corwin numeric				1		
Corwin numeric 0				1		
Effective Tick	1		1			
Effective Tick 2	1		1			
Amihud		1	1			
Amivest		1	1			
Closing Spread					1	
Gibbs	1					
LIX	1		1	1		
FHT		1				
Gamma		1	1			1
Holden	1		1			
Abdi Monthly	1			1		
Abdi 2-day	1			1		
Tobek Corwin				1		
Tobek Corwin 0				1		
Tobek FHT			1	1		
VoV High-Low			1			
VoV Sigma		1	1			
VoV daily			1	1		

Table 2: Filtering Procedure
This table shows the filtering procedure applied to our dataset. "Common stock" refers to CRSP share codes 10 and 11. Stock splits are identified by changes in the "cfacpr"-item. "Unusual status" refers to unexpected trading or security status of a stock; we call a trading status unusual if it is "halted", "suspended" or "unknown"; we call a security status unusual if it is "When Issued", "Ex-Distributed" or "Bankruptcy". Rows NYSE, AMEX, NASDAQ show the distribution across the different exchanges after the last filtering step applied to the CRSP daily stock sample.
The last three lines show the sample after it is aggregated to the stock-month level, i.e. after low-frequency estimators have been calculated. They are merged to the monthy average spreads derived from the TAQ data. Finally, the "12 days/month"-filter assures that every estimator is estimated from at least 12 days of non-missing observations.

	Observations	**Firms**
CRSP Raw Data	38,198,992	19,449
Common Stock	28,324,608	14,157
Exchange NYSE, AMEX, or NASDAQ	27,873,362	14,157
Eliminate months with stock splits	27,545,856	14,127
Eliminate months with listing changes	27,515,228	14,127
Eliminate months with ticker changes	27,442,650	14,127
Eliminate months with cusip changes	27,348,380	14,126
Eliminate months with unusual status	27,338,272	14,126
NYSE	8,646,671	3,400
AMEX	3,501,152	1,707
NASDAQ	19,356,468	10,410
After estimator calculation (from daily to monthly frequency)	1,307,133	14,122
After joining with liquidity data	1,083,680	13,599
If 12 days/month filter is applied	1,079,509	13,578

Table 3: Sample Overview
This table shows the number of distinct firms per year and exchange in our sample. Firms are identified by CRSP permno. The last line shows the total number of distinct firms in our sample.

Year	NYSE	AMEX	NASDAQ
1993	2041	731	3979
1994	2102	744	4387
1995	2152	758	4701
1996	2233	789	5259
1997	2255	801	5338
1998	2198	795	5098
1999	2102	735	4955
2000	1948	730	4957
2001	1759	555	3859
2002	1650	515	3381
2003	1605	517	3302
2004	1627	546	3446
2005	1636	571	3317
2006	1615	583	3289
2007	1604	613	3226
2008	1497	537	2828
2009	1400	435	2422
2010	1443	428	2530
2011	1425	404	2455
2012	1393	331	2296
Total	**3364**	**1617**	**10038**

Table 4: Descriptive Statistics
The table provides monthly Mean (equally-weighted and value-weighted), Median, Standard Deviation, 5% and 95% percentile of several variables for our sample of US stocks. The unit of measurement is usually provided in brackets.
The first 3 lines provide statistics for variables derived from the CRSP daily file: *Market Value* is the respective firm's market value at the end of each month in mio \$. *Daily Turnover* is measured as the daily dollar-volume traded divided by the firm's market capitalization.
Lines 4-9 are based on the TAQ dataset and starting from line 10 the different low-frequency spread estimators are depicted. For a derivation of those estimators see Section 2.

	N	**Mean**	**Median**	**Sd**	**5%**	**95%**
Market Value (in mio)	1083665	1839.685	248.136	5269.770	18.043	8969.699
Daily Return (in %)	1083276	0.081	0.057	0.697	-1.003	1.217
Daily Turnover (in %)	1083681	0.660	0.380	0.811	0.036	2.283
Daily Trades	1083684	1396.319	84.727	4162.555	3.267	7238.143
Relative Effective Spread (in %)	1083684	1.471	0.798	1.753	0.069	5.092
Dollar Effective Spread	1083684	0.178	0.118	0.200	0.017	0.557
Relative Quoted Spread (in %)	1083684	1.875	1.000	2.278	0.068	6.644
Dollar Quoted Spread	1083684	0.230	0.156	0.265	0.017	0.729
5min Price Impact (in %)	820695	0.002	0.001	0.004	0.000	0.008
Closing Spread CRSP (in %)	1078424	2.164	1.236	2.766	0.059	7.332
Roll 0 (in cents)	1070594	29.610	16.811	41.103	0.000	109.849
Roll 0 (ret) (in %)	1082508	6.509	5.232	4.828	1.588	15.693
Gibbs (in cents)	1082909	35.497	27.856	26.329	9.400	89.620
Zero (in %)	1083276	10.554	5.000	12.736	0.000	36.842
LOT (in %)	1083150	3.226	2.372	3.594	0.000	9.442
LOT y-split (in %)	1083111	1.549	0.805	2.390	0.000	5.376
FHT (in %)	1082823	0.823	0.351	1.263	0.000	3.271
Tobek FHT (in %)	1081005	1.707	0.997	2.250	0.350	5.796
Effective Tick (in %)	1083632	0.913	0.441	1.286	0.031	3.241
Corwin 0 (in %)	1058265	1.200	0.909	1.041	0.210	3.151
Tobek Corwin 0 (in %)	1058265	0.971	0.706	0.935	0.109	2.696
Abdi 2-day (in %)	1083573	1.701	1.254	1.500	0.384	4.489
Abdi monthly (in %)	1083573	1.607	1.048	1.984	0.000	5.383
VoV High-Low (in %)	1081005	2.042	1.692	1.409	0.560	4.732
VoV daily (in %)	1081005	2.533	2.192	1.558	0.725	5.509
VoV Sigma (in %)	1082858	2.154	1.763	1.523	0.577	5.106
Amihud (in 10^6)	1083252	8.334	0.407	25.550	0.007	44.203
(-) Amivest (in mio)	1083175	71.917	6.107	210.563	0.085	372.739
(-) LIX	1081005	6.258	6.205	0.930	4.861	7.887
(-) Gamma 1 (in 10^9)	1082881	-42.978	0.042	1547.062	-1131.191	1177.963

Table 5: Overview of Existing Papers and Applied Procedures
This table provides an overview over the samples and procedures used by the different papers that tried to compare different spread proxies. *Crosssection, Pooled, Timeseries, RMSE* equal one if the respective analysis is conducted in the paper. *TS: weighting* equals "e"/"v" if assets were equally-/value-weighted. *TS: procedure* equals s/p if the timeseries analysis was done on a stock-by-stock/portfolio basis.

Paper	Year	Market	Period	Frequency	Aggregation	observation number drop	Crosssection	Pooled	Timeseries	TS: weighting	TS: procedure	RMSE
Roll	1984	US	1963 - 1982	Yearly		21 days/year	1	0	0			0
Lesmond 1999	1999	US (NYSE / Amex)	1963 - 1990	Yearly		25 days/year	0	0	0			0
Corwin/Schultz 2011	2011	US	1993 - 2006	Monthly	trade-weighted	12 days/month	1	0	1	e	s	0
Danyliv et al. 2014	2014					not empirically tested						
Fong et al. 2011	2011	International (38 countries)	1996 - 2007	Monthy	local-currency-volume weighted	5 days/month (nonzero volume / nonzero return)	1	0	1	e	p	1
Amihud 2002	2002	US (NYSE)	1963 - 1997	Monthly		200 days/year	1	0	0			0
Pastor/Stambaugh 2003	2003	US (NYSE / Amex)	1963 - 1999	Monthly		15 days/month	0	0	0			0
Amivest	2002					no paper						
Holden 2009	2009	US	1993 - 2005	Monthly	volume-/time-weigthed	2 days/month (implicit)	1	0	1	e	p	1
Hasbrouck 2009	2009	US	1993 - 2005	Yearly	trade-weighted	2 days/month (implicit)	0	1	0			0
Abdi/Ranaldo 2016	2016	US	2003 - 2014	Monthly	dollar-weighted	12 days/month	1	0	1	e	s	1
Tobek 2016	2016	US	1993 - 2015	Monthly		11 days / month	1	0	0			1
Goyenko et al. 2009	2009	US	1993 - 2005	Monthly / Yearly	dollar-volume-weighted	2 days/month (implicit)	1	0	1	e	p	1
This paper	2016	US	1993 - 2012	Monthly / Yearly	volume-/time-weigthed	12 days/month as robustness check	1	0	1	e/v	s/p	1

Table 6: Monthly Cross-Sectional Correlations (Level)

Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. Each **month** the cross-sectional correlation of those with the respective spread proxies is calculated.
N gives the average number of firms included in each monthly correlation. *Correlation* gives the mean cross-sectional correlation in percent and its significance (t-statistic after Fama and MacBeth (1973); *** 1%, ** 5% and * 10%) is indicated. *% ≥ 0* gives the percentage of monthly correlations that where larger or equal to 0 and *% ≥ 0 (5%)* gives the percentage of monthly correlations that where significantly greater than zero at 5% significance level. We repeat the analysis, but now correlate estimators to a 5-minute price impact measure. *N (PI)* and *Correlation (PI)* are calculated analogous to *N* and *Correlation*.

Measure	N	Correlation	> 0	> 0 (5%)	N (PI)	Correlation (PI)	> 0 (PI)	> 0 (5%) (PI)
Closing Spread CRSP	4493	87.42***	100.00	100.00	4550	36.25***	100.00	100.00
Roll 0	4461	1.30**	47.92	39.17	4521	-7.63	12.78	4.44
Roll 0 (ret)	4510	43.54***	99.58	99.58	4573	21.92***	100.00	100.00
Gibbs	4512	0.25	44.17	37.92	4574	-12.27	6.67	0.56
Zero	4514	39.52***	100.00	100.00	4576	16.29***	96.11	91.11
LOT	4513	46.97***	100.00	100.00	4575	22.33***	100.00	100.00
LOT y-split	4513	37.03***	100.00	100.00	4575	14.80***	99.44	98.33
FHT	4512	55.54***	100.00	100.00	4574	24.43***	100.00	100.00
Tobek FHT	4504	73.52***	100.00	100.00	4568	30.04***	100.00	99.44
Effective Tick	4515	57.91***	100.00	100.00	4577	29.01***	100.00	100.00
Corwin 0	4409	38.67***	99.58	99.17	4473	20.88***	100.00	98.89
Tobek Corwin 0	4409	33.11***	97.08	94.58	4473	17.11***	99.44	98.89
Abdi 2-day	4515	70.51***	100.00	100.00	4577	32.89***	100.00	100.00
Abdi monthly	4515	66.57***	100.00	100.00	4577	28.33***	100.00	100.00
VoV High-Low	4504	87.61***	100.00	100.00	4568	38.24***	100.00	100.00
VoV daily	4504	86.94***	100.00	100.00	4568	40.32***	100.00	100.00
VoV Sigma	4512	86.39***	100.00	100.00	4574	35.74***	100.00	100.00
Amihud	4514	64.79***	100.00	100.00	4576	20.78***	100.00	96.67
(-) Amivest	4513	25.25***	100.00	100.00	4575	13.20***	98.89	89.44
(-) LIX	4504	72.14***	100.00	100.00	4568	32.41***	100.00	99.44
(-) Gamma	4512	8.50***	95.00	87.92	4574	1.46***	62.78	29.44

Table 7: Monthly Cross-Sectional Correlations (First Differences)

Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. Each **month** the cross-sectional correlation of the first difference of those spreads with the first difference of the respective spread proxies is calculated.
N gives the average number of firms included in each monthly correlation. *Correlation* gives the mean cross-sectional correlation in percent and its significance (t-statistic after Fama and MacBeth (1973); *** 1%, ** 5% and * 10%). *% ≥ 0* gives the percentage of monthly correlations that where larger or equal to 0 and *% ≥ 0 (5%)* gives the percentage of monthly correlations that where significantly greater than zero at 5% significance level. *insignificantly different from* gives a list of those estimators that can not be significantly (t-test with 5%) distinguished from the regarded estimator in terms of their correlation with effective spreads. We repeat the analysis, but now correlate estimators to a 5-minute price impact measure. *N (PI)* and *Correlation (PI)* are calculated analogous to *N* and *Correlation*.

Measure	N	Correlation	> 0	> 0 (5%)	N (PI)	Correlation (PI)	> 0 (PI)	> 0 (5%) (PI)
Closing Spread CRSP	4319	58.34***	100.00	100.00	4388	6.65***	87.71	68.16
Roll 0	4276	6.00***	95.82	84.94	4344	0.19	50.84	5.59
Roll 0 (ret)	4344	17.86***	99.58	99.58	4422	3.02***	87.15	44.13
Gibbs	4344	9.89***	99.16	96.23	4420	0.39**	56.42	10.61
Zero	4348	-5.48	6.28	0.84	4424	-1.45	33.52	3.91
LOT	4347	8.91***	93.31	83.68	4424	1.65***	69.83	27.37
LOT y-split	4346	1.46***	59.83	46.03	4423	0.51**	59.22	16.20
FHT	4346	9.92***	91.63	81.17	4423	1.11***	62.01	26.82
Tobek FHT	4333	30.37***	100.00	100.00	4411	5.80***	87.71	66.48
Effective Tick	4349	4.97***	73.64	62.76	4425	0.18	49.72	23.46
Corwin 0	4209	21.67***	100.00	100.00	4278	3.64***	82.68	48.04
Tobek Corwin 0	4209	18.17***	100.00	97.49	4278	2.82***	74.86	40.78
Abdi 2-day	4349	31.19***	100.00	100.00	4426	4.84***	92.74	61.45
Abdi monthly	4349	22.64***	100.00	100.00	4426	3.17***	86.59	45.81
VoV High-Low	4333	42.70***	100.00	100.00	4411	8.41***	94.41	76.54
VoV daily	4333	46.77***	100.00	100.00	4411	9.87***	97.21	83.24
VoV Sigma	4346	41.35***	100.00	100.00	4423	7.08***	94.41	78.77
Amihud	4347	23.96***	100.00	99.58	4424	4.13***	82.12	53.07
(-) Amivest	4347	0.63***	74.90	0.84	4424	0.05	55.31	0.00
(-) LIX	4333	30.94***	100.00	100.00	4411	8.42***	97.21	85.47
(-) Gamma	4346	3.17***	76.99	51.05	4423	-0.12	47.49	17.88

Table 8: Monthly Portfolio Time-Series Correlations (Level)
Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. Each **month** the cross-section of firms is aggregated (equally- or market value-weighted) to one entity. The time-series correlation of the *aggregated* firm's effective spread with the respective spread proxies is calculated. N gives the number of monthly observations, *Correlation* gives the correlation and significance for an equally-weighted cross-section, *Correlation (vw)* for a market value-weighted cross-section, respectively.
We repeat the analysis, but now correlate estimators to a 5-minute price impact measure.

Measure	N	Correlation	Correlation (vw)	N (Price Impact)	Correlation (Price Impact)	Correlation (vw) (Price Impact)
Closing Spread CRSP	240	98.39***	87.31***	180	45.75***	40.24***
Roll 0	240	32.76***	4.23	180	32.74***	49.00***
Roll 0 (ret)	240	59.99***	3.87	180	80.48***	73.64***
Gibbs	240	70.25***	30.92***	180	30.25***	49.10***
Zero	240	96.76***	93.01***	180	10.89	-11.96
LOT	240	97.21***	94.80***	180	42.92***	25.66***
LOT y-split	240	98.39***	92.17***	180	28.79***	-17.25
FHT	240	98.66***	98.19***	180	30.42***	5.78
Tobek FHT	240	86.59***	13.73**	180	69.28***	71.53***
Effective Tick	240	97.70***	98.77***	180	40.80***	14.38*
Corwin 0	240	85.73***	2.95	180	69.48***	74.08***
Tobek Corwin 0	240	88.34***	-0.00	180	65.88***	74.35***
Abdi 2-day	240	88.80***	12.57*	180	71.52***	75.09***
Abdi monthly	240	90.84***	17.11***	180	68.72***	65.02***
VoV High-Low	240	87.52***	83.93***	180	69.58***	50.28***
VoV daily	240	90.39***	85.25***	180	67.13***	50.70***
VoV Sigma	240	90.13***	85.13***	180	67.15***	49.44***
Amihud	240	83.35***	91.76***	180	63.16***	8.40
(-) Amivest	240	89.89***	95.48***	180	27.13***	15.73**
(-) LIX	240	94.32***	97.56***	180	41.92***	15.39**
(-) Gamma	240	82.70***	7.53	180	24.43***	-1.23

Table 9: Monthly Portfolio Time-Series Correlations (First Differences)
Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. Each **month** the cross-section of firms is aggregated (equally- or market value-weighted) to one entity. The time-series correlation of the first-difference of *aggregated* firm's effective spread with the first-difference of the respective spread proxy is calculated.
N gives the number of monthly observations, *Correlation* gives the correlation and significance for an equally-weighted cross-section, *Correlation (vw)* for market value-weighted firms, respectively.
We repeat the analysis, but now correlate estimators to a 5-minute price impact measure.

Measure	N	Correlation	Correlation (vw)	N (Price Impact)	Correlation (Price Impact)	Correlation (vw) (Price Impact)
Closing Spread CRSP	239	78.85***	37.44***	179	55.62***	28.66***
Roll 0	239	21.93***	25.28***	179	3.20	11.72
Roll 0 (ret)	239	68.84***	61.17***	179	49.59***	38.77***
Gibbs	239	41.20***	45.01***	179	7.34	11.41
Zero	239	-27.97	-22.11	179	2.98	-2.94
LOT	239	27.24***	19.94***	179	33.37***	12.39*
LOT y-split	239	41.76***	-12.69	179	51.14***	3.10
FHT	239	56.09***	48.41***	179	46.85***	18.35**
Tobek FHT	239	78.95***	75.09***	179	49.90***	38.35***
Effective Tick	239	54.18***	62.16***	179	44.22***	25.15***
Corwin 0	239	67.40***	62.87***	179	55.47***	35.86***
Tobek Corwin 0	239	64.34***	59.08***	179	54.30***	35.16***
Abdi 2-day	239	76.17***	54.94***	179	59.01***	35.07***
Abdi monthly	239	55.12***	15.05**	179	48.34***	6.02
VoV High-Low	239	87.13***	81.98***	179	52.53***	36.37***
VoV daily	239	87.84***	80.32***	179	56.19***	40.33***
VoV Sigma	239	83.80***	75.15***	179	49.73***	39.47***
Amihud	239	45.14***	12.71**	179	23.48***	10.51
(-) Amivest	239	39.13***	23.88***	179	45.53***	30.60***
(-) LIX	239	73.09***	58.01***	179	47.11***	31.55***
(-) Gamma	239	10.55	0.02	179	3.40	-4.92

Table 10: Monthly Stock-by-Stock Time-Series Correlations (Level)
Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. For each firm, we calculate the time-series correlation of the firm's effective spread with the respective spread proxies. Then the crosssectional average of those correlations is taken.
N gives the number of firms for which we were able to calculate a correlation, $\overline{\rho}$ gives the equally-weighted cross-sectional average correlation and significance (from a simple t-test), $\overline{\rho}$ *(by Size)* the market value-weighted average, respectively. $\overline{\rho}$ *(by Obs)* weights each firm by the number of monthly observations used to determine the time-series correlation. > 0 shows the fraction of firms that had a correlation above 0 and $> 0(5\%)$ the fraction that had a correlation that was significantly above 0 (at the 5% level).
We repeat the analysis, but now correlate estimators to a 5-minute price impact measure.

	N	$\bar{\rho}$	$\bar{\rho}$ (by Size)	$\bar{\rho}$ (by Obs)	> 0	> 0 (5%)	N (PI)	$\bar{\rho}$ (PI)	$\bar{\rho}$ (PI by Size)	$\bar{\rho}$ (PI by Obs)	> 0 (PI)	> 0 (5%) (PI)
Closing Spread CRSP	13446	79.65***	68.06	81.33	97.97	94.00	11873	23.35***	32.24	24.75	75.84	44.72
Roll 0	13385	8.46***	1.49	6.99	60.58	22.86	11826	-1.84	2.62	-1.55	42.29	6.98
Roll 0 (ret)	13449	34.20***	23.10	32.36	87.79	60.23	11880	17.54***	21.49	19.28	74.87	34.53
Gibbs	13451	9.46***	7.35	12.17	61.40	31.75	11879	-3.59	7.52	-1.73	42.28	11.68
Zero	13406	21.20***	37.73	37.75	71.19	43.37	11845	0.93***	6.69	4.65	50.20	15.16
LOT	13449	33.12***	36.08	42.58	87.70	59.64	11877	10.31***	13.55	12.67	67.39	24.50
LOT y-split	13412	24.27***	28.17	34.85	78.54	47.44	11852	5.08***	5.36	6.44	56.24	17.37
FHT	13409	40.08***	43.76	51.41	89.48	65.39	11849	9.11***	11.97	11.17	63.63	23.63
Tobek FHT	13439	45.21***	33.97	43.58	92.38	72.88	11875	21.36***	27.34	22.91	76.29	40.65
Effective Tick	13451	51.75***	69.68	63.05	94.13	76.51	11880	15.99***	24.58	18.08	70.52	34.62
Corwin 0	13392	43.58***	29.55	40.61	90.28	70.23	11831	17.38***	20.38	18.94	70.89	35.90
Tobek Corwin 0	13386	38.79***	25.55	36.25	88.08	64.86	11828	14.81***	17.75	16.25	68.40	32.55
Abdi 2-day	13447	53.82***	30.64	51.27	94.45	79.70	11878	20.63***	22.81	22.26	76.96	40.31
Abdi monthly	13426	41.16***	15.54	40.53	89.65	66.06	11851	12.40***	8.68	13.65	70.15	27.16
VoV High-Low	13440	70.20***	69.33	72.97	98.21	92.29	11874	29.14***	36.49	32.66	82.71	53.07
VoV daily	13439	74.52***	71.96	76.80	98.46	93.70	11875	29.70***	37.02	33.03	83.33	53.58
VoV Sigma	13444	63.22***	62.56	67.90	97.60	88.18	11875	26.43***	34.83	30.15	81.73	49.89
Amihud	13448	53.91***	61.38	59.82	95.79	80.77	11877	18.90***	26.59	21.40	71.47	38.55
(-) Amivest	13445	40.34***	51.49	40.65	94.27	73.02	11879	13.43***	23.44	16.11	77.28	28.50
(-) LIX	13438	64.40***	67.51	70.21	96.91	88.33	11876	23.57***	30.23	27.75	79.92	46.62
(-) Gamma	13447	-0.64	0.90	-1.00	48.38	9.19	11879	-0.10	0.64	0.06	50.11	6.47

Table 11: Monthly Stock-by-Stock Time-Series Correlations (First Differences)
Relative Effective Spreads are equally-weighted averages calculated from every trade of the TAQ dataset. For each firm, we calculate the time-series correlation of the first-difference of the firm's effective spread with the respective first-difference of spread proxies. Then the crosssectional average of those correlations is taken.
N gives the number of firms for which we were able to calculate a correlation, $\overline{\rho}$ gives the equally-weighted cross-sectional average correlation and significance (from a simple t-test), $\overline{\rho}$ *(vw)* the market value-weighted average, respectively. $\overline{\rho}$ *(by Obs)* weights each firm by the number of monthly observations used to determine the time-series correlation. > 0 shows the fraction of firms that had a correlation above 0 and $> 0(5\%)$ the fraction that had a correlation that was significantly above 0 (at the 5% level).
We repeat the analysis, but now correlate estimators to a 5-minute price impact measure.

	N	$\bar{\rho}$	$\bar{\rho}$ (by Size)	$\bar{\rho}$ (by Obs)	> 0	> 0 (5%)	N (PI)	$\bar{\rho}$ (PI)	$\bar{\rho}$ (PI by Size)	$\bar{\rho}$ (PI by Obs)	> 0 (PI)	> 0 (5%) (PI)
Closing Spread CRSP	13252	57.61***	33.45	52.60	97.59	85.76	11720	11.39***	7.63	7.79	67.24	20.64
Roll 0	13188	8.39***	6.07	7.92	69.38	15.68	11665	0.42**	0.34	0.19	50.14	5.21
Roll 0 (ret)	13252	23.10***	27.46	23.07	87.28	46.05	11721	7.35***	9.49	6.55	63.41	14.85
Gibbs	13254	10.44***	11.32	10.81	72.44	20.67	11722	0.69***	1.68	0.57	50.80	6.03
Zero	13223	-4.22	-2.11	-3.84	40.28	4.93	11699	-1.59	-1.35	-0.95	47.00	5.70
LOT	13254	9.63***	6.60	8.19	68.04	19.64	11723	3.32***	2.52	2.42	55.38	9.74
LOT y-split	13228	2.82***	-0.05	1.53	53.35	10.68	11706	1.34***	0.30	0.87	50.85	8.34
FHT	13224	10.35***	4.95	9.33	68.50	22.22	11701	2.21***	0.47	1.67	53.41	10.11
Tobek FHT	13244	33.27***	47.76	33.91	91.62	62.03	11718	11.45***	15.15	10.23	66.74	22.94
Effective Tick	13257	14.27***	14.24	12.50	74.99	28.46	11727	4.29***	3.18	2.67	55.43	11.89
Corwin 0	13154	29.26***	30.33	28.91	89.64	57.85	11638	6.81***	7.05	6.17	61.66	15.43
Tobek Corwin 0	13153	24.82***	25.64	24.70	86.92	50.58	11637	5.35***	5.66	4.92	59.47	13.23
Abdi 2-day	13257	34.70***	29.10	32.68	93.67	65.78	11727	8.38***	8.12	7.17	64.71	16.76
Abdi monthly	13244	20.85***	8.74	18.30	82.35	40.29	11712	3.37***	0.51	2.54	56.15	9.91
VoV High-Low	13244	49.93***	48.78	48.44	96.93	83.50	11717	14.33***	12.47	12.22	72.06	27.14
VoV daily	13244	56.86***	52.84	55.26	97.70	87.88	11718	15.86***	13.48	13.36	73.37	29.64
VoV Sigma	13252	38.80***	35.42	37.55	95.34	72.31	11720	11.83***	11.57	10.06	70.03	22.79
Amihud	13253	28.91***	21.52	26.28	88.66	54.41	11724	8.56***	5.89	6.22	62.76	18.14
(-) Amivest	13254	15.84***	8.11	9.81	87.01	19.28	11723	4.50***	2.67	2.96	65.87	5.64
(-) LIX	13242	40.74***	28.19	37.98	95.83	75.29	11717	11.16***	5.18	9.22	70.78	20.63
(-) Gamma	13252	0.31*	0.66	0.34	50.75	10.14	11723	-0.03	0.58	0.07	50.84	8.93

Table 12: RMSE / MAE
Every firm-month / firm-year, we calculate the absolute error between the effective spread and the respective spread proxy. We call this number AE (Absolute Error) and its square SE (Square Error). Foreach firm, we take the mean of these two figures. In case of SE, we additionally take the square root of that mean. We calculate the crosssectional mean of these numbers and call them MAE (Mean Average Error) and RMSE (Root Mean Square Error).
MAE and *RMSE* provide the MAE/RMSE in %.

Measure	MAE monthly	RMSE monthly	MAE yearly	RMSE yearly
Closing Spread CRSP	1.108	1.428	1.393	1.671
Roll 0 (ret)	5.857	7.016	7.122	7.665
LOT	2.821	3.889	3.139	3.631
LOT y-split	1.564	2.235	1.370	1.735
FHT	1.094	1.359	0.901	1.058
Tobek FHT	1.209	1.532	1.444	1.711
Effective Tick	0.995	1.258	0.947	1.105
Corwin 0	1.128	1.359	1.092	1.229
Tobek Corwin 0	1.210	1.443	1.161	1.293
Abdi 2-day	0.861	1.098	0.846	0.997
Abdi monthly	1.092	1.439	1.049	1.285
VoV High-Low	0.906	1.046	0.879	0.969
VoV daily	1.190	1.301	1.188	1.259
VoV Sigma	0.968	1.119	0.951	1.047

Table 13: Correlations: Monthly vs. Yearly
We repeat the analyses from tables 6, 8 and 10 respectively. However with yearly aggregated data instead of monthly data.
monthly repeats the correlation from above mentioned tables, *yearly* shows the correlations for the same approach using yearly data. In all three cases, results from the respective equal-weighted approach are shown here. The asterisks behind the yearly column show whether the difference between monthly and yearly data is significant. This test is conducted as a t-test assuming unequal variances for the Crosssection and Timeseries Stock-by-Stock and as a Fisher z-test for the Timeseries portfolio analysis.
Row *Spearman Rank Correlation* provides the rank correlation coefficient between columns *Correlation monthly* and *Correlation yearly*.

	Crosssection		**Timeseries Portfolio**		**Timeseries Stock-by-Stock**	
Measure	monthly	yearly	monthly	yearly	monthly	yearly
Roll 0	1.30	12.78***	32.76	58.14	8.46	16.27***
Roll 0 (ret)	43.54	53.85***	59.99	57.97	34.20	36.15***
Gibbs	0.24	13.69***	70.21	90.92***	9.51	19.41***
Zero	39.52	57.14***	96.76	98.35	21.20	39.89***
LOT	46.97	60.65***	97.21	97.20	33.12	48.18***
LOT ysplit	37.03	48.31***	98.39	99.42**	24.27	39.89***
FHT	55.54	68.26***	98.66	99.05	40.08	57.89***
Tobek FHT	73.52	84.36***	86.59	88.84	45.21	51.63***
Effective Tick	57.91	71.97***	97.70	85.16***	51.75	64.93***
Corwin 0	38.67	46.47*	85.73	91.21	43.58	50.53***
Tobek Corwin 0	33.11	41.69*	88.34	93.46	38.79	47.12***
Abdi 2-day	70.51	78.26***	88.80	91.77	53.82	60.79***
Abdi monthly	66.57	78.98***	90.84	92.54	41.16	50.42***
VoV High-Low	87.61	90.60***	87.52	87.81	70.20	69.96
VoV daily	86.94	90.32***	90.39	91.84	74.52	76.27***
VoV Sigma	86.39	88.98***	90.13	88.80	63.22	63.50
Closing Spread CRSP	87.42	88.64	98.39	98.80	79.65	79.76
Amihud	64.79	71.65***	83.35	61.82*	53.91	65.55***
(-) Amivest	25.25	24.39	89.89	89.64	40.34	56.83***
(-) LIX	72.14	74.77***	94.32	94.57	64.40	71.47***
(-) Gamma	8.50	-17.55***	82.70	-76.37***	-0.64	-12.47***
Spearman Rank Correlation		98.18		77.79		95.97

Table 14: Crosssectional Correlation (sorted by effective spread)
This table shows the same as column *Correlation* of table 6. However, the sample is split into quintiles based on relative effective spreads. Here we show the correlation for quintiles 1, 3 and 5.
Column *Small* shows the correlation for the firms with smallest relative spread levels in each given month, *Medium* for the middle and *Large* for the highest spread levels. *Large - Small* gives the difference in correlations between groups 1 and 5 and its significance (based on a t-test) is indicated.
Below the *Spearman Rank Correlation* between the different columns is provided.

Measure	Small	Medium	Large	Large - Small
Roll 0	-7.70	-2.17	12.99	20.69***
Roll 0 (ret)	24.71	8.43	39.40	14.68***
Gibbs	-13.55	-5.31	22.83	36.38***
Zero	11.72	7.91	16.56	4.85***
LOT	13.48	8.78	33.83	20.35***
LOT y-split	5.09	6.59	24.69	19.60***
FHT	17.31	11.49	38.55	21.24***
Tobek FHT	37.88	14.95	62.95	25.07***
Effective Tick	37.79	19.08	28.24	-9.55***
Corwin 0	29.73	16.90	21.68	-8.05***
Tobek Corwin 0	25.95	15.13	17.65	-8.30***
Abdi 2-day	27.40	18.88	58.45	31.05***
Abdi monthly	9.74	13.63	55.32	45.58***
VoV High-Low	59.05	35.75	70.62	11.57***
VoV daily	61.37	40.07	67.40	6.03***
VoV Sigma	52.71	28.23	71.80	19.09***
Closing Spread CRSP	38.49	33.26	78.49	40.00***
Amihud	24.95	16.56	54.19	29.24***
(-) Amivest	36.78	13.96	13.25	-23.52***
(-) LIX	49.48	27.28	55.67	6.18***
(-) Gamma	0.67	-0.40	9.80	9.13***
Spearman Rank Correlation				
Small		93.90	69.35	
Medium			73.64	

Table 15: Timeseries Portfolio Correlation (sorted by effective spread)
This table shows the same as column *Correlation* of table 8. However, the sample is split into quintiles based on relative effective spreads. Here we show the correlation for quintiles 1, 3 and 5.
Column *Small* shows the correlation for the firms with smallest relative spread levels in each given month, *Medium* for the middle and *Large* for the highest spread levels. *Large - Small* gives the difference in correlations between groups 1 and 5 and its significance (based on a Fisher z-test) is indicated.
Below the *Spearman Rank Correlation* between the different columns is provided.

Measure	Small	Medium	Large	Large - Small
Roll 0	-10.59	12.65	91.93	102.52***
Roll 0 (ret)	5.15	12.92	94.87	89.72***
Gibbs	22.61	57.20	92.88	70.27***
Zero	93.02	97.68	88.85	-4.17***
LOT	95.38	94.94	94.53	-0.85
LOT ysplit	91.89	98.90	92.47	0.58
FHT	97.76	98.72	93.63	-4.13***
Tobek FHT	6.82	62.86	92.42	85.60***
Effective Tick	98.41	99.40	93.06	-5.34***
Corwin 0	-11.28	48.42	95.93	107.20***
Tobek Corwin 0	-17.42	48.23	94.38	111.80***
Abdi 2-day	5.23	61.27	99.13	93.90***
Abdi monthly	0.92	73.51	99.24	98.32***
VoV High-Low	88.26	91.47	87.35	-0.92
VoV daily	88.83	93.47	95.60	6.77***
VoV Sigma	90.15	91.65	87.30	-2.85
Closing Spread CRSP	89.38	99.46	99.28	9.91***
Amihud	94.69	96.74	48.24	-46.46***
(-) Amivest	91.25	82.40	74.39	-16.87***
(-) LIX	96.50	94.58	82.70	-13.80***
(-) Gamma	-5.74	-16.91	37.94	43.68***
Spearman Rank Correlation				
Small		85.58	-26.36	
Medium			4.55	

Table 16: Timeseries Stock-by-Stock Correlation (sorted by effective spread)
This table shows the same as column $\bar{\rho}$ of table 10. However, the sample is split into quintiles based on relative effective spreads. Here we show the correlation for quintiles 1, 3 and 5.
Column *Small* shows the correlation for the firms with smallest relative spread levels in each given month, *Medium* for the middle and *Large* for the highest spread levels. *Large - Small* gives the difference in correlations between groups 1 and 5 and its significance (based on a t-test) is indicated.
Below the *Spearman Rank Correlation* between the different columns is provided.

Measure	Small	Medium	Large	Large - Small
Roll 0	3.56	5.21	16.21	12.65***
Roll 0 (ret)	21.97	19.67	34.32	12.35***
Gibbs	12.33	12.85	21.74	9.41***
Zero	17.38	18.54	13.76	-3.61***
LOT	17.57	20.06	29.50	11.93***
LOT y-split	10.75	17.03	21.47	10.73***
FHT	28.77	28.56	31.63	2.86***
Tobek FHT	34.52	31.17	40.42	5.90***
Effective Tick	51.16	41.58	31.72	-19.44***
Corwin 0	25.51	30.54	35.33	9.82***
Tobek Corwin 0	21.05	26.60	31.31	10.26***
Abdi 2-day	27.57	35.53	51.61	24.04***
Abdi monthly	7.19	20.94	44.11	36.92***
VoV High-Low	60.93	51.81	54.16	-6.77***
VoV daily	64.65	58.73	59.75	-4.90***
VoV Sigma	53.02	42.51	49.89	-3.12***
Closing Spread CRSP	58.01	68.01	72.49	14.48***
Amihud	52.53	41.33	38.91	-13.62***
(-) Amivest	42.41	35.82	28.29	-14.12***
(-) LIX	56.53	47.37	46.64	-9.89***
(-) Gamma	1.13	-1.55	0.76	-0.37
Spearman Rank Correlation				
Small		96.23	78.18	
Medium			85.19	

Table 17: Correlations: NYSE vs. NASDAQ (matched)
We repeat the analysis from table **??** but only for stocks matched between NYSE and NASDAQ based on the effective spread and the size of those companies. The matching approach is described in detail in the text section.
We compare the crosssectional and stock-by-stock timeseries correlation. Results from the respective equal-weighted approach are shown here. *NYSE* and *NASDAQ* show the correlation in the respective market. *NASDAQ > NYSE* shows the fraction of observations with a NASDAQ correlation greater than the NYSE correlation. *Significance* indicates whether the difference between NYSE and NASDAQ data is significant. This test is conducted as a t-test assuming unequal variances.
Row *Spearman Rank Correlation* provides the rank correlation coefficient between columns *NYSE* and *NASDAQ* and row *N* the number of observations.

	Crosssection				Timeseries Stock-by-Stock			
Measure	NYSE	NASDAQ	NASDAQ > NYSE	Significance	NYSE	NASDAQ	NASDAQ > NYSE	Significance
Closing Spread CRSP	81.29	89.41	92.50	***	66.82	79.00	63.60	***
Roll 0	-8.37	2.10	93.75	***	-1.19	8.26	63.93	***
Roll 0 (ret)	34.23	33.19	44.58		23.75	33.97	62.65	***
Gibbs	-12.41	-3.25	88.33	***	1.06	7.67	56.37	***
Zero	38.04	33.73	32.08	***	33.74	14.86	31.96	***
LOT	39.63	37.98	40.00		34.32	25.77	37.53	***
LOT y-split	30.58	30.38	50.42		27.80	16.73	37.71	***
FHT	49.61	46.67	29.17	**	44.20	30.52	34.32	***
Tobek FHT	63.24	67.50	74.58	***	36.89	45.65	59.26	***
Effective Tick	63.77	54.57	14.58	***	63.87	46.23	27.26	***
Corwin 0	38.77	32.97	25.42	***	35.16	50.25	63.36	***
Tobek Corwin 0	33.68	28.25	26.25	**	30.50	45.99	64.11	***
Abdi 2-day	54.84	63.49	81.25	***	38.67	52.72	63.81	***
Abdi monthly	42.86	59.00	99.17	***	23.47	36.82	65.03	***
VoV High-Low	82.35	87.00	89.17	***	66.34	70.28	54.29	***
VoV daily	83.24	86.40	81.25	***	71.15	75.24	55.12	***
VoV Sigma	77.94	84.35	94.58	***	58.06	59.87	49.58	***
Amihud	61.85	58.86	42.50	***	56.18	52.48	41.85	***
(-) Amivest	24.96	26.58	67.50	***	42.79	40.24	46.64	***
(-) LIX	65.50	72.06	93.33	***	63.60	62.87	44.32	
(-) Gamma	-2.16	4.81	69.17	***	-0.38	-1.10	47.50	
N				240				3341
Spearman Rank Correlation				96.94				88.64

Table 18: Explaining Timeseries Correlations

We use the results from Table 10 to explain the differences in correlations between a given estimator and the effective spread across different firms:
Dependent variable is the timeseries stock-by-stock correlation between an estimator and the effective spread. For each independent variable the timeseries mean is calculated. *Size* is measured in billions and *Firm Age* is measured in log(years) between a given date and the CRSP begin date of that firm. AMEX, NASDAQ, NYSE are dummy variables that are equal to 1 if the fund was listed on that exchange.
We then conduct a simple crosssectional WLS regression with heteroskedasticity robust standard errors. As weight, the number of monthly observations in the stock-by-stock timeseries of the respective firm is used.

	(1) Roll 0 (ret)	(2) Zero	(3) Corwin 0	(4) FHT	(5) Amihud	(6) (-) Gamma	(7) Effective Tick	(8) CRSP Closing Spread	(9) LOT y-split	(10) (-) LIX	(11) Abdi 2-day	(12) VoV High-Low
Effective Spread	14.04^{***}	-2.479^{***}	12.76^{***}	6.147^{***}	-10.01^{***}	1.434^{***}	-4.138^{***}	1.562^{***}	2.860^{***}	-3.538^{***}	21.41^{***}	2.330^{***}
	(25.41)	(-3.62)	(19.25)	(10.86)	(-20.68)	(3.33)	(-7.97)	(4.70)	(4.94)	(-7.33)	(39.97)	(5.54)
Squared Effective Spread	-146.5^{***}	-44.64^{***}	-221.4^{***}	-96.73^{***}	94.34^{***}	6.618	-25.03^{***}	-58.77^{***}	-46.89^{***}	-13.79^{*}	-308.4^{***}	-69.17^{***}
	(-19.88)	(-4.66)	(-23.36)	(-12.43)	(13.67)	(1.13)	(-3.43)	(-12.35)	(-5.83)	(-1.90)	(-41.29)	(-11.12)
Size	0.00229^{***}	-0.00278^{***}	-0.00105	-0.00350^{***}	0.00115^{*}	0.00142^{***}	0.00170^{***}	-0.00487^{***}	-0.00352^{***}	0.00118^{**}	-0.00213^{***}	0.00253^{***}
	(3.63)	(-4.42)	(-1.50)	(-6.46)	(1.94)	(3.50)	(3.69)	(-8.40)	(-6.48)	(2.01)	(-3.04)	(4.09)
Turnover	-0.697	7.701^{***}	2.076^{**}	6.309^{***}	3.228^{***}	0.349	7.931^{***}	4.634^{***}	7.834^{***}	2.053^{***}	-3.186^{***}	4.885^{***}
	(-0.99)	(9.22)	(2.52)	(8.87)	(5.71)	(0.62)	(13.90)	(11.02)	(10.85)	(3.63)	(-4.42)	(11.16)
ln(Firm Age)	-0.0452^{***}	0.161^{***}	-0.0256^{***}	0.117^{***}	0.0285^{***}	-0.00814^{***}	0.0988^{***}	0.0148^{***}	0.130^{***}	0.0400^{***}	-0.0248^{***}	0.00215
	(-17.49)	(53.87)	(-8.26)	(46.28)	(12.07)	(-4.44)	(43.40)	(7.13)	(49.66)	(17.43)	(-9.44)	(1.02)
Return Std.Dev.	-0.158^{***}	0.0622^{***}	0.0847^{***}	-0.0596^{***}	0.0410^{***}	-0.0607^{***}	0.0676^{***}	0.0219^{***}	0.100^{***}	0.0804^{***}	0.0169^{*}	0.0436^{***}
	(-15.98)	(4.99)	(7.19)	(-5.69)	(4.66)	(-7.36)	(7.17)	(3.67)	(9.16)	(9.50)	(1.70)	(6.11)
AMEX	0.332^{***}	-0.192^{***}	0.169^{***}	0.0566^{***}	0.586^{***}	0.0434^{***}	0.239^{***}	0.560^{***}	-0.242^{***}	0.519^{***}	0.353^{***}	0.603^{***}
	(21.03)	(-9.37)	(9.46)	(3.44)	(44.02)	(4.02)	(15.70)	(40.48)	(-14.03)	(39.97)	(25.16)	(52.97)
NASDAQ	0.416^{***}	-0.0000814	0.352^{***}	0.216^{***}	0.616^{***}	0.00936	0.393^{***}	0.835^{***}	-0.0646^{***}	0.634^{***}	0.437^{***}	0.680^{***}
	(35.03)	(-0.01)	(24.95)	(17.32)	(56.90)	(1.02)	(35.36)	(100.42)	(-5.10)	(61.22)	(38.35)	(74.01)
NYSE	0.327^{***}	-0.0469^{***}	0.182^{***}	0.180^{***}	0.539^{***}	0.0435^{***}	0.376^{***}	0.622^{***}	-0.106^{***}	0.537^{***}	0.279^{***}	0.592^{***}
	(29.62)	(-3.59)	(13.66)	(16.17)	(55.17)	(5.19)	(37.32)	(68.87)	(-9.32)	(57.15)	(25.48)	(69.05)
N	12115	12072	12058	12075	12114	12113	12116	12112	12078	12103	12112	12105
Adjust. R2	0.730	0.697	0.744	0.841	0.902	0.026	0.912	0.964	0.698	0.934	0.871	0.950

t statistics in parentheses

Table 19: Predicting Timeseries Correlations

We repeat the analysis from table 18 and just replace effective spread and squared effective spread by CRSP closing spread and squared CRSP closing spread.

	(1) Roll 0 (ret)	(2) Zero	(3) Corwin 0	(4) FHT	(5) Amihud	(6) (-) Gamma	(7) Effective Tick	(8) CRSP Closing Spread	(9) LOT y-split	(10) (-) LIX	(11) Abdi 2-day	(12) VoV High-Low
CRSP Closing Spread	6.013***	-2.432***	4.598***	2.542***	-4.881***	1.330***	-2.396***	-0.568***	1.432***	-3.238***	7.351***	-0.211
	(25.20)	(-8.43)	(15.08)	(10.63)	(-23.10)	(7.32)	(-10.68)	(-3.68)	(5.88)	(-15.90)	(29.54)	(-1.16)
Squared CRSP Closing Spread	-32.17***	2.756	-39.62***	-17.02***	23.10***	-4.047***	1.551	-6.195***	-8.359***	8.087***	-53.40***	-7.468***
	(-19.68)	(1.44)	(-17.62)	(-10.41)	(14.93)	(-3.39)	(1.02)	(-5.40)	(-5.10)	(5.25)	(-28.95)	(-5.50)
Size	0.00202***	-0.00321***	-0.00158**	-0.00365***	0.00114*	0.00165***	0.00160***	-0.00532***	-0.00351***	0.000645	-0.00318***	0.00206***
	(3.22)	(-5.10)	(-2.28)	(-6.74)	(1.90)	(4.07)	(3.42)	(-9.29)	(-6.43)	(1.10)	(-4.44)	(3.39)
Turnover	-2.523***	8.608***	1.739**	6.830***	3.720***	0.386	9.732***	3.562***	8.743***	1.489***	-5.846***	4.036***
	(-3.71)	(10.27)	(2.14)	(9.66)	(6.72)	(0.70)	(16.73)	(8.67)	(12.13)	(2.70)	(-8.17)	(9.35)
ln(Firm Age)	-0.0489***	0.164***	-0.0265***	0.117***	0.0315***	-0.00930***	0.102***	0.0157***	0.130***	0.0426***	-0.0277***	0.00296
	(-18.53)	(53.99)	(-8.44)	(46.18)	(13.31)	(-5.06)	(43.42)	(7.60)	(49.59)	(18.57)	(-10.18)	(1.39)
Return Std.Dev.	-0.166***	0.0757***	0.0848***	-0.0716***	0.0606***	-0.0701***	0.0741***	0.0461***	0.0859***	0.111***	0.0285***	0.0632***
	(-16.12)	(5.83)	(6.89)	(-6.61)	(6.71)	(-8.19)	(7.44)	(7.27)	(7.65)	(12.65)	(2.69)	(8.41)
AMEX	0.383***	-0.180***	0.228***	0.0775***	0.561***	0.0368***	0.233***	0.589***	-0.238***	0.537***	0.462***	0.634***
	(24.88)	(-8.95)	(13.43)	(4.95)	(44.53)	(3.57)	(15.27)	(44.33)	(-14.37)	(43.05)	(33.90)	(58.73)
NASDAQ	0.505***	-0.0327**	0.402***	0.238***	0.557***	0.0186**	0.339***	0.843***	-0.0604***	0.613***	0.557***	0.691***
	(48.76)	(-2.50)	(32.22)	(21.88)	(57.66)	(2.31)	(33.79)	(116.34)	(-5.35)	(67.10)	(54.68)	(86.33)
NYSE	0.341***	-0.0472***	0.189***	0.181***	0.534***	0.0409***	0.366***	0.629***	-0.108***	0.543***	0.305***	0.600***
	(30.38)	(-3.57)	(14.06)	(16.36)	(55.07)	(4.85)	(35.47)	(71.81)	(-9.58)	(58.44)	(27.04)	(70.92)
N	12113	12070	12056	12073	12112	12111	12114	12112	12076	12101	12110	12103
Adjust. R2	0.725	0.691	0.740	0.841	0.902	0.024	0.909	0.964	0.698	0.934	0.863	0.949

t statistics in parentheses

Table 20: Predicted Best Estimator

We use predicted correlation values based on 19. using average firm characteristics as input.
Panel A shows the number/fraction of firms for which the respective estimator is predicted to perform best. The row *Predicted Mean Correlation* shows the predicted timeseries stock-by-stock correlation if one would always pick this best estimator.
Panel B shows the equal-, size- and observation-weighted average timeseries stock-by-stock correlation of the closing spread and a newly defined estimator. For each stock, this estimator is defined as the estimator that is predicted to perform best (see Panel A). Row *P-value* is the p-value of a ttest that tests whether the correlation of the combined estimator is higher than that of the Closing Spread. Row *Actual Max = Predicted Max* shows the fraction of firms for which the predicted best estimator is actually the best estimator.

PANEL A

Measure	**N**	**%**
Closing Spread CRSP	10027	78.44
Roll 0 (ret)	7	0.05
LOT y-split	1	0.01
FHT	16	0.13
Effective Tick	1035	8.10
Abdi 2-day	266	2.08
VoV High-Low	1019	7.97
Amihud	282	2.21
(-) LIX	48	0.38
Zero	0	0.00
Corwin 0	0	0.00
(-) Gamma	0	0.00
Missing	82	0.64
Predicted Mean Correlation		83.12

PANEL B

Measure	**Rho (equal-weighted)**	**Rho (size-weighted)**	**Rho (obs-weighted)**
Closing Spread CRSP	79.65	68.06	81.33
Maximum Predicted Correlation	80.77	79.59	84.60
P-value	.018		
Actual Max = Predicted Max	60.78		

Table 21: Composite Spread Estimator

Panel A shows the main results for the Principal Component Analysis:
We identify the first principal component in our monthly dataset using the below listed estimators. *RHO PCA* shows what fraction of the variation in the estimators can be explained by the first Principal Component. We next calculate the correlation between effective spreads and the principal component on a stock-by-stock basis. As benchmark for the performance of the Principal Component, we correlate effective spreads with the Closing Spread from CRSP. Panel A of this table shows the average Correlation Coefficient across all firms. The *Overall* specification shows the results for the entire sample. In specifications *Firm* and *Time* we halved the sample into two equally-sized groups either by time (first period / second period) or by randomly assigning firms to each group. We then calibrate the PCA based on sample 1 and test the correlation in sample 2. Variable definitions are identical to those in table 10.
Panel B shows results for the Maximum Correlation Estimator:
Based on the equal- or value weighted (*Weighting*) Portfolio Dataset (see Table 8), we calculate the correlation between the relative effective spread and a combined estimator $\hat{x_t} = \sum w_i x_{i,t}$ that is a linear combination of the below listed (standardized) estimators $x_{i,t}$.
We then try to maximize the thus calculated correlation by adjusting the weights, we put on each estimator: $\max_{w_i} \rho(s_t, \hat{x_t}) s.t \sum w_i = 1$.
The table shows the thus calculated weights for different specifications (as above).
While the weights are calibrated using the Portfolio Dataset (due to computational limitations), we test the performance of the Maximum Correlation Estimator in the same stock-by-stock sample as the Principal component.

Panel A: PCA Estimator

Subsample	RHO PCA	$\bar{\rho}$	$\bar{\rho}$ (by Obs)	$\bar{\rho}$ (by Size)	$\bar{\rho}(CRSP)$	$\bar{\rho}$ (by Obs) (CRSP)	$\bar{\rho}$ (by Size) (CRSP)	Weights Roll 0 (ret)	Zero	Corwin 0	Closing Spread CRSP	Effective Tick	FHT	Amihud	(-) Gamma	LIX	Abdi 2-day	VoV High-Low	LOT y-split
Overall	56.37	70.33	77.63	71.13	79.65	81.33	68.06	8.82	7.36	9.07	10.57	9.71	10.47	5.84	0.30	7.83	10.66	9.94	9.44
by Time	57.73	71.01	75.03	74.65	76.88	77.04	61.17	8.99	6.38	9.16	10.31	9.69	10.50	6.50	0.29	7.93	10.69	10.19	9.39
by Firm	56.37	69.92	77.59	70.69	79.28	81.21	67.81	8.84	7.36	9.10	10.58	9.71	10.48	5.79	0.28	7.81	10.67	9.92	9.45

Panel B: Maximum Correlation Estimator

Subsample	Weighting	$\bar{\rho}$	$\bar{\rho}$ (by Obs)	$\bar{\rho}$ (by Size)	$\bar{\rho}(CRSP)$	$\bar{\rho}$ (by Obs) (CRSP)	$\bar{\rho}$ (by Size) (CRSP)	Weights Roll 0 (ret)	Zero	Corwin 0	Closing Spread CRSP	Effective Tick	FHT	Amihud	Gamma	LIX	Abdi 2-day	VoV High-Low	LOT y-split
Overall	equal	48.47	61.57	58.16	79.65	81.33	68.06	4.00	47.00	12.00	16.00	0	17.00	0	0	0	0	3.00	0
Overall	value	61.64	70.46	64.18	79.65	81.33	68.06	0	0	0	1.00	0	59.00	1.00	0	14.00	0	20.00	6.00
by Time	equal	38.95	42.18	30.10	73.36	71.89	59.50	14.35	46.87	0	1.21	0	8.42	0	0	29.15	0	0	0
by Time	value	54.81	55.63	40.98	73.36	71.89	59.50	0	0	0	2.55	0	42.59	0	0.59	25.42	0	17.76	11.09
by Firm	equal	52.30	64.44	61.76	80.02	81.45	68.32	5.40	46.01	9.41	20.23	0	15.01	0	0	3.94	0	0	0
by Firm	value	63.46	71.71	66.16	80.02	81.45	68.32	0	0	0	2.60	0	62.12	0.90	0.75	17.33	0	16.31	0

Table 22: Event Compare Diff-in-Diff
We conduct a difference-in-differences analysis of five events: Nasdaq Order Handling Rule (*NASDAQ OHR* below), Nyse Tick Size change from 1/16 to decimals (*NYSE TICK* below), Nasdaq Tick Size change from 1/16 to decimals (*NASDAQ TICK* below), Nyse Open Book (*NYSE OB*) and *NYSE HYBRID*. The diff-in-diff approach is applied to the cross-section (*CS*) of liquidity estimators. For details of the methodology see Section 4.9. This table shows the $(Treatment_{Post} - Control_{Post}) - (Treatment_{Pre} - Control_{Pre})$-term. Asterisks indicate the results of a t-test which tests whether the diff-in-diff-term is significantly different from zero. N provides the number of firms in the sample.

	NASDAQ OHR	NYSE TICK	NASDAQ TICK	NYSE OB	NYSE HYBRID
Roll 0 (ret)	0.07**	-0.16**	0.01	-0.02	-0.02
Zero	0.15***	-0.01	0.02	-0.02	-0.02
Corwin 0	0.09***	-0.17*	0.05	-0.02	0.02
Closing Spread CRSP	-0.05***	-0.25**	0.07	-0.10	-0.08
Effective Tick	0.05**	0.21***	0.38***	-0.06*	0.02
FHT	0.12***	-0.11***	-0.01	-0.07	-0.02
Amihud	-0.03	0.02	0.02	-0.01	-0.15**
(-) Gamma	-0.05	0.05	-0.04	0.11	-0.10
LIX	0.04***	0.03	0.04***	0.03	0.05**
Abdi 2-day	0.12***	-0.21**	0.08	-0.05	0.01
VoV High-Low	0.01	-0.01	0.06*	-0.02	0.04
LOT y-split	0.11***	-0.12***	-0.03	-0.02	0.03
N	1415	1336	1307	1259	1252