Consumption-Based Asset Pricing with Rare Disaster Risk

A Simulated Method of Moments Approach

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Abstract

The rare disaster hypothesis suggests that the extraordinarily high postwar U.S. equity premium resulted because investors ex ante demanded compensation for unlikely but calamitous risks that they happened not to incur. Although convincing in theory, empirical tests of the rare disaster explanation are scarce. We estimate a disaster-including consumption-based asset pricing model (CBM) using a combination of the simulated method of moments and bootstrapping. We consider several methodological alternatives that differ in the moment matches and the way to account for disasters in the simulated consumption growth and return series. Whichever specification is used, the estimated preference parameters are of an economically plausible size, and the estimation precision is much higher than in previous studies that use the canonical CBM. Our results thus provide empirical support for the rare disaster hypothesis, and help reconcile the nexus between real economy and financial markets implied by the consumption-based asset pricing paradigm.

Key words: equity premium, rare disaster risk, asset pricing, simulated method of moments

JEL: G10, G12, C58

1 Introduction

A paradigm in asset pricing theory asserts that positive expected excess returns result as a form of risk compensation. The very high empirical equity premium for postwar U.S. stocks implies that there must be considerable risk for which to compensate. The methodological lynchpin of this view is Hansen and Singleton's (1982) consumption-based asset pricing model (CBM). Assuming additive power utility, it implies the asset pricing equation

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1}\right] = 1, \tag{1.1}$$

for a gross return R_{t+1}, where β denotes the subjective discount factor, γ captures relative risk aversion, and C_t is consumption in period t. As demonstrated by Mehra and Prescott (1985) though, the canonical CBM cannot explain the high equity premium at plausible values of relative risk aversion, leading to a widespread belief that the model is strong in theory, but weak in application.[1] As Rietz (1988) first noted, the reason for the empirical failure of the CBM may be rare but extreme contractions in consumption. That is, investors demand compensation for the risk of sharp downturns in their consumption that occur with only a very small probability. This compensation is reflected in the high expected returns for assets whose payoffs covary positively with consumption. In a sample without such contractions, the average returns of those assets can be high.

With this paper we provide an empirical assessment of Rietz's (1988) rare disaster hypothesis by estimating a disaster-including CBM using a combination of the simulated method of moments and bootstrapping. We consider several method-

[1] Cochrane (1996) uses Hansen's (1982) generalized method of moments (GMM) to estimate the parameters of Equation (1.1) and obtains $\hat{\gamma} = 241$, which implies an implausibly high risk aversion. Mehra and Prescott (1985) and Rietz (1988) consider a range for γ between 1 to 10 as plausible, whereas Cochrane (2005) suggests 1 to 5.

ological approaches that differ in the moment matches and the way to account for disasters in the simulated consumption growth and return series. The estimation is performed using various sets of test assets. In all instances, the parameter estimates are economically plausible, which vindicates the consumption-based asset pricing paradigm.

In the related literature, Barro (2006) draws on Rietz's (1988) approach and provides a model that permits a calibration of equity premia. He uses information about disastrous GDP contractions, assumes plausible values of relative risk aversion and time preferences, and shows that the calibrated equity premia are in the range of the empirically observed counterparts. These seminal contributions laid the foundation for a growing literature, though empirical tests of the rare disaster hypothesis remain rare. Calibrations such as Barro's certainly are useful, but the question remains: How do asset pricing models that account for rare disasters – and in particular the CBM – perform when econometric techniques get applied to estimate the model parameters?

The empirical analysis of asset pricing models that account for the possibility of rare disasters is hampered because extreme consumption contractions are rare by definition. How can the estimation of a disaster-including CBM be accomplished without observing any sharp downturns in the first place? We tackle this epistemological problem by pursuing an econometric approach inspired by Cochrane's (2005, p. 461) remark:

> We had no banking panics, and no depressions; no civil wars, no constitutional crises; we did not lose the Cold War, no missiles were fired over Berlin, Cuba, Korea, or Vietnam. If any of these things had happened, we might well have seen a calamitous decline in stock values, and I would not be writing about the equity premium puzzle.

In line with this view, we posit that the extremely high risk aversion estimates result from a sample selection effect, i.e. the consumption and return data that the U.S. economy produced over the past 65 years represent a single, lucky itinerary of the histories that could have been.[2] In that case, the empirical deficiencies of the CBM emerge because the available data do not represent the possible and disaster-including scenarios that investors have anticipated. If disastrous contractions in consumption were possible but did not occur, then we have to account for them by traveling (metaphorically) the roads that the U.S. postwar economy did not take. We must consider histories marked by banking panics and depressions, in which the U.S. did lose the Cold War, in short, alternative histories in which we would not write about the equity premium puzzle.

We combine the simulated method of moments (SMM) and non-parametric and parametric bootstrapping to facilitate such journeys within frequentist statistics' concept of repeated sampling. All methodological alternatives rely on simulated disaster-including consumption growth and return data. Adopting the disaster identification scheme proposed by Barro (2006) and using GDP data collected by Bolt and van Zanden (2013), we identify contractions that exceed a specified threshold. We draw consumption shrinkage factors from a Double Power Law distribution to allow for sharp contractions of the bootstrapped "regular" consumption growth series. Following Barro and Jin (2011), we use the sample of identified contractions and estimate the Double Power Law parameters by maximum likelihood. We consider four ways to simulate disaster-including financial returns, and we propose three variants of moment matches that facilitate the estimation of a disaster-including CBM. When applied to four different sets of test assets all variants deliver comparable, economically plausible, and precise estimates of the relative risk aversion coefficient

[2] By invoking the "peso problem hypothesis," Veronesi (2004) offers a similar argument.

and the subjective discount factor. The results thus provide empirical support for the rare disaster hypothesis and help restore the nexus between real economy and financial markets that is implied by the consumption-based asset pricing paradigm.

Our study accordingly contributes to a growing literature on rare disaster risk in asset pricing. Barro and Jin (2011) present a calibration of the Barro (2006) model with Epstein-Zin preferences and Double Power Law-distributed extreme contractions. As a possible solution to the volatility puzzle, Wachter (2013) extends Barro's (2006) model by recursive preferences and time-varying disaster probabilities. Gabaix (2012) formulates a model in which the severity of disasters varies with time, and challenges ten prominent puzzles in macro-finance. Backus et al. (2011) rely on equity index options to obtain the distribution of consumption growth disasters, and Gourio (2012) includes time-varying disaster risk in a business cycle model. Julliard and Ghosh (2012) fit a non-parametric distribution to disastrous GDP contraction data and argue that the equity premium puzzle itself emerges as a rare event. Weitzman (2005) uses a Bayesian approach that focuses on learning about consumption volatility, which implies fat-tailed posterior distributions of future consumption growth. The paper by Posch and Schrimpf (2012) is closest to the present study. They consider an alternative approach to evaluate the rare disaster hypothesis by simulating consumption and return data of economies potentially hit by disasters. Using the simulated data, Posch and Schrimpf estimate the CBM on samples that do not include disasters and analyze the implied Euler equation errors. They report that the parameter estimates obtained from such a procedure are comparably implausible to those computed from empirical data. Our results complement theirs, in that we explicitly focus on potentially disaster-including consumption series and the plausible estimates of relative risk aversion and subjective discount factor obtained after accounting for rare disaster risk.

From a broader perspective, our study contributes to a literature that attempts to vindicate the CBM, while retaining its core paradigm reflected in Equation (1.1). These studies only partially succeed in providing plausible and precise estimates of the CBM preference parameters. Yogo (2006), for example, proposes a structural model that differentiates between the consumption of durable and nondurable goods. This model can explain cross-sectional and time series variation in expected stock returns but at a level of risk aversion that is still very high. The smallest relative risk aversion estimate obtained with the unconditional model version amounts to $\hat{\gamma} = 191.4$. The estimated subjective discount factor is plausible ($\hat{\beta} = 0.9$). Savov (2011) relies on waste data as a measure of consumption, fixes $\beta = 0.95$, and (using the excess market return as the single test asset) obtains a considerably lower estimate of the parameter of risk aversion, $\hat{\gamma} = 17.0$ with s.e.$(\hat{\gamma}) = 9.0$. Julliard and Parker (2005) analyze the ultimate risk of consumption, defined as the covariance of returns and consumption growth aggregated over current and future periods. They fix $\beta = 1$ and estimate $\hat{\gamma} = 9.1$. However, the estimate has a relatively high standard error (s.e.$(\hat{\gamma}) = 17.2$). Savov's (2011) and Julliard and Parker's (2005) studies resonate with ours in the sense that they modify the consumption data used in the empirical analysis, instead of elaborating investors' preferences.

The remainder of the paper is structured as follows. Section 2 motivates a disaster-including CBM and outlines the econometric methodology. Section 3 contains a description of the data. In Section 4 we present the estimation results, before wrapping up the discussion and concluding in Section 5.

2 Econometric methodology

2.1 Rare disasters in a consumption-based asset pricing model

There are two rational explanations for the high equity premia measured in U.S. postwar return data, and both are compatible with the CBM. The first is that investors are extremely risk averse and demand massive compensation for carrying little risk, a reasoning that is in line with the high risk aversion estimates reported by Cochrane (1996) and Yogo (2006). The second is the rare disaster hypothesis, which states that high equity premia result because investors are compensated for the risk of calamitous contractions that (luckily) did not happen. An empirical assessment of this explanation is hampered when the historical data do not contain enough information about disastrous contractions of consumption and asset prices. In the following, we propose econometrically testing the rare disaster hypothesis within the CBM framework.

Barro (2006) considers a disaster-including consumption process that he uses to obtain closed form solutions of equity premia, conditional and unconditional on disaster periods. We adopt his specification and assume that consumption evolves as[3]

$$C_{t+1} = C_t e^{u_{t+1}} e^{v_{t+1}}, \tag{2.1}$$

where $u_{t+1} \sim (\mu, \sigma^2)$ and $v_{t+1} = \ln(1 - b_{t+1})d_{t+1}$. The binary disaster indicator d_{t+1} is equal to 1 if a disaster occurs in $t+1$ and 0 otherwise. If $d_{t+1} = 1$, consumption contracts by a random factor $b_{t+1} \in [q, 1]$, where q refers to the disaster threshold, such that

$$\frac{C_{t+1}}{C_t} = e^{u_{t+1}}(1 - b_{t+1})^{d_{t+1}}. \tag{2.2}$$

[3] In Barro's (2006) endowment economy, consumption equals output A_t, where $\ln A_{t+1} = \ln A_t + \mu + u_{t+1} + v_{t+1}$, with $u_{t+1} \sim N(0, \sigma^2)$. For convenience, we modify Barro's specification by allowing for a non-zero mean of u_{t+1} and discarding the drift parameter μ.

Accordingly, $e^{u_{t+1}}$ denotes regular, non-disastrous consumption growth, and $(1 - b_{t+1})^{d_{t+1}}$ accounts for the effect of a potential disaster on consumption.

Substituting the right-hand side of Equation (2.2) into Equation (1.1), we can write the basic asset pricing equation as it applies to a gross return:

$$\begin{aligned}\mathbb{E}_t\left[\beta\left(e^{u_{t+1}}e^{v_{t+1}}\right)^{-\gamma}R_{t+1}\right] &= p\,\underbrace{\mathbb{E}_t\left[\beta\left(e^{u_{t+1}}(1-b_{t+1})\right)^{-\gamma}R_{t+1}|d_{t+1}=1\right]}_{\text{expect. cond. on disaster in } t+1} \\ &\quad + (1-p)\,\underbrace{\mathbb{E}_t\left[\beta\left(e^{u_{t+1}}\right)^{-\gamma}R_{t+1}|d_{t+1}=0\right]}_{\text{expect. cond. on no disaster in } t+1} \\ &= 1,\end{aligned} \tag{2.3}$$

where the disaster probability $p = \mathbb{P}(d_{t+1} = 1)$ is assumed to be time-invariant. Applying the law of total expectation to Equation (2.3) and rearranging, we obtain

$$\mathbb{E}\left[\beta\left(e^{u_t}\right)^{-\gamma}R_t|d_t=0\right] = \frac{1}{1-p}\left[1 - p\mathbb{E}\left[\beta\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t|d_t=1\right]\right]. \tag{2.4}$$

For the pricing of an excess return R_t^e, the analogue of Equation (2.4) is

$$\mathbb{E}\left[\left(e^{u_t}\right)^{-\gamma}R_t^e|d_t=0\right] = -\frac{p}{1-p}\left[\mathbb{E}\left[\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t^e|d_t=1\right]\right]. \tag{2.5}$$

In this case, β is not identified.

If a sample with disaster observations were available, we could write the sample counterparts of the population moments in Equation (2.4) as

$$\frac{1}{T-D_T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}(1-d_t) = \frac{1}{1-\frac{D_T}{T}}\left[1 - \frac{D_T}{T}\left[\frac{1}{D_T}\sum_{t=1}^{T}\beta cg_{d,t}^{-\gamma}R_{d,t}d_t\right]\right], \tag{2.6}$$

where $D_T = \sum_{t=1}^{T} d_t$ counts the number of disasters in a series of length T; $cg_{nd,t}$ and $R_{nd,t}$ are regular consumption growth and return; $cg_{d,t}$ and $R_{d,t}$ denote disaster consumption growth and gross return, respectively. As $T \to \infty$ and when a law of large numbers holds, $\frac{D_T}{T} \underset{p}{\to} p$ and $\frac{T-D_T}{T} \underset{p}{\to} 1-p$. Furthermore, assuming that a uniform law of large numbers holds,

$$\frac{1}{D_T}\sum_{t=1}^{T}\beta cg_{d,t}^{-\gamma}R_{d,t}d_t \underset{p.u.}{\longrightarrow} \mathbb{E}\left[\beta\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t|d_t=1\right], \tag{2.7}$$

and

$$\frac{1}{T-D_T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}(1-d_t) \underset{p.u.}{\longrightarrow} \mathbb{E}\left[\beta\left(e^{u_t}\right)^{-\gamma}R_t|d_t=0\right], \tag{2.8}$$

where $\underset{p.u.}{\longrightarrow}$ denotes uniform convergence in probability. Analogous expressions can be given for the sample counterparts of Equation (2.5).

Suppose we have access to disaster-including consumption and return data. We then might use GMM and match the sample moments in Equation (2.6) with their population counterparts in Equation (2.4). However, this strategy would be impeded, because even for long time series, the quality of the moment matches would be poor, with huge parameter standard errors. Rare disasters are, well, rare, and T must be very large to ensure moderate estimation precision.

For the U.S. postwar data, used by all the studies mentioned in the introduction, the problem becomes aggravated. These data do not incorporate any disaster observations, such that $d_t = 0\ \forall\ t$, and thus $D_T = 0$, and $\hat{p} = 0$. To apply GMM, we would use the disaster-free consumption growth $cg_{nd,t}$ and return series $R_{nd,t}$ (with excess returns, $R^e_{nd,t}$), and match the left-hand side of Equation (2.4) (with excess returns, Equation (2.5)) with their sample counterparts $\frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}$ (with excess returns, $\frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R^e_{nd,t}$). However, the right-hand side of Equation (2.4) is equal to 1, and the right hand side of Equation (2.5) is equal to 0, only if

$p = 0$. The usual moment matches for GMM estimation

$$\mathbb{G}_T(\beta, \gamma) \equiv \frac{1}{T} \sum_{t=1}^{T} \beta c g_{nd,t}^{-\gamma} \mathbf{R}_{nd,t} - 1, \tag{2.9}$$

using the gross returns of N test assets, $\mathbf{R}_{nd,t} = [R_{nd,t}^{1}, \ldots, R_{nd,t}^{N}]'$, and

$$\mathbb{G}_T(\gamma) \equiv \frac{1}{T} \sum_{t=1}^{T} c g_{nd,t}^{-\gamma} \mathbf{R}_{nd,t}^{e}, \tag{2.10}$$

using excess returns, $\mathbf{R}_{nd,t}^{e} = [R_{nd,t}^{e1}, \ldots, R_{nd,t}^{eN}]'$, thus are valid only if disastrous consumption contractions are impossible. Yet, it is hard to imagine that investors in 1946, after World War II and the Great Depression, and at the onset of the Cold War, should have assigned a probability of zero to states in which their consumption may suffer from extreme contractions.

Does this imply that an empirical assessment of the rare disaster hypothesis and the estimation of a disaster-including CBM cannot be performed due to a lack of suitable data? We tackle this problem with an estimation strategy that consists of a mix of parametric and non-parametric bootstrapping and SMM. Our approach is inspired by a quote of Singleton (2006, p. 254), with which he advocates the simulated method of moments:

> More fully specified models allow experimentation with alternative formulations of economies and, perhaps, analysis of processes that are more representative of history for which data are not readily available.

We propose three SMM estimation strategies along that line, each of which implies matching sample moments and simulated theoretical moments. The latter account for the possibility of consumption disasters. None of them requires the availability of disaster-including consumption and return data.

2.2 Moment Matches for estimating a disaster-including CBM

For the first approach to estimate a disaster-including CBM, we derive moment matches from Equations (2.4) and (2.5). Using time series of length T of regular consumption and (excess) returns, sample counterparts of the left-hand side conditional expectations can be computed as $\frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}$ and $\frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}^{e}$. The right-hand side moments of Equations (2.4) and (2.5) instead can neither be expressed as functions of parameters (which would facilitate GMM), nor can the sample counterparts be computed using disaster-free data. However, if it is possible to specify processes that are more representative of history, in the spirit of Singleton's (2006) quote, i.e. series that would include disaster observations, these moments can be simulated, viz

$$\begin{aligned}\frac{1}{1-p}\left[1-p\mathbb{E}\left[\beta\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t|d_t=1\right]\right]&\approx\frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}(T)}}\left(1-\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}R_sd_s\right)\\ \frac{p}{1-p}\left[\mathbb{E}\left[\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t^e|d_t=1\right]\right]&\approx\frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}(T)}}\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}cg_s^{-\gamma}R_s^ed_s,\end{aligned}\tag{2.11}$$

where $D_{\mathcal{T}}=\sum_{s=1}^{\mathcal{T}(T)}d_s$ denotes the number of disasters in the simulated sample of size $\mathcal{T}(T)$. Using the gross risk-free rate R^f and a vector of excess returns $\mathbf{R}^e$ as test assets, we can apply the moment matches,

$$\mathbb{G}_T(\boldsymbol{\theta})=\begin{bmatrix}\frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}R_{nd,t}^{f}-\frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}(T)}}\left(1-\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}R_s^fd_s\right)\\ \frac{1}{T}\sum_{t=1}^{T}\beta cg_{nd,t}^{-\gamma}\mathbf{R}_{nd,t}^{e}+\frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}(T)}}\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}\mathbf{R}_s^ed_s\end{bmatrix},\tag{2.12}$$

where $\boldsymbol{\theta} = [\beta, \gamma]'$.[4] SMM estimates then can be obtained by

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\arg\min}\ \mathbb{G}_T(\boldsymbol{\theta})' \mathbf{W}_T \mathbb{G}_T(\boldsymbol{\theta}), \tag{2.13}$$

where $\mathbf{W}_T$ is a symmetric and positive definite weighting matrix. The analysis has to be based on a large $\mathcal{T}(T)$ to ensure that the simulated data contain enough disasters, and the approximations in Equation (2.11) are sufficiently accurate. We refer to this estimation strategy as MAD-SMM (*M*oments *A*ccounting for *D*isasters).

In the next section, we explain in detail how we simulate the disaster-including consumption growth, $\{cg_s\}_{s=1}^{\mathcal{T}(T)}$, and return series, $\{R_s\}_{s=1}^{\mathcal{T}(T)}$. But first we propose an alternative set of moment matches that results from a reformulation of the basic asset pricing equation advocated by Julliard and Parker (2005). They relate the expected excess return to the covariance of the excess return and the stochastic discount factor (SDF), $m_t = \beta cg_t^{-\gamma}$,

$$\operatorname{cov}(m_t, R_t^e) = \mathbb{E}[(\beta cg_t^{-\gamma} - \mu_m) R_t^e], \tag{2.14}$$

where $\mu_m = \mathbb{E}[\beta cg_t^{-\gamma}] = \mathbb{E}\left[\frac{1}{R_t^f}\right]$. In particular, they propose using the moment condition

$$\mathbb{E}\left[R_t^e + \frac{\mathbb{E}[(\beta cg_t^{-\gamma} - \mu_m) R_t^e]}{\mu_m}\right] = 0 \tag{2.15}$$

for GMM. Again, we seek to account for the effect of calamitous, yet unobserved consumption contractions on risk compensations, which is reflected in $\operatorname{cov}(m_t, R_t^e)$. This population moment cannot be expressed analytically as a function of parameters, and using the sample covariance based on non-disastrous data is not helpful either. We therefore resort to an approximation by simulated moments that allows for the

[4] If we were only interested in the γ estimate and an analysis of the equity premium, the second moment match in Equation (2.18) can be omitted, as Julliard and Parker (2005) did. Then, β is not identified, and can conveniently be set equal to 1.

possibility of disasters in the generated series, viz

$$\begin{aligned}\mathbb{E}[(\beta cg_t^{-\gamma}-\mu_m)R_t^e] &\approx \frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\left(\beta cg_s^{-\gamma}-\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}\right)R_s^e \\ \mu_m = \mathbb{E}[\beta cg_t^{-\gamma}] &\approx \frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}.\end{aligned} \tag{2.16}$$

Recognizing that the risk-free rate and the mean of SDF are related by

$$\mathbb{E}\left[\frac{1}{R_t^f}\right]=\mu_m, \tag{2.17}$$

we can obtain estimates of β, γ, and μ_m using the moment matches

$$\mathbb{G}_T(\beta,\gamma,\mu_m)=\frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix}\mathbf{R}_t^e+\dfrac{\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}(\beta cg_s^{-\gamma}-\mu_m)\mathbf{R}_s^e}{\mu_m}\\ \frac{1}{R_t^f}-\mu_m\\ \mu_m-\frac{1}{\mathcal{T}(T)}\sum_{s=1}^{\mathcal{T}(T)}\beta cg_s^{-\gamma}\end{bmatrix}, \tag{2.18}$$

in the SMM objective function in Equation (2.13). Julliard and Parker (2005) point out that last moment match must be exact. We refer to estimates obtained in this fashion as JPM-SMM (Julliard-Parker Moments) estimates.

2.3 Simulating disaster-including consumption and return data

2.3.1 Disastrous contractions and the Double Power Law distribution

To apply the two SMM estimation strategies, we must simulate disaster-including consumption growth $\{cg_s\}_{s=1}^{\mathcal{T}(T)}$, the risk-free rate $\{R_s^f\}_{s=1}^{\mathcal{T}(T)}$, and the excess return series $\{\mathbf{R}_s^e\}_{s=1}^{\mathcal{T}(T)}$. Our starting point is Barro's (2006) disaster-including consumption process in Equation (2.2), rewritten as

$$cg_s = cg_{nd,s}(1 - b_s)^{d_s}, \tag{2.19}$$

which suggests separating the simulated consumption growth cg_s into a regular component $cg_{nd,s}$ and the shrinkage factor $(1 - b_s)$, which contracts $cg_{nd,s}$ (only) if a disaster occurs ($d_s = 1$). Our empirical strategy thus consists of a two-way bootstrap, where $cg_{nd,s}$ is drawn with replacement from regular consumption growth data, and b_s is drawn from a Double Power Law distribution (DPL) that is fitted to a sample of macroeconomic disasters. For that purpose, we adopt a procedure proposed by Barro (2006), and identify calamitous GDP contractions using cross-country panel data.[5] Defining a disaster as a contraction that exceeds the pre-specified threshold q, yields a sample of disaster observations, from which we could bootstrap b_s. However, the number of disasters will be small for reasonable values of q, which limits the range of possible contractions.[6] We therefore follow Barro and Jin (2011) and apply the maximum likelihood method to fit a DPL to the transformed contractions,[7] $z_c = \frac{1}{1-b}$. The support of the DPL is thus $\left[\frac{1}{1-q}; \infty\right)$.

As in Barro (2006) we estimate the annualized disaster probability p_a by dividing the number of identified disasters by the number of country-years in the data. The quarterly disaster probability is then estimated by

$$\hat{p}_q = 1 - (1 - \hat{p}_a)^{1/4}. \tag{2.20}$$

[5] We rely on data collected by Bolt and van Zanden (2013). Details on the data and disaster identification procedure are provided in Section 3.
[6] Barro (2006) uses $q = 14.5\%$ and Barro and Jin (2011) also consider $q = 9.5\%$ and $q = 19.5\%$.
[7] We collect useful information about the mean, density and quantile functions of the DPL distribution in Appendix A.

2.3.2 Copula-based approaches to simulate disaster-including data

The available data that can be used to identify disastrous contractions do not contain information about associated financial returns. To simulate such returns, we transfer the notion of a disaster-including consumption growth process in Equation (2.19) to a gross return of some asset, viz

$$R_s = (1 - \tilde{b}_s)^{d_s} R_{nd,s}, \tag{2.21}$$

and bootstrap $R_{nd,s}$ from the regular data. We consider three options to obtain the contraction factor $\tilde{b}_s$, which allow for different degrees of dependence between the consumption growth and return contractions. In all cases, we draw transformed contraction factors $z_{c,s}$ and $z_{R,s}$ from their joint distribution and then translate them via

$$b_s = 1 - \frac{1}{z_{c,s}} \qquad \text{and} \qquad \tilde{b}_s = 1 - \frac{1}{z_{R,s}}. \tag{2.22}$$

We assume that the marginal distributions of z_c and z_R can be described by the DPL distribution fitted to the GDP contractions and use a copula function $C(\cdot, \cdot)$ to model the dependence between z_c and z_R. The joint cumulative distribution function (c.d.f.) is then given by

$$F(z_c, z_R) = C\left(F_{DP}(z_c), F_{DP}(z_R)\right), \tag{2.23}$$

where $F_{DP}(\cdot)$ refers to the c.d.f. of the DPL. Using the Gaussian copula, Equation (2.23) becomes:

$$F(z_c, z_R) = C_G\left(u_c, u_R; \rho\right), \tag{2.24}$$

where $u_c = F_{DP}(z_c)$ and $u_R = F_{DP}(z_R)$, and where ρ denotes the copula correlation, which determines the dependence of z_c and z_R. To simulate consumption and return contractions, we first draw two independent standard normal variates $\omega_{c,s}$ and $\omega_{R,s}$ and use them to generate two standard normally distributed variables $y_{c,s}$ and $y_{R,s}$, which have correlation ρ. This can be achieved by setting $y_{c,s} = \omega_{c,s}$ and $y_{R,s} = \omega_{c,s} \cdot \rho + \omega_{R,s}\sqrt{1-\rho^2}$, such that we obtain $u_{c,s} = \Phi(y_{c,s})$ and $u_{R,s} = \Phi\left(y_{R,s}\right)$, where $\Phi(\cdot)$ denotes the c.d.f. of the standard normal. The simulated contraction factors b_s and $\tilde{b}_s$ then result from

$$b_s = 1 - \frac{1}{F_{DP}^{-1}(u_{c,s})} \qquad \text{and} \qquad \tilde{b}_s = 1 - \frac{1}{F_{DP}^{-1}\left(u_{R,s}\right)}. \tag{2.25}$$

We focus on three prominent choices for the copula correlation ρ. The first approach is to estimate ρ by the empirical correlation of regular consumption growth and return of asset i. We refer to this return generating procedure as *EmpCorr* (*Emp*irical *Corr*elation). In a second specification, we set $\rho = 0.99$ for all test assets. This approach is motivated by empirical evidence that indicates that the correlations between financial returns increase in the tails of the joint distribution (Longin and Solnik, 2001). We refer to it as *TailCorr* (*Tail Corr*elation). The third option is to use $\rho = 0$ for all test assets, which amounts to drawing b_s and $\tilde{\mathbf{b}}_s$ independently, but from the same DPL distribution. This is our *ZeroCorr* (*Zero Corr*elation) approach. We also perform a sensitivity analysis, in which we vary ρ between 0 and 0.99.

2.3.3 An alternative approach to simulate disaster-including data

A fourth strategy to simulate disaster-including data is based on the assumption that the log-consumption growth c_d and the log-return r_d in the disaster state can be described by a bivariate Gaussian distribution:

$$\begin{bmatrix} c_d \\ r_d \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_{c,d} \\ \mu_{r,d} \end{bmatrix}, \begin{bmatrix} \sigma_{c,d}^2 & \sigma_{cr,d} \\ \sigma_{cr,d} & \sigma_{r,d}^2 \end{bmatrix}\right), \tag{2.26}$$

which implies that

$$\begin{aligned} \mathbb{E}\left[r_d|c_d\right] &= \mu_{r,d} + \psi_d \sigma_{c,d} \sigma_{r,d}\left[c_d - \mu_{c,d}\right] \\ \operatorname{Var}\left[r_d|c_d\right] &= (1-\psi_d^2)\sigma_{c,d}^2, \end{aligned} \tag{2.27}$$

where $\psi_d = \frac{\sigma_{cr,d}}{\sigma_{c,d}\,\sigma_{r,d}}$. If the parameters of the bivariate normal distribution in Equation (2.26) were known, given a simulated disaster log-consumption $c_{d,s} = \ln[(1-b_s)cg_{nd,s}]$, we could simulate a disaster log-return $r_{d,s}$ by drawing from a Gaussian distribution with mean and variance given in Equation (2.27).

To estimate the five distributional parameters $(\mu_{r,d},\ \mu_{c,d},\ \sigma_{c,d},\ \sigma_{r,d},\ \psi_d)$, we proceed as follows. First, $\mu_{c,d}$ and $\sigma_{c,d}$ are estimated by the sample mean and standard deviation of a very long simulated disaster-including consumption growth series, obtained as described previously. Second, to estimate ψ_d, we assume that the correlation of log-consumption growth and log-returns, conditional on $d = 1$, is the same as that conditional on $d = 0$ (ψ_{nd}):

$$\psi_d = \psi_{nd} = \frac{\sigma_{cr,nd}}{\sigma_{c,nd}\,\sigma_{r,nd}}. \tag{2.28}$$

Here, $\sigma_{cr,nd}$, $\sigma_{c,nd}$ and $\sigma_{r,nd}$ denote covariance and standard deviations conditional on $d = 0$, which can be estimated using the regular consumption and return data.

To provide estimates of $\mu_{r,d}$ and $\sigma_{r,d}$, we further assume that the expected value of a gross return in the disaster state equals the expected value of that gross return in the regular state, scaled by 1 minus the mean contraction size:

$$\mathbb{E}[R_d] = (1 - \mathbb{E}[b])\mathbb{E}[R_{nd}]. \tag{2.29}$$

By the properties of the log-normal distribution, we then have

$$\mu_{r,d} = \ln(1 - \mathbb{E}[b]) + \ln(\mathbb{E}[R_{nd}]) - \frac{\sigma^2_{r,d}}{2}. \tag{2.30}$$

To estimate the mean contraction size, we replace, in the analytical expression for $\mathbb{E}[b]$ (see Appendix A), the Double Power Law parameters with their maximum likelihood estimates.

The final parameter to account for is $\sigma^2_{r,d}$, which we do by assuming constant "Sharpe ratios",

$$\frac{\mathbb{E}[R_d]}{\sqrt{\mathrm{Var}(R_d)}} = \frac{\mathbb{E}[R_{nd}]}{\sqrt{\mathrm{Var}(R_{nd})}}, \tag{2.31}$$

and using the properties of the log-normal, which imply that

$$\sigma^2_{r,d} = \ln\left(1 + \frac{\mathrm{Var}(R_{nd})}{\mathbb{E}[R_{nd}]^2}\right). \tag{2.32}$$

Then $\mathrm{Var}(R_{nd})$ and $\mathbb{E}[R_{nd}]$ can be estimated by sample moments of the regular gross return data.

We can now replace all right-hand side parameters of Equation (2.27) by their estimates and simulate log-returns $r_{d,s}$, conditional on log-consumption growth in the disaster state $c_{d,s}$. We refer to this procedure as *G-Draw* (*G*aussian *Draw*s).

2.3.4 Alternative Histories Bootstrap

In summary, the simulation procedure to generate $\{cg_s\}_{s=1}^{\mathcal{T}(T)}$, $\left\{R_s^f\right\}_{s=1}^{\mathcal{T}(T)}$, and $\{\mathbf{R}_s^e\}_{s=1}^{\mathcal{T}(T)}$ works as follows:

- For every $s = 1, \dots, \mathcal{T}(T)$, decide by drawing from a Bernoulli distribution with probability $\hat{p}_q$ whether $d_s = 0$ or $d_s = 1$. Regardless of the outcome, draw $cg_{nd,s}$, $R^f_{nd,s}$, and $\mathbf{R}_{nd,s}$ with replacement from the regular consumption and return data. To maintain the covariance structure of consumption and returns, the draws must be performed simultaneously.

- If $d_s = 0$, set $cg_s = cg_{nd,s}$, $R^f_s = R^f_{nd,s}$, and $\mathbf{R}^e_s = \mathbf{R}_{nd,s} - R^f_{nd,s}$.

- If $d_s = 1$, provide disaster-including consumption growth $cg_{d,s}$ and returns $\mathbf{R}_{d,s}$ according to the four variants of simulating disaster-including data using either the copula assumption (*ZeroCorr*, *EmpCorr*, and *TailCorr*) or the alternative approach (*G-Draw*). All variants use $cg_s = (1 - b_s)cg_{nd,s}$. Finally, $\mathbf{R}^e_s = \mathbf{R}_{d,s} - R^f_{nd,s}$ and $R^f_s = R^f_{nd,s}$.

This two-way bootstrap suggests an alternative approach to estimating the parameters of a disaster-including CBM, which complements the two SMM procedures described in Section 2.2. We have argued that the moment matches in Equations (2.9) and (2.10) should not be used if disasters are possible but not observed in the data. As the simulated consumption growth and return series include disaster observations, these moment matches can be reconsidered and used for GMM estimation. We refer to this approach as *A*lternative *H*istories *B*ootstrap (AHB), a term that echoes Cochrane's remark from the introduction.

The input for the AHB procedure are H independent disaster-including simulated samples ("alternative histories") of size $\mathcal{T}(T)$, which we generate as just described. Let $\left\{cg_s^{(h)}\right\}$, $\left\{R_s^{f(h)}\right\}$, and $\left\{R_s^{e(h)}\right\}$ denote the simulated data from replication h. For each $h = 1, \ldots, H$, we estimate β and γ by GMM, using the moment matches

$$\mathbb{G}_T^{(h)}(\beta, \gamma) = \begin{bmatrix} \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta \left(cg_s^{(h)} \right)^{-\gamma} R_s^{f(h)} - 1 \\ \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta \left(cg_s^{(h)} \right)^{-\gamma} \mathbf{R}_s^{e(h)} \end{bmatrix}. \tag{2.33}$$

AHB estimates of β and γ can be obtained by averaging the resulting estimates across ensembles:

$$\hat{\beta} = \frac{1}{H} \sum_{h=1}^{H} \hat{\beta}^{(h)} \qquad \text{and} \qquad \hat{\gamma} = \frac{1}{H} \sum_{h=1}^{H} \hat{\gamma}^{(h)}, \tag{2.34}$$

where $\hat{\gamma}^{(h)}$ and $\hat{\beta}^{(h)}$ refer to the estimates obtained in the h^{th} replication. We thus obtain four sets of AHB estimates, using the alternative ways to generate disaster-including data described in Section 2.3. We use the empirical distribution of $\hat{\gamma}^{(h)}$ and $\hat{\beta}^{(h)}$ to provide standard errors, quantiles, and kernel density estimates. By varying $\mathcal{T}(T)$, we can quantify the considerations in Section 2.1 regarding the size of the disaster-including samples and the implications for estimation precision.

3 Data

Our procedure to obtain an empirical distribution of disaster sizes and to estimate the disaster probability is based on the cross-country GDP panel data set assembled by Bolt and van Zanden (2013). They extend the data collected by Angus Maddison, which was used by Barro (2006) and Barro and Jin (2011), and provide annual GDP information about 35 countries between 1900 and 2010.[8] For our main analysis, we follow Barro (2006) and set the disaster threshold q to 14.5%, but we also consider $q = 9.5\%$ and $q = 19.5\%$ as in Barro and Jin (2011).

[8] The data are available at:
http://www.ggdc.net/maddison/oriindex.net accessed 06/26/2014.

Barro's (2006) disaster identification procedure provides the blueprint for our study. Specifically, it does not matter whether a GDP contraction larger than q accrued over one period or more, and the length of the contraction is measured up to the year before the rebound of the economy. Contractions in GDP that represent the aftermaths of war, which allegedly are not related to a drop in consumption, are neglected.[9] Barro (2006) is not specific about how he deals with short periods of positive growth amidst a disaster. We ignore such one-period intermezzos if the positive growth does not offset the negative growth in the next period. Moreover, the size of the identified contraction must not decrease when ignoring the intermediate positive growth period.

[insert Figure 1 about here]

Figure 1 depicts the resulting disaster data. One can see that disastrous contractions in GDP are clustered during WWI, the Great Depression, WWII, and turmoils in South America during 1980 – 2000.

To bootstrap from regular consumption growth data, we use quarterly real personal consumption expenditures per capita on services and nondurable goods in chained 2009 Dollars provided by the Federal Reserve Bank of Saint Louis.[10] The sample spans the time period 1947:Q1 – 2013:Q3.

Financial data on a monthly frequency come from Kenneth French's financial data library.[11] We use as test assets the ten size-sorted portfolios (*size dec*), the ten industry portfolios (*industry*), and the market portfolio (*mkt*) comprised of NYSE,

[9] The excluded contractions are: Canada (1917-1921) -30%, Italy (1918-1921) -25%, U.K. (1918-1921) -19%, U.K. (1943-1947) -15%, and U.S.A. (1944-1947) -28%.

[10] For services: http://research.stlouisfed.org/fred2/series/A797RX0Q048SBEA, and for nondurable goods: http://research.stlouisfed.org/fred2/series/A796RX0Q048SBEA, accessed 04/30/2014.

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html, accessed 04/30/2014. Due to frequent changes in the underlying CRSP data, newer or older downloads may results in different series.

AMEX, and NASDAQ traded stocks, as well as the Treasury bill. All portfolios are value-weighted. Nominal monthly returns are converted to real returns on a quarterly frequency, using the growth of the consumer price index of all urban consumers.[12] The quarterly real T-bill return represents the risk-free rate proxy. Excess returns for the portfolios result from subtracting the risk-free rate proxy from the respective portfolio returns.

We also use the quarterly U.S. postwar data base collected by Cochrane (1996), which includes consumption growth, gross returns of ten size-sorted portfolios and a risk-free rate proxy, spanning the time period 1947:Q2 – 1993:Q4. These data are particularly convenient, in that various CBM-type asset pricing models have been estimated on them. The previously reported results provide useful reference points for our study.[13]

[insert Table 1 about here]

Table 1 reports descriptive statistics of the data.

4 Empirical results

All variants to estimate a disaster-including CBM rely on simulated disaster sizes drawn from a Double Power Law distribution that is fitted to the sample of identified disasters shown in Figure 1. Maximum likelihood estimates of the distributional parameters and their standard errors are reported in the caption of Figure 2, which depicts the empirical distribution function of the disaster sizes and the fitted DPL c.d.f. The point κ indicates the switch from one Power Law density to another.

[12] These data are provided by the Federal Reserve Bank of Saint Louis http://research.stlouisfed.org/fred2/series/CPIAUCSL, accessed 04/30/2014.

[13] The data are available on John Cochrane's web-site: http://faculty.chicagobooth.edu/john.cochrane/research/Data_and_Programs/JPE_cross%20 sectional_test_of_investment_based/Data/ accessed 04/10/2013.

As Figure 2 shows, the DPL fits the empirical distribution function well for small contraction sizes but also leaves more room for severe downturns.

[insert Figure 2 about here]

The two SMM variants (MAD and JPM) use $\mathcal{T}(T) = 10,000,000$. For AHB, we vary $\mathcal{T}(T)$ and perform $H = 400$ replications. The longest simulated AHB history is $\mathcal{T}(T) = 16,000$, and it should help compare SMM and AHB results. The varying AHB time series lengths provide a means to study the effect of an increasing sample size on parameter estimates and their precision. The GMM and SMM objective functions use the identity matrix for $\mathbf{W}_T$; for JPM-SMM, we make sure that the moment condition that invokes the mean of the SDF is exactly matched, as requested by Julliard and Parker (2005).

Table 2, using excess returns of the portfolios in the sets of test assets *mkt*, *size dec*, *industry*, or *Cochrane*, each augmented by the risk-free rate, and Table 3, using excess returns only, collect the estimation results. The respective panels break down the results by the four different sets of test assets, procedure used to simulate disaster-including data (*G-Draw*, *TailCorr*, *EmpCorr*, or *ZeroCorr*), and type of moment match (MAD-SMM, JPM-SMM, or AHB). All in all, we consider $4 \times 4 \times 3$ cases, for which we report the preference parameter estimates and asymptotic standard errors (for SMM) or standard deviations across replications (for AHB), the p-values of the J-statistics (percentage), and the root mean squared pricing errors ($\times 10^4$), computed as

$$RMSE = \sqrt{\frac{1}{N}\mathbb{G}_T(\hat{\beta},\hat{\gamma})'\mathbb{G}_T(\hat{\beta},\hat{\gamma})}, \tag{4.1}$$

where N denotes the number of rows of $\mathbb{G}_T(\hat{\beta},\hat{\gamma})$. Figure 3 illustrates the estimation results using kernel density estimates, focusing on the $H = 400$ AHB ensembles.

[insert Tables 2 and 3 about here]

[insert Figure 3 about here]

All variants to estimate a disaster-including CBM yield economically plausible estimates for the preference parameters.[14] For example, when using the excess return of the market portfolio and the risk-free rate as test assets, we obtain estimates of the coefficient of relative risk aversion that range between 3.459 (JPM-SMM/*TailCorr*) and 6.961 (MAD-SMM/*G-Draw*). Estimates of the subjective discount factor range between 0.930 (JPM-SMM/*G-Draw*) and 0.994 (MAD-SMM/*TailCorr*). Only the MAD-SMM/*TailCorr* combination applied to the *Cochrane* data yields a β estimate slightly (yet not significantly) above 1. In 47 of the 48 cases we obtain estimates of the subjective discount factor between $\hat{\beta} = 0.925$ (JPM-SMM/*G-Draw*/size deciles) and $hat\beta = 0.994$ (MAD-SMM/*TailCorr*/market portfolio). Both asymptotic inference (for SMM) and the bootstrap inference (for AHB) yield small parameter standard errors and narrow confidence bounds for the preference parameters.[15] The kernel density estimates in Figure 3 illustrate these findings using the AHB estimates.

[insert Figure 4 about here]

Figure 4 is a graphical representation of the estimation results presented in Table 2. The general picture is that neither simulation procedures nor moment matches or choice of test assets yield qualitatively different results. We note some interesting variation across the estimates, though. MAD-SMM delivers the highest, and JPM-SMM the smallest γ estimates, with AHB in between. Relative risk aversion estimates across the 4×4 combinations of data simulation procedures and test assets range between 3.459 and 4.345 for JPM-SMM, 4.149 and 5.059 for AHB, and 5.853 and

[14] For the risk aversion parameter γ, values from 1 to 10 are considered reasonable (more rigorous limits cap the interval at 5); the subjective discount factor β should be less than 1.

[15] We note the caveat, though, that the reported standard errors do not take estimation uncertainty regarding the disaster probability or parameters of the DPL distribution into account.

7.023 for MAD-SMM. The *TailCorr* simulation implies somewhat smaller relative risk aversion estimates than the other simulation procedures, such that the range is spanned by $\hat{\gamma} = 3.459$ (JPM-SMM/*TailCorr*/market portfolio) and $\hat{\gamma} = 7.023$ (MAD-SMM/*G-Draw*/size deciles). Holding test assets and moment matches constant, the relative risk aversion estimates do not change much with respect to the procedure used to simulate disaster data. For example, comparing the JPM-SMM estimates obtained using the market portfolio and the risk-free rate as test assets, we obtain $\hat{\gamma} = 3.459$ when applying the *TailCorr* simulation procedure, $\hat{\gamma} = 3.870$ when the *EmpCorr* procedure is employed, $\hat{\gamma} = 4.074$ based on the *ZeroCorr* procedure, and $\hat{\gamma} = 4.262$ with *G-Draw*.

Figure 4b graphically illustrates that the size of the γ estimates depend more on the type of moment match than on the data simulation procedure. Note how the solid symbols labeling MAD-SMM cases are located consistently above the small and large blank symbols, which represent AHB and JPM-SMM estimates, respectively. The variation of $\hat{\gamma}$ across type of symbol (representing the simulation procedure) is much smaller, an observation that is also illustrated by the right-hand side panels of Figure 3. They show that the kernel densities for the AHB estimates of γ are very similar across simulation procedures and test assets. In contrast, Figure 4a shows that regarding $\hat{\beta}$, the choice of the data simulation procedure is more important than the moment matches. Diamond symbols (*TailCorr*) tend to be found at the top, triangles (*InDraw*) and squares (*G-Draw*) at the bottom of Figure 4a . The effect of the simulation procedure on AHB estimates of β is also depicted in the left-hand side panels of Figure 3. They show that the kernel densities for the AHB estimates of β are clearly more different across simulation procedures than across test assets.

JPM-SMM tends to provide higher p-values of the J-statistic than MAD-SMM; only when using *EmpCorr* on the industry portfolios can the JPM moment conditions be rejected at the 5% level. Yet, also the MAD conditions are not rejected at the 5% level for most combinations of simulation procedure and test assets. JPM-SMM delivers the smallest RMSEs, but they cannot be compared directly to the MAD-SMM and AHB RMSEs, because the moment matches are in a different dimension. What can be compared in terms of RMSE, however, are the simulation procedures within each set of moment matches. Here we find that the *G-Draw* and *TailCorr* procedures yield the smallest RMSE.

[insert Figure 5 about here]

Figure 5 shows the effects of varying the copula correlation on the parameter estimates of β and γ. For all data simulation procedures $\hat{\gamma}$ decreases and $\hat{\beta}$ gets larger with increasing copula correlation, while all estimates remain of an economically plausible size and exhibit small confidence bounds. Figure 5 shows the results using the excess return of the market portfolio and the risk-free rate as test assets, but it is representative for the other test assets, too.

[insert Figure 10 about here]

In order to ensure that our results do not depend strongly on the pre-selected disaster threshold $q = 14.5\%$, we perform additional robustness checks using $q = 9.5\%$ and $q = 19.5\%$. These values are in accordance with Barro and Jin's (2011) choices and Figure 10 provides an illustration of the resulting parameter estimates. We find that although a lower disaster threshold somewhat increases the variation in the estimates of the subjective discount factor, the estimates of the coefficient of relative risk aversion are barely affected. Indeed, for all choices of q, it is only the MAD-SMM/*TailCorr* combination using the *Cochrane* data that yields a $\hat{\beta}$ slightly above 1. The choice of the disaster threshold is not crucial for the plausibility of the parameter estimates.

We have argued in Section 2 that the quality of the standard CBM moment matches is affected when using short time series that contain too few if any disaster observations to be representative of the possible paths of history that investors imagined. Using the AHB approach, we can assess what sample size would be needed to achieve a reasonable estimation precision. We can also study the distribution of the estimates when the simulated sample size is as small as in the empirical data, but some simulated histories do include disaster observations. A comparison with the empirical results using disaster-free data serves as a plausibility check for our methodology.

In addition to $\mathcal{T}(T) = 16,000$, we therefore also perform AHB estimations with simulated histories of lengths $\mathcal{T}(T) = 187$, 267, 1 000, and 5 000. These choices are motivated as follows. The lengths of the shortest simulated series are equal to the lengths of the original data set. In particular, we use $\mathcal{T}(T) = 267$ for the sets of test assets for which we have observations ranging until 2013:Q3 and $\mathcal{T}(T) = 187$ for the *Cochrane* data, so the ensembles have the same length, but potentially include disasters. For $\mathcal{T}(T) = 1\,000$, the simulated data span roughly three successive investor generations, assuming a life-span of 80 years. For $\mathcal{T}(T) = 5\,000$, they would overlap approximately fifteen generations.

[insert Tables 4 and 5 about here]

[insert Figures 6 - 9 about here]

For each $\mathcal{T}(T)$, we perform separate AHB estimations using the *EmpCorr*, *G-Draw*, *ZeroCorr*, and *TailCorr* simulation procedures. The results are reported in Table 4 (test assets include the risk-free rate) and Table 5 (using only excess returns as test assets). Figures 6 - 9 illustrate the findings using kernel densities and the excess return of the market portfolio and the risk-free rate as test assets.

The AHB estimates using the small simulated sample sizes $\mathcal{T}(T) = 187$ and $\mathcal{T}(T) = 267$ exhibit properties that are well-known from empirical applications. Regardless of the simulation procedure, $\hat{\beta}$ is greater than 1, and $\hat{\gamma}$ is far beyond the upper plausibility limit. Furthermore, the estimates are imprecise, as indicated by the huge standard errors and the shape of the kernel density estimates (see the left panels of Figures 6 - 9).

Increasing the sample size to $\mathcal{T}(T) = 1\,000$, the point estimates take on more plausible values. Estimation precision improves, but is still low as indicated by the standard errors and the kernel density estimates. At $\mathcal{T}(T) = 5\,000$, the estimation results are satisfactory, in the sense that estimation precision is good and the β and γ point estimates are economically plausible. This simulation exercise shows that the apparent failure of the CBM comes as no surprise, and is not at odds with the rare disaster hypothesis. If the rare disaster hypothesis is true, and using conventional estimation techniques, we would have to wait for a long time – with unpleasant intermezzos of consumption contractions – before we can expect sufficient estimation precision. Our simulation-based methods thus provide a shortcut.

5 Discussion and conclusion

Financial economics and econometrics alike use Hansen and Singleton's (1982) CBM with additive power utility SDF as reference point and springboard for theoretical extensions and methodological developments. When applying the canonical CBM to empirical data however, its performance has been notoriously disappointing. The estimates of the CBM preference parameters tend to be implausible and imprecise. However, the CBM framework is not easily discarded, because it represents the rational link between the real economy and financial markets. Accordingly, attempts to vindicate the CBM have been manifold. Scaled factors have been employed to account for time-varying risk aversion, alternative measures for the errors-in-variables-prone macroeconomic consumption data have been proposed, investor heterogeneity has been accounted for, and more flexible specifications of intertemporal utility functions have been tested. Although these studies can claim some empirical success, the problem of imprecise and implausible preference parameter estimates has been, at best, only mitigated.

Our study probes an alternative explanation to vindicate the CBM: the rare disaster hypothesis, associated with seminal work by Rietz (1988) and Barro (2006). We retain the stochastic discount factor of Hansen and Singleton's (1982) CBM, but we account for the suspicion that the U.S. data, which have been used extensively to test the CBM, may not be representative. Consumers and investors born and living after WWII in the United States and other Western countries, which have collected consumption and financial data for more than 60 years, have experienced unprecedented periods of peace, prosperity, and progress. A lucky path of history spared them from calamitous contractions of GDP and aggregate consumption. Those investors and data tell the story of survivors, which is always a pleasant, but often a misleading narrative. Statistics and econometrics classes center around sample selection problems and the danger of interpreting self-selected data. In empirical macro-finance, we sometimes ignore these caveats.

Adopting Barro's (2006) specification of a disaster-including consumption process, we propose two alternative moment matches that we use to estimate the CBM preference parameters by SMM. To simulate disaster-including consumption growth and return processes, we perform a non-parametric bootstrap from regular U.S. postwar data, combined with a parametric bootstrap from a Double Power Law distribution that is fitted to calamitous contractions data. An alternative estimation strategy entails repeating the bootstrap simulation and applying GMM to the simulated alternative histories. Here, point estimates of the preference parameters result from averaging over the replications. The alternative methods we adopted to estimate a disaster-including CBM rely on four specifications to simulate disaster-including financial returns and on five different portfolio choices. Moreover, we perform the estimation with and without the risk-free rate included in the test assets and account for different disaster thresholds.

Whichever approach and data are used, the results remain qualitatively the same: The estimated preference parameters are economically plausible in size, and the estimation precision is much higher than in previous studies that have used the canonical CBM. In particular, the estimates of the relative risk aversion parameter are smaller than 5 in most specifications and always much smaller than 10, so the estimates are in a range considered consistent with reasonably risk-averse investors. The parameter standard errors are small, the confidence bounds narrow. A comparable combination of plausibility and estimation precision has not been provided previously in related literature.

We also show that the size and precision of the parameter estimates reported in previous studies are realistic under the rare disaster hypothesis. Decades would have to pass before standard econometric techniques could yield precise estimation results with empirical data. The simulation-based estimation approaches that we apply in our study provide a shortcut to empirically assessing the effect of consumption disasters on asset prices. They come at the cost of assumptions, which may be questioned but can be modified, and one can study the sensitivity of the estimation results. In our study, the variation of assumptions did not change the results qualitatively.

Our findings should encourage those who believe that rational investor behavior prevails in financial markets. Yes, the CBM can explain the equity premium at reasonable levels of risk aversion, once the latent risk of rare disasters is accounted for. The nexus between finance and the real economy postulated by the CBM is, after all, empirically not refuted.

A Power Law distribution: useful results

Following Barro and Jin (2011), we use a Double Power Law distribution to model the distribution of disastrous contraction sizes b. For that purpose, we use the transformation of disaster sizes into the random variable $z = \frac{1}{1-b}$, for which we assume the Double Power Law density

$$f_Z(z) = \begin{cases} 0 & \text{if } z < z_0 \\ Bz^{-(\theta+1)} & \text{if } z_0 \le z < \delta \\ Az^{-(\alpha+1)} & \text{if } \delta \le z \end{cases}, \tag{A.1}$$

where $B = A\delta^{(\theta-\alpha)}$ and $A = \left[\frac{\delta^{(\theta-\alpha)}}{\theta-1}\left(z_0^{(1-\theta)} - \delta^{(1-\theta)}\right) + \frac{\delta^{(1-\alpha)}}{\alpha-1}\right]^{-1}$. In turn, z_0 is defined as $z_0 = \frac{1}{1-q}$, where q denotes the disaster threshold.

A draw from the Double Power Law density can be performed by drawing a standard uniform random variable ν and inserting it in the quantile function, which is given by

$$z_{[\nu]} = \begin{cases} \sqrt[-\theta]{z_0^{-\theta} - \frac{\theta}{B}\nu} & \text{if} \quad \nu \le F_{\delta \le z}(\delta) \\ \sqrt[-\alpha]{\delta^{-\alpha} - \frac{\alpha}{A}\left(\nu - \frac{B}{\theta}\left(z_0^{-\theta} - \delta^{-\theta}\right)\right)} & \text{if} \quad \nu > F_{\delta \le z}(\delta). \end{cases} \tag{A.2}$$

The realizations of the random variables z drawn using the quantile function in Equation (A.2) must be retransformed into contraction sizes by $b = 1 - \frac{1}{z}$.

Using the density for $z = \frac{1}{1-b}$, the expected value of the contraction size b (which we need for the *G-Draw* return simulation) is given by:

$$\mathbb{E}[b] = \mathbb{E}\left[1 - \frac{1}{z}\right] = 1 + A\delta^{-(\alpha+1)}\left(\frac{1}{\theta+1} - \frac{1}{\alpha+1}\right) - \frac{A}{(\theta+1)}\delta^{(\theta-\alpha)}z_0^{-(\theta+1)}. \tag{A.3}$$

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Tables and Figures

Table 1: **Descriptive statistics of regular consumption growth and (size-sorted) return data used for bootstrapping**

This table contains the descriptive statistics of consumption growth and gross returns of the four sets of test assets. For Panels A (size-sorted portfolios and risk-free rate, *size dec*), C (industry portfolios and risk-free rate, *industry*), and D (market portfolio and risk-free rate, *mkt*) the data range is 1947:Q1 – 2013:Q3. For Panel B (Cochrane's (1996) size deciles and risk-free rate, *Cochrane*), the range is 1947:Q2 – 1993:Q4. Consumption growth is denoted $\frac{C_{t+1}}{C_t}$, and R^f is the risk-free rate proxy. In Panels A and B 1^{st}, 2^{nd}, and so on refer to the respective return deciles for ten size-sorted portfolios. The column ρ gives information on the autocorrelation of the variables and std refers to the standard deviation. The remaining columns report the correlations between the variables.

								Panel A: size dec						
	mean	std	ρ	correlation										
$\frac{C_{t+1}}{C_t}$	1.002	0.002	0.307	$\frac{C_{t+1}}{C_t}$	R^f	10^{th}	9^{th}	8^{th}	7^{th}	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}
1^{st}	1.029	0.126	0.061	0.234	-0.022	0.711	0.818	0.857	0.883	0.895	0.910	0.931	0.949	0.963
2^{nd}	1.027	0.118	0.002	0.237	0.029	0.782	0.872	0.916	0.934	0.948	0.961	0.975	0.981	
3^{rd}	1.029	0.112	-0.021	0.224	0.027	0.820	0.908	0.943	0.957	0.969	0.975	0.985		
4^{th}	1.027	0.108	-0.015	0.232	0.026	0.831	0.914	0.948	0.962	0.976	0.983			
5^{th}	1.027	0.104	0.017	0.240	0.047	0.856	0.937	0.968	0.972	0.982				
6^{th}	1.026	0.097	0.022	0.231	0.029	0.868	0.946	0.971	0.978					
7^{th}	1.026	0.097	0.045	0.232	0.029	0.893	0.966	0.982						
8^{th}	1.024	0.093	0.026	0.226	0.057	0.907	0.976							
9^{th}	1.023	0.085	0.072	0.222	0.048	0.935								
10^{th}	1.019	0.077	0.123	0.253	0.117									
R^f	1.002	0.008	0.570	0.179										
								Panel B: Cochrane						
	mean	std	ρ	correlation										
$\frac{C_{t+1}}{C_t}$	1.004	0.006	0.206	$\frac{C_{t+1}}{C_t}$	R^f	10^{th}	9^{th}	8^{th}	7^{th}	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}
1^{st}	1.031	0.100	0.290	0.288	0.090	0.754	0.858	0.893	0.910	0.936	0.939	0.948	0.960	0.971
2^{nd}	1.027	0.090	0.293	0.288	0.131	0.797	0.895	0.928	0.946	0.965	0.968	0.975	0.982	
3^{rd}	1.026	0.087	0.307	0.285	0.132	0.812	0.905	0.940	0.955	0.969	0.976	0.981		
4^{th}	1.026	0.083	0.313	0.267	0.129	0.836	0.923	0.956	0.969	0.980	0.979			
5^{th}	1.024	0.080	0.308	0.259	0.158	0.848	0.936	0.966	0.973	0.979				
6^{th}	1.025	0.078	0.288	0.238	0.144	0.863	0.952	0.974	0.979					
7^{th}	1.024	0.074	0.332	0.242	0.165	0.896	0.973	0.985						
8^{th}	1.023	0.070	0.291	0.239	0.173	0.906	0.976							
9^{th}	1.022	0.067	0.302	0.201	0.193	0.930								
10^{th}	1.018	0.058	0.376	0.232	0.224									
R^f	1.002	0.008	0.700	0.130										

Table 1: **Descriptive statistics of regular consumption growth and return data used for bootstrapping (continued)**

				Panel C: industry										
	mean	std	ρ	correlation										
$\frac{C_{t+1}}{C_t}$	1.002	0.002	0.307	$\frac{C_{t+1}}{C_t}$	R^f	Other	Utils	Hlth	Shops	Telcm	HiTec	Enrgy	Manuf	Durbl
NoDur	1.023	0.082	0.053	0.171	0.213	0.837	0.680	0.800	0.872	0.656	0.642	0.445	0.827	0.681
Durbl	1.024	0.116	0.103	0.250	0.040	0.798	0.481	0.517	0.772	0.574	0.690	0.497	0.832	
Manuf	1.022	0.090	0.087	0.235	0.036	0.900	0.583	0.744	0.825	0.642	0.807	0.639		
Enrgy	1.026	0.088	0.045	0.120	-0.189	0.600	0.540	0.429	0.431	0.430	0.500			
HiTec	1.025	0.117	0.067	0.228	0.055	0.758	0.477	0.663	0.734	0.651				
Telcm	1.018	0.081	0.157	0.220	0.174	0.690	0.638	0.570	0.666					
Shops	1.023	0.096	0.040	0.208	0.177	0.836	0.563	0.705						
Hlth	1.025	0.092	0.062	0.187	0.198	0.726	0.548							
Utils	1.019	0.071	0.093	0.168	0.158	0.657								
Other	1.021	0.099	0.079	0.261	0.106									
R^f	1.002	0.008	0.570	0.179										
				Panel D: mkt										
	mean	std	ρ	correlation										
$\frac{C_{t+1}}{C_t}$	1.002	0.002	0.307	$\frac{C_{t+1}}{C_t}$	R^f									
market	1.021	0.082	0.086	0.249	0.084									
R^f	1.002	0.008	0.570	0.179										

Table 2: **Estimation results using excess returns and the risk-free rate**

This table presents the SMM and AHB estimates of the preference parameters β and γ. Asymptotic standard errors of the SMM estimates are in parentheses. The numbers in brackets are standard deviations of the AHB estimates across H = 400 replications. The table also reports the p-values (percentage) of Hansen's J-statistic, and the RMSE ($\times 10^4$). The RMSE is computed using the average pricing errors, RMSE= $\sqrt{\frac{1}{N}\mathbb{G}_T(\hat{\beta},\hat{\gamma})'\mathbb{G}_T(\hat{\beta},\hat{\gamma})}$, where N denotes the number of rows of $\mathbb{G}_T(\hat{\beta},\hat{\gamma})$. For the AHB method, the reported RMSE is obtained by averaging over the 400 replications. Panels A1-D4 break down the results by the choice of test assets, procedure used to simulate disaster-including data (*G-Draw*, *TailCorr*, *EmpCorr*, or *ZeroCorr*), and type of moment match (MAD-SMM, JPM-SMM, or AHB). The estimations use the excess returns of the portfolios in the sets of test assets *mkt*, *size dec*, *industry*, or *Cochrane*, which in each case are augmented by the risk-free rate.

	A1: *G-Draw*/mkt		**A2: *G-Draw*/size dec**				**A3: *G-Draw*/industry**				**A4: *G-Draw*/Cochrane**			
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE
MAD-SMM	0.950	6.961	0.941	7.023	43.5	18	0.948	6.973	64.5	24	0.960	6.994	50.6	22
	(0.016)	(0.115)	(0.017)	(0.110)			(0.014)	(0.099)			(0.016)	(0.108)		
JPM-SMM	0.934	4.262	0.925	4.345	57.3	16	0.932	4.277	69.7	25	0.938	4.311	62.0	21
	(0.019)	(0.186)	(0.020)	(0.174)			(0.016)	(0.158)			(0.018)	(0.170)		
AHB	0.930	4.932	0.929	5.059		22	0.936	4.940		27	0.944	5.000		26
	[0.067]	[1.029]	[0.012]	[0.953]			[0.017]	[0.936]			[0.010]	[0.931]		
	B1: *TailCorr*/mkt		**B2: *TailCorr*/size dec**				**B3: *TailCorr*/industry**				**B4: *TailCorr*/Cochrane**			
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE
MAD-SMM	0.994	6.050	0.989	5.903	34.9	25	0.993	5.853	52.4	24	1.004	5.876	36.5	28
	(0.005)	(0.109)	(0.006)	(0.094)			(0.005)	(0.085)			(0.005)	(0.092)		
JPM-SMM	0.976	3.459	0.971	3.584	33.9	26	0.974	3.505	60.0	24	0.980	3.549	41.5	28
	(0.006)	(0.170)	(0.007)	(0.147)			(0.006)	(0.136)			(0.006)	(0.144)		
AHB	0.974	4.149	0.967	4.320		26	0.972	4.176		24	0.978	4.258		28
	[0.003]	[0.903]	[0.004]	[0.912]			[0.003]	[0.877]			[0.002]	[0.894]		
	C1: *EmpCorr*/mkt		**C2 *EmpCorr*/size dec**				**C3: *EmpCorr*/industry**				**C4: *EmpCorr*/Cochrane**			
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE
MAD-SMM	0.967	6.413	0.966	6.154	6.7	48	0.972	6.102	0.0	58	0.985	6.105	3.0	51
	(0.012)	(0.109)	(0.011)	(0.092)			(0.009)	(0.086)			(0.010)	(0.091)		
JPM-SMM	0.955	3.870	0.945	3.948	48.1	26	0.951	3.880	4.4	33	0.958	3.895	26.7	28
	(0.013)	(0.170)	(0.014)	(0.140)			(0.011)	(0.132)			(0.012)	(0.138)		
AHB	0.950	4.743	0.944	4.821		53	0.951	4.690		47	0.958	4.745		50
	[0.017]	[0.979]	[0.007]	[0.968]			[0.006]	[0.929]			[0.006]	[0.944]		
	D1: *ZeroCorr*/mkt		**D2: *ZeroCorr*/size dec**				**D3: *ZeroCorr*/industry**				**D4: *ZeroCorr*/Cochrane**			
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE	$\hat{\beta}$	$\hat{\gamma}$	J	RMSE
MAD-SMM	0.935	6.628	0.948	6.282	4.6	60	0.958	6.213	0.0	62	0.967	6.241	2.4	58
	(0.020)	(0.109)	(0.016)	(0.092)			(0.012)	(0.086)			(0.014)	(0.090)		
JPM-SMM	0.936	4.074	0.927	4.105	49.4	29	0.937	4.020	8.3	30	0.941	4.062	29.1	32
	(0.019)	(0.175)	(0.019)	(0.141)			(0.015)	(0.131)			(0.017)	(0.138)		
AHB	0.937	4.953	0.931	5.016		54	0.941	4.861		47	0.946	4.956		52
	[0.023]	[0.981]	[0.011]	[0.982]			[0.008]	[0.939]			[0.010]	[0.954]		

Table 3: **Estimation results using excess returns only**

This table presents the SMM and AHB estimates of the relative risk aversion parameter γ (using $\beta = 1$). Asymptotic standard errors of the SMM estimates are in parentheses. The numbers in brackets are standard deviations of the AHB estimates across $H = 400$ replications. The table also reports the p-values (percentage) of Hansen's J-statistic, and the RMSE ($\times 10^4$). The RMSE is computed using the average pricing errors, RMSE= $\sqrt{\frac{1}{N}\mathbb{G}_T(\hat{\gamma})'\mathbb{G}_T(\hat{\gamma})}$, where N denotes the number of rows of $\mathbb{G}_T(\hat{\gamma})$. For the AHB method, the reported RMSE is obtained by averaging over the 400 replications. Panels A1-D4 break down the results by the choice of test assets (*mkt*, *size dec*, *industry*, or *Cochrane*), procedure used to simulate disaster-including data (*G-Draw*, *TailCorr*, *EmpCorr*, or *ZeroCorr*) and type of moment match (MAD-SMM, JPM-SMM, or AHB). The estimations use the excess returns of the portfolios in the sets of test assets *mkt*, *size dec*, *industry*, or *Cochrane*.

	A1: *G-Draw*/mkt	**A2: *G-Draw*/size dec**			**A3: *G-Draw*/industry**			**A4: *G-Draw*/Cochrane**		
	$\hat{\gamma}$	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE
MAD-SMM	6.961	7.023	47.1	20	6.973	65.2	26	6.993	53.9	25
	(0.115)	(0.110)			(0.099)			(0.108)		
JPM-SMM	4.262	4.345	57.3	17	4.277	69.7	26	4.311	62.0	22
	(0.186)	(0.174)			(0.158)			(0.170)		
AHB	4.920	5.058		25	4.937		31	4.999		28
	[1.021]	[0.952]			[0.935]			[0.931]		
	B1: *TailCorr*/mkt	**B2: *TailCorr*/size dec**			**B3: *TailCorr*/industry**			**B4: *TailCorr*/Cochrane**		
	$\hat{\gamma}$	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE
MAD-SMM	6.050	5.903	36.4	27	5.853	52.6	25	5.876	37.8	29
	(0.109)	(0.094)			(0.085)			(0.092)		
JPM-SMM	3.459	3.584	33.9	27	3.505	60.0	25	3.549	41.5	30
	(0.170)	(0.147)			(0.136)			(0.144)		
AHB	4.149	4.319		28	4.176		26	4.257		31
	[0.903]	[0.912]			[0.876]			[0.894]		
	C1: *EmpCorr*/mkt	**C2: *EmpCorr*/size dec**			**C3: *EmpCorr*/industry**			**C4: *EmpCorr*/Cochrane**		
	$\hat{\gamma}$	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE
MAD-SMM	6.413	6.153	9.7	53	6.100	0.1	62	6.104	4.5	54
	(0.109)	(0.092)			(0.087)			(0.091)		
JPM-SMM	3.870	3.948	48.1	27	3.880	4.4	34	3.895	26.7	29
	(0.170)	(0.140)			(0.132)			(0.138)		
AHB	4.743	4.817		59	4.686		52	4.742		54
	[0.979]	[0.968]			[0.928]			[0.944]		
	D1: *ZeroCorr*/mkt	**D2: *ZeroCorr*/size dec**			**D3: *ZeroCorr*/industry**			**D4: *ZeroCorr*/Cochrane**		
	$\hat{\gamma}$	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE	$\hat{\gamma}$	J	RMSE
MAD-SMM	6.628	6.280	8.2	67	6.211	0.1	68	6.239	4.3	63
	(0.109)	(0.092)			(0.086)			(0.090)		
JPM-SMM	4.074	4.105	49.4	30	4.020	8.3	32	4.062	29.1	34
	(0.175)	(0.141)			(0.131)			(0.138)		
AHB	4.953	5.009		61	4.855		53	4.950		58
	[0.981]	[0.982]			[0.939]			[0.954]		

Table 4: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates**
This table reports the AHB estimates of the subjective discount factor and the coefficient of relative risk aversion for a varying $\mathcal{T}(T)$. The 95% quantiles of the parameter estimates from the $H = 400$ simulated histories are underlined and standard deviations are reported in brackets. The RMSE ($\times 10^4$) is computed using the average pricing errors, RMSE$= \frac{1}{H}\sum_{h=1}^{H}\sqrt{\frac{1}{N}\mathbb{G}_T^{(h)}(\hat{\beta}^{(h)},\hat{\gamma}^{(h)})'\mathbb{G}_T^{(h)}(\hat{\beta}^{(h)},\hat{\gamma}^{(h)})}$, where N denotes the number of rows of $\mathbb{G}_T^{(h)}(\hat{\beta}^{(h)},\hat{\gamma}^{(h)})$. Panels A1-D4 break down the results by the choice of test assets, procedure used to simulate disaster-including data (*G-Draw*, *TailCorr*, *EmpCorr*, or *ZeroCorr*) and length of the simulated series. The estimations use the excess returns of the portfolios in the sets of test assets *mkt*, *size dec*, *industry*, or *Cochrane*, which in each case are augmented by the risk-free rate. The smallest $\mathcal{T}(T)$ is 267 for the *mkt*, *size dec*, and *industry* data sets and 187 when using *Cochrane*.

	A1: *G-Draw*/mkt		**A2: *G-Draw*/size dec**			**A3: *G-Draw*/industry**			**A4: *G-Draw*/Cochrane**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE
187/267	1.061	104.588	1.066	107.045	33	1.073	103.003	56	1.052	82.736	54
	[0.245]	[168.655]	[0.225]	[169.182]		[0.230]	[161.619]		[0.173]	[93.337]	
	1.531	455.404	1.531	476.968		1.533	432.972		1.364	248.723	
1 000	0.944	17.969	0.945	18.988	30	0.953	18.111	38	0.962	13.304	31
	[0.123]	[62.119]	[0.078]	[65.921]		[0.085]	[61.279]		[0.045]	[29.284]	
	0.996	21.839	0.971	22.590		0.981	21.753		1.005	22.661	
5 000	0.933	5.663	0.928	5.877	24	0.935	5.725	31	0.945	5.795	26
	[0.051]	[1.719]	[0.015]	[1.664]		[0.020]	[1.628]		[0.014]	[1.637]	
	0.967	8.681	0.949	8.583		0.959	8.377		0.965	8.401	
16, 000	0.930	4.932	0.929	5.059	22	0.936	4.940	27	0.944	5.000	26
	[0.067]	[1.029]	[0.012]	[0.953]		[0.017]	[0.936]		[0.010]	[0.931]	
	0.962	6.577	0.946	6.617		0.955	6.433		0.959	6.514	
	B1: *TailCorr*/mkt		**B2: *TailCorr*/size dec**			**B3: *TailCorr*/industry**			**B4: *TailCorr*/Cochrane**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE
187/267	1.072	110.664	1.062	117.380	39	1.077	113.361	58	1.062	86.785	50
	[0.233]	[182.623]	[0.244]	[182.793]		[0.233]	[174.403]		[0.172]	[93.912]	
	1.537	479.848	1.547	510.573		1.547	481.513		1.394	240.628	
1 000	0.962	10.399	0.957	14.135	31	0.966	13.638	34	0.973	11.601	31
	[0.067]	[27.449]	[0.072]	[43.837]		[0.069]	[42.785]		[0.028]	[26.149]	
	0.979	17.951	0.974	21.405		0.978	20.058		0.985	20.972	
5 000	0.972	4.668	0.962	5.203	27	0.968	5.013	26	0.976	5.125	29
	[0.005]	[1.557]	[0.007]	[1.581]		[0.006]	[1.522]		[0.004]	[1.565]	
	0.979	7.542	0.972	8.195		0.976	7.911		0.981	8.121	
16, 000	0.974	4.149	0.967	4.320	26	0.972	4.176	24	0.978	4.258	28
	[0.003]	[0.903]	[0.004]	[0.912]		[0.003]	[0.877]		[0.002]	[0.894]	
	0.979	5.723	0.974	6.003		0.977	5.804		0.982	5.936	

Table 4: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates (continued)**

	C1: *EmpCorr*/mkt		**C2: *EmpCorr*/size dec**			**C3: *EmpCorr*/industry**			**C4: *EmpCorr*/Cochrane**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE
187/267	1.067	110.477	1.080	116.381	75	1.090	112.395	82	1.069	86.201	75
	[0.239]	[182.711]	[0.229]	[183.334]		[0.224]	[174.936]		[0.167]	[94.356]	
	1.537	479.848	1.547	510.573		1.547	481.513		1.394	240.628	
1 000	0.948	10.487	0.955	13.972	75	0.963	13.513	68	0.971	11.452	69
	[0.064]	[27.050]	[0.070]	[43.763]		[0.068]	[42.715]		[0.030]	[26.021]	
	0.982	17.724	0.974	18.699		0.978	17.823		1.003	18.514	
5 000	0.950	5.250	0.944	5.667	60	0.951	5.487	53	0.960	5.573	55
	[0.022]	[1.558]	[0.009]	[1.540]		[0.007]	[1.475]		[0.007]	[1.520]	
	0.971	8.226	0.957	8.470		0.962	8.171		0.971	8.502	
16, 000	0.950	4.743	0.944	4.821	53	0.951	4.690	47	0.958	4.745	50
	[0.017]	[0.979]	[0.007]	[0.968]		[0.006]	[0.929]		[0.006]	[0.944]	
	0.968	6.453	0.956	6.482		0.961	6.293		0.968	6.363	
	D1: *ZeroCorr*/mkt		**D2: *ZeroCorr*/size dec**			**D3: *ZeroCorr*/industry**			**D4: *ZeroCorr*/Cochrane**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE	$\hat{\beta}$	$\hat{\gamma}$	RMSE
187/267	1.066	110.418	1.080	116.273	76	1.090	112.282	82	1.068	86.127	77
	[0.241]	[182.738]	[0.229]	[183.389]		[0.224]	[174.995]		[0.168]	[94.408]	
	1.537	479.848	1.547	510.573		1.547	481.513		1.394	240.628	
1 000	0.941	10.612	0.950	14.056	76	0.960	13.587	68	0.966	11.538	70
	[0.078]	[27.018]	[0.071]	[43.735]		[0.069]	[42.691]		[0.034]	[25.979]	
	0.983	17.731	0.973	18.272		0.979	17.455		1.007	18.194	
5 000	0.938	5.461	0.934	5.858	60	0.943	5.652	53	0.950	5.773	57
	[0.031]	[1.547]	[0.013]	[1.530]		[0.010]	[1.463]		[0.011]	[1.508]	
	0.966	8.359	0.951	8.608		0.957	8.265		0.967	8.677	
16, 000	0.937	4.953	0.931	5.016	54	0.941	4.861	47	0.946	4.956	52
	[0.023]	[0.981]	[0.011]	[0.982]		[0.008]	[0.939]		[0.010]	[0.954]	
	0.961	6.663	0.947	6.696		0.954	6.472		0.960	6.566	

Table 5: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates (only excess-returns)**
This table reports the AHB estimates of the coefficient of relative risk aversion for a varying $\mathcal{T}(T)$ and $\beta = 1$. The 95% quantiles of the parameter estimates from the $H = 400$ simulated histories are underlined and standard deviations are reported in brackets. The RMSE ($\times 10^4$) is computed using the average pricing errors, RMSE= $\frac{1}{H}\sum_{h=1}^{H}\sqrt{\frac{1}{N}\mathbb{G}_T^{(h)}(\hat{\gamma}^{(h)})'\mathbb{G}_T^{(h)}(\hat{\gamma}^{(h)})}$, where N denotes the number of rows of $\mathbb{G}_T^{(h)}(\hat{\gamma}^{(h)})$. Panels A1-D4 break down the results by the choice of test assets, procedure used to simulate disaster-including data (*G-Draw*, *TailCorr*, *EmpCorr*, or *ZeroCorr*) and length of the simulated series. The estimations use the excess returns of the portfolios in the sets of test assets *mkt*, *size dec*, *industry*, or *Cochrane*. The smallest $\mathcal{T}(T)$ is 267 for the *mkt*, *size*, and textitindustry data sets and 187 when using *Cochrane*.

	A1: *G-Draw*/mkt	**A2: *G-Draw*/size dec**		**A3: *G-Draw*/industry**		**A4: *G-Draw*/Cochrane**	
	$\hat{\gamma}$	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE
187/267	103.016	105.253	34	100.918	54	74.588	52
	[164.105]	[164.849]		[155.407]		[80.754]	
	455.404	466.337		421.633		214.472	
1 000	17.880	18.831	34	17.888	42	12.910	34
	[62.124]	[64.693]		[59.777]		[26.840]	
	21.306	22.326		21.746		22.456	
5 000	5.639	5.874	27	5.714	35	5.793	29
	[1.718]	[1.663]		[1.626]		[1.636]	
	8.681	8.581		8.351		8.400	
16,000	4.920	5.058	25	4.937	31	4.999	28
	[1.021]	[0.952]		[0.935]		[0.931]	
	6.530	6.616		6.431		6.513	
	B1: *TailCorr*/mkt	**B2: *TailCorr*/size dec**		**B3: *TailCorr*/industry**		**B4: *TailCorr*/Cochrane**	
	$\hat{\gamma}$	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE
187/267	109.392	113.668	41	109.515	57	79.950	48
	[177.576]	[172.577]		[163.416]		[83.725]	
	479.848	487.708		456.751		212.543	
1 000	10.260	14.118	34	13.643	37	11.148	33
	[27.178]	[43.830]		[42.857]		[22.818]	
	17.951	21.237		20.022		20.964	
5 000	4.668	5.202	29	5.012	28	5.124	31
	[1.557]	[1.581]		[1.521]		[1.564]	
	7.542	8.194		7.909		8.118	
16,000	4.149	4.319	28	4.176	26	4.257	31
	[0.903]	[0.912]		[0.876]		[0.894]	
	5.723	6.002		5.803		5.934	

Table 5: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates (only excess-returns, continued)**

	C1: *EmpCorr*/mkt	**C2: *EmpCorr*/size dec**		**C3: *EmpCorr*/industry**		**C4: *EmpCorr*/Cochrane**	
	$\hat{\gamma}$	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE
187/267	109.205	112.678	79	108.554	81	79.355	75
	[177.665]	[173.127]		[163.962]		[84.186]	
	479.848	487.708		456.751		212.543	
1 000	10.487	13.942	83	13.503	74	10.985	75
	[27.050]	[43.762]		[42.789]		[22.670]	
	17.724	18.671		17.735		18.481	
5 000	5.250	5.659	66	5.480	59	5.566	60
	[1.558]	[1.538]		[1.474]		[1.519]	
	8.226	8.461		8.162		8.491	
16,000	4.743	4.817	59	4.686	52	4.742	54
	[0.979]	[0.968]		[0.928]		[0.944]	
	6.453	6.475		6.288		6.359	
	D1: *ZeroCorr*/mkt	**D2: *ZeroCorr*/size dec**		**D3: *ZeroCorr*/industry**		**D4: *ZeroCorr*/Cochrane**	
	$\hat{\gamma}$	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE	$\hat{\gamma}$	RMSE
187/267	109.146	112.565	80	108.445	82	79.281	76
	[177.692]	[173.186]		[164.020]		[84.238]	
	479.848	487.708		456.751		212.543	
1 000	10.563	14.023	84	13.575	74	11.068	76
	[27.014]	[43.735]		[42.766]		[22.625]	
	17.498	18.245		17.436		18.164	
5 000	5.461	5.847	68	5.643	59	5.763	62
	[1.547]	[1.529]		[1.463]		[1.508]	
	8.359	8.603		8.256		8.667	
16,000	4.953	5.009	61	4.855	53	4.950	58
	[0.981]	[0.982]		[0.939]		[0.954]	
	6.663	6.688		6.468		6.561	

Figure 1: **Disastrous GDP contractions identified from Bolt and van Zanden's (2013) data (1900-2010)**

The figure illustrates the 67 cases for which the contraction in GDP exceeded $q = 14.5\%$. The analysis is based on per capita GDP data for 35 countries: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Finland, France, Germany, Greece, India, Indonesia (1942-1948), Italy, Japan, Malaysia (1900-1910 and 1943-1946), Mexico, the Netherlands, New Zealand, Norway, the Philippines (1900-1901 and 1941-1945), Peru, Portugal, South Korea (1900-1910 and 1941-1949), Spain, Sri Lanka, Sweden, Switzerland, Taiwan (1900 and 1941-1949), U.K., U.S.A., Uruguay, and Venezuela. Numbers in parentheses indicate missing data. Black lines refer to European countries, red ones to South America and Mexico, green indicates Western offshores (i.e. Australia, Canada, New Zealand, U.S.A.), and blue denotes Asian countries. The average contraction size is 27.27%. The standard deviation of contractions is 13.24%. The smallest disaster found in the data is 14.52% (India, 1916-1918), whereas the biggest one equals 66.14% (Greece, 1937-1945). Computed as proposed by Barro (2006), these data imply an estimated quarterly disaster probability of $\hat{p}_q = 0.44\%$.

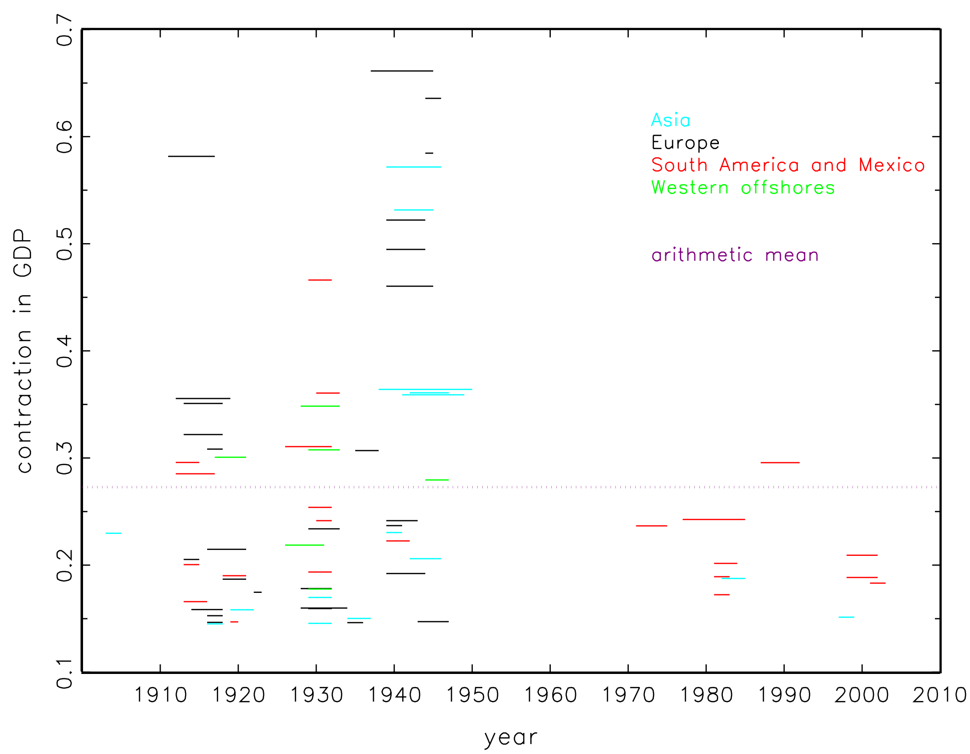


Figure 2: **Comparison of Double Power Law and empirical distribution function for disaster sizes**

The figure illustrates the empirical distribution function (solid blue line) and the fitted cumulative distribution function (dashed red line) of the disastrous contractions identified in Bolt and van Zanden's (2013) macroeconomic data using a disaster threshold of 14.5%. We estimate the parameters by means of maximum likelihood as $\hat{\alpha} = 3.956$ (0.427), $\hat{\theta} = 11.395$ (1.620), and $\hat{\delta} = 1.365$ (0.025). Standard errors are reported in parentheses. κ denotes the threshold at which one Power Law density transfers into the other, linked to $\hat{\delta}$ by means of $\kappa = 1 - \frac{1}{\hat{\delta}}$=0.267.

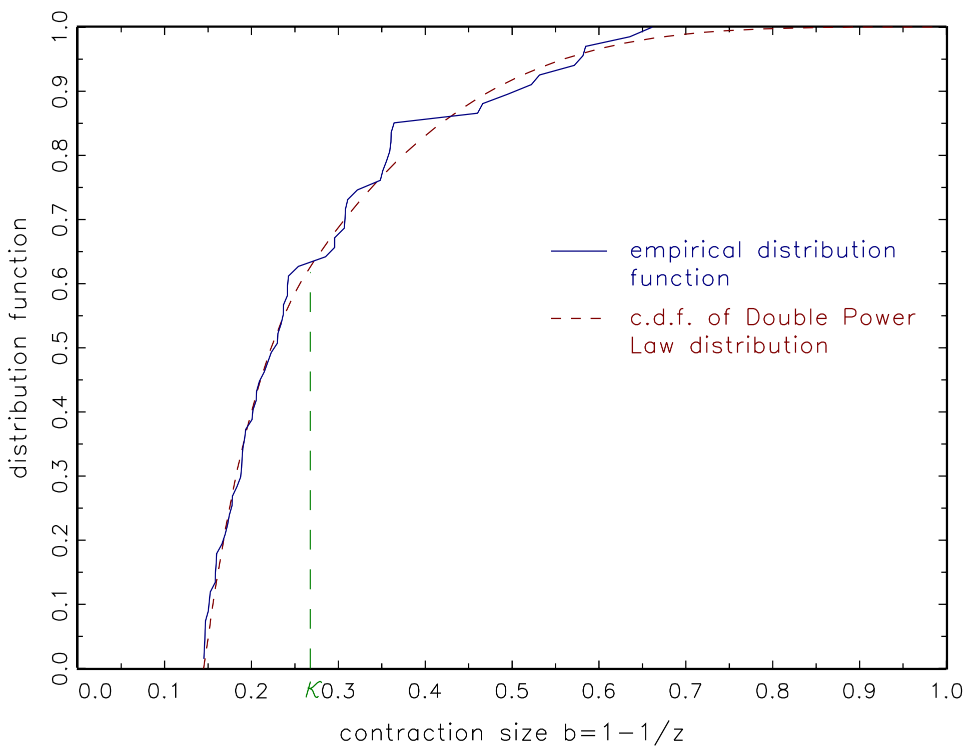
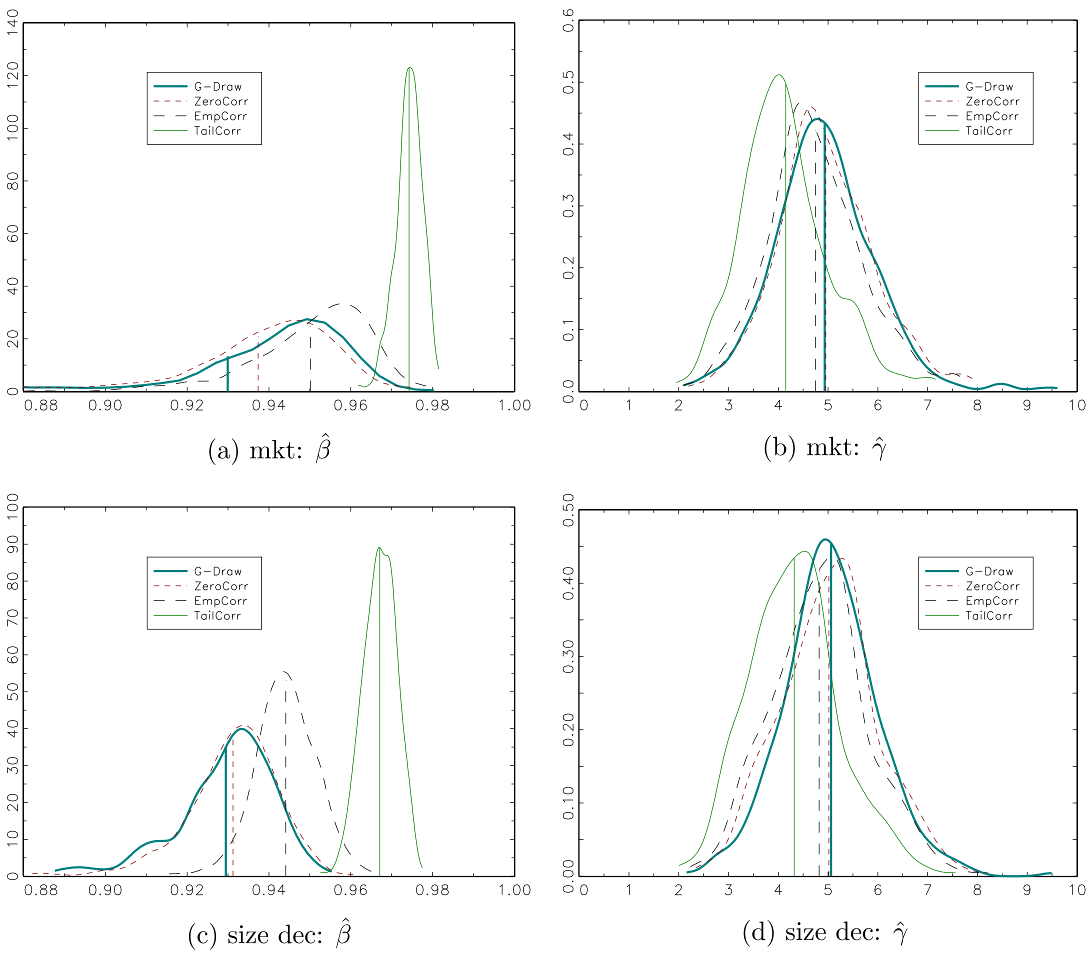


Figure 3: **Kernel densities for AHB estimates**

The figure depicts kernel densities of the AHB estimates of the subjective discount factor, $\hat{\beta}$, and the coefficient of relative risk aversion, $\hat{\gamma}$. We use $H = 400$ and $\mathcal{T}(T) = 16,000$. The panels break down the results for the four sets of test assets: *mkt* (Panels 3a and 3b), *size dec* (Panels 3c and 3d), *industry* (Panels 3a and 3b), and *Cochrane* (Panels 3c and 3d). The thick solid (cyan) density belongs to estimates based on the *G-Draw* simulation procedure, and the short-dashed (red) density refers to *ZeroCorr*. The (black) density with long dashes is for parameter estimates that rely on the *EmpCorr* procedure, and the thin solid (green) density refers to estimates that use the *TailCorr* procedure. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with bandwidth as suggested by Silverman's (1986) rule of thumb.



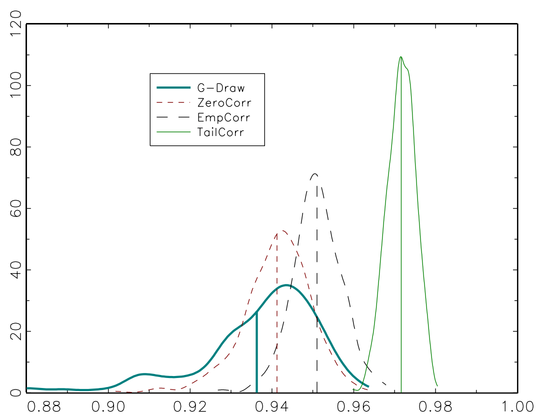
(a) mkt: $\hat{\beta}$

(b) mkt: $\hat{\gamma}$

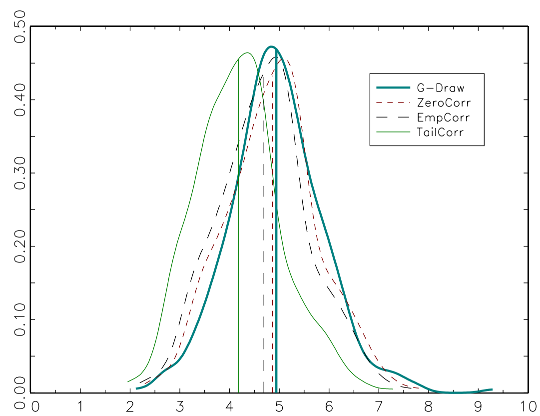
(c) size dec: $\hat{\beta}$

(d) size dec: $\hat{\gamma}$

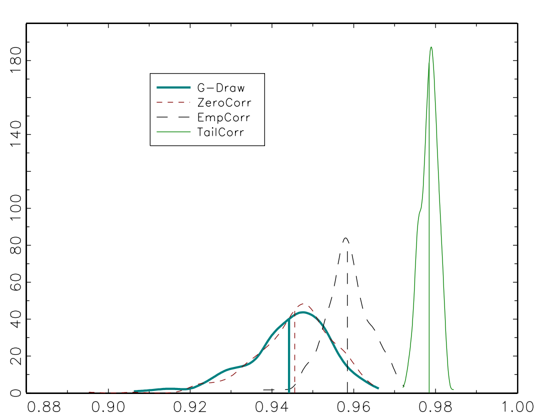
Figure 3: **Kernel densities for AHB estimates (continued)**



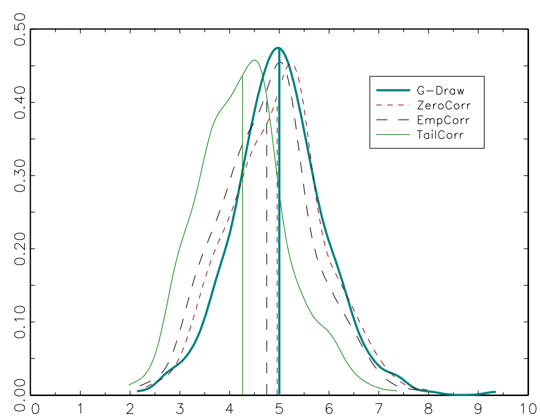
(a) industry: $\hat{\beta}$



(b) industry: $\hat{\gamma}$

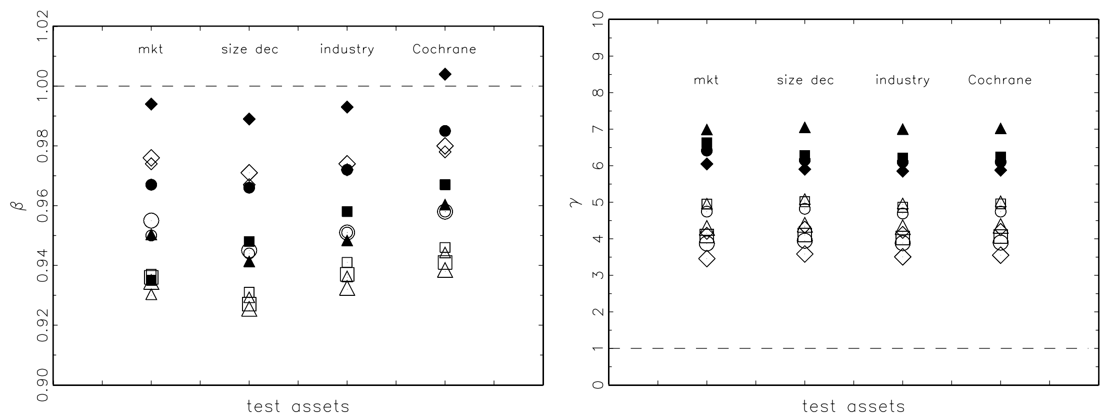


(c) Cochrane: $\hat{\beta}$



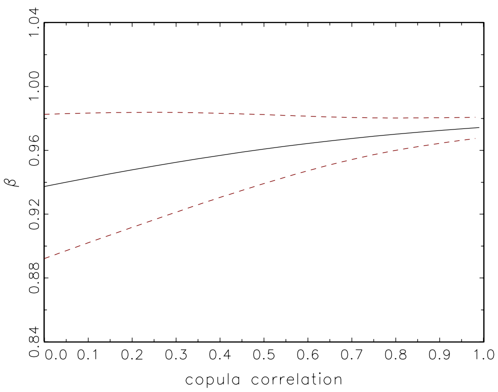
(d) Cochrane: $\hat{\gamma}$

Figure 4: **Comparison of estimation results**
The figure depicts estimates of the subjective discount factor, $\hat{\beta}$, and the coefficient of relative risk aversion, $\hat{\gamma}$ for varying test assets, data generating procedures and type of moment match (MAD-SMM, JPM-SMM, or AHB). For AHB estimates, we use $H = 400$ and $\mathcal{T}(T) = 16,000$, for SMM estimates, we set $\mathcal{T}(T) = 10,000,000$. Panel 4a illustrates the estimates of the subjective discount factor and Panel 4b refers to the coefficient of relative risk aversion. Square symbols belong to estimates based on the *G-Draw* simulation procedure, and triangles refer to *ZeroCorr*. Circles are for parameter estimates that rely on the *EmpCorr* procedure, and the diamond symbol refers to estimates that use the *TailCorr* procedure. Furthermore, small solid symbols belong to MAD-SMM, small blank symbols refer to AHB estimates and large blank symbols belong to JPM-SMM. Limits of the plausible parameter range are indicated by horizontal lines. Estimates labeled *mkt* refer to the excess return of the market portfolio, those labeled *size dec* use the excess returns of the ten size-sorted portfolios, *industry* refers to the excess returns of the ten industry portfolios, and those labeled *Cochrane* use the excess returns of the ten size-sorted portfolios presented in Cochrane's (1996) study. All sets of test assets also include the risk-free rate proxy.

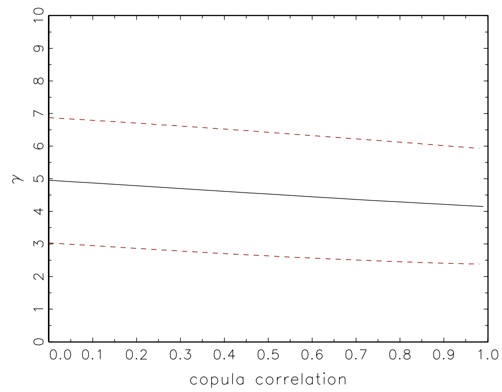


(a) subjective discount factor (b) relative risk aversion

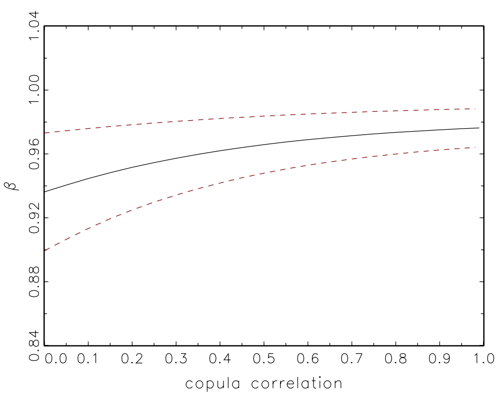
Figure 5: **Effect of varying copula correlation on parameter estimates**
The figure depicts the estimates of the preference parameters, β (Panels 5a, 5c, and 5e) and γ (Panels 5b, 5d, and 5f), for varying copula correlation. Test assets are the excess return of the market portfolio (1947:Q1 - 2013:Q3) and the risk-free rate. Panels 5a and 5b refer to AHB estimates, Panels 5c and 5d to the JPM-SMM estimates, and Panels 5e and 5f to the MAD-SMM estimates. The dashed (red) lines capture the 95% confidence bounds.



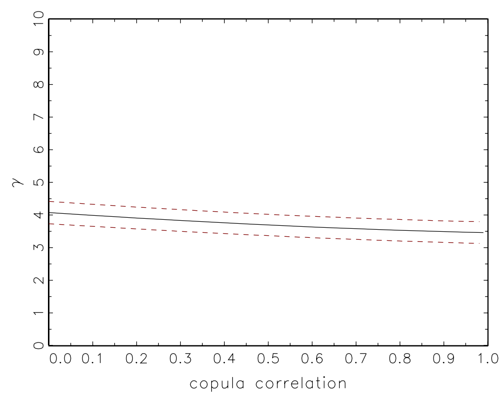
(a) AHB: $\hat{\beta}$



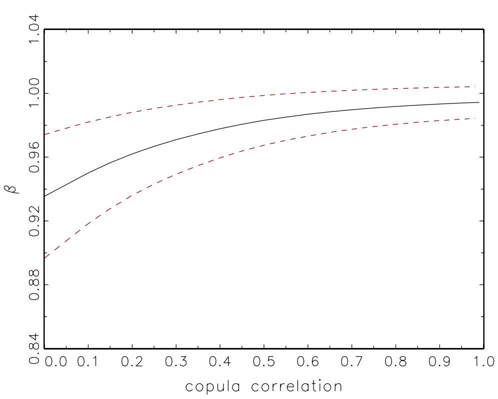
(b) AHB: $\hat{\gamma}$



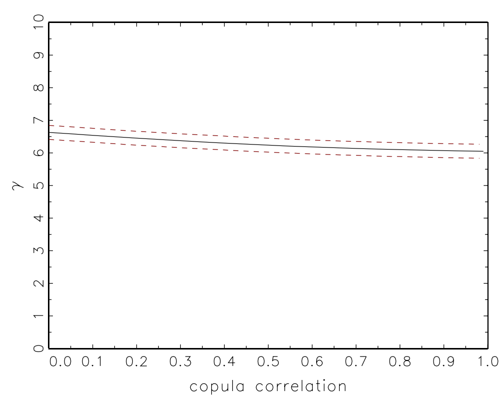
(c) JPM-SMM: $\hat{\beta}$



(d) JPM-SMM: $\hat{\gamma}$



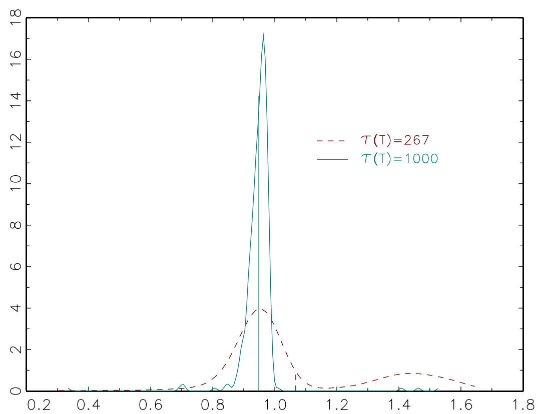
(e) MAD-SMM: $\hat{\beta}$



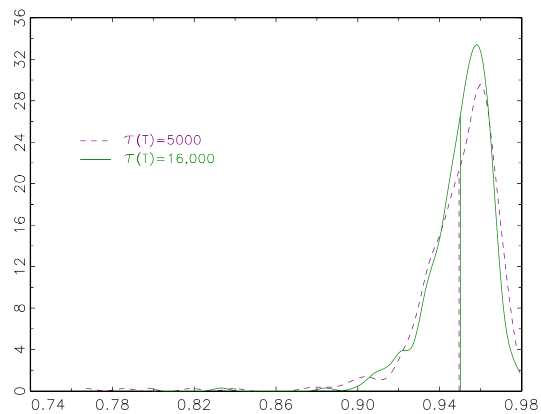
(f) MAD-SMM: $\hat{\gamma}$

Figure 6: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates using the *EmpCorr* simulation procedure**

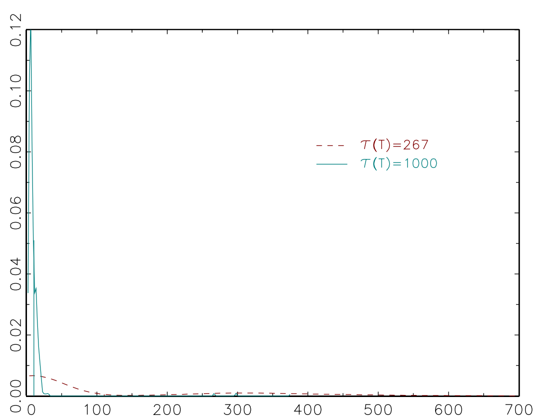
The four panels depict kernel densities of the AHB/*EmpCorr* estimates of the subjective discount factor, $\hat{\beta}$ (Panels 6a and 6b), and the coefficient of relative risk aversion, $\hat{\gamma}$ (Panels 6c and 6d). We use $H = 400$, and $\mathcal{T}(T)$ varies between 267 and 1 000 (Panels 6a and 6c) and 5 000 and 16,000 (Panels 6b and 6d). Test assets are the excess return of the market portfolio (1947:Q1 - 2013:Q3) and the risk-free rate. The dashed (red) densities in Panels 6a and 6c use $\mathcal{T}(T) = 267$, whereas the solid (cyan) densities use $\mathcal{T}(T) = 1\,000$. The dashed (purple) densities in Panels 6b and 6d use $\mathcal{T}(T) = 5\,000$, and the solid (green) densities use $\mathcal{T}(T) = 16,000$. The AHB point estimates are indicated by vertical lines in the respective colors. We use a Gaussian kernel with bandwidth as suggested by Silverman's (1986) rule of thumb.



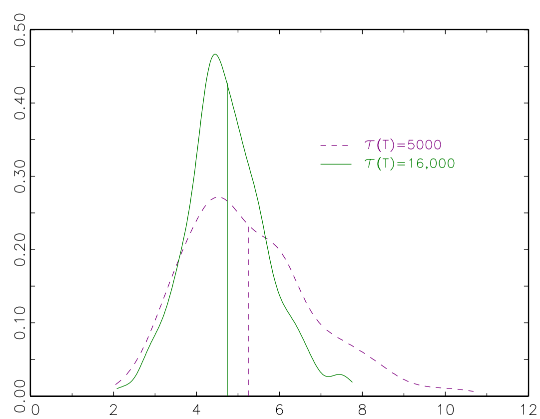
(a) $\hat{\beta}$ for small $\mathcal{T}(T)$



(b) $\hat{\beta}$ for large $\mathcal{T}(T)$



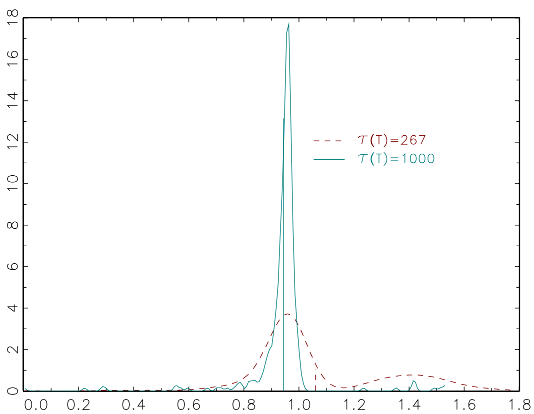
(c) $\hat{\gamma}$ for small $\mathcal{T}(T)$



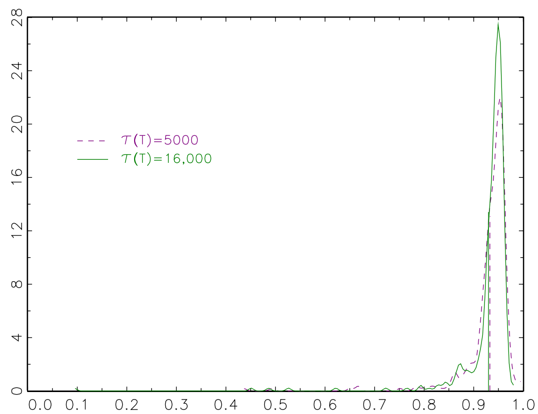
(d) $\hat{\gamma}$ for large $\mathcal{T}(T)$

Figure 7: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates using the *G-Draw* simulation procedure**

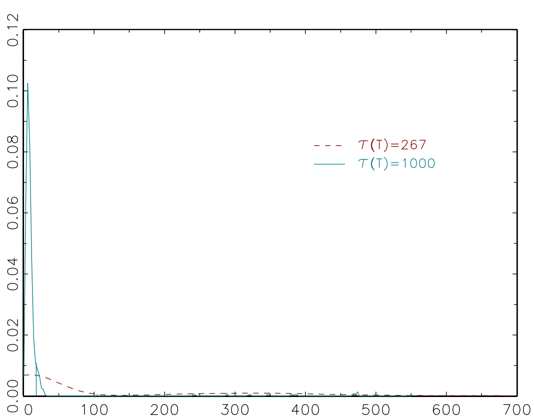
The four panels depict kernel densities of the AHB/*G-Draw* estimates of the subjective discount factor, $\hat{\beta}$ (Panels 7a and 7b), and the coefficient of relative risk aversion, $\hat{\gamma}$ (Panels 7c and 7d). We use $H = 400$, and $\mathcal{T}(T)$ varies between 267 and 1 000 (Panels 7a and 7c) and 5 000 and 16,000 (Panels 7b and 7d). Test assets are the excess return of the market portfolio (1947:Q1 - 2013:Q3) and the risk-free rate. The dashed (red) densities in Panels 7a and 7c use $\mathcal{T}(T) = 267$, whereas the solid (cyan) densities use $\mathcal{T}(T) = 1\,000$. The dashed (purple) densities in Panels 7b and 7d use $\mathcal{T}(T) = 5\,000$, and the solid (green) densities use $\mathcal{T}(T) = 16,000$. The AHB point estimates are indicated by vertical lines in the respective colors. We use a Gaussian kernel with bandwidth as suggested by Silverman's (1986) rule of thumb.



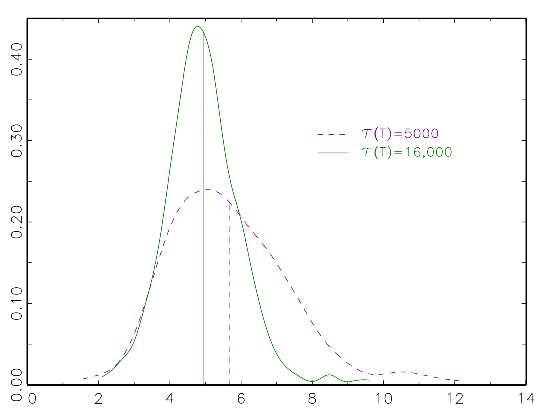
(a) $\hat{\beta}$ for small $\mathcal{T}(T)$



(b) $\hat{\beta}$ for large $\mathcal{T}(T)$



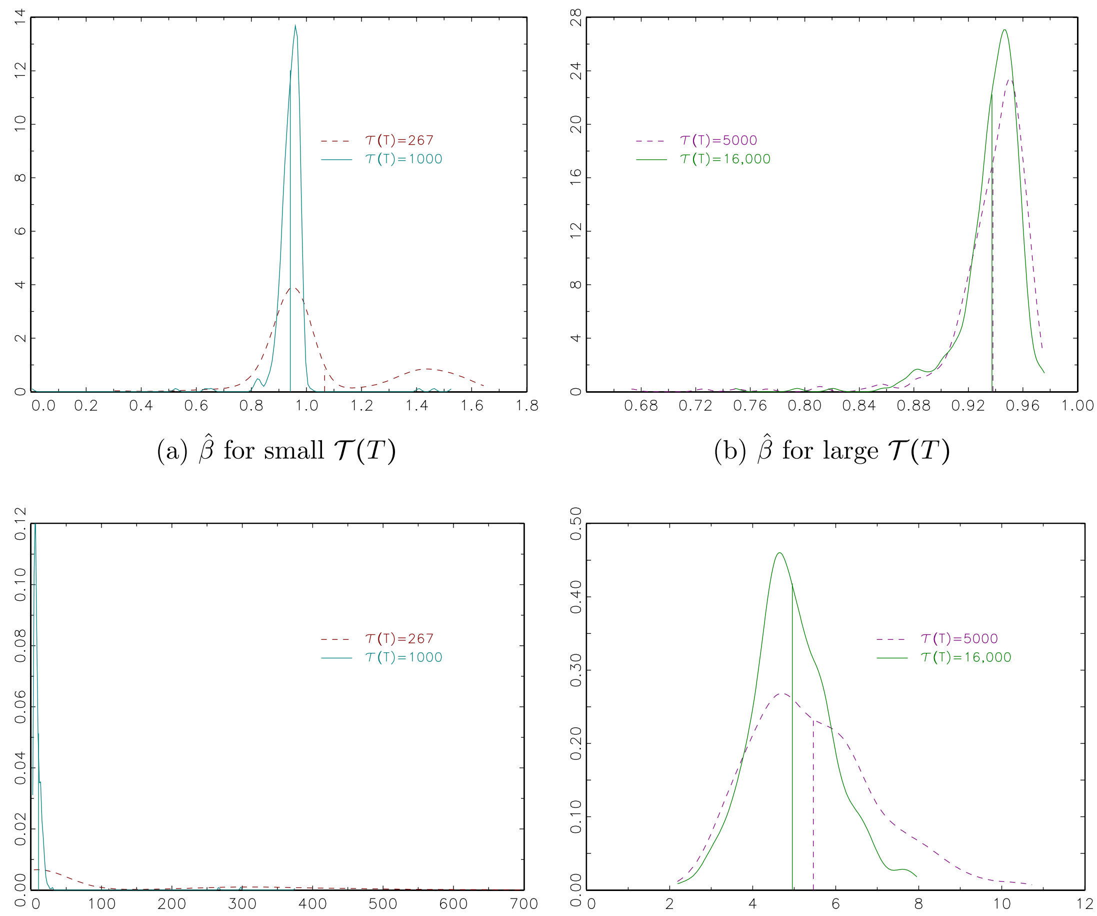
(c) $\hat{\gamma}$ for small $\mathcal{T}(T)$



(d) $\hat{\gamma}$ for large $\mathcal{T}(T)$

Figure 8: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates using the *ZeroCorr* simulation procedure**

The four panels depict kernel densities of the AHB/*ZeroCorr* estimates of the subjective discount factor, $\hat{\beta}$ (Panels 8a and 8b), and the coefficient of relative risk aversion, $\hat{\gamma}$ (Panels 8c and 8d). We use $H = 400$, and $\mathcal{T}(T)$ varies between 267 and 1 000 (Panels 8a and 8c) and 5 000 and 16,000 (Panels 8b and 8d). Test assets are the excess return of the market portfolio (1947:Q1 - 2013:Q3) and the risk-free rate. The dashed (red) densities in Panels 8a and 8c use $\mathcal{T}(T) = 267$, whereas the solid (cyan) densities use $\mathcal{T}(T) = 1\,000$. The dashed (purple) densities in Panels 8b and 8d use $\mathcal{T}(T) = 5\,000$, and the solid (green) densities use $\mathcal{T}(T) = 16,000$. The AHB point estimates are indicated by vertical lines in the respective colors. We use a Gaussian kernel with bandwidth as suggested by Silverman's (1986) rule of thumb.



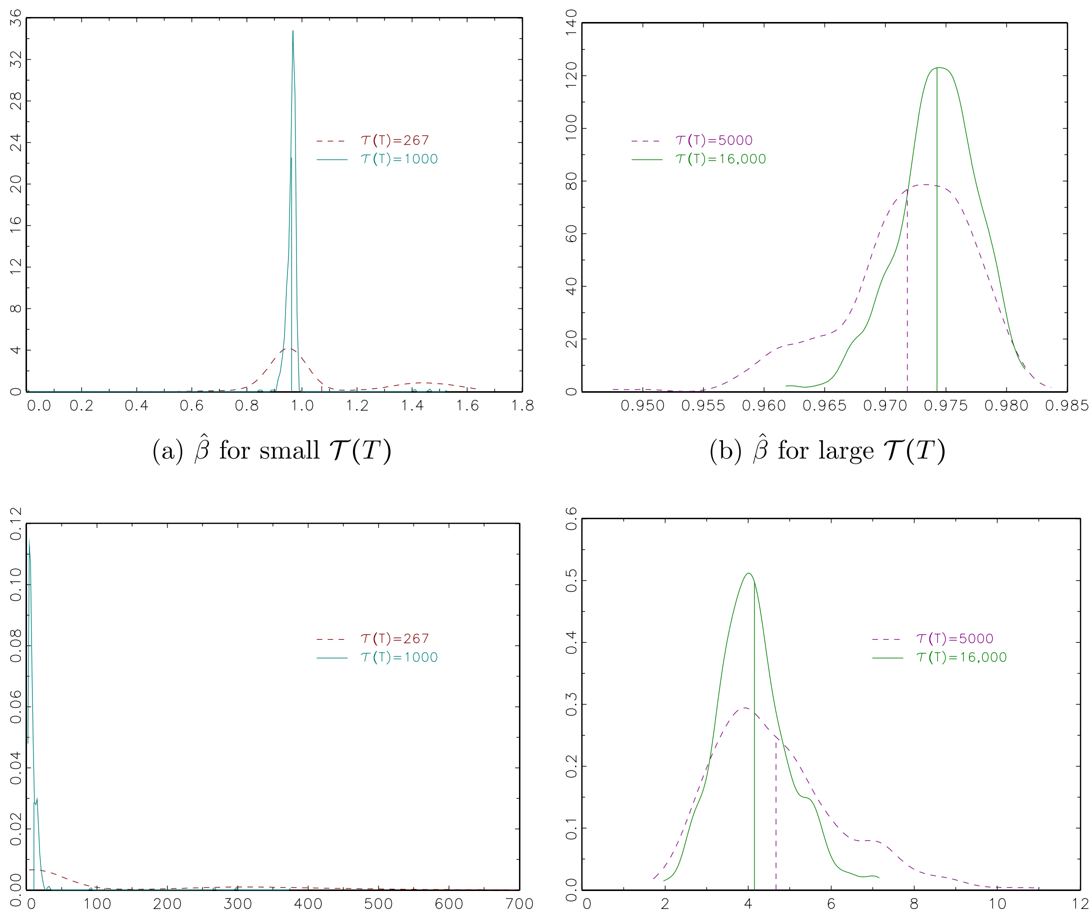
(a) $\hat{\beta}$ for small $\mathcal{T}(T)$

(b) $\hat{\beta}$ for large $\mathcal{T}(T)$

(c) $\hat{\gamma}$ for small $\mathcal{T}(T)$

(d) $\hat{\gamma}$ for large $\mathcal{T}(T)$

Figure 9: **Effect of varying $\mathcal{T}(T)$ on AHB parameter estimates using the *TailCorr* simulation procedure**
The four panels depict kernel densities of the AHB/*TailCorr* estimates of the subjective discount factor, $\hat{\beta}$ (Panels 9a and 9b), and the coefficient of relative risk aversion, $\hat{\gamma}$ (Panels 9c and 9d). We use $H = 400$, and $\mathcal{T}(T)$ varies between 267 and 1 000 (Panels 9a and 9c) and 5 000 and 16,000 (Panels 9b and 9d). Test assets are the excess return of the market portfolio (1947:Q1 - 2013:Q3) and the risk-free rate. The dashed (red) densities in Panels 9a and 9c use $\mathcal{T}(T) = 267$, whereas the solid (cyan) densities use $\mathcal{T}(T) = 1\,000$. The dashed (purple) densities in Panels 9b and 9d use $\mathcal{T}(T) = 5\,000$, and the solid (green) densities use $\mathcal{T}(T) = 16,000$. The AHB point estimates are indicated by vertical lines in the respective colors. We use a Gaussian kernel with bandwidth as suggested by Silverman's (1986) rule of thumb.



(a) $\hat{\beta}$ for small $\mathcal{T}(T)$ (b) $\hat{\beta}$ for large $\mathcal{T}(T)$

(c) $\hat{\gamma}$ for small $\mathcal{T}(T)$ (d) $\hat{\gamma}$ for large $\mathcal{T}(T)$

Figure 10: **Effect of varying disaster threshold q on parameter estimates** The figure illustrates the effect of a varying disaster threshold on the estimates of the subjective discount factor and the coefficient of relative risk aversion. The black plus symbol refers to the base case of our analysis, which is $q = 14.5\%$ ($\hat{p}_q = 0.44\%$). The cyan squares use a lower disaster threshold, $q = 9.5\%$ ($\hat{p}_q = 0.69\%$) and the red circles refer to a higher threshold, $q = 19.5\%$ ($\hat{p}_q = 0.28\%$).