Tree-Based Conditional Portfolio Sorts: The Relation Between Past and Future Stock Returns[*]

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Abstract

Which variables provide independent information about the cross-section of future returns? Portfolio sorts and Fama-MacBeth regressions cannot easily answer this question when the number of candidate variables is large and when cross-terms might be important. We introduce a new method based on ideas from the machine learning literature that can be used in this context. Applying the method to past-return-based prediction of future returns, short-term returns become the most important predictors. A trading strategy based on our findings has an information ratio twice as high as a Fama-MacBeth regression accounting for two-way interactions. Transaction costs do not explain the results.

Keywords: Cross-sectional asset pricing, Stock market anomalies, Momentum, Machine Learning

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1 Introduction

Consider the challenge of a portfolio manager who wants to use past information to estimate expected returns at the firm level. He has at his disposal an overwhelming set of potentially correlated predictor variables, as documented by several recent survey papers. Subrahmanyam (2010) surveys 50 earnings-based return predictive signals, McLean and Pontiff (2012) document 82, and Harvey et al. (2013) and Green et al. (2013) both extend the list to around 330. These variables range from classic accounting-based variables, such as book-to-market, to return-based variables, such as the stock return over the previous year. They may even include more exotic variables such as the creativity of a stock's ticker. Many of these diverse variables might interact in nontrivial ways, increasing the set even more. Furthermore, the literature suggests stand-ins for many variables (for example, value or quality). Which should the manager pick? Beyond these many challenges lurks the risk of overfitting the data with any estimation method that the manager might use, rendering the analysis worthless for new observations. How then should one go about estimating expected returns while taking all these issues into account?

The literature in empirical asset pricing provides a few methods. We show, however, that two of them — portfolios sorts and Fama-MacBeth regressions — can deal only with some of the challenges outlined above. We suggest an alternative approach, motivated by the method of *conditional* portfolio sorts but extending easily to large sets of predictor variables and flexibly dealing with their interactions. In contrast to how conditional portfolio sorts are usually applied, we use the data to estimate both the optimal conditioning variables and associated optimal thresholds.

Our contribution to the literature is threefold: First, we import ideas from the machine learning literature and tailor them to a financial application in order to produce a model that is suited to evaluate the independent information in the entirety of many cross-sectional predictor variables and their potential interactions. While these methods are data-driven, we are careful to develop valid out-of-sample validations of the model. As the machine learning literature is often criticized for producing black box predictions, we especially emphasize new measures to extract interpretable information about the structure of the estimated prediction function.

Second, we apply our methodology to the prediction of future returns from past stock returns, and we recover short-term returns (that is, the past six most recent one-month returns) as the most important predictors. Implementable trading strategies based on our findings have a risk-adjusted monthly return of around 2 percent per month, with an information ratio that is about three times as high as the information ratio achievable in a linear framework that does not account for nonlinearities and variable interactions. The information ratio is about twice as high as in a Fama-MacBeth framework that accounts for two-way interactions. Transaction costs cannot account for our results.

Third, we trace the improved predictions to several sources. We find that recent past returns, not more distant past returns, contain almost all information about future returns when information is exploited more efficiently than in a linear model. This finding addresses the tension between the superior return of intermediate momentum in Novy-Marx (2012) and Goyal (2011), who cannot find this effect in other countries. Reassuringly, our model also reproduces previous facts in U.S. equity returns data. Our model finds short-term reversal, the seasonal return effect of Heston and Sadka (2008), and momentum for longer-dated returns. But the model also suggests a few new facts: A nonlinear relationship between relatively recent past returns and future returns and important interactions among past returns at different horizons.

These results pose a challenge for existing methodologies when the goal is to evaluate many variables in a joint framework. The portfolio sort methodology, a dominant method in analyzing cross-sectional predictor variables,[1] sorts stocks into 3 to 10 portfolios each month (or year) based on the value of a particular variable. In the next step, subsequent returns for each portfolio are calculated and it is checked whether there is a monotone relation between the sorting variable and these subsequent portfolio returns. In addition, researchers often compute the equal- or value-weighted hedge return of going long (short) in the highest quantile portfolio and going short (long) in the lowest quantile portfolio. The relevance of the sorting variable is then assessed by comparing the hedge return to some equilibrium model of asset prices (for example, the capital asset pricing model) and/or by assessing the monotonicity of the returns over deciles. The portfolio sort methodology is a powerful, nonparametric tool that works best in low dimensional cases. Problems arise if returns are to be sorted on more than two or three predictor variables, as there will typically be few stocks in each portfolio. But this makes controlling for information

[1] See the survey of Green et al. (2013).

contained in other variables challenging, or, as Fama and French (2008) put it, "sorts are awkward for drawing inference about which anomaly variables have unique information about average returns."

Multivariate Fama-MacBeth regressions (Fama and MacBeth (1973)) are able to address this concern by showing the marginal effect of each predictor variable once all others are controlled for. The methodology is based on estimating a cross-sectional regression in each period and averaging the coefficient estimates over time. This works well with a larger number of predictor variables. But when we include interactions between predictor variables, this methodology reaches its limit, too: Even if only 50 variables are considered jointly, the total number of regression coefficients that include all two-way interactions (and no higher-order interactions) is 1275, higher than the number of companies in early months of the sample and higher than the number of companies throughout the entire sample if the sample is split by firm size first, as in Fama and French (2008). Second, as Fama and French note, results can be vulnerable to influential observations of extreme returns. With this in mind, Green et al. (2014) "view it as infeasible to examine non-linearities in RPS-returns relations in the manner undertaken in Fama and French (2008)."

Our method addresses the aforementioned challenges by accounting for arbitrary interaction terms.[2] Conditional portfolio sorts arrange firms into groups based on the value of some variable (e.g. book-to-market). Within each group, stocks are then sorted again based on the value of some other variable. Sorting variables and sorting values are typically chosen based on a priori reasoning. We start from the assumption that neither the sorting variable nor the sorting value are known and therefore need to be estimated. Furthermore, conditional sorts are typically conducted for no more than two levels (that is, stocks are sorted twice) and the same sorting variable is used in all branches on the second level. We estimate sorts at deeper levels and allow for flexible variable selection at each branch.

This approach incurs two well-known and related problems. First, since the model can only be estimated stepwise, the variables and sorting values that are selected at each point need not be globally optimal. But since the sorting rule is discrete, any error in the estimation of the sorting variable and sorting value could have a large impact on the model's predictions. Second, the

[2] The finance literature is somewhat imprecise about the distinction between interaction terms and nonlinearities, often using both terms interchangeably. We reserve "interactions" for cross-products between two variables, and we think of "nonlinearities" as higher-order polynomial terms with respect to a single variable.

approach is data-driven, has many degrees of freedom, and easily overfits the data. We therefore must make sure that the estimates are valid out of sample.

How do we deal with these problems? Our solution is based on model-averaging. We estimate tree-based conditional portfolio sorts many times, with different subsets of regressors and on different subsets of the data, and combine the estimates from all models into a final prediction. The rationale is that by averaging estimates from models that are de-correlated in this manner, one can obtain different but related signals about the true underlying process, even if the simple underlying models are not entirely correct. At the same time, model-averaging helps with the overfitting problem because only subsets of the data and predictor variables are used in each model. The idea is grounded in the computer science literature and has been successfully applied in many contexts. We find that the idea can be applied to accurately predict expected returns.

The main drawback of averaging over many models is that results are not as easy to interpret as a single conditional sort. In order to shed light on the mechanism, we suggest a number of evaluation measures. We compute a measure of predictor variable importance that can be interpreted similarly to t-statistics in regressions. In addition, we compute partial derivatives for each predictor variable to assess directional effects of specific variables. We also run diagnostic checks to see whether the predictions from the model can be explained by a simple linear regression on our predictor variables (which would speak against the importance of interaction effects).

The method takes into account that a predictor variable's influence might vary over time.[3] We set up the out-of-sample tests in such a way that they lend themselves naturally to investigate time variation of the importance of particular variables. In each year, we estimate the model with data over the past years. For the next twelve months, one-month expected returns are then projected by fixing the model estimates and making predictions based on the new data that were reported only after the estimation period. Not only are our trading results strong in this exercise, but the approach also enables us to look at which variables come out as important in the search procedure in which period.

We apply our method to contribute to the debate about whether past returns contain infor-

[3] As Harvey et al. (2013) note "it is possible that a particular factor is very important in certain economic environments and not important in other environments. The unconditional test might conclude the factor is marginal."

mation about future returns and, if so, which past returns matter the most. This debate has recently regained interest after Novy-Marx (2012) found that medium-term momentum — that is, a stock's return over the 7-12 months prior to portfolio formation — can be a better indicator of future return than momentum calculated over the entire previous year (excluding the most recent month). Goyal and Wahal (2013) cannot find this effect in 37 other markets outside the United States. Other recent articles have looked at a moving average strategy derived from past prices (Han et al. (2011)) or construct a trend factor from daily to annual returns that outperforms the standard momentum factor (Han and Zhou (2013)). We therefore regard the relation between past and future returns as a natural laboratory for our method.

We construct a set of decile rankings for the non-overlapping one-month returns over the two years before portfolio formation as predictors. Our model combines these predictors into an optimal forecast of the next period's returns and uses the aforementioned measures to evaluate which one-month returns are most important for the prediction.

Various checks underscore the robustness of our findings. We construct another set of predictor variables that includes many possible past returns with different horizons and gaps to the portfolio formation date to see how our methodology performs when many of the predictor variables (a total of 126) are highly correlated. In this setting, abnormal returns are again high and a similar return structure, with similar partial derivatives for specific predictor variables, is estimated. Our results are also unaffected by including 86 additional firm characteristics from the literature. Here, results for abnormal returns are actually a bit stronger because of the additional information in accounting variables and other characteristics, and the return structure results still hold. We then make sure that our results are not entirely driven by illiquid stocks by re-performing all computations for large, small, and micro firms (in the terminology of Fama and French (2008)) separately. While we find that results are stronger in small stocks and strongest in micro stocks, our main conclusions hold throughout all size categories. We conclude that more recent past returns are more relevant than intermediate past returns for prediction of future returns, and more generally that past returns are related to future returns in a more complex way than can be captured by any single past return.

Before we continue, we provide a short overview of the related literature. In his presidential address, Cochrane (2011) describes the "factor zoo" of stock market anomalies and how it has developed over the years. Subrahmanyam (2010), Goyal (2011), Green et al. (2013) and Harvey et al. (2013) review as many as 330 anomalies that have been found by academic research and call for a synthesis of the existing literature. While early attempts in this direction focused on smaller sets of characteristics were undertaken by Haugen and Baker (1996), Daniel and Titman (1997) and Brennan et al. (1998), Cochrane (2011) argues that different methods might be required to find the independent information for average returns in the entirety of documented predictor variables. Our paper can be read as an attempt to provide just such a new method.

Green et al. (2014) investigate the mutual information in 100 signals and find that up to 24 of them have predictive power for returns when used jointly. They suggest an alternative to the standard three-factor model by Fama and French (1992) that is based on 10 different characteristics. The paper notes the potential relevance of interactions but does not investigate them in detail.[4] Lewellen (2013) investigates the power of 15 different firm characteristics to predict variation in the cross-section. He finds that expected stock returns derived from the model are strongly predictive of actual stock returns for as long as 12 months.

Fama and French (2013) follow an alternative approach that attempts to capture variation in returns by a (small) factor model. They extend the three-factor model by proxies for profitability and investment, which appears to capture contemporaneous variation in cross-sectional returns well, except for small stocks. The paper uses a quadruple sorting strategy to address interactions between size, value, profitability, and investment opportunities. Kogan and Tian (2012) construct all combinations of three-and four-factor models from a set of 27 firm characteristics. They find that the best performing models are unstable across time periods.

The literature on momentum and reversal is too large to review comprehensively here, but we note a few key articles. If stock prices systematically over- or underreact, future stock returns should be predictable from past returns data alone. de Bondt and Thaler (1985) test overreaction by sorting stocks based on the return in the previous three years (the portfolio formation period). They find that losers (the bottom decile of returns in the formation period) outperforms winners by about 25 percent over three years. Jegadeesh (1990) and Lehmann (1990) find a "reversal" effect for portfolios that are formed based on short-term (one week to one month) prior returns.

[4] They write, "fundamental valuation type measures and market trading type measures appear to matter across firm size. In large-cap firms the important RPS can be broadly classified as fundamental valuation measures or trading type measures. For mid-cap and small-cap firms the themes appear slightly different."

Jegadeesh and Titman (1993), on the other hand, find evidence for a "momentum effect" when portfolios are sorted on medium-term (3 to 12 months) prior return. The momentum finding survives the analysis in Fama and French (1996), who use the three-factor model as a model of equilibrium returns. Long-term reversal disappears as an anomaly once normal returns are approximated by the three-factor model. For much more on momentum, we refer to Asness et al. (2014), who use simple analysis and survey published studies to show that momentum returns are (among other things) not too volatile, are not only a small firm phenomenon, and are not dwarfed by tax considerations or transaction costs.

A related literature in machine learning that tries to predict stock returns has developed largely unnoticed by the finance literature. The machine learning literature has focused on predicting stock returns from a few return-based and accounting-based variables jointly but has then largely ignored the structure of the prediction equation, instead analyzing the quality of the prediction itself.[5][6] This article can also be viewed as an attempt to connect the two and to provide a synthesized framework that can be used in either field.

The paper is organized as follows. Section 2 discusses the data, sets up a motivating framework and investigates two standard methods, portfolio sorts and simple Fama-MacBeth regressions, that a portfolio manager could employ to predict future returns. Section 3 explains tree-based conditional portfolio sorts in detail. Section 4 applies the method to past return predictor variables and section 5 has further results on transaction costs, the performance during the recent financial crisis, medium-term momentum, and a risk factor vs characteristics interpretation. Section E in the appendix illustrates robustness of our results along many dimensions. Section 6 concludes.

2 Data, motivating framework and standard methods

Before we introduce tree-based conditional portfolio sorts, we analyze a few standard approaches that an investor might try: single variable selection, that is, investing based on the single best-performing variable in historical data over a certain time window; standard Fama-MacBeth

[5] For example, Tsai et al. (2011) or Huerta et al. (2013).

[6] The variables that this literature uses for prediction are typically not motivated by results from the finance literature, but they are chosen based on their availability in different datasets (convenience samples). In analyzing the predictions itself, the joint hypothesis problem (Fama (1965, 1970)) is usually ignored and the evaluation is conducted for raw return estimates.

regressions, that is, a multivariate prediction that combines historically important signals; and Fama-MacBeth regressions that include variable interactions.

2.1 Data

Since we will use the relation between past returns and future returns as a running example throughout the article, we start by describing the data and the variable construction first.

The basis for our analysis is the universe of monthly U.S. stock returns from the Center for Research in Security Prices (CRSP) from 1963 to 2012. Since we use firm characteristics from Compustat and IBES in some robustness checks, we match stock price data to those datasets first, and we focus our analysis on those firms that can be linked in all datasets. Firm characteristics include traditional variables like size, book-to-market, dividend yield, gross profitability, and 82 others that are described in more detail in appendix E.1. The number of firms in our sample varies over time between 1182 and 6626. Size, value, momentum factors, and the risk-free interest rate are taken from Kenneth French's data library.[7]

Figure 1 illustrates how return-based predictor variables are constructed. Suppose that the investor wants to form a portfolio at the formation time, t_f. Return-based predictor variables can be defined by two parameters: the *gap* between the time of portfolio formation and the most recent month that is included in the return calculation, and the *length* of the return computation horizon. We denote the former by g, the latter by l and a return function by $R_{i,t_f}(g,l)$ which maps returns into cross-sectional decile ranks. For example, $R_{i,t_f}(1,11) = 10$ implies that firm i is in the highest decile of returns at time t_f for the return that is computed over the previous 12 months and that leaves out the most recent one.

Our benchmark set of predictors contains all one-month returns over the two years before portfolio formation — that is, $R_{i,t}(g,1), g = 0,\ldots,24$. Much of the related literature is based on sorting firms into 1 of 10 deciles depending on the values of a sorting variable. When we consider return-based strategies below, we refer to buying the upper decile and selling the lower decile based on $R_{i,t}(g,l)$. As in Novy-Marx (2012), we will use the notation $R_{i,t}(g,l)$ to denote both the return for portfolio formation and the strategy return based on that simple sorting strategy.[8]

[7] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[8] We have checked that results are robust when future returns are computed over the next future month, but we skip

The problem of predicting future returns based on past returns has the ingredients that make it difficult for an investor to find the relevant signals: Should momentum be measured over the most recent 6 or 12 months? What if the signals go in opposite directions? Should one leave out the most recent month? Or the most recent six (Novy-Marx (2012))? Degrees of freedom in choosing the gap and length parameters above contribute to the fact that these questions do not have definitive answers yet.

2.2 Motivating framework

In each time period, an investor has access to information Θ_{it} about firm i to model the conditional expectation of next period's return, as in equation (1), a general version of the model[9] that is typically estimated in the literature:

$$E_t[r_{i,t+1}|\Theta_{it}] = f_t(\Theta_{it}). \tag{1}$$

Here, the expectation of $r_{i,t+1}$ is formed at time t (we take a period to be one month in what follows), and the function $f_t()$ that maps the information set into expected returns can be time-varying. The information set Θ_{it} can contain data on the firm's past earnings, balance sheet information, past stock return movements, and many other variables. Since we will focus on the relation between past and future returns in this paper, and in line with the sorting-based literature, we assume that the information set consists of decile rankings of companies over the past two years — that is, $\Theta_{it} = \{R_{i,t}(0,1), \ldots, R_{i,t}(24,1)\}$. In other words, we consider decile rankings for each of the most recent twenty-five one-month returns.

With that information set, adding an additive error term and choosing the common specification of a linear form (see, for example, Haugen and Baker (1996), Daniel and Titman (1997) or

a day to make sure that the return would actually be implementable.

[9] At a greater level of generality, one could write the model as

$$E_t[r_{i,t+1}|\Theta_{it}] = f_t(z_{i,t}, z_{i,t-1}, \ldots, \lambda_t, \lambda_{t-1}, \ldots),$$

which would also include risk factors, and z_{it} and λ_t and their histories are subsumed in the information set $\Theta_{it} = \{z_{i,t}, \ldots, \lambda_t, \ldots\}$ at time t. We disregard this aspect for now but note that our framework easily extends to the case where all returns are interpreted as excess returns over risk factors.

Brennan et al. (1998)) for the function $f_t()$, equation (1) can be written as

$$r_{i,t+1} = a + \sum_{g=0}^{24} \beta_g^t R_{i,t}(g,1) + \epsilon_{i,t}, \tag{2}$$

which is usually estimated via a Fama-MacBeth procedure or by a cross-sectional regression. In general, the model can be viewed as a joint test of the relevance of characteristics and of the linearity assumption.

How could an investor predict returns using standard methods? Appendix C shows results for portfolio sorts and Fama-MacBeth regressions. We measure performance by computing the hypothetical return of a trading strategy that goes long in the decile of firms with the highest return predictions each month and that goes short in the decile with the lowest predictions (as in Lewellen (2013)). The best-performing model — a Fama-MacBeth regression that accounts for two-way interactions of past returns — generates average excess returns over the four-factor model of 1.13 percent per month at an information ratio of 1.3.

Of course, this begs the question whether we have captured all information in past returns for future returns or whether we should estimate the prediction equation more flexibly. For instance, if we are interested in exploring all systematic variation, why would we stop at two-way interactions?

Interactions among different anomalies can arise quite naturally from simple economic models. Chen et al. (2002) test the theory of gradual information diffusion to explain momentum. They argue that the rate of information diffusion could be different for firms which would result in different extents of momentum profits. They find that momentum interacts with firm size and with analyst coverage, and that the effect of analyst coverage on momentum profits is largest in small firms (a triple interaction). Vassalou and Xing (2004) illustrate a complex interaction among size, value, and default risk. They show that small stocks earn higher returns than big stocks only if they have higher default risk and that the same holds for the return of value over growth stocks. Complementary, high-default-risk firms earn higher returns than low-default-risk firms if they are small or value stocks. Expected return-relevant two-way interactions have been demonstrated between size and value (Fama and French (1992)), between size and seasonal effects (Daniel and Titman (1997)), and between stock exchange and volume (Brennan et al. (1998)).

Some authors have also considered interactions between past returns and other characteristics (see, for example, Asness (1997) for the interaction between value and momentum or (Lee and Swaminathan (2000)) for the interaction between volume and momentum). Interactions among different past-return variables are rare in the literature, with Grinblatt and Moskowitz (2004) who consider the consistency of return patterns and Han and Zhou (2013) who construct a trend-factor from past returns of different frequencies being two exceptions.

Appealing as the Fama-MacBeth method might seem, it quickly becomes infeasible when we want to analyze the entirety of potential interactions. Considering only two-way interactions, the number of terms to include when p candidate predictors are included is $\frac{p(p+1)}{2}$, which starts to become greater than 1000 at a mere 45 predictor variables. This prevents the use of Fama-MacBeth regressions in the early years of our sample if all firms are considered, and over the entire sample if the sample is divided first by, say, firm size. With higher-order interactions, estimation becomes difficult for even fewer candidate predictors.

Consider a model that allows for arbitrary three-way interactions

$$\begin{aligned} r_{i,t+1} = a &+ \sum_{g=0}^{G} \beta_g^t R_{i,t}(g,1) + \sum_{g=0}^{G} \sum_{j>g} \gamma_{gj}^t R_{i,t}(g,1) R_{i,t}(j,1) \\ &+ \sum_{g=0}^{G} \sum_{j>g} \sum_{k>j} \delta_{gjk}^t R_{i,t}(g,1) R_{i,t}(j,1) R_{i,t}(k,1) + \epsilon_{i,t}. \end{aligned} \tag{3}$$

Even if we consider a small set of $G = 20$ firm characteristics, 190 two-way interactions and 1140 three-way interactions would need to be considered. In the application of Green et al. (2014) with $G = 100$, these numbers amount to 4950 and 161700, which is prohibitively large for statistical analysis.[10]

Given the difficulties that stem from comprehensively investigating the interactions among characteristics using these standard methodologies, the existing evidence is restricted to the low-dimensional cases that have been and can be considered, and we may not learn the full extent to which interactions are relevant. Our approach below provides one way to address this challenge.

[10] In general, all k-way interactions are given by $\binom{G}{k}$.

3 Estimation strategy

3.1 Conditional Portfolio Sorts

Our goal is to estimate the conditional expectation in equation (1) more flexibly than can be achieved by a globally linear model like Fama-MacBeth regressions, or by portfolio sorts that allow for nonlinearities but that, in their usual form, are restricted to one- or two-dimensional cases.

Our estimation is based on the well-known concept of *conditional* portfolio sorts, which are illustrated schematically in figure 2. Consider sorting stocks into two portfolios based on sorting variable $R(g^{(1)}, 1)$ and threshold $\tau^{(1)}$, such that all stocks with $R(g^{(1)}, 1) \leq \tau^{(1)}$ are pooled together into one portfolio, and stocks with $R(g^{(1)}, 1) > \tau^{(1)}$ are pooled together into another portfolio. For instance, if $\tau^{(1)} = 5$ and $g^{(1)} = 0$, we would sort all stocks with returns below the cross-sectional median in the previous month into one portfolio and all stocks with returns above the cross-sectional median in the previous month into another portfolio.[11] The expected stock returns in each portfolio are now $E[r_{i,t+1}|R(g^{(1)}, 1) \leq \tau^{(1)}]$ and $E[r_{i,t+1}|R(g^{(1)}, 1) > \tau^{(1)}]$, respectively, and if the expected return is modeled as a constant within each portfolio, the prediction is just the average of realizations of next month's returns within each group. Sorting stocks within each portfolio again by another (or the same) characteristic with associated thresholds $\tau^{(2a)}$ and $\tau^{(2b)}$ results in four different portfolios, S_1 to S_4; for example, the stocks in portfolio S_1 in the figure have expected return $E[r_{i,t+1}|R(g^{(1)}, 1) \leq \tau^{(1)}, R(g^{(2a)}, 1) \leq \tau^{(2a)}]$.

A simple way to test whether $R(g^{(2a)}, 1)$ provides additional information over $R(g^{(1)}, 1)$ would be to compare the sorts on $R(g^{(2a)}, 1)$ within each portfolio sorted on $R(g^{(1)}, 1)$.[12] One could also test whether $R(g^{(2a)}, 1)$ creates a return spread only in the portfolio of, say, low $R(g^{(1)}, 1)$ firms, thereby testing for a potential interaction between characteristics $R(g^{(2a)}, 1)$ and $R(g^{(1)}, 1)$. In supplementary appendix A.1, we illustrate a basic conditional portfolio sort with a few standard firm characteristics.

[11] The literature usually considers one-variable sorts of stocks into 10 different portfolios. However, our sort into two portfolios is not restrictive because a one-variable sort into multiple portfolios can always be achieved by a repeated sort into two portfolios.

[12] This kind of test is, for example, applied in Bandarchuk and Hilscher (2012).

3.2 Tree-based Conditional Portfolio Sorts

We suggest extending the method of conditional portfolio sorts along the following dimensions. First, unlike in our earlier example that had thresholds and sorting variables chosen ex-ante, we will choose thresholds and sorting variables optimally (where "optimally" will be defined below) within each portfolio in a data-driven way. Second, we apply the procedure to levels deeper than the two levels that are usually considered, which gives rise to what we call a *tree-based conditional portfolio sort*. Third, since conditional sorts involve hard thresholds that are sensitive to small changes in the data, their predictions do not work very well out of sample. Following Kleinberg (1990, 1996), Ho (1998), and Breiman (2001), we average over many tree-based conditional portfolio sorts to smooth out the decision boundary, which improves predictions significantly, as explained below in more detail.

Our approach draws on parallel concepts from the machine learning literature. The techniques that we use to estimate tree-based conditional portfolio sorts mirror those that are used to estimate a so-called decision tree in computer science.[13] Model averaging or ensemble methods are also developed in that literature, and they are successfully applied to areas as diverse as biology (DNA sequencing), psychology, and motion sensing. Applications in economics are rare,[14] and our paper can also be read as an attempt to investigate whether these techniques provide value-added to academic research in finance and economics. This is the first paper that interprets conditional portfolio sorts from a machine learning perspective, tailors the methodology to similar approaches well-known in finance, and applies it to a comprehensive financial dataset.

3.2.1 Estimation

We start by describing how variables are selected and how thresholds are estimated. The goal is to estimate the expectation of the return of firm i in period $t + 1$ conditional on information in period t, as in equation (1).

To illustrate the method, start out with the conditional portfolio sort in figure 2. Consider the

[13] For further reading on decision-trees, see Hastie et al. (2009), Zhang and Ma (2012), Murphy (2012), or Criminisi and Shotten (2013).

[14] A few examples in a macroeconomic context use decision trees to analyze currency crises (Kaminsky (2006)), sovereign debt crises (Manasse and Roubini (2009)), banking crises (Duttagupta and Cashin (2011)) or to develop early warning indicators for, say, excessive credit growth (Alessi and Detken (2014)).

portfolio S_1 in that figure, which is defined by variable $R(g^{(1)}, 1)$ being less than threshold $\tau^{(1)}$ and variable $R(g^{(2a)}, 1)$ being smaller than threshold $\tau^{(2a)}$. Other portfolios can be defined similarly by their relations between sorting variables and associated thresholds. Within each portfolio S_l, the predicted expected return is modeled as the average return, μ_l, of all firms in the portfolio; that is,

$$\hat{\mu}_l = \text{Mean}(r_{i,t+1} | \text{Firm i } \in S_l \text{ in period t}). \tag{4}$$

In other words, analogously to a linear regression, we are interested in approximating the conditional mean of the outcome variable at a value of the regressor by the average of the outcome variable over observations with close values of the regressors. The conditional portfolio sort therefore generates subsets of firm observations that are more homogenous. Suppose for a moment that we have found such a homogenous allocation of firms into portfolios. The prediction function could then be written as

$$\hat{r}_{i,t+1} = \sum_{l=1}^{L} \hat{\mu}_l \mathbb{1}(\text{Firm i } \in S_l \text{ in period t}), \tag{5}$$

giving a portfolio-specific expected return prediction for each observation. What we have described so far is nothing more than a formal definition of the common conditional sorting methodology that we carried out in the previous section.

Of course, the conditional sort does not need to end after two levels but can be computed at greater depth. We consider the case in which the depth of the conditional sort, the sorting variables, and associated thresholds are not preselected but need to be identified from the data.

While it can be shown that finding the optimal solution to this problem requires solving an optimization problem for which a computationally fast solution does not exist (see Hyafil and Rivest (1976)), there exist feasible approximations. We use a standard algorithm that proceeds step-wise and that was first suggested in Breiman et al. (1984). For the interested reader, the supplementary appendix A.2 explains the algorithm in detail and discusses related methods.

Figure 3 illustrates the results of the procedure using the data and variables described in section 2.1. Rather than showing the entire iterative sort, the figure only shows the first few nodes.

The first selected split variable is R(0,1), the return over the previous month. The associated threshold is 6; that is, all firms with a return over the previous month in the lowest 6 deciles are sorted into one portfolio, and the remaining ones are sorted into the other. Conditional on this split, R(0,1) is selected again in the left branch at the next level and R(2,1), the one-month return two months ago, is selected in the right branch. The actual iterative sort goes deeper but, for illustration, we have computed the one-month-ahead returns in each of the four subsets. Differences are already pretty stark: The subset S_1, which is the set of companies that were in the lower of the two R(0,1) groups, display the highest return, indicating short-term reversal. The right branch illustrates a momentum effect: Stocks with higher values of R(2,1) have a higher subsequent return on average.

3.2.2 Model averaging

Constructing tree-based conditional portfolio sorts in the way we have described results in a few challenges. First, as described earlier, because of the complexity of the optimization, we have to use a greedy algorithm to estimate the model. This algorithm, however, does not guarantee that thresholds and split variables are selected optimally at each node. Second, the threshold rule is discrete, and any error in the estimation of the threshold could greatly distort the correct path for any expected return that is supposed to be predicted from the estimated model. Third, our initial results showed that a single estimated tree-based conditional portfolio sort summarizes the estimation data well, but the model does not extend well to new observations. In other words, because there are so many degrees of freedom (variables and thresholds) at each step, the tree-based conditional portfolio sort can often overfit the estimation sample.

These problems are well known in the machine learning literature, and we adopt a common solution suggested in Breiman (2001). The idea is to estimate a tree-based conditional portfolio sort a number of times using only a random subset of variables each time. The resulting models are less prone to overfitting because they are arguably less complex. At the same time, we also only use subsets of the data to estimate each model. We then compute estimates for expected returns from each model and average over all models' estimates to get a final prediction.

The idea of combining many predictions to construct a more accurate one can be illustrated in a simple voting setup in which people use majority voting to make a decision or to determine the (objective) value of an object. If everyone has the same information set, then nothing can be learned from aggregating individual votes; instead, every single vote is a sufficient statistic for the outcome. Only if voters differ in their information can aggregation lead to a more precise estimate. Using subsets of data and variables induces just such different information sets.

More formally, let B be the number of tree-based conditional sorts that are computed, and let $\hat{f}_b(\Theta_{it})$ be the predicted expected return for stock i at time t that is based on model b. The final expected return estimate is given by

$$\hat{r}_{i,t+1} = \frac{1}{B} \sum_{b=1}^{B} \hat{f}_b(\Theta_{i,t}). \tag{6}$$

In all results that follow, we construct 200 tree-based conditional portfolio sorts (that is, $B = 200$) and we use 8 out of 25 regressors (that is, roughly 30 percent of the number of regressors) in each of them. We have tried other values for the share of sampled regressors (between 20 and 40 percent) and also larger values for the number of estimated tree-based conditional portfolio sorts but have found that results do not vary much with these choices. We settled on the share of 30 percent of regressors because it is a standard recommendation in the random forest literature, and we chose $B = 200$ because higher values did not have any apparent benefit for the estimation but are more costly in terms of computation.

3.2.3 Discussion

Our ultimate goal is to provide a new method that is capable of tracing out which firm characteristics predict the cross-section of stock returns well. Tree-based conditional portfolio sorts are potentially interesting because they can account for both the correlation and the interactions of candidate characteristics. Model averaging as described earlier protects against the risk of in-sample overfitting and deals with the hard thresholds that sorting induces.

The flexibility of our approach does not come without costs: Model averaging loses the simple interpretation from a single tree-based conditional portfolio sort. Moreover, we cannot summarize our model as a simple linear equation in the space of firm characteristics and factors. One reason for the popularity of linear regression methods certainly lies in their apparent transparency. Our approach draws on methods from computer science that are sometimes criticized for producing black box predictions that cannot easily be interpreted. One contribution of this paper is to introduce measures with which the relation between model predictions and regressors can nevertheless be evaluated transparently.

Variable importance Since the relevance of a variable is determined by both its level and its potential interactions with other variables, summarizing statistical significance via a simple t-test is not appropriate. Instead, we rely on a relative variable importance measure that can be interpreted similarly to t-statistics in simple regressions.

For each predictor variable and each tree-based conditional portfolio sort, we compute the mean squared error (MSE) of the prediction when the values of that variable are randomly permuted, and we express its MSE relative to the model's MSE when all variables are at their original values. This fraction is then averaged over all iterative conditional sorts and predictor variables are ranked by this measure, where higher values imply that random permutations of a predictor variable cause higher increases in mean squared error, and the predictor variable is therefore considered more relevant.

Results are typically displayed relative to the predictor variable that causes the highest increase in mean squared error when it is permuted, a convention that we follow. For example, a value of .8 for a predictor variable means that this variable is associated with an MSE increase equal to 80 percent of the variable with the highest MSE increase.

Partial derivatives To assess directional effects of particular predictor variables on the prediction, we define a measure of partial derivatives that can be applied to tree-based conditional portfolio sorts. Define $R_{it}(g^{-},1)$ as the vector of past return variables that does not include past return g. We approximate a partial derivative of the prediction with respect to past return ranking $R_{it}(g,1)$ as follows. Recall that we construct past return rankings as the cross-sectional decile ranks — that is, $R_{it}(g,1) \in \{1,\ldots,10\}$. For each of the 10 values, counterfactually set $R_{it}(g,1) = d, \forall d = 1,\ldots,10$ for all observations and compute the average prediction over firms, time, and bootstrap samples:

$$\hat{r}^{g,d}_{i,t+1} = \frac{1}{N}\frac{1}{T}\frac{1}{B}\sum_{i,t,b} \hat{f}_b(R_{it}(g,1); R_{it}(g^-,1)).$$

Repeat this for all values of d and graph the results for each past return g and each value of d. Our method can easily be extended to varying two (or more) variables at the same time. In section 4, we also report partial derivatives for two-way interactions of return variables.

Return predictions Finally, we address the question of whether tree-based conditional portfolio sorts really work in the sense that they make superior return predictions. Based on our model estimates, we predict stock returns for each firm in each month and we sort stocks into deciles each month based on those predictions. We then compute the mean return spread that is generated across deciles. In addition, we employ a simple trading strategy: Each month, we go long the highest decile of predicted returns and we go short the lowest decile of predicted returns, therefore earning an equal-weighted hedge return.

It is, of course, essential to test the model out of sample. While an actual out-of-sample test is difficult to implement, we follow a standard pseudo-out-of-sample procedure, as illustrated in figure 4. Tree-based conditional portfolio sorts are re-estimated each year with data over the past five years. Predicted returns are then calculated for the next 12 months. In each of these months, we trade on our predicted returns as described in the previous paragraph. This approach takes into account the potential time-varying importance of different regressors and answers whether averaged tree-based conditional portfolio sorts could, in principle, be used for trading purposes.

4 Empirics

We apply our method to the prediction of future returns based on past returns. We will provide evidence for the following results. First, tree-based conditional portfolio sort works well in this setting in the sense that expected return predictions are ordinally accurate. Strategy returns and information ratios based on the model's predictions are much higher than those from alternative models. Second, among return-functions, the most important ones refer to the more recent past. Third, superior predictive ability can be traced to flexibly dealing with nonlinear relations between past and future returns, and interaction effects between past return functions. The relation between past and future returns is more complex (and more predictable) than can be captured by any one summary return.

4.1 Strategy returns

We first show that a strategy that buys high predicted expected returns and that sells low predicted expected returns makes robust and strong risk-adjusted excess returns.We proceed as described in section 3.2.3; that is, we estimate the model with five years of data up to period t and use the estimated model to predict returns for $t+1, \ldots, t+12$. This procedure is repeated for every year between 1968 and 2012. We sort returns into 10 deciles from the lowest to the highest predictions each month.

Figure 5 shows that the annual strategy return would have been positive for each of the past 45 years. Returns tend to be lower after the year 2000, which is consistent with the observation that momentum strategies have not performed well recently (see Lewellen (2013)). Figure 6 shows the return to investing $1 in the long portfolio and the short portfolio and illustrates that the tree-based conditional portfolio sort works well in both portfolios.

More generally, figure 7 illustrates that the tree-based conditional portfolio sort manages to spread returns more accurately across the entire distribution of firm-months than common past return sorting strategies. It plots the average decile performance for predictions based on the rolling model estimation. The tree-based sorting performs consistently better than a simple sorting on a single past return. Although this is not surprising, it is not self-evident that a larger set of explanatory variables will perform better in these dimensions. Recall that we evaluate all performances out of sample for 12 months by fixing the prediction function based on past estimates.

Table 1 regresses the return to the long-short strategy on the CAPM, the three-factor model, and the four-factor model. The raw average monthly return in column (1) is 2.3 percent. The strategy is significantly positively correlated with the market return with a very low factor loading; however, projecting the strategy return on the market return does not have a strong effect on the average abnormal return. The strategy does not load highly on the size or value factors.

Overall, results for the CAPM and the three-factor model are very similar, with almost no increase in R^2. As is not surprising, time-variation in the strategy return can partially be explained by the momentum factor, but the intercept is still strongly significant and large with a value of 2 percent per month. The R^2 goes up to 0.13, which still leaves a large part of the strategy variation unexplained by the equilibrium model. We observe very high information ratios at a value of around 2.9 throughout all specifications. While averaged tree-based conditional portfolio sorts produce mean excess returns that are somewhat, if not greatly, above those of the standard methods in section C, the method seems to do so with a large reduction in variance.

Table 2 sheds more light on the decile portfolios that are formed based on the models' predictions. They show the factor loadings of each decile portfolio return for one of four risk models. The returns of all decile portfolios appear to correlate one to one with the market return, with the extreme portfolios experiencing a slightly higher covariance. Second, there is no apparent spread in factor loadings for the size and the value factor. The extreme portfolios load slightly higher on the size factor (an issue that we come back to in appendix E) and slightly lower on the value factor. Third, there is a monotone relationship of decile returns with respect to loadings on the momentum factor. Quantitatively, however, these differences are small. Fourth, even though none of these portfolios differ much in their loadings on risk factors, there is a strong monotone relation between the portfolios and their (risk-adjusted) average returns. This stands in stark contrast to the seemingly very similar portfolios in terms of risk loadings. What is more, this relation is not only driven by the extreme portfolios (although it is particularly strong in those portfolios), but it exists across all 10 portfolios.[15]

While the strategy returns in our tree-based conditional portfolio sort appear high, they could still disappear after taking transaction costs into account. Strategies that are based on past returns generally have been found to have relatively high turnover (see de Groot et al. (2012) or Frazzini et al. (2013)), especially so when they are based on recent past returns. As the tree-based conditional portfolio sort mainly exploits variation in the most recent past returns, we expect turnover to be high as well.

Appendix D shows that this expectation is correct: An equal-weighted hedge strategy that goes

[15] In unreported monotonicity tests based on Patton and Timmermann (2010), we confirm that raw and risk-adjusted returns are monotonically increasing in deciles at all levels of significance (available on request).

long $1 and short $1 in the extreme portfolios has an average monthly turnover of 318 percent. Turnover is also high using the less extreme hedge returns that go long the ninth or eighth decile and that go short the second or third decile, respectively.[16] However, appendix D also shows that our strategy has positive excess return after transaction costs based on an extrapolation of transaction cost estimates from Frazzini et al. (2013).

Tree-based conditional portfolio sorts appear to work well in our application in the sense that they produce high and stable excess returns out of sample that are not explained by standard factor models. This begs the question what the method finds that researchers have not paid attention to. We discuss the discovered structure of predictor variables next.

4.2 Exploring the mechanism

4.2.1 Predictor variable importance

Recall that we re-estimate the model each year for a total of 45 different estimated models over time. When we compute our measure of predictor variable importance for each year, this gives us a ranking of the importance of each variable in each year. As a first summary, we rank past returns by their median rank in these 45 models. Table 3 shows the median rank as well as the upper and lower quartile of ranks for each of the top 10 past returns.

The top four return functions are related to the most recent six months of returns; all return functions over the most recent six months enter the top 10. In addition, some returns that show up provide information about the intermediate return between 6 and 12 months before the formation date. In particular, it is interesting and reassuring to see past return functions considered in the preceding literature to rank highly in the list. R(0,1), the return over the most previous month, is the return function of Jegadeesh (1990) and many other papers, while R(11,1), the one-month return exactly 12 months ago, is the seasonal effect documented by Heston and Sadka (2008).

There is also considerable time variation in the exact ranks as illustrated by the interquartile range of ranks for each past return. All of them were in the top half for more than 50 percent of the time, and 7 out of the 10 return functions are in the top 10 for at least half of the years. On the other hand, each variable also had periods during which it appears less relevant to the prediction,

[16] These numbers are similar to those reported in de Groot et al. (2012) or Frazzini et al. (2013) for strategies based on short-term returns.

as expressed in the last column of the table. We computed the rank correlation of past returns' importance between subsequent years and found it to be around 0.7, which points to the fact that the structure is relatively stable over time.

The fact that the pattern of more recent returns being more relevant than more distant past returns comes out of an agnostic search procedure is intriguing. We find that it is a quite robust fact in the data throughout various specifications. For instance, we find very similar results for past-return-based variables when we include other firm characteristics in the estimation (appendix E.1). In appendix E.2, we consider an expanded set of predictor variables that uses 126 past return functions of different gaps and different lengths such that standard past return functions like R(0,6) (the return over the most recent six months) are also part of the set of regressors. In that exercise, all 10 predictor variables are related to the most recent 6 months of returns and, what is more, the top 6 return functions are returns of length one that, taken together, summarize the most recent six-month return. The fact that a standard return like R(0,6) is not chosen but its components are illustrates that using the return over the previous year alone (and not the one-month returns that it is based on) leads to a loss of relevant information. One-month returns contain important information that is neglected when summary returns such as R(0,6) or R(1,11) are considered. For both sets of past-return functions, we repeat the estimations by firm size in appendix E.3 and again find similar results.

Our first intermediate result is, thus, that tree-based conditional portfolio sorts work because they effectively exploit variation in relatively recent one-month returns. The next sections look at how these variables are combined.

4.2.2 Average partial derivatives

Next, we consider our measure of an approximated partial derivative that we introduced in section 3.2. For each one-month return ranking over the previous half year, for all observations, we vary its value from the lowest (1) to the highest (10), and compute the counterfactual predictions. This allows us to trace out whether a variable is monotonically related to returns and to evaluate the sign of the average derivative going from the lowest to the highest value of the predictor variable. We focus on the most recent half year before portfolio formation because our results so far suggest that these returns are the most important for return prediction.

Figure 8 shows results. Focus on the first row for now (we will get to the second row in section E.1), which corresponds to the tree-based conditional portfolio sort that we have considered so far. Each column shows results for one of the most recent past one-month returns. Each panel varies the respective predictor from low to high and averages the prediction for each of 10 values. We observe that short-term reversal, the most recent one-month return, is negatively related to the return predictions; that is, higher values of the most recent one-month return predict lower returns. For the next return function, R(1,1), the one-month return over the second-to-last month, both high and low values are associated with lower returns. The next return functions are monotonically related to predictions, but in a nonlinear way: Low realizations have a large negative effect on the prediction, but high realizations do not have as much of a positive effect. These returns, thus, help to identify stocks with low expected returns but do not necessarily help much to identify stocks with high expected returns. It is only when we consider one-month returns that are in the more distant past (more than four months out) that we find a standard momentum effect — that is, a monotonically positive and close-to-linear relation between past and predicted returns.

The literature has not paid much attention to nonlinear relations between past and future returns. However, given that a) predictions from our tree-based conditional portfolio sorts make high risk-adjusted excess returns, b) short-term return functions have high values in our predictor variable importance calculations, and c) the partial effects of these variables cannot all be linearly related to returns, it appears that nonlinearities should be investigated further in future research.

Figure 9 shows contour plots for all two-way interactions of the most recent one-month return functions. In each panel, darker areas represent lower return predictions and brighter areas represent higher return predictions. A couple of interesting results stand out: First, many return variables interact in nonlinear ways. For example, the upper-left panel shows the interaction of R(0,1), the most recent one-month return, and R(1,1), the return over the preceding month. Return predictions generally decrease in the value of R(0,1), reflecting short-term reversal. However, within high values of R(0,1), return predictions *increase* in R(1,1), while they *decrease* in R(1,1) within low values of R(0,1). This type of nonlinearity holds, to a varying extent, in many panels involving R(0,1). Second, for some return variables, we find monotonically increasing predictions within both return variables, mostly for those that involve returns from four or more months ago. Third, some return predictions neither decrease nor increase monotonically in the predictor variable range but are nonlinearly related to return predictions, once one variable is fixed. For instance, from figure 8 we know that R(1,1) is nonlinearly related to returns. In figure 9, we see that this nonlinearity is more pronounced when R(1,1) is interacted with intermediate returns like R(3,1) or R(4,1).

Finally, we find evidence that the estimate average partial derivatives are time-varying. Figures 10 and 11 illustrate this for two different variables. Figure 10 shows average partial derivatives in eight different years, evenly spaced over the sample period, for R(0,1), the return over the previous month. Short-term reversal is detectable across all years, but its strength varies over time. While our model estimates indicate relatively monotone (or regular) short-term reversal across all 10 deciles for the first half of the sample, short-term reversal is more apparent in the extreme deciles in the second half of the sample. Similar conclusions can be drawn from figure 11, which shows the same calculations for R(5,1), the one-month return six months before portfolio formation. In the first half of the sample, momentum is apparent and robust across all deciles. In the second half, however, differences in average partial derivatives are more pronounced between extreme deciles than between intermediate deciles. Interestingly, recently (in 2012, the lower-right panel), the average partial derivative of R(5,1) has reversed such that lower values of R(5,1) are associated with higher returns in the model estimates. Recall that the estimation period for this panel is 2006 to 2011, which coincides with an episode of a momentum crash as documented by Daniel and Moskowitz (2015). As we have shown in the previous section, a trading strategy based on our model estimates has not suffered the strong crash that a standard momentum strategy has experienced in this period. The average partial derivative at that time indicates that the model has picked up the weakness of standard momentum and that the estimated relationship was adjusted (in that case, reversed) accordingly.

An extensive discussion of the time-variation of all predictor variables is beyond the scope of the paper. The examples above, however, serve to illustrate its importance. The question of whether time-variation in past-return signals can be related to, e.g., the macroeconomic or financial cycle is left for future research.

Appendix D.1 contrasts our results to results from Fama-MacBeth regressions with and without interaction terms between past returns. Here we provide a brief summary of our findings.

The raw and factor-adjusted returns of the tree-based conditional portfolio sort are about 0.5 percentage points higher than in the Fama-MacBeth regressions and, more interestingly, the information ratios are generally roughly three times as high. Even if we include all two-way interactions in a Fama-MacBeth regression as in table 11, average excess returns and information ratios are generally much lower than in our results for the tree-based conditional sort.

Interaction terms turn out to be significant in the Fama-MacBeth regressions as well. Factor-adjusted strategy returns based on predictions from Fama-MacBeth regressions that include two-way past return interactions are higher than returns based on predictions from Fama-MacBeth regressions that do not include those interactions. Remarkably, the information ratio increases by about 50 percent, indicating that the strategy return is earned at a much better risk-return tradeoff.

These results let tree-based conditional sorts shine in two dimensions. If regarded as a kitchen sink method (that just uses all available information), the tree-based conditional sort leads to better performance than the Fama-MacBeth analogue, although both perform well. If regarded as a variable selection device (that selects a variable at each split), the Fama-MacBeth method mostly recovers momentum as an important determinant of expected returns, whereas the structure discovered by the tree-based conditional sort cannot be explained by (simple) factor models.

5 Discussion and robustness

In this section, we present several robustness checks and investigate related questions. Our main result thus far is that more recent past returns are the most important predictors among past return variables to predict returns one month ahead. In section 5.1, we show that this structure is also discovered in subsamples of different firm sizes and for a larger set of correlated past return functions. Section 5.2 focuses on the year 2009, when momentum strategies did not perform well (see Daniel and Moskowitz (2015) or Barroso and Santa-Clara (2015)). We document how our strategy has adjusted to avoid the momentum crash during that period. In order to investigate the debate initiated by Novy-Marx (2012) and Goyal (2011) about the importance of intermediate momentum in more detail, section 5.3 re-estimates the model when we use only recent past returns and compares the results to a model that uses only intermediate past returns. In section

D.1, we compare our results to those of a Fama-MacBeth regression that uses recent past returns and two-way interactions. Section 5.4 addresses the issue of whether the discovered structure should be given a characteristics or risk-factor interpretation.

5.1 Robustness of the discovered predictor variable structure

We investigate whether the discovered structure of predictor variables, with more recent past returns being more important than more distant ones, is stable across several specifications. This section provides a summary of our findings, while detailed results can be found in appendix E.

Our first specification adds a set of 86 firm characteristics as additional predictor variables. We construct firm characteristics as in Green et al. (2014) for this exercise. Table 13 in appendix E shows that the most important variables among past returns are still the more recent ones. Nine out of the top 10 variables are the same that show up in the benchmark case in table 3, with the exact order slightly altered. We also find that the partial derivatives (second row in figure 8 and figure 14 in the appendix) are very similar to the benchmark case.

Our second specification uses a broad set of correlated past return functions. These include not only the one-month returns over the past 2 years but also returns over up to 18 months during the past 2 years (Details in appendix E.2). For instance, the standard notion of momentum — the return over the previous year excluding the most previous month — is part of this expanded set. Overall, the set includes 126 past-return-based variables. Our findings, presented in table 14 in the appendix, are intriguing in this case: The most important variables are one-month returns over the most recent six months, followed by two-month returns over the same horizon. Note that standard momentum could have been chosen in the conditional sorts. The fact that standard momentum does not appear to be important but its components are illustrates that using standard momentum alone leads to a loss of information.

Finally, our third specification splits the sample by firm size and then estimates tree-based conditional portfolio sorts in each sample. We sort firms into three groups of micro, small, and large stocks based on NYSE breakpoints, as in Fama and French (2008), and we estimate tree-based conditional sorts for both the benchmark set and the expanded set of past returns described above. Table 15 in the appendix paints a consistent picture across all size categories for both sets of predictor variables: More recent past returns are consistently more important than more distant ones across all specifications.

5.2 The momentum crash of 2009

The traditional momentum strategy has experienced sharp losses during the recent financial crisis, a fact that can be attributed to a momentum strategy's time-varying exposure to aggregate risk (Daniel and Moskowitz (2015) and Barroso and Santa-Clara (2015)). Although our strategy is also based on predictions of future returns from past returns, our strategy did not lose money during that time (see figure 5). It is thus informative to consider our strategy during that period in more detail.

Figure 12 shows the time-variation in the CAPM beta for the long- and the short-portfolio of the strategy. The loadings do not display systematic variation over time and the difference between the betas fluctuates around zero. As such, it is no surprise that the strategy return is less affected by the aggregate market.

The solid lines in figure 13 show the time-varying performance of typical momentum strategies between 2000 and 2010: Both R(11,1), the strategy that is based on the return from exactly 12 months ago, and R(5,1), the strategy that is based on the return from exactly 6 months ago, display a sharp performance decrease in 2008-2009. The figure illustrates that both variables indicate reversal, especially in crisis times.

The dashed lines in figure 13 show that this is how both return variables were incorporated into the strategy during that time period: While the strategy is typically positively related to both variables (a value greater than zero means that the strategy is long stocks that had positive exposure to the simple past returns), exposure became negative or close to zero right around the financial crisis, such that the portfolio positions were reversed relative to a standard momentum strategy based on these variables. This illustrates the algorithm picked up on changes in the underlying relationship between past and future returns and was therefore able to avoid the drawdown in 2009.

5.3 Medium-term momentum

Our results suggest that the most important predictor variables are related to the most recent six months before portfolio formation. One could therefore suspect that short-horizon returns are generally better predictors of future returns than intermediate-horizon returns.

We address this question by defining two more sets of return-based functions that split the regressors into those that provide information about the most recent six months and those that provide information about returns 7 to 12 months before portfolio formation. Formally, our split is based on the sum of the gap and length parameters. One set includes all return-based functions for which the sum of gap and length is smaller than 7 months (we call this the *short-term set*), and our second set includes all return-based functions for which the sum of gap and length is between 7 and 12 months (the *intermediate-term set*). The latter set of functions includes the function suggested by Novy-Marx (2012) and other functions that are correlated with it.

Table 4 provides the factor loadings of the equal-weighted hedge return strategy that goes long the highest predicted decile and that goes short the lowest predicted decile based on the predictions derived from each set of predictor variables. The first five columns report loadings for the strategy based on the short-term set and the remaining columns report loadings for the strategy based on the intermediate-term set. There are a couple of intriguing results. First, we see that both strategies make high and robust excess returns relative to the CAPM, and the three- and four-factor models. Second, as is immediately apparent, alpha is lower for the medium-term strategy than for the short-term strategy throughout all specifications. In column (5), we add the strategy return of the intermediate-term set to the factors. As indicated by the t-statistic on the coefficient and the increase in R^2, the two strategies are correlated, and the excess return of the short-term strategy decreases to 1 percent per month.

In column (10), we do the same and add the short-term strategy return to the factor regression for the intermediate-strategy return. Interestingly, alpha disappears almost entirely once the short-term strategy return is accounted for.

We interpret this as evidence that the most important variation for return prediction purposes stems from short-term variation in returns rather than intermediate-term variation once interactions and confounding returns are included in the estimation. This reconciles the result in Novy-Marx (2012) with Goyal and Wahal (2013), who cannot find the intermediate-term momentum effect in 37 out of 38 markets.

5.4 Risk factors or return characteristics?

Our results in section 4.1 indicate that portfolios that are based on forecasted returns from the estimated model have similar loadings on the Fama-French factors and the momentum factor, and yet they consistently have different expected returns. While, on the surface, this seems to be a challenge to the four-factor model, we investigate the issue further in this section.

Table 7 shows the bivariate correlation between the strategy return, formed from the extreme portfolios, and the four standard risk factors. The strategy return displays low correlation with the market return and the value factor. It is displays somewhat higher correlation with the size factor and, as one would expect from a past-return-based strategy, with the momentum factor. In general, however, these correlations are low relative to the correlations between the other factors, which makes the strategy return a potentially suitable candidate factor.

This motivates table 8, which mirrors the analysis in Haugen and Baker (1996). It shows the average values of various firm characteristics in each decile of expected returns. The first panel of the table shows measures of risk across the ten deciles with no clear (monotone) pattern. Average market beta is higher in the extreme deciles. The same holds for the profitability measures in the second panel. Interestingly, gross profitability is very similar in each decile, but expected returns are very different. This illustrates that our sorting is not driven by the Novy-Marx (2013) measure of gross profitability. Panel 3 shows that book-to-market is balanced across deciles, as one would expect from the balanced factor loadings in table 2. The last panel shows that the firms in the extreme deciles are, on average, smaller.

More intriguingly, since the strategy is based on the extreme deciles, it is worthwhile to compare the average values within these two deciles. Note that the values of most firm characteristics are very similar in these two deciles. The strategy appears to be based on riskier, less profitable, and smaller companies, on average. Yet, within the set of these firms, there are stark differences in returns that can be systematically predicted.

Recall that alternative strategies that are based on buying the second (third) highest decile and selling the second (third) lowest decile rather than the extreme portfolios also make robust excess returns. Comparing deciles two and nine, or deciles three and eight, illustrates that the two corresponding portfolios are again very balanced throughout characteristics. As a sole characteristic, return-on-equity is lower in decile nine (eight) than in decile two (three), but other measures of profitability indicate that the portfolios are comparable along this dimension. While the non-extreme decile portfolios display similar characteristics, their excess returns vary and (see table 2) can be predicted from past returns. Since the portfolios based on the tree-based conditional portfolio sort appear not to be discernible based on many characteristics, we would not expect the strategy return that is based on it to help explain other anomalies.

As the strategy return itself appears to be unpriced by equilibrium models and unrelated to standard characteristics, the return could, in principle, be added as an additional risk factor to standard equilibrium models. However, in unreported results, we found that the strategy return, as expected, only weakly helps to explain other asset pricing anomalies, which is why we prefer the interpretation from a characteristics' rather than a risk factor perspective.

6 Conclusion

Some 50 years after the Capital Asset Pricing Model of Sharpe (1964), Lintner (1965), and Mossin (1966), and some 20 years after the three-factor model of Fama and French (1992), there is still a remarkable lack of consensus about which variables can be related to expected stock returns. To date, the literature has found more than 300 variables that spread returns in a way that is unaccounted for by the standard equilibrium models. This has led Green et al. (2013) to conclude that "either U.S. stock markets are pervasively inefficient, or there exist a much larger number of rationally priced sources of risk in equity returns than previously thought." Surely, many of these variables contain correlated information, and some will not hold up out of sample, but, so far, the literature has not rigorously identified which ones are fundamentally important. Furthermore, we have illustrated that some variables interact in nontrivial ways, making it more challenging to single out the important ones with standard methodologies.

We introduce a framework — tree-based conditional portfolio sorts — that can deal with a large number of variables and their potential nonlinearities and interactions. It also puts emphasis on systematic out-of-sample testing of all results. It connects model evaluation in finance to the machine learning literature in computer science and can serve to bridge the two fields.

We apply our framework to find information in past returns that can be related to future returns. A simple, linear Fama-MacBeth framework finds moderate excess returns relative to the four-factor model. Using the same variables in the tree-based conditional portfolio sort framework, on the other hand, yields high and stable excess returns, indicating that the linear framework does not exploit all relevant information in the data.

Finance has criticized machine learning for producing black box predictions without any possibility to "get insights into the underlying structure of the data" (Breiman, 2002). We show that, even though the structure does not come in the form of simple equations, one can still extract interpretable information from the resulting tree-based conditional sorts. First, we find that, among the prior two years of one-month return functions before portfolio formation, the more recent ones are the most important for accurate return predictions. Second, some of these return-functions are nonlinearly related to future returns, mostly returns between two and four months before portfolio formation. For instance, both high and low values of the return over the second-to-last-month forecast lower returns. Third, many of the return functions display nontrivial interactions. For instance, the one-month return over the second-to-last month, is positively related to returns for stocks with low returns last month, but it is positively related to returns with high returns last month. At a minimum, our results indicate that the relation between past and future returns is more complex than can be captured by any one summary return, such as standard momentum or intermediate momentum. Our results are robust to including a larger set of correlated return functions and to the inclusion of other firm characteristics. Similar structures are also discovered within different size-sorted portfolios.

Lastly, tree-based conditional portfolio sorts can accommodate the inclusion of new predictor variables quite easily. Starting from the observation that if a predictor variable is relevant, it should show up among the most important variables that the method finds, one could just add the variable in question to the existing set of variables. Running the estimation on this extended set effectively controls for correlations with other variables and takes potential interactions and nonlinearities into account. Our hope is that a framework around tree-based conditional portfolio sorts can significantly speed up the process of scientific discovery in this literature.

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A Tables

Table 1: *Strategy factor loadings: tree-based conditional portfolio sort*

	(1)	(2)	(3)	(4)
Intercept	2.30	2.23	2.25	2.05
	(16.75)	(16.04)	(16.51)	(14.54)
MKT		0.07	0.05	0.09
		(2.14)	(1.53)	(2.78)
SMB			0.08	0.09
			(1.40)	(1.69)
HML			-0.03	0.04
			(-0.39)	(0.61)
UMD				0.20
				(5.57)
R^2		0.02	0.03	0.13
IR		2.90	2.93	2.82
SR	2.96			
N	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. Strategies are based on the predictions of a tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length l equal to 1 and gap g between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation). Predictions are based on the model in section 3.2. Strategies go long the highest predicted return decile and go short the lowest predicted return decile. The sample period covers 1968 to 2012, and all results are based on rolling out-of-sample estimates of the models. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

Table 2: *Factor loadings of decile portfolios: tree-based conditional portfolio sort*

	Low	2	3	4	5	6	7	8	9	High	High-Low
Average return	-0.53	0.21	0.44	0.58	0.68	0.80	0.94	1.01	1.22	1.76	2.30
	(-1.74)	(0.77)	(1.66)	(2.29)	(2.61)	(3.10)	(3.52)	(3.78)	(4.27)	(5.54)	(16.75)
						CAPM					
Intercept	-1.52	-0.73	-0.48	-0.32	-0.23	-0.13	0.01	0.06	0.23	0.72	2.23
	(-8.59)	(-4.97)	(-3.60)	(-2.47)	(-1.81)	(-1.03)	(0.08)	(0.42)	(1.55)	(3.97)	(16.04)
MKT	1.12	1.08	1.06	1.04	1.04	1.06	1.06	1.09	1.14	1.20	0.07
	(27.40)	(30.01)	(31.94)	(32.94)	(30.65)	(31.83)	(30.62)	(28.90)	(29.10)	(25.30)	(2.14)
						Three-factor model					
Intercept	-1.64	-0.87	-0.63	-0.48	-0.38	-0.28	-0.13	-0.07	0.10	0.61	2.25
	(-13.12)	(-7.71)	(-6.80)	(-5.58)	(-4.66)	(-3.45)	(-1.42)	(-0.71)	(0.98)	(4.51)	(16.51)
MKT	0.99	0.97	0.97	0.96	0.97	0.98	0.98	0.99	1.02	1.04	0.05
	(27.70)	(29.22)	(34.92)	(36.04)	(38.43)	(38.76)	(32.98)	(29.14)	(32.43)	(27.55)	(1.53)
SMB	0.87	0.74	0.69	0.67	0.65	0.66	0.69	0.71	0.80	0.95	0.08
	(8.64)	(8.49)	(8.76)	(8.51)	(8.80)	(8.69)	(8.51)	(8.84)	(10.21)	(12.08)	(1.40)
HML	0.23	0.25	0.27	0.30	0.29	0.28	0.27	0.25	0.25	0.21	-0.03
	(2.84)	(3.58)	(4.44)	(4.73)	(4.64)	(4.44)	(4.24)	(3.50)	(3.68)	(2.52)	(-0.39)
						Four-factor model					
Intercept	-1.37	-0.67	-0.48	-0.36	-0.29	-0.21	-0.07	-0.02	0.13	0.69	2.05
	(-12.77)	(-7.10)	(-6.32)	(-4.96)	(-4.11)	(-3.10)	(-0.90)	(-0.25)	(1.40)	(5.13)	(14.54)
MKT	0.94	0.94	0.94	0.93	0.95	0.97	0.96	0.98	1.02	1.03	0.09
	(30.12)	(31.11)	(37.89)	(39.00)	(41.63)	(44.66)	(36.30)	(31.58)	(35.46)	(27.00)	(2.78)
SMB	0.86	0.73	0.69	0.67	0.65	0.65	0.69	0.71	0.80	0.95	0.09
	(11.69)	(11.04)	(10.88)	(10.14)	(10.15)	(9.58)	(9.15)	(9.38)	(10.52)	(13.16)	(1.69)
HML	0.15	0.19	0.23	0.26	0.25	0.26	0.25	0.23	0.24	0.18	0.04
	(2.47)	(3.39)	(4.40)	(4.91)	(4.96)	(4.65)	(4.53)	(3.62)	(3.86)	(2.34)	(0.61)
UMD	-0.27	-0.20	-0.15	-0.12	-0.10	-0.07	-0.06	-0.05	-0.03	-0.08	0.20
	(-7.71)	(-6.29)	(-4.87)	(-3.82)	(-2.90)	(-2.26)	(-1.86)	(-1.49)	(-0.93)	(-2.34)	(5.57)

This table shows time-series regressions of decile portfolio returns on factors. Returns are specified in percent per month. Each decile is formed on the predicted returns of a tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length *l* equal to 1 and gap *g* between 0 and 24 months. Predictions are based on the model in section 3.2. *Low* denotes the lowest decile of predicted returns and *High* denotes the highest decile of predicted returns. The first panel reports the average return, the second panel reports CAPM estimates, the third reports the three-factor model estimates and the fourth panel adds momentum. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. The sample period covers 1968 to 2012. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

Table 3: *Most important past return variables: Rank statistics*

	Median	75th percentile	25th percentile
R(0,1)	1	1	1
R(1,1)	4	2	15
R(2,1)	4	3	8
R(3,1)	6	4	12
R(11,1)	7	3	9
R(4,1)	8	6	11
R(5,1)	8	5	14
R(8,1)	11	7	18
R(10,1)	11	7	16
R(9,1)	12	8	15

This table shows the ten most important past returns (by median rank) in the tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length *l* equal to 1 and gap *g* between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation). The model is estimated each year between 1968 and 2012 for a total of 45 different rankings. The table reports the median, and the upper and the lower quartile for the top ten past returns (by median rank) over the 45 estimations.

Table 4: *Strategy factor loadings: Short-term and intermediate-term return functions*

	Dependent variable									
	Return of short-term strategy					Return of intermediate-term strategy				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept	2.15	2.12	2.17	2.04	1.05	1.78	1.74	1.78	1.44	0.16
	(19.09)	(18.47)	(19.35)	(17.37)	(9.11)	(15.01)	(14.90)	(15.52)	(13.33)	(1.65)
MKT		0.03	0.01	0.03	-0.04		0.05	0.04	0.10	0.08
		(1.38)	(0.46)	(1.64)	(-2.37)		(1.40)	(1.32)	(4.25)	(4.39)
SMB			0.04	0.04	0.06			-0.05	-0.04	-0.06
			(0.65)	(0.85)	(1.95)			(-0.74)	(-0.93)	(-2.27)
HML			-0.08	-0.04	-0.06			-0.08	0.03	0.05
			(-1.52)	(-0.85)	(-1.95)			(-1.15)	(0.78)	(1.85)
UMD				0.13	-0.11				0.35	0.27
				(3.80)	(-4.28)				(11.11)	(10.71)
MT/ST strategy					0.69					0.63
					(15.63)					(15.24)
R^2		0.00	0.02	0.08	0.48		0.01	0.02	0.37	0.64
IR		3.38	3.46	3.37	2.29		2.40	2.47	2.49	0.37
SR	3.42					2.45				
N	540	540	540	540	540	540	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. Strategies are based on the predictions of a tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Strategies go long the highest predicted return decile and go short the lowest predicted return decile. The short-term strategy is based on predictions from a tree-based conditional portfolio sort that only uses the most recent six months of past return rankings, while the intermediate-term strategy is based on predictions that use past return rankings from seven to twelve months before portfolio formation. Predictions are based on the model in section 3.2. The row "MT/ST strategy" adds the intermediate-term strategy return to the factor regressions for the short-term strategy, and adds the short-term strategy return when the intermediate-term strategy is the dependent variable. The sample period covers 1968 to 2012, and all results are based on rolling out-of-sample estimates of the models. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

Table 5: *Strategy factor loadings: Fama-MacBeth predictions using the six most recent one-month returns*

	Levels only				plus relevant two-way interactions				plus all two-way interactions			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	1.27	1.19	1.23	0.81	1.61	1.47	1.60	1.38	1.58	1.45	1.57	1.35
	(6.57)	(6.04)	(7.21)	(4.22)	(9.39)	(8.73)	(9.67)	(7.28)	(9.29)	(8.60)	(9.44)	(7.08)
MKT		0.09	0.09	0.17		0.16	0.09	0.13		0.14	0.08	0.13
		(1.36)	(1.30)	(3.10)		(3.23)	(1.91)	(3.14)		(3.11)	(1.92)	(3.18)
SMB			-0.03	-0.03			0.10	0.10			0.09	0.09
			(-0.35)	(-0.38)			(1.10)	(1.38)			(1.01)	(1.27)
HML			-0.07	0.06			-0.23	-0.16			-0.21	-0.14
			(-0.48)	(0.61)			(-2.33)	(-2.12)			(-2.21)	(-1.91)
UMD				0.41				0.22				0.22
				(4.13)				(2.49)				(2.59)
R^2		0.01	0.01	0.22		0.04	0.09	0.17		0.04	0.08	0.16
IR		1.02	1.06	0.78		1.48	1.65	1.49		1.49	1.65	1.48
SR	1.09				1.59				1.60			
N	540	540	540	540	540	540	540	540	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. Strategies are based on the predictions of a Fama-MacBeth regressions of future returns on past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length equal to 1 and gaps between 0 and 6 months, that is, predictions are based on the equation

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{6} \beta^t_g R_{it}(g,1) + \epsilon_{it},$$

or, when two-way interactions are included,

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{24} \beta^t_g R_{i,t}(g,1) + \sum_{g=0}^{24} \sum_{j>g} \gamma^t_{gj} R_{i,t}(g,1) R_{i,t}(j,1) + \epsilon_{i,t}.$$

The first four columns include only the levels of past returns, the next four columns include relevant two-way interactions as identified from the tree-based conditional portfolio sort and the last four columns include all two-way interactions between those returns. Strategies go long the highest predicted return decile and go short the lowest predicted return decile. The sample period covers 1968 to 2012, and all results are based on rolling out-of-sample estimates of the models. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

Table 6: *Fama-MacBeth regression coefficients and t-statistics: Using the six most recent one-month returns*

	Coefficients			t-stats		
	Levels only	Relevant Int	All Int	Levels only	Relevant Int	All Int
R(0, 1)	-1.63	-6.29	-6.32	-9.20	-16.30	-16.52
R(1, 1)	0.06	-2.87	-2.82	0.43	-8.83	-8.69
R(2, 1)	0.64	-0.93	-1.48	4.76	-4.65	-4.80
R(3, 1)	0.42	-0.87	-1.06	3.44	-4.25	-3.67
R(4, 1)	0.43	-0.42	-0.47	3.26	-1.99	-1.66
R(5, 1)	0.62	-0.11	-0.20	5.19	-0.60	-0.72
R(6, 1)	0.39	-0.33	-0.70	3.51	-1.80	-2.50
R(0, 1) X R(1, 1)		2.33	2.33		12.45	12.42
R(0, 1) X R(2, 1)		2.19	2.16		11.84	11.81
R(0, 1) X R(3, 1)		1.55	1.54		8.83	8.88
R(0, 1) X R(4, 1)		0.88	0.87		5.10	5.11
R(0, 1) X R(5, 1)		0.82	0.86		5.20	5.48
R(0, 1) X R(6, 1)		0.77	0.79		4.85	5.00
R(1, 1) X R(2, 1)		0.60	0.53		3.64	3.21
R(1, 1) X R(3, 1)		0.73	0.69		4.40	4.18
R(1, 1) X R(4, 1)		0.61	0.60		3.73	3.66
R(1, 1) X R(5, 1)		0.48	0.49		2.95	3.04
R(1, 1) X R(6, 1)		0.54	0.55		3.25	3.27
R(2, 1) X R(3, 1)			0.27			1.59
R(2, 1) X R(4, 1)			0.29			1.82
R(2, 1) X R(5, 1)			0.36			2.24
R(2, 1) X R(6, 1)			0.20			1.20
R(3, 1) X R(4, 1)			-0.01			-0.08
R(3, 1) X R(5, 1)			-0.10			-0.66
R(3, 1) X R(6, 1)			0.22			1.41
R(4, 1) X R(5, 1)			-0.26			-1.61
R(4, 1) X R(6, 1)			0.10			0.65
R(5, 1) X R(6, 1)			0.08			0.49

This table shows coefficient estimates and t-statistics for the three regression models in table 5. Past returns include return-based functions R(g,l) with length equal to 1 and gaps between 0 and 6 months. The sample period covers 1968 to 2012. "Levels only" only includes the levels of past return functions and is based on the equation

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{6} \beta^t_g R_{it}(g,1) + \epsilon_{it}.$$

"Relevant Int" includes relevant two-way interaction terms and "All Int" includes all two-way interaction terms between the six most recent past returns. The first three columns show the coefficient estimates times 100, and the last three columns show t-statistics.

Table 7: *Strategy return correlations with four factors*

	MKT	SMB	HML	UMD	DCPS
MKT	1.000	0.306	-0.320	-0.140	0.125
SMB		1.000	-0.241	-0.032	0.129
HML			1.000	-0.146	-0.081
UMD				1.000	0.291
DCPS					1.000

Bivariate correlations between the market return (MKT), the size (SMB) and value (HML) factors, the momentum factor (UMD) and the strategy return from an estimated tree-based conditional portfolio sort (DCPS).

Table 8: *Firm characteristics: Portfolios based on tree-based conditional portfolio sort*

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10
Risk										
Debt to Equity	2.61	2.33	2.75	2.47	3.29	2.52	2.70	3.62	2.60	3.55
Long-term debt to Equity	1.43	0.78	1.15	0.81	1.61	0.74	0.92	1.98	0.90	2.09
Debt Ratio	0.51	0.52	0.53	0.53	0.54	0.54	0.54	0.54	0.54	0.54
Beta	1.18	1.09	1.07	1.05	1.05	1.05	1.06	1.08	1.12	1.19
Profitability										
Gross Profitability	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
Return on Assets	-0.02	0.01	0.02	0.02	0.02	0.03	0.02	0.02	0.01	-0.02
Return on Equity	-0.15	0.03	-0.07	0.05	-0.15	0.04	0.04	-0.25	-0.18	-0.33
Profit Margin	-2.79	-1.31	-1.20	-1.04	-1.11	-1.60	-1.11	-0.75	-1.20	-2.41
Gross Margin	-1.31	-0.63	-0.26	-0.31	-0.19	-0.22	-0.27	-0.16	-0.37	-1.04
Earnings per Share	0.87	1.31	1.45	1.58	1.63	1.64	1.59	1.55	1.34	0.93
Basic Earnings Power Ratio	0.04	0.06	0.07	0.07	0.08	0.08	0.08	0.07	0.07	0.03
Price level										
Price Earnings Ratio	4.40	5.18	4.68	6.68	6.15	5.46	3.87	7.04	3.99	4.52
Book to Market	0.79	0.81	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.87
Price Sales Ratio	2.00	1.35	0.91	0.88	0.84	0.88	0.88	0.71	0.93	1.42
Dividend Yield	0.04	0.03	0.03	0.04	0.04	0.03	0.04	0.03	0.03	0.03
Activity										
Current Ratio	3.17	2.94	2.83	2.78	2.78	2.74	2.78	2.77	2.82	2.93
Quick Ratio	2.05	1.88	1.81	1.77	1.76	1.74	1.76	1.77	1.79	1.86
Net Working capital Ratio	0.30	0.29	0.28	0.27	0.27	0.27	0.27	0.28	0.28	0.29
Cash Ratio	1.49	1.23	1.13	1.07	1.07	1.04	1.07	1.06	1.11	1.22
Assets - Turnover Ratio	1.17	1.17	1.16	1.15	1.15	1.15	1.15	1.16	1.18	1.21
Inventory-Turnover Ratio	20.60	19.24	20.12	23.77	22.12	23.97	23.85	21.34	22.33	20.25
RandD	0.09	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.09
Others										
Size	681.95	976.24	1113.54	1177.19	1180.10	1177.22	1156.71	1085.86	967.02	633.43

Each month, all stocks are ranked by their estimated expected return based on a tree-based conditional portfolio sort that is based on all one-month return functions over the two years before portfolio formation. The table reports the average value of each firm characteristic in each decile over time.

B Figures

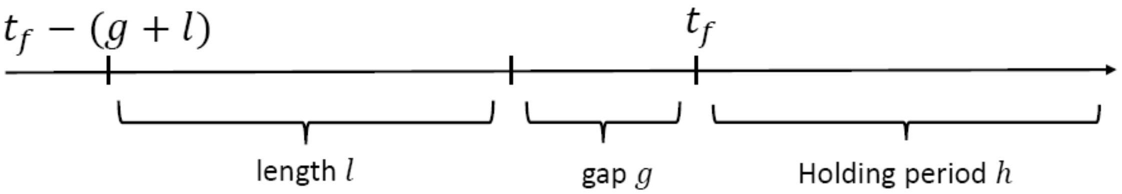


Figure 1: *Construction of past return-based characteristics: The investor forms a portfolio at time t_f. Return-based predictor variables can be defined by two parameters; the* gap *between the time of portfolio formation and the most recent month that is included in the return calculation, and the* length *of the return computation horizon. We denote the former by g, the latter by l and a return function by $R_{i,t_f}(g, l)$ maps returns into cross-sectional decile ranks.*

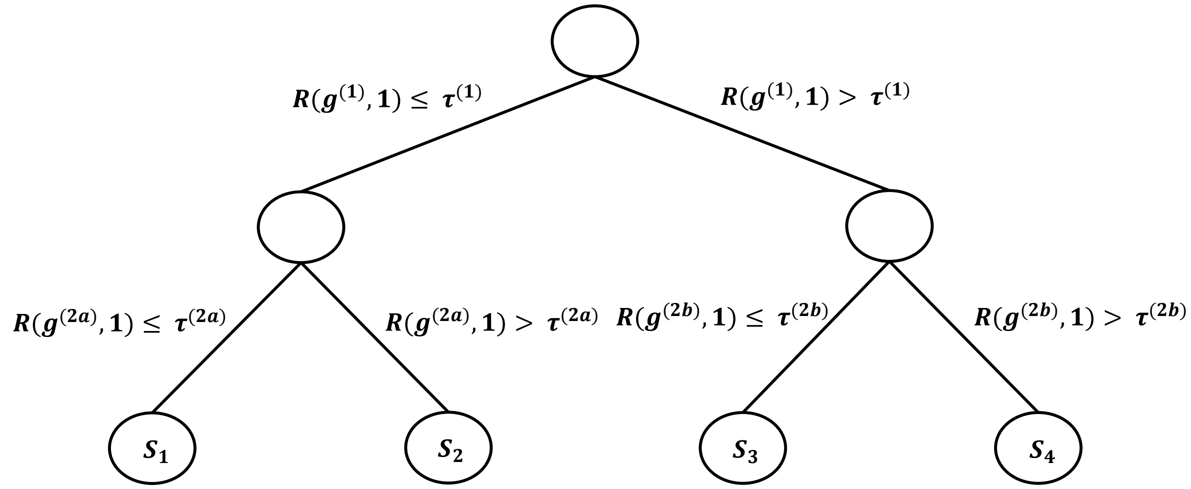

$R(g^{(1)}, 1) \leq \tau^{(1)}$
$R(g^{(1)}, 1) > \tau^{(1)}$
$R(g^{(2a)}, 1) \leq \tau^{(2a)}$
$R(g^{(2a)}, 1) > \tau^{(2a)}$
$R(g^{(2b)}, 1) \leq \tau^{(2b)}$
$R(g^{(2b)}, 1) > \tau^{(2b)}$
S_1
S_2
S_3
S_4

Figure 2: *Schematic representation of a conditional portfolio sort: First, observations are sorted into two portfolios based on past return $R(g^{(1)}, 1)$ and threshold $\tau^{(1)}$. The resulting portfolios are then sorted again on variables $R(g^{(2a)}, 1)$ and $R(g^{(2b)}, 1)$ with thresholds $\tau^{(2a)}$ and $\tau^{(2b)}$ for a total of four portfolios S_1, S_2, S_3 and S_4.*

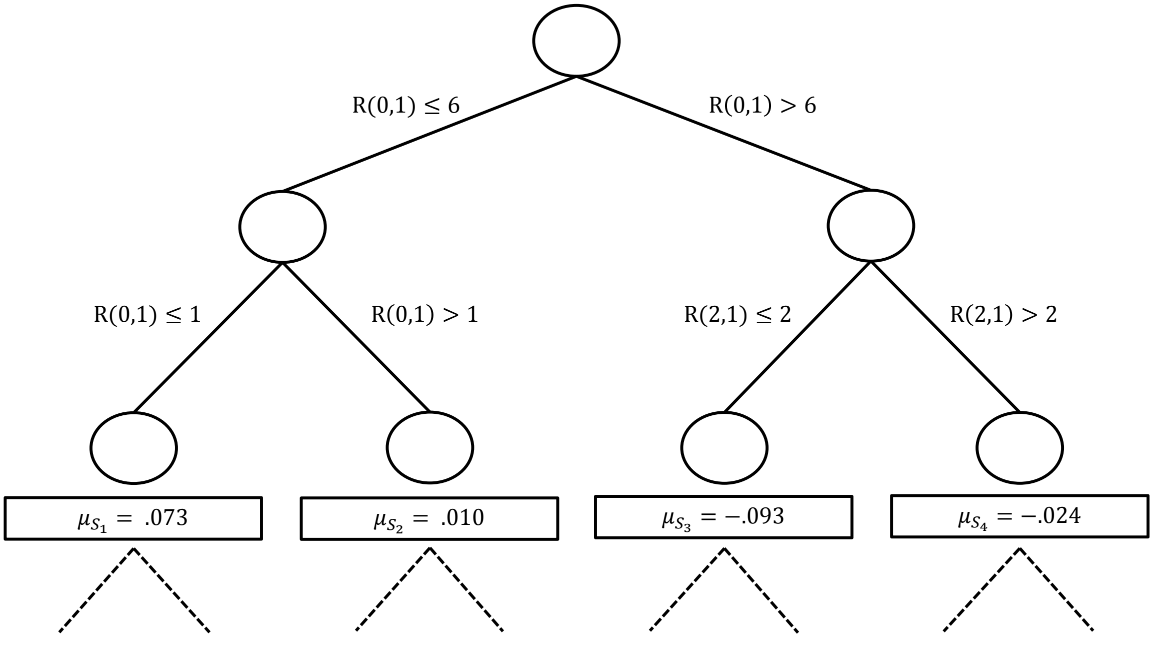


Figure 3: *tree-based conditional portfolio sort using the entire data set: First nodes*

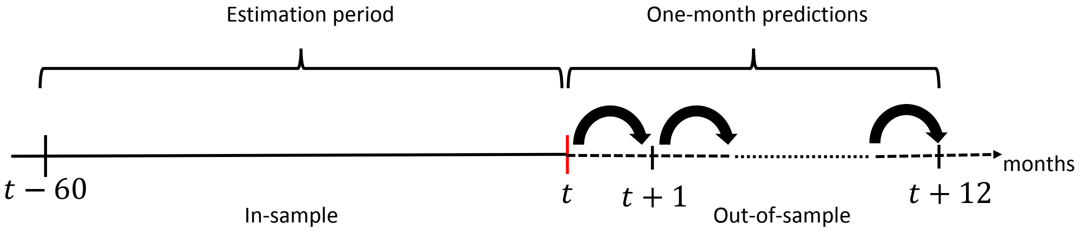


Figure 4: *Out-of-sample testing: tree-based conditional portfolio sorts are re-estimated every year with data over the past sixty months. Predicted returns are then calculated for the next twelve months. The strategy is go long (short) the highest (lowest) decile of those predictions each month.*

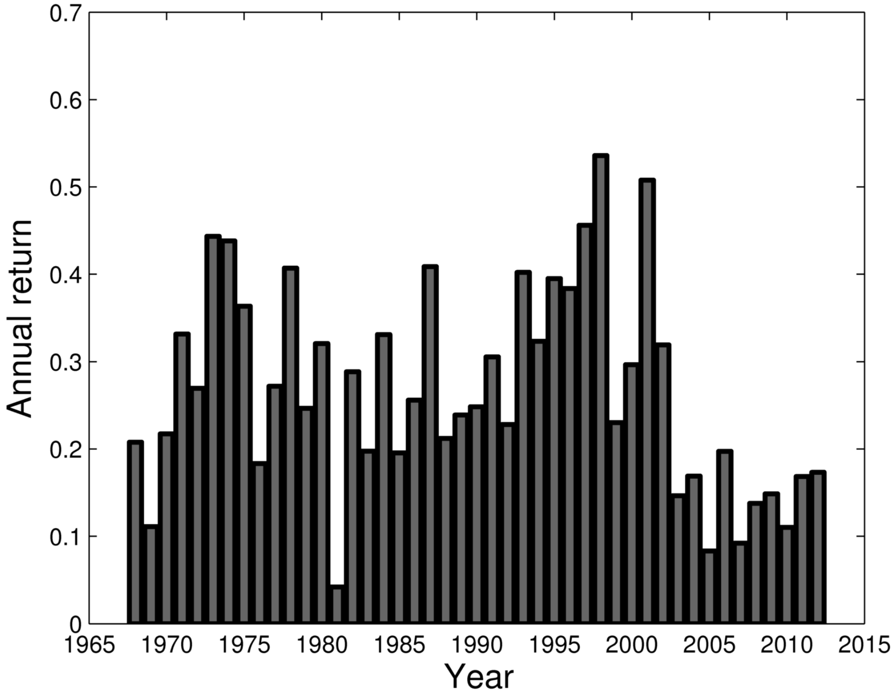


Figure 5: *Annual strategy return: The strategy is based on the predictions of tree-based conditional portfolio sorts that relate future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length l equal to 1 and gap g between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation). The strategy goes long the highest decile of predictions and goes short the lowest decile of predictions each month. The figure shows the annual return for each of forty-five out-of-sample predictions.*

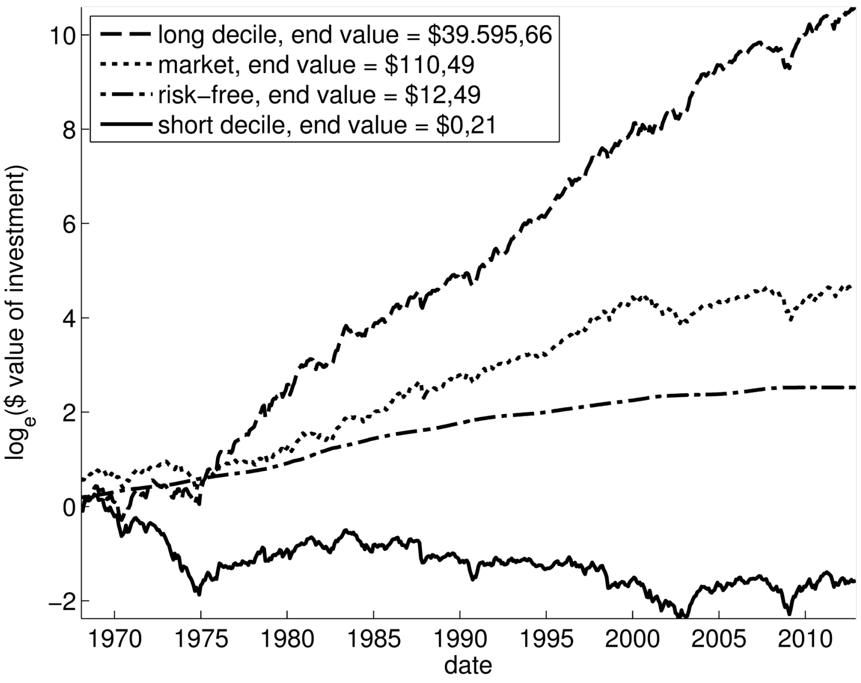


Figure 6: *Earned profit from investing $1 in the strategy in 1968: The strategy is based on the predictions of a tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length l equal to 1 and gap g between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation). The strategy goes long the highest decile of predictions and goes short the lowest decile of predictions each month. The figure shows the earned profit from investing $1 in the long and the short portfolio, respectively. For reference, the figure also includes the returns to investing $1 at the riskfree rate and for investing at the rate of the market return over the same horizon.*

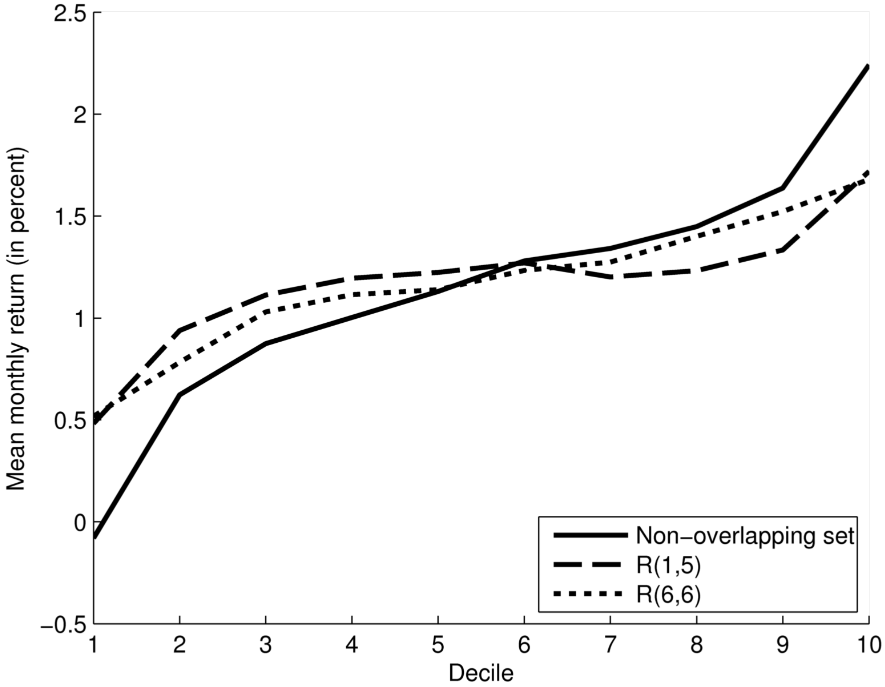


Figure 7: *Average monthly decile return for strategy return and simple return strategies: The strategy is based on the predictions of a tree-based conditional portfolio sort that relates future returns to past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length equal to 1 and gaps between 0 and 24 months. The strategy goes long the highest decile of predictions and goes short the lowest decile of predictions each month. Simple return strategies are plotted for comparison. R(1,5) is the strategy that goes long (short) the highest (lowest) decile of returns over the past six months, leaving out the most recent one. R(6,6) is Novy-Marx (2012)'s intermediate return strategy that goes long (short) the highest (lowest) decile of returns that are computed over the six months that skip the most recent six months.*

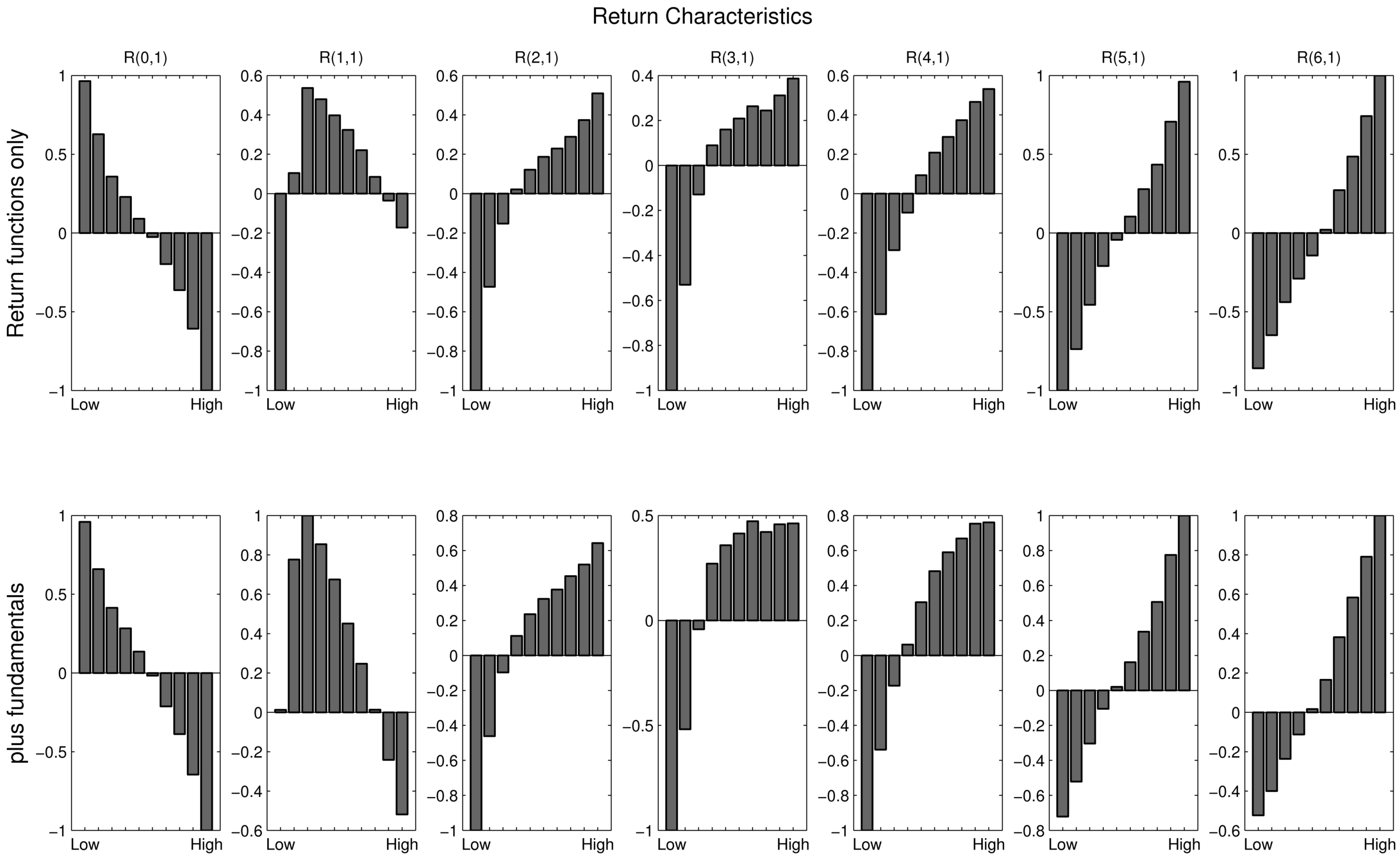


Figure 8: *Average partial derivatives for return characteristics: The figure shows the average prediction when a characteristic is counterfactually varied from low to high values. Details are in section 3.2.3. The first row shows results when we use only twenty-five past one-month returns as predictors. The second row shows results for the same one-month return functions when other firm characteristics (defined in appendix E.1) are included in the estimations as additional variables. Each column shows one return characteristic and predictions are averaged over the sample period.*

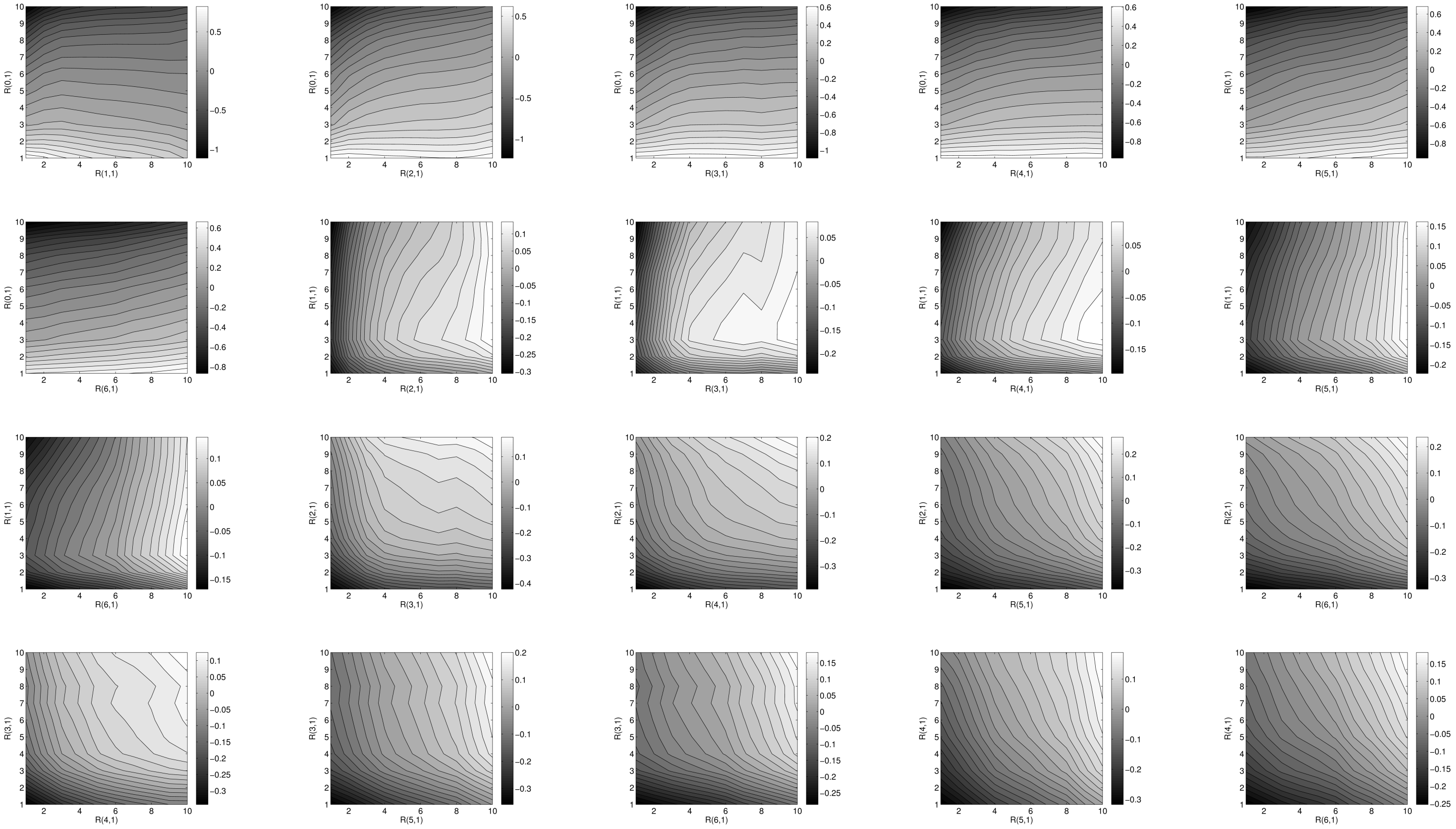


Figure 9: *Average double partial derivatives. The figure shows the average prediction when two characteristics are counterfactually varied from low to high values. Results are based on rolling optimization of the model and predictions are averaged over the sample period. Details are in section 3.2.3.*

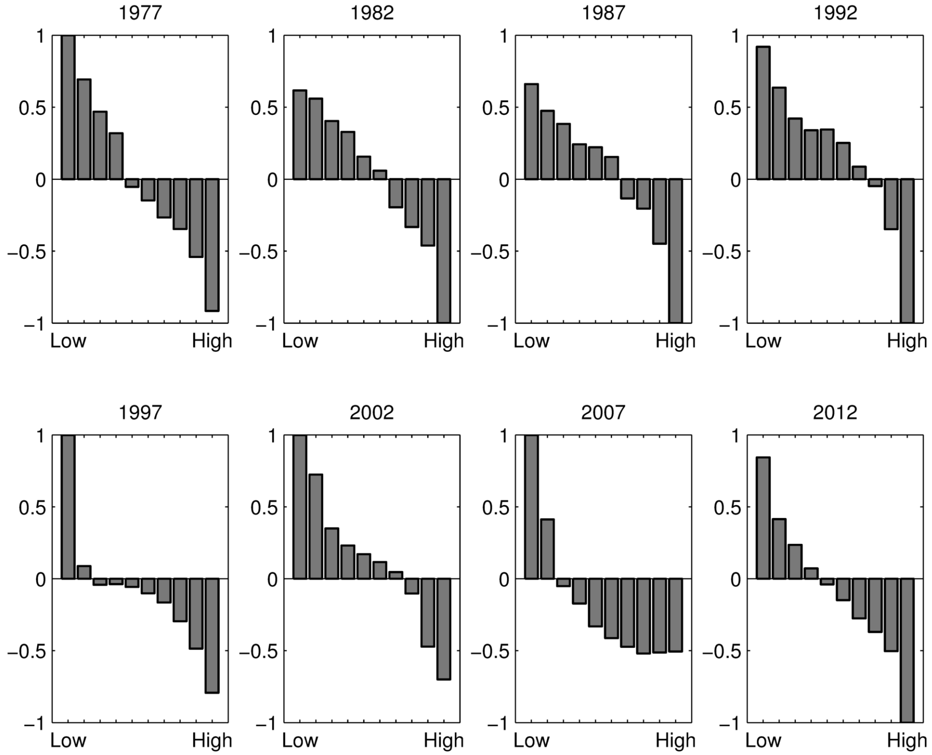


Figure 10: *Average partial derivatives in different years. The figure shows the average prediction when R(0,1), the return over the previous month, is counterfactually varied from low to high values, and results are displayed for different years to illustrate time-variation. Results are based on rolling optimization of the model, details can be found in section 3.2.3.*

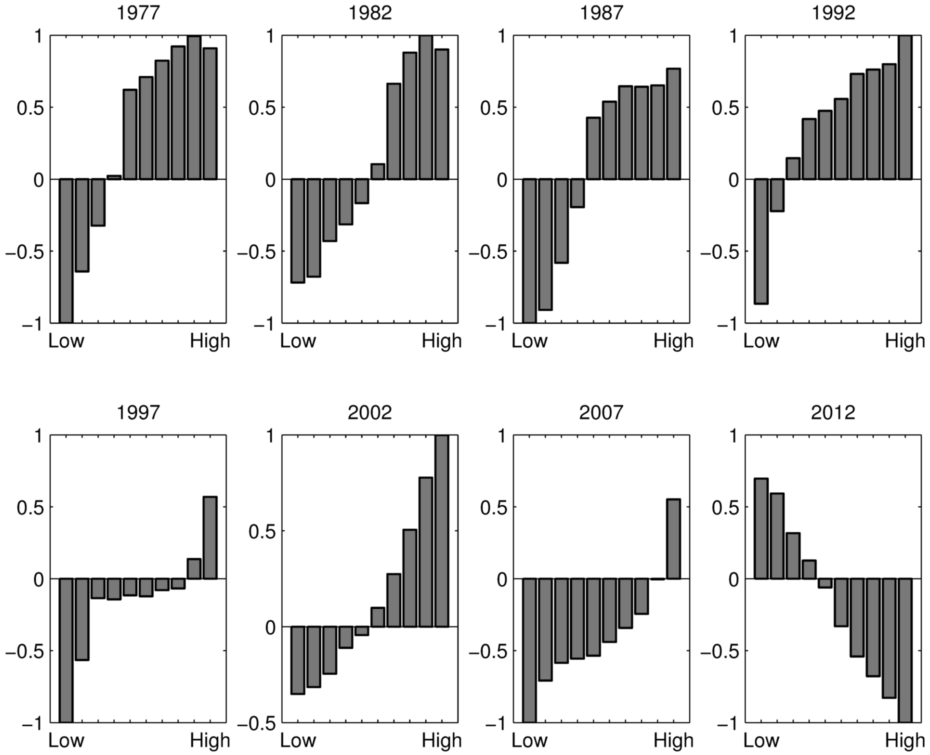


Figure 11: *Average partial derivatives in different years. The figure shows the average prediction when R(5,1), the one-month return six months before portfolio formation, is counterfactually varied from low to high values, and results are displayed for different years to illustrate time-variation. Results are based on rolling optimization of the model, details can be found in section 3.2.3.*

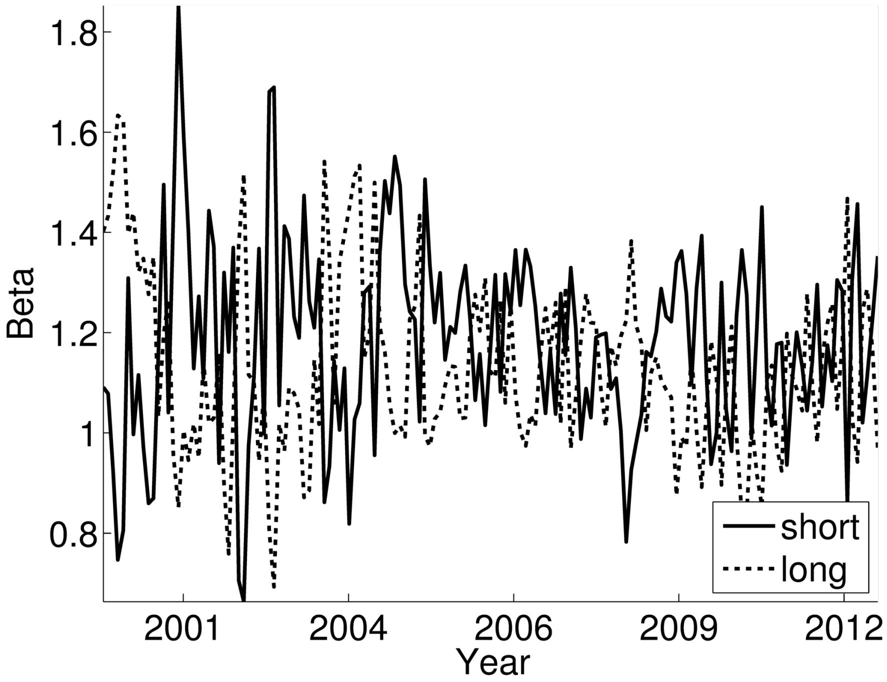


Figure 12: *Beta exposure of the long- and short-portfolios: Loadings of the top (solid line) and bottom (dashed line) decile portfolios on the market factor. Deciles are based on predicted returns of the tree-based conditional portfolio sort in section 4.*

C Estimating expected returns with standard methods

C.1 Portfolio sort

The potentially simplest strategy is to evaluate one variable at a time, then base forecasts on the single variable that has performed best in the past. More specifically, we suggest the following simple strategy: In each month, compute the m month trailing average return for each sorting variable, pick the one with the best performance (in terms of the Sharpe ratio), and base the subsequent long and short orders on values of that variable.

Table 9 shows that the return to such a strategy is 0.71 percent per month with an information ratio (relative to the four factor model) of 0.89, when the trailing performance is computed over the 60 months that precede the portfolio formation date. While this is already a good result, each month's returns are based on the values of a single sorting variable. The question remains whether the investor can do even better by combining information from different variables. While a few more variables can be incorporated (for example, double sorts), the number of observations in each portfolio decreases quickly such that estimates become unreliable.

Table 9: *Strategy factor loadings: Portfolio Sort*

	(1)	(2)	(3)	(4)
Intercept	0.71	0.74	0.71	0.72
	(6.45)	(6.77)	(6.60)	(6.67)
MKT		-0.03	-0.03	-0.03
		(-1.51)	(-1.28)	(-1.28)
SMB			0.03	0.03
			(0.73)	(0.73)
HML			0.04	0.04
			(1.17)	(1.16)
UMD				-0.00
				(-0.11)
R^2		0.00	0.01	0.01
IR		0.92	0.89	0.89
SR	0.88			
N	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. The strategy is to go long (short) the highest (lowest) decile of firms based on a single past return variable from the set of the most recent twenty-five past one-month returns. In each month, the past return that would have produced the highest strategy Sharpe ratio over the sixty preceding months is selected as the sorting variable. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. The sample period covers 1968 to 2012. T-statistics are in parentheses, and standard errors are clustered using Newey-West's adjustment for serial correlation.

C.2 Fama-MacBeth regressions

With that question, the investor turns to a multivariate regression setup that we describe in some detail. We suggest two approaches: The first one is a "kitchen sink" Fama-MacBeth estimation that throws in all past return variables and that uses them for prediction regardless of their individual significance. The second one bases predictions solely on the relevant variables where we define "relevant" as variables that are selected in a LASSO regression.[17] While we report

[17] Least absolute shrinkage and selection operator (LASSO), originally introduced by Tibshirani (1996), is a method that regularizes regressions by putting a penalty on the size of regression coefficients. Due to the nature of the penalty term (the sum of the absolute values of individual coefficients), the optimum will typically set many coefficients to

results for the LASSO regression, we have tried other model selection methods (general-to-specific, specific-to-general) and obtained similar results.

Our general implementation for the Fama-MacBeth-framework works as follows. In each cross-section, the investor fits the regression

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{24} \beta^t_g R_{it}(g,1) + \epsilon_{it} \tag{7}$$

and keeps either all coefficients (kitchen sink) or uses LASSO to select the relevant variables.

His period $t+1$ forecast is computed based on the rolling average of the coefficient estimates up to period $t-1$ and then applying the linear model to $R_{it}(g,1)$, that is,

$$\hat{r}_{i,t+1} = \overline{\beta}^{t-1}_{cons} + \sum_{g=0}^{24} \overline{\beta}^{t-1}_g R_{it}(g,1), \tag{8}$$

where $\overline{\beta}^{t-1}_g = \frac{1}{m} \sum_{j=t-1-m}^{t-1} \hat{\beta}^t_g$. We initially use a rolling window of 120 months but, as in Lewellen (2013), have found that results are robust to varying that parameter.

Lewellen (2013) uses a set of 15 predictor variables that are well established in the literature. In contrast, we consider an investor who faces substantial uncertainty about which variables he should include and, therefore, has to cast a wide net. Consistent with our running example, the investor considers all one-month returns over the two years before portfolio formation. Each period, he computes return predictions based on past model estimates and sorts predictions into 10 deciles. He constructs an equal-weighted hedge portfolio that goes long the highest decile of predicted returns and that goes short the lowest decile of predicted returns, analogous to the strategies described earlier.

Starting with the kitchen sink model, the first four columns of table 10 show the strategy's factor loadings from time-series regressions on the market, size, value, and momentum factors. The strategy has a positive and significant average return of 1.51 percent per month and loads mostly on the market and the momentum factor. The alpha relative to the four-factor model is

exact zeros, which is why the method can be viewed as a variable selection device.

about 1 percent per month, with an information ratio of about 1.

When we use the LASSO in the Fama-MacBeth framework as described earlier, results remain almost unchanged. The last four columns of table 10 show that the average strategy return is again around 1.5 percent per month, and the four-factor alpha is 1 percent per month. The information ratio is close to 1, as in the kitchen sink regression. The reason that these results are very similar is that many irrelevant regressors have coefficients close to zero in the kitchen sink case.

Note that the approaches so far have not included variable interactions. The Fama-MacBeth regression framework lends itself to a simple implementation of additionally including interactions of predictor variables. Equation (9) shows the regression equation that adds all two-way interactions among past return rankings:

$$r_{i,t+1} = a + \sum_{g=0}^{24} \beta_g^t R_{i,t}(g,1) + \sum_{g=0}^{24} \sum_{j>g} \gamma_{gj}^t R_{i,t}(g,1) R_{i,t}(j,1) + \epsilon_{i,t}. \tag{9}$$

Table 11 shows strategy returns that are based on predictions from equation (9).[18] At 1.13 percent per month, the average excess return relative to the four-factor model is slightly higher than in the levels-only version above. The information ratio, however, experiences a much stronger increase to 1.3-1.4. Hence, the main benefit to including two-way interactions appears to be a reduction in variance rather than an improved mean return.

D Transaction costs

While the strategy returns in our tree-based conditional portfolio sort appear high, they could still disappear after taking transaction costs into account. Strategies that are based on past returns generally have been found to have relatively high turnover (see de Groot et al. (2012) or Frazzini et al. (2013)), especially so when they are based on recent past returns. As the tree-based conditional portfolio sort mainly exploits variation in the most recent past returns, we expect turnover to be high as well.

The first row of table 12 shows that this expectation is correct: An equal-weighted hedge

[18] Since the model with two-way interactions has 325 regressors, we focus on results based on variable selection.

Table 10: *Strategy factor loadings: Fama-MacBeth predictions using all variables*

	Kitchen sink regression				LASSO regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.51	1.33	1.30	1.00	1.50	1.31	1.28	1.00
	(8.93)	(7.87)	(8.06)	(6.23)	(8.63)	(7.60)	(7.79)	(6.16)
MKT		0.20	0.20	0.26		0.21	0.21	0.26
		(3.07)	(3.08)	(4.34)		(3.29)	(3.34)	(4.51)
SMB			0.05	0.06			0.05	0.05
			(0.58)	(0.51)			(0.53)	(0.47)
HML			0.05	0.14			0.05	0.14
			(0.40)	(1.35)			(0.44)	(1.31)
UMD				0.30				0.27
				(4.25)				(3.76)
R^2		0.06	0.06	0.18		0.06	0.07	0.16
IR		1.23	1.20	0.98		1.20	1.17	0.97
SR	1.36				1.33			
N	540	540	540	540	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. Strategies are based on the predictions of a Fama-MacBeth regressions of future returns on past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length l equal to 1 and gap g between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation), that is, predictions are based on the equation

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{24} \beta^t_g R_{it}(g,1) + \epsilon_{it}.$$

The kitchen sink Fama-MacBeth model uses all variables in each period regardless of their significance, and the LASSO model selects a set of relevant variables each period based on a penalty function approach. Both procedures are described in section C.2. Strategies go long the highest predicted return decile and go short the lowest predicted return decile. The sample period covers 1968 to 2012, and all results are based on rolling out-of-sample estimates of the models. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

Table 11: *Strategy factor loadings: Fama-MacBeth predictions using all variables and two-way interactions*

	(1)	(2)	(3)	(4)
Intercept	1.46	1.27	1.28	1.13
	(9.62)	(8.29)	(8.35)	(7.55)
MKT		0.21	0.18	0.21
		(4.38)	(3.82)	(4.40)
SMB			0.12	0.12
			(1.63)	(1.48)
HML			-0.03	0.02
			(-0.33)	(0.20)
UMD				0.15
				(2.91)
R^2		0.09	0.11	0.15
IR		1.43	1.46	1.32
SR	1.57			
N	540	540	540	540

This table shows time-series regressions of strategy returns on factors. Returns are specified in percent per month. Strategies are based on the predictions of a Fama-MacBeth regressions of future returns on past decile sorts of returns. Past return sorts include decile rankings R(g,l) with length *l* equal to 1 and gap *g* between 0 and 24 months (i.e. all one-month returns over the two years before portfolio formation) and their two-way interactions, that is, predictions are based on the equation

$$r_{i,t+1} = \beta^t_{cons} + \sum_{g=0}^{24} \beta^t_g R_{i,t}(g,1) + \sum_{g=0}^{24} \sum_{j>g} \gamma^t_{gj} R_{i,t}(g,1) R_{i,t}(j,1) + \epsilon_{i,t}.$$

LASSO estimation is applied to select relevant variables each period, described in more detail in section C.2. Strategies go long the highest predicted return decile and go short the lowest predicted return decile. The sample period covers 1968 to 2012, and all results are based on rolling out-of-sample estimates of the models. MKT is the market return, SMB and HML are the Fama-French factors for size and value, and UMD is the momentum factor. SR is the Sharpe ratio and IR is the information ratio. T-statistics are in parentheses, and standard errors were clustered using Newey-West's adjustment for serial correlation.

strategy that goes long $1 and short $1 in the extreme portfolios has an average monthly turnover of 318 percent. Turnover is also high using the less extreme hedge returns that go long the ninth or eight decile and that go short the second or third decile, respectively.[19]

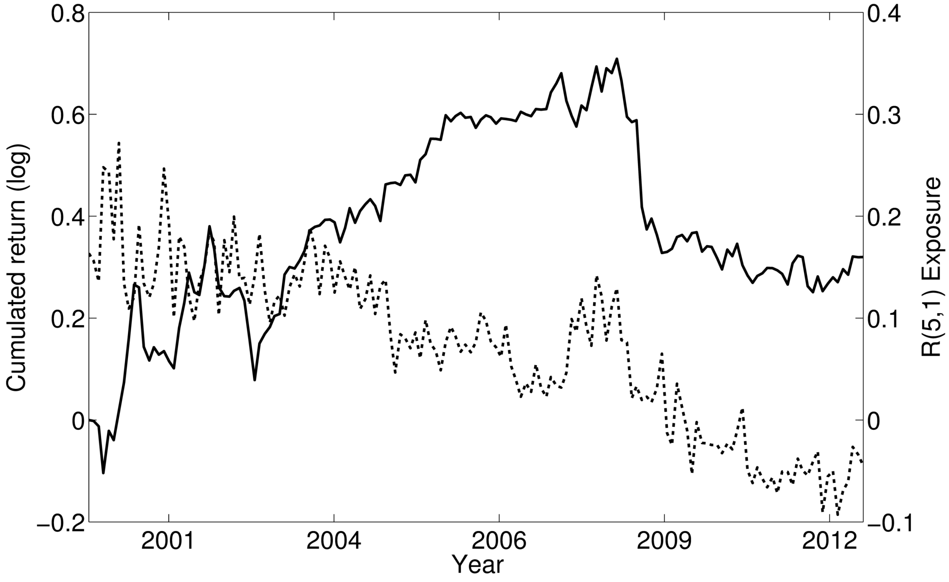
Recent research has noted the large heterogeneity of trading costs across different types of investors. Keim and Madhavan (1997) suggest a simple model to estimate transaction costs for a sample of institutional traders. However, as de Groot et al. (2012) note, this model can give rise to negative transaction costs in recent years. We therefore considered a rough approximation that extrapolates transaction costs from the turnover estimates in Frazzini et al. (2013). Even though the numbers might not apply to our sample exactly, they should be of similar magnitude, given the similarities of the data sample.

Using this approximation, we find that trading costs are around 7 to 8 percent per year (second row of table 12), close to the trading costs of the standard short-term reversal strategy investigated in the aforementioned papers. The last row of the table subtracts the approximate trading costs from the gross annual returns that we reported in table 1. After adjusting for trading costs, the hedge strategy that trades the extreme portfolios has an excess return of 24 percent per year. Trading the ninth minus the second decile (recall that these companies are larger and therefore probably more suited to the extrapolation from Frazzini et al. (2013)) yields an excess return of 5 percent per year. The excess return of trading the eighth versus the third decile is insignificant and slightly negative. In other words, the iterative conditional portfolio sort manages to profitably spread 40 percent of the companies, even after adjusting for transaction costs.

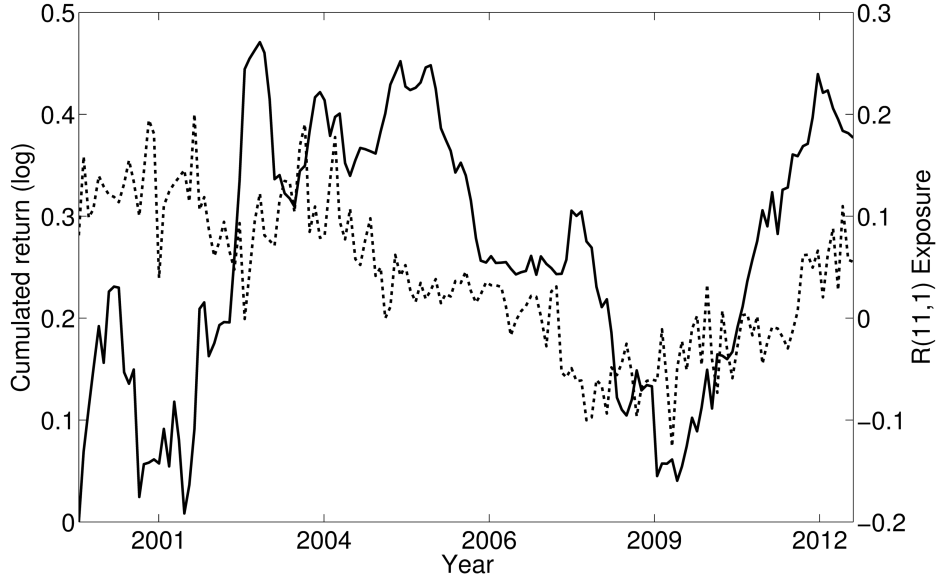
While our strategy implementation is standard in the stock market anomalies literature, more sophisticated variants could be designed for trading purposes when transaction costs are taken into account. de Groot et al. (2012) suggest reducing turnover of the short-term reversal strategy by holding onto the position in stocks even when they are not ranked in the extreme portfolios. We do not pursue their implementation here, but, given the return spread in the less extreme portfolios, it is plausible that such an implementation could be constructed here as well in order to reduce turnover and trading costs further.[20]

[19] These numbers are similar to those reported in de Groot et al. (2012) or Frazzini et al. (2013) for strategies based on short-term returns.

[20] For instance, Novy-Marx and Velikov (2014) find that many anomalies can be exploited by following an (s,S)-type strategy that, e.g. buys stocks when they are in the highest decile but only sells them if they drop out of the highest quintile.



(a) *R(5,1)*



(b) *R(11,1)*

Figure 13: *Simple strategy returns and exposure of the tree-based strategy: Solid lines show the performance of simple long-short strategies that are based on stock returns from 6 months (R(5,1) or 12 months (R(11,1) ago. Dashed lines show the exposure of the tree-based strategy to the simple return strategies.*

Table 12: *Turnover and trading costs*

	Low	2	3	4	5	6	7	8	9	High	High-Low	9-2	8-3
Turnover (monthly)	1.56	1.74	1.76	1.78	1.78	1.8	1.78	1.76	1.76	1.62	3.18	3.5	3.52
Trading cost (annual)	3.71	4.09	4.13	4.17	4.17	4.21	4.17	4.13	4.13	3.83	7.13	7.81	7.85
Gross return (annual)	-6.18	2.55	5.41	7.19	8.47	10.03	11.88	12.82	15.66	23.29	31.37	12.82	7.06
Net return (annual)	-9.88	-1.54	1.28	3.02	4.30	5.82	7.71	8.69	11.53	19.46	24.24	5.01	-.79

The first row (turnover) shows monthly turnover for the ten decile portfolios and for the equal-weighted hedge strategies. Turnover is computed for a strategy that goes $1 long and $1 short. Trading costs are extrapolated using the results in Frazzini et al. (2013).

D.1 Fama-MacBeth with recent returns only

In this section, we briefly contrast the results from the tree-based conditional portfolio sorts to the Fama-MacBeth results in section C.2. Tree-based conditional portfolio sorts can be viewed as either a kitchen sink regression or as a variable-selection method (since a variable is selected for each split). An initial interesting comparison can thus be conducted between the performance of the tree-based conditional sort in table 2 and the Fama-MacBeth regressions in table 10. The raw and factor-adjusted returns are about 0.5 percentage points higher than in the Fama-MacBeth regressions and, more interestingly, the information ratios are generally roughly three times as high. Even if we include all two-way interactions in a Fama-MacBeth regression, as in table 11, average excess returns and information ratios are generally much lower than in our results for the tree-based conditional sort.

These results let tree-based conditional sorts shine in two dimensions. If regarded as a kitchen sink method, the tree-based conditional sort leads to better performance than the Fama-MacBeth analogue, although both perform well. If regarded as a variable selection device, the Fama-MacBeth method mostly recovers momentum as an important determinant of expected returns, whereas the structure discovered by the tree-based conditional sort is more stable and cannot be explained by (simple) factor models.

In table 5, we contrast this to the case in which only variables that are considered important based on our tree-based conditional sorts are included in the Fama-MacBeth estimations. In particular, as a consistent set, we focus on the six most recent months of past returns, since our results above indicate that the most recent returns are most important for estimating expected returns. We abstract from variable selection and therefore act as if variable selection had already been conducted based on the tree-based conditional sort's results.

The first four columns of table 5 use only the return functions themselves and no interactions. The long-short strategy has an average return of 1.27 percent per month, and a four-factor alpha of 0.81 percent per month, with an information ratio of 0.78. Results based on using all past return functions are slightly higher, indicating that there is some information to be gained by including more distant past returns as well.

The next four columns of table 5 additionally include the most relevant two-way interactions among the six most recent return functions based on our previous results. The average strategy return is 1.61 percent per month and the four-factor alpha is 1.38 percent per month, both higher than in the kitchen sink regression above and also than in the estimations without interactions. Most remarkably, we observe an approximately 50 percent increase in the information ratio relative to the kitchen sink regression, from 1 to 1.49, indicating that the strategy return is earned at a much better risk-return tradeoff.

The last four columns of the table use all (and not only relevant) two-way interactions among the six most recent return functions. Results are almost identical to including the relevant interactions only. Table 6 shows that this can be attributed to the fact that the non-relevant interactions have small and insignificant coefficients and therefore do not have a big impact on the predictions.

Note that the returns in table 5 are still lower than the strategy returns in the original tree-based conditional portfolio sort. The Fama-MacBeth regressions only include two-way interactions.[21] While the Fama-MacBeth regression with two-way interactions goes some way to achieve similarly sized returns, the remaining differences can be attributed to the actual return structure being more involved than can be captured by including levels and two-way interactions of past returns alone.

Table 6 shows the Fama-MacBeth coefficient estimates averaged over the entire sample and corresponding t-statistics for the three regression models in table 5.[22] The second column shows coefficients in the levels-only regression. We observe the short-term reversal effect while all other past return variables enter with a positive sign. This is in line with the standard reversal and momentum effects in the literature.

Column 3 illustrates how these results completely flip when interaction terms are introduced in the regression. All level effects are on average negatively associated with expected returns while interaction terms are positive. This result is robust to including further (less relevant) interaction terms in column 4. A possible interpretation of this finding is that momentum is more likely to exist when returns are more consistent. For instance, we find that the effect of high returns in either the last month or in the second-to-last month indicate low returns. When both

[21] In unreported tests, we find that two-way interactions explain only around 10 percent of the variance of the estimated expected returns of the tree-based conditional sort. This implies that much of the predictive power of tree-based sorts comes from higher-order interactions.

[22] Note that for the prediction exercise we based predictions on rolling estimates of past coefficients as described above, while table 6 gives an average over the entire sample period.

returns are high, however, the interaction effect of this consistently high return works against the reversal effect of the two individual returns. Return consistency effects in momentum have been documented before by, among others, Watkins (2003) and Grinblatt and Moskowitz (2004).

How do the estimated Fama-MacBeth interactions compare to the average double partial derivatives in figure 9?[23] We find both similarities and differences. When we calculate the same average derivatives for the Fama-MacBeth model, we find that interactions of returns display the aforementioned consistency effect; that is, consistently high past returns predict high returns. These patterns coincide with the ones in figure 9. We also see that in two-way interactions that involve R(0,1), returns are less sensitive to the more distant returns, as in the top row of figure 9. On the other hand, in the Fama-MacBeth results, the interactions sometimes overturn the reversal effect, unlike in the tree-based conditional portfolio sort. Owing to their simplicity, the Fama-MacBeth regressions do not capture the more involved interaction patterns between R(1,1) and more distant returns that are apparent in the second row figure 9.

To summarize, we have emphasized the flexibility to control for variable interactions as one of the strengths of tree-based conditional portfolio sorts before. Now we see that the (two-way) interactions could have been discovered in a Fama-MacBeth regression framework, too. The tree-based conditional portfolio sort, however, is an efficient way to screen out the irrelevant interactions when the set of candidates is potentially large. At the same time, it also allows controlling for more involved interactions.

E Robustness of the discovered structure

In section E.1, we start by adding a set of 86 additional firm characteristics to the estimation and show that, again, the most recent returns are discovered as the most important ones. The same holds true when we consider a much larger set of correlated past return variables; results are presented in section E.2. In all cases, we find that the derivatives and interactions are similar to our main results.

We then turn to the question of how our return term structure result varies across firm size categories. We repeat the analysis for three groups of stocks that are sorted by firm size first in

[23] Note that since we do not include higher-order polynomials of the past decile ranks, the average partial derivatives with respect to each variable will be linear and therefore cannot capture nonlinear effects.

section E.3.

E.1 Including firm characteristics

We investigate whether our results on the structural relation between future and past returns are robust to including other firm characteristics. For this paper, we focus on the changing nature of the return term structure result, although the effect of firm characteristics (and the question of which of them can be found by our agnostic procedure) is an interesting one in itself.[24]

Going back to our original set of one-month return functions, we add 86 common firm characteristics, including size, book-to-market, gross profitability, earnings surprises, leverage, and many more. The full set is described in detail in the appendix to Green et al. (2014); we generate the same set of variables from annual and quarterly data on firm fundamentals from Compustat, daily and monthly stock price data from CRSP, and earnings expectations and firm recommendations data from IBES.

Table 13 reports the top 10 return-based predictor variables. The top 10 return functions for rolling and entire period optimization, again, evolve around the most recent returns and, apart from some changes in the exact order of predictor variables, are largely unaffected by the inclusion of other firm characteristics.

When comparing the top 10 variables to the top 10 in table 3, we see that 9 out of 10 show up in either list, with the exact ordering sometimes slightly altered. Recent returns are again the most important predictors.

The second row of average partial derivatives for the most recent one-month return functions in figure 8 mirror the patterns from the first row that did not include firm characteristics. In particular, we observe stable linear relationships between past performance and prediction for returns that are further than four months in the past and for the most recent past return, but we also observe non-monotone or nonlinear relationships for recent past returns in between.

Figure 14 shows double partial derivatives for return characteristics when firm characteristics are included and corresponds to figure 9. In both cases, we observe patterns that are qualitatively very similar and only differ in details — for example, the interaction between R(1,1) and R(3,1) is

[24] In ongoing work, a companion paper focuses exclusively on a large set of (mostly accounting- and earnings-based) firm characteristics.

Table 13: *Most important predictor variables: Including firm characteristics*

Variable	Importance
R(0,1)	1
R(1,1)	0.5
R(2,1)	0.45
R(5,1)	0.44
R(8,1)	0.41
R(4,1)	0.4
R(3,1)	0.39
R(11,1)	0.39
R(6,1)	0.38
R(7,1)	0.37

This table shows the most important return functions for the tree-based conditional portfolio sorts that use all one-month returns over the two years before portfolio formation and 86 additional firm characteristics. Return functions are sorted by their median importance over forty-five years. Variable importance is measured as described in section 3.2.3.

somewhat more pronounced.

Overall, we conclude that the discovered structure among return characteristics is largely unaffected by the inclusion of additional firm characteristics.

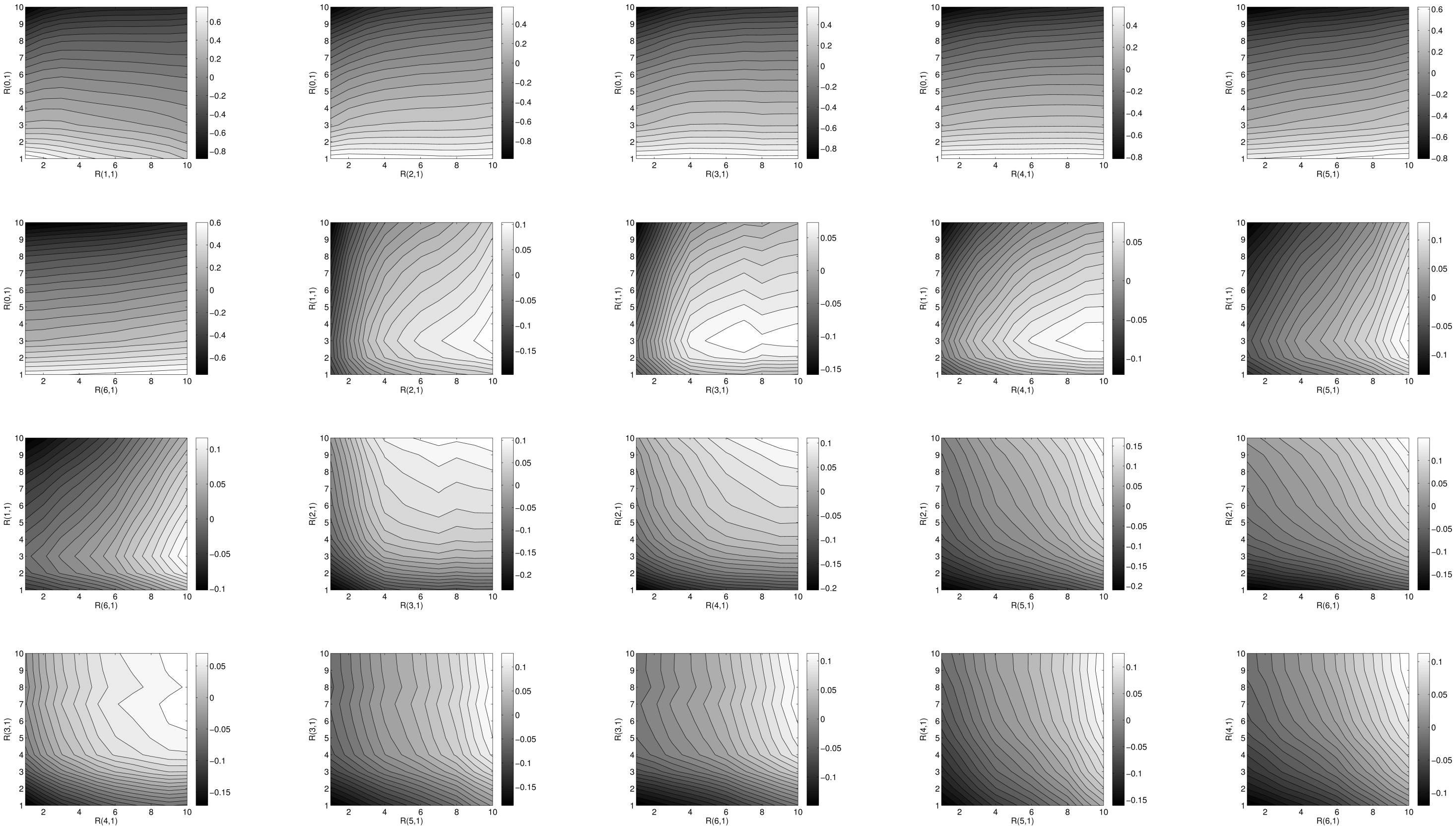


Figure 14: *Average double partial derivatives: Firm characteristics included. The figure shows the average prediction when two characteristics are counterfactually varied from low to high values. The figure shows results for return functions when 86 additional firm characteristics are included in the tree-based conditional portfolio sort. Results are based on rolling optimization of the model and predictions are averaged over the sample period. Details are in section 3.2.3.*

E.2 Expanded set of return functions

In this section, we directly give the algorithm access to standard notions of momentum, R(1,11) in our notation, and other past return functions. More precisely, we define an *expanded* set of past return functions that includes return-based characteristics $\{R(g,l)\}, g = 0,\ldots,6; l = 1,\ldots,18$; that is, the set includes a total of 126 return-based predictor variables that are often highly correlated. Our main findings show that the algorithm derives its predictive power from optimally using the variation in relatively short-term returns. Is the algorithm just trying to recreate standard momentum? Or is there more information in the individual returns than in a summary return like R(1,11)?

Turning to predictor variable importance in table 14, the tree-based conditional portfolio sorts recover recent past returns as the most important ones. The 10 most important return functions are all related to the most recent six months of returns and, what is more, the top 7 return functions are returns of length one that, taken together, summarize the most recent six-month return. Notice that all one-month return-based functions in the expanded set actually show up as the most important functions.

Recall that the return R(0,6) — that is, the total return over the most recent six months — could have been chosen by the algorithm in the expanded set. The fact that this return is not chosen but its components are illustrates that using the return over the previous year alone (and not the one-month returns that it is based on) leads to a loss of relevant information.

E.3 Estimation by size categories

We re-estimate the model for three separate size categories of firms. Following Fama and French (2008), we divide the sample of firms into three size categories based on NYSE breakpoints. Micro stocks are defined as the smallest 20 percent of companies by market value, small companies are the next 30 percent of companies, and the upper 50 percent make up the category of large firms. We repeat our analysis within each size category and compute the most relevant predictor variables for both our standard set of one-month return variables and for the expanded set of return functions from appendix E.2. Table 15 shows that the most important predictor variables are remarkably consistent across size categories, apart from some variations in exact rank of each

Table 14: *Most important predictor variables: Expanded set of return functions*

Variable	Importance
R(0,1)	1
R(1,1)	0.88
R(2,1)	0.69
R(6,1)	0.69
R(3,1)	0.66
R(4,1)	0.66
R(5,1)	0.61
R(0,2)	0.52
R(1,2)	0.45
R(1,3)	0.43

This table shows the most important return functions for a tree-based conditional portfolio sort that uses past returns functions R(g,l) with length $g = 1, \ldots, 18$ and gaps $g = 0, \ldots, 6$. Return functions are sorted by their median importance over forty-five years. Variable importance is measured as described in section 3.2.

predictor variable. Furthermore, most predictor variables in both sets relate to relatively recent returns.

Table 15: *Most important predictor variables: Within size category*

One-month returns						Expanded set					
Micro		Small		Big		Micro		Small		Big	
R(0,1)	1	R(0,1)	1	R(0,1)	0.99	R(0,1)	1	R(0,1)	1	R(1,1)	1
R(2,1)	0.54	R(2,1)	0.75	R(3,1)	0.72	R(1,1)	0.72	R(1,1)	0.65	R(2,1)	0.7
R(5,1)	0.51	R(4,1)	0.6	R(4,1)	0.6	R(3,1)	0.63	R(0,2)	0.65	R(3,1)	0.66
R(1,1)	0.5	R(1,1)	0.59	R(8,1)	0.6	R(5,1)	0.62	R(2,1)	0.63	R(5,1)	0.66
R(3,1)	0.5	R(3,1)	0.59	R(1,1)	0.51	R(2,1)	0.61	R(5,1)	0.56	R(6,1)	0.66
R(6,1)	0.49	R(5,1)	0.58	R(2,1)	0.5	R(6,1)	0.59	R(6,1)	0.56	R(4,1)	0.6
R(4,1)	0.42	R(6,1)	0.55	R(5,1)	0.5	R(0,2)	0.56	R(3,1)	0.52	R(0,1)	0.57
R(11,1)	0.4	R(7,1)	0.52	R(9,1)	0.5	R(4,1)	0.51	R(4,1)	0.5	R(0,2)	0.42
R(7,1)	0.39	R(8,1)	0.47	R(11,1)	0.5	R(1,2)	0.42	R(2,2)	0.43	R(6,4)	0.34
R(8,1)	0.38	R(23,1)	0.43	R(18,1)	0.5	R(0,3)	0.38	R(3,2)	0.43	R(6,6)	0.34

This table shows predictor variable importance in portfolios that are first sorted on size. Micro stocks are defined as the smallest 20% of companies by market value, small companies are the next 30% of companies, and the upper 50% make up the category of large firms. One-month returns include all one-month returns over the two years before portfolio formation. The expanded set consists of 126 return-based characteristics R(g,l) over the two years before portfolio formation with length $g = 1, \ldots, 18$ and gaps $g = 0, \ldots, 6$. Results are shown for both the rolling model estimation and for optimization over the entire horizon. For rolling estimates, return functions are sorted by their median importance over forty-five years. Variable importance is measured as described in section 3.2.

A Supplementary web appendix

A.1 Illustration of a conditional portfolio sort

Our results complement Fama and French (2008), who sort stocks into three size portfolios first and then sort each portfolio subsequently on a further firm characteristic. In our illustration, we consider conditional portfolio sorts that are each based on two of the following variables: short-term reversal, momentum, intermediate momentum, size, gross profitability, and book-to-market.

In our illustration, we consider conditional portfolio sorts that are each based on two of the following variables: short-term reversal, momentum, intermediate momentum, size, gross profitability, and book-to-market. Our results complement Fama and French (2008) who sort stocks into three size portfolios first and then sort each portfolio subsequently on a further firm characteristic. We follow the same approach with a few modifications:

Each month, we sort stocks into one of three portfolios based on the value of a sorting variable from the following list: short-term reversal, momentum, intermediate momentum, size, gross profitability, and book-to-market. Short-term reversal is defined as the return over the most recent prior month, momentum is the return over the past twelve months (excluding the most recent month) and intermediate momentum is the return over the past twelve months excluding the most recent six months. Accounting-based variables are constructed in a standard fashion (see appendix E.1).

Each portfolio is then further divided into ten portfolios based on a second sorting variable from the same list and we compute the equal weighted hedge return based on the second sorting within each of the three portfolios. Table 16 reports the equal weighted hedge returns, their associated t-statistics, and the test of Patton and Timmermann (2010) for the monotonicity of returns over the decile sort.[25] Columns labeled "Low" contain estimates based on firms in the lowest tercile of the first sorting variable, "Middle" and "High" denote the next two terciles, and "All" uses all observations without a sort on the first sorting variable for comparison.[26]

First, note that all of the sorting variables achieve significant returns in the equal-weighted

[25] Since the Patton and Timmermann test for monotonicity is not (yet) standard, here is a brief summary: It computes the pairwise difference between the average returns of adjacent decile portfolios and then tests whether the minimum of these differences is greater than zero (if the research hypothesis is that returns are increasing over deciles). If the test rejects, this provides support for the research hypothesis.

[26] In other words, the column "All" gives the results for an unconditional sort on the row variable.

hedge portfolios and pass the monotonicity test of Patton and Timmermann. To delve into the details of the results: When firms are sorted on short-term reversal first, momentum, intermediate momentum and value still manage to pass the t-test and the monotonicity test within each short-term reversal tercile portfolio. Size does not work in the top tercile, and gross profitability passes the t-test, but fails to provide monotone returns throughout all terciles. A similar picture emerges when returns are sorted on momentum first. Value and short-term reversal work throughout all terciles, while intermediate momentum yields (weakly) significant t-statistics in each tercile but does not pass the monotonicity test for the low and middle groups of momentum-sorted returns. Size passes all t-tests, but fails to provide monotone returns in the lowest two terciles. Interestingly, momentum sorts continue to work well when firms are sorted on intermediate momentum first but the reverse is not true: Intermediate momentum sorts do not consistently give a significant hedge return (only in low momentum stocks) or monotone returns (only in the middle tercile of momentum stocks). Initial sorts on value or size leave the monotonicity of return sorts intact, but interfere with the monotonicity and t-tests of gross profitability. When firms are sorted on gross profitability first, equal-weighted hedge returns are significant for all variables in all terciles, but the returns to medium gross profitability firms is not monotone when sorted by value.

The overall picture that emerges is that of return sorts being relatively stable while accounting-based sorts are less robust to initial sorts on some other return- or accounting-based variable. The results illustrate the potential relevance of correlated return- and accounting-based characteristics, and the necessity to consider conditional returns when the objective is to evaluate the importance of a new candidate predictor variable. Variable interactions can also be relevant as is evident from the fact conditional sorts often work only in some of the tercile portfolios.[27]

[27] It is also possible to condition on more than one variable in this setting by first doubly sorting all stocks on two variables into, say, three categories each for a total of nine portfolios. Within each portfolio, one could then compute the same statistics as above, and discuss the effects of conditioning on levels and interactions of variables. While, in principle, feasible for a few variables, the approach does not lend itself to an easy interpretation in higher dimensions.

Table 16: *Conditional portfolio sorts: Average returns, t-statistics and p-values of monotonicity tests*

First sort on: R(0,1)												
	Average return				t-statistics				p-value PT test			
Sorting on	Low	Mid	High	All	Low	Mid	High	All	Low	Mid	High	All
R1 11	0.59	1.51	2.78	1.56	2.88	7.46	12.95	8.54	0.13	0.00	0.00	0.00
R6 6	0.71	1.13	1.74	1.19	3.98	6.32	9.48	7.69	0.00	0.00	0.00	0.00
size	-1.50	-0.73	-0.21	-1.28	-5.84	-2.87	-0.86	-5.37	0.00	0.02	0.57	0.00
gross profitability	0.60	0.32	0.77	0.48	4.24	2.32	5.52	3.99	0.39	0.59	0.94	0.09
booktomarket	1.27	1.09	1.00	1.09	7.00	6.02	4.96	6.36	0.00	0.01	0.00	0.00
First sort on: R(1,11)												
R0 1	-3.29	-1.02	-0.85	-1.79	-15.17	-6.01	-4.79	-10.60	0.00	0.04	0.00	0.00
R6 6	0.37	0.27	0.25	1.19	2.65	1.90	1.52	7.69	0.97	0.07	0.43	0.00
size	-0.75	-1.04	-0.95	-1.28	-3.07	-4.41	-4.04	-5.37	0.62	0.67	0.00	0.00
gross profitability	0.66	0.36	0.57	0.48	4.03	2.82	4.32	3.99	0.05	0.06	0.13	0.09
booktomarket	1.39	1.23	0.67	1.09	7.50	7.25	3.52	6.36	0.00	0.00	0.00	0.00
First sort on: R(6,6)												
R0 1	-2.58	-1.46	-1.38	-1.79	-12.62	-8.58	-7.15	-10.60	0.00	0.00	0.00	0.00
R1 11	1.35	1.03	1.24	1.56	7.15	6.43	6.88	8.54	0.09	0.02	0.01	0.00
size	-0.92	-1.03	-0.78	-1.28	-3.72	-4.34	-3.38	-5.37	0.08	0.05	0.00	0.00

	Average return				t-statistics				p-value PT test			
Sorting on	Low	Mid	High	All	Low	Mid	High	All	Low	Mid	High	All
gross profitability	0.68	0.38	0.50	0.48	4.32	3.22	3.75	3.99	0.00	0.04	0.68	0.09
booktomarket	1.44	1.14	0.83	1.09	7.65	6.12	4.97	6.36	0.00	0.00	0.14	0.00
First sort on: Book to Market												
R0 1	-1.74	-1.84	-2.10	-1.79	-9.09	-10.19	-10.33	-10.60	0.12	0.00	0.00	0.00
R1 11	1.98	1.35	1.10	1.56	9.39	6.54	6.11	8.54	0.00	0.00	0.00	0.00
R6 6	1.60	1.00	0.88	1.19	9.17	5.75	5.37	7.69	0.03	0.00	0.08	0.00
size	-1.03	-0.73	-1.33	-1.28	-3.49	-3.21	-5.14	-5.37	0.22	0.08	0.07	0.00
gross profitability	0.79	0.66	0.37	0.48	4.69	4.53	2.85	3.99	0.32	0.00	0.19	0.09
First sort on: Gross Profitability												
R0 1	-1.69	-1.95	-1.93	-1.79	-9.81	-9.74	-10.67	-10.60	0.00	0.00	0.02	0.00
R1 11	1.75	1.40	1.47	1.56	8.37	6.72	8.00	8.54	0.00	0.00	0.00	0.00
R6 6	1.48	0.99	1.02	1.19	8.28	5.45	6.49	7.69	0.00	0.00	0.01	0.00
size	-1.30	-1.35	-1.18	-1.28	-4.64	-5.55	-4.40	-5.37	0.08	0.06	0.04	0.00
booktomarket	1.49	1.11	1.00	1.09	7.03	6.36	5.44	6.36	0.00	0.27	0.00	0.00
First sort on: Size												
R0 1	-3.09	-1.74	-1.02	-1.79	-11.92	-9.13	-6.58	-10.60	0.00	0.00	0.09	0.00
R1 11	1.44	1.85	1.08	1.56	8.58	8.87	4.82	8.54	0.00	0.00	0.30	0.00

	Average return				t-statistics				p-value PT test			
Sorting on	Low	Mid	High	All	Low	Mid	High	All	Low	Mid	High	All
R6 6	0.96	1.27	1.04	1.19	5.71	7.20	5.42	7.69	0.06	0.00	0.00	0.00
gross profitability	0.14	0.68	0.40	0.48	0.87	4.43	2.95	3.99	0.06	0.23	0.23	0.09
booktomarket	0.63	1.05	0.53	1.09	3.74	5.44	2.81	6.36	1.00	0.02	0.00	0.00

A.2 Greedy algorithm

Before we describe the details of the algorithm, we give a high-level summary. The algorithm starts out with all observations and splits them into two subsets. From a given set of variables, it finds the variable and the associated threshold value that minimize the mean squared error over all observations if predictions are computed as in equation (4). The algorithm is called greedy because it solves the minimization problem in a brute-force fashion by trying every combination of variable and threshold values. The same procedure is then repeated in each subset until the number of observations in a subset becomes small or if no further split can meaningfully improve upon the mean squared error. The result is a tree-based conditional portfolio sort, that is, a conditional portfolio sort with many levels.

Formally, let $S_1(g,\tau)$ and $S_2(g,\tau)$ be two portfolios that are defined by a firm's past return decile ranking $R(g,1)$ and a threshold value τ such that, as before, all observations for which $R(g,1) \leq \tau$ are in portfolio S_1, and all observations for which $R(g,1) > \tau$ are in portfolio S_2. At each node, all observations that are members of that node are split into two such portfolios. The greedy algorithm finds the past return characteristic $R(g,1)$ and the threshold value τ such that

$$(g^*,\tau^*) = \arg\min_{g,\tau} SC(g,\tau), \tag{10}$$

where $SC(g,\tau)$ is a *split criterion function* which we adopt from the related machine learning literature. The split criterion function selects the predictor variable and the associated threshold that minimize the sum of mean squared errors in the resulting portfolios with respect to the expected returns, that is,

$$SC(g,\tau) = \min_{\mu_1}\left(\sum_{R_{it}(g,1)\in S_1(g,\tau)} (r_{i,t+1}-\mu_1)^2\right) + \min_{\mu_2}\left(\sum_{R_{it}(g,1)\in S_2(g,\tau)} (r_{i,t+1}-\mu_2)^2\right) \tag{11}$$

and the inner minimizations are solved by equation (4). This algorithm reduces a complex non-linear estimation problem into subsets of simpler linear ones. The problem is solved in a brute-force fashion where the value of the split criterion function is computed for each firm characteristic and each threshold value. The optimization is repeated in each of the resulting portfolios until a. the number of observations in a node gets too small for further splits, or b. no variable provides a sufficient improvement of the mean squared error in equation (11). The result is a conditional portfolio sort with many levels.[28]

Before we move on, we want to point out a few links to other estimation methods in the literature. The greedy algorithm introduced in this section bears some resemblance to forward-selection methods in regression models. Forward-selection starts out with the smallest possible linear model, estimates bivariate regressions of the outcome variable on each candidate regressor separately, and keeps the one with the highest t-statistic (or some other selected performance criterion). The procedure is then repeated for all of the remaining variables with the best-performing variable joining the regression each round until no further variables are significant. As tree-based conditional sorts, forward-selection works when there are more regressors than observations. On the other hand, forward selection is global in nature in the sense that one regression function is fitted for the entire sample and variable selection is based on performance over the entire sample. In addition, interaction terms would need to be added one-by-one as well, leading to a large set of candidate variables whereas the set of candidate variables is always less than the number of main signals in tree-based conditional portfolio sorts.

Kernel regression is based on approximating an outcome variable by a (kernel-) weighted average of the outcome at each value of the regressor. Tree-based conditional portfolio sorts approximate the outcome by the average value of the outcome for a regressor region defined by split points and threshold values. Kernel regressions are very flexible but do not extend easily beyond the bivariate case. A small practical issue is the difficulty to display results in higher dimensions. More importantly, since kernel regression is based on using local averages, there are few observations in each subspace over which an average is taken as the number of regressors becomes large. This is known as the curse of dimensionality and one can show that the convergence rate for kernel regressions deteriorates sharply with the dimensionality of the regressors. Local linear regressions run into analogous problems in high dimensions.

[28] The question of when to stop adding new levels to the conditional sort relates to a standard bias-variance tradeoff. Using many levels potentially results in overfitting, which would worsen the predictive power of equation (5) out of sample. Estimating only a few levels might miss important aspects of the data leading to bias. Within this sphere the number of levels can be chosen. We stop when the number of firms in a portfolio is smaller than 100 and make sure to validate all our estimates out of samples as described in section 3.2.3.