Ambiguous Long Run Risks

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Abstract

This paper provides empirical and model-theoretic evidence that stock returns comprehend a large positive premium for ambiguity about macroeconomic volatility. We use the cross-section of analysts' interval forecasts of aggregate output growth to construct a measure of ambiguity about volatility. This measure predicts stock returns over short horizons and explains time-variation in the variance premium. We use a long run risks asset pricing model in which the investor raises concerns about possible misspecification of the variance dynamics to explain these findings in a general equilibrium context. Given the investor is ambiguity averse, the model explains unconditional moments and the predictability pattern of the variance premium. The key feature is a large ambiguity premium which corroborates the empirical findings.

Keywords: Ambiguity, ambiguous volatility, asset pricing, long run risks

JEL: G12, E44, D81

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1 Introduction

This paper studies the link between ambiguity about macroeconomic volatility and asset prices. The seminal work of Andersen et al. (2000) spurred on a whole series of papers that analyze the impact of investors' aim to choose portfolios that are robust to model misspecification on prices of stocks and bonds. Most of these papers assume that growth rates of macroeconomic quantities are ambiguous. Recently, Epstein and Ji (2013) provide a model that allows for an investor's concern with ambiguity about volatility. In their conclusion the authors state

> *A question that remains to be answered more broadly and thoroughly is "does ambiguity about volatility and possibility matter empirically?"*

In this paper we aim to answer that question. For this purpose, we construct a measure of ambiguity about macroeconomic volatility from analysts' interval forecasts of aggregate output growth. With this measure at hand, we perform time series regressions and find strong evidence for a large positive premium for ambiguous volatility. Our measure predicts excess returns on the CRSP stock market index over short horizons of up to two years. Moreover, there is a positive contemporaneous relation between ambiguous volatility and the variance premium.

In the second part of the paper, we explain these findings within a discrete time general equilibrium asset pricing model. Our model corroborates the existence of a substantial premium for ambiguity about volatility: The high predictability of stock returns by the variance premium can only be explained if a major part of the equity premium is a compensation for ambiguous volatility. The model is an extension of the long run risks model of Bansal and Yaron (2004). One usual assumption in the long run risks literature is that the representative investor knows the structure of the model, observes the state variables, and is hence aware of the conditional distribution of consumption and cash-flows in the following periods. Among others, this assumption is put into question by Collard et al. (2011), Jahan-Parvar and Liu (2014), Ju and Miao (2012), and Miao et al. (2012), who all assume that trend growth rates are ambiguous. More generally, the literature on model uncertainty and its implications for asset markets is reviewed by Epstein and Schneider (2010), Etner et al. (2012), and Guidolin and Rinaldi (2013). The papers cited above however have in common that they all use the recursive smooth ambiguity model of Klibanoff et al. (2005) and Klibanoff et al. (2009) to model the investor's attitudes towards risk and ambiguity. We use the same model, since it allows a clear separation of ambiguity from ambiguity attitudes which is difficult in other models such as the *maxmin*-model of Gilboa and Schmeidler (1989). Moreover, the model's parameters are estimated by Thimme and Völkert (2014), which enables a realistic calibration.

As opposed to the papers cited above, we consider ambiguity about consumption growth volatility, i.e. about the state variable σ_t^2 in Bansal and Yaron (2004). Our model features one state variable that describes the volatility level that is implied by the reference model which is considered as the most likely by the investor. She however casts doubts on whether this model is correctly specified. We introduce a further state variable that describes the magnitude of possible deviations from that reference volatility. This magnitude may be time-varying due to a time-varying model set that the investor deems possible, or due to time-varying differences in implications of the same candidate models for the volatility level.

Apart from time-series regressions, we use our empirical proxies of the state variables to estimate and calibrate our model. We only use these proxies, together with data on aggregate consumption and dividend growth, to estimate the endowment dynamics which dissipates the concern the parameters are engineered to fit asset pricing moments. The obtained calibration produces a large equity premium, a low and volatile risk-free rate, volatile price-dividend ratios, and a sizable variance premium. Our model lacks a highly persistent uncertainty process, which is a key ingredient of the model of Bansal and Yaron (2004). The difficulty to detect such a process in the data is well-known (see e.g. Constantinides and Ghosh (2011)). Thus, the model is not able to explain the predictive power of the price dividend ratio over long horizons. It does, however, perfectly match the predictability pattern of the variance premium, which is affine in ambiguity about volatility in our model. We show that a large ambiguity premium, in combination with a rather low risk premium, is necessary to generate the high return predictability over short horizons that is observed in the data.

Our paper is technically related to the literature about asset pricing in models with sophisticated volatility structures, such as Bollerslev et al. (2012), Bollerslev et al. (2009), Jin (2013) and Zhou and Zhu (2014). The state variables in these models, for example the vol-of-vol in Bollerslev's papers, are however interpreted as observable quantities. We consider ambiguity about the level of volatility which stems from the difficulty to access the evolution of volatility in the future. One may argue that current volatility can be observed (for example from high frequency stock return data), such that the dynamics of the volatility process can be estimated quite precisely. Carr and Lee (2009) however argue that "noise in the data generates noise in the estimate, raising doubts that a modeler can correctly select any parametric stochastic process from the menu of consistent alternatives." A large menu may lead to high ambiguity about volatility if the different alternatives imply different volatility levels in the future.

The remainder of this paper is organized as follows. Section 2 demonstrates the construction of uncertainty measures using data from the Survey of Professional Forecasters. We study the explanatory power of these measures in contemporaneous and predictive regressions in Section 3. In Section 4, we introduce our asset pricing model and discuss approximate analytic solutions. In Section 5, we estimate and calibrate the model and study its implications for asset prices. Section 6 concludes.

2 A measure of ambiguous volatility

We assume throughout the paper that log growth Δc_{t+1} in aggregate endowment is Gaussian with conditional mean x_t and conditional variance σ_t^2, i.e.

$$\Delta c_{t+1} = x_t + \sigma_t \varepsilon_{t+1}^c,$$

where $\varepsilon_t^c \sim$ i.i.d. $\mathcal{N}(0,1)$. The goal of this section is to find approximate time series of the state variables x_t and σ_t^2, as well as the amount of possible ambiguity about x_t and especially σ_t^2. For this purpose, we rely on analysts' forecasts of GDP growth probabilities.

2.1 Extracting time series of growth moments

The Survey of Professional Forecasters (SPF henceforth), conducted at a quarterly frequency by the Philadelphia Fed, comprehends the table PRGDP (Probability of Changes in Real GDP) - Individual Responses, which contains a number of analysts' estimations of the probability of the annual-average over annual-average real GDP growth falling in various ranges.[1] We use forecasts of real GDP growth probabilities, although we are rather interested in real consumption growth moments. Unfortunately, the SPF only surveys the analysts' forecasts of mean consumption growth but no interval forecasts. Since we are particularly interested in the analysts' assessment of the magnitude of deviation from the mean, we have to rely on real GDP. However, a comparison of the single analysts' assessment of mean GDP growth with their assessment of mean consumption growth shows that both quantities are very close to each other. Note that in endowment economy-models such as the model we study in Section 4, the investor has to consume the exogenous endowment instantly, hence there is no difference between consumption and output. Moreover, using GDP

[1] Zarnowitz and Braun (1993) study the SPF in detail.

growth data to approximate consumption growth is common in the literature. Bansal and Shaliastovich (2010), Ulrich (2011, 2012), and Colacito et al. (2012) use GDP growth forecasts.

Let J_t be the number of analysts featured in time t's survey. Each analyst is asked for her assessment of the probabilities that annual GDP-growth falls in the intervals $(-\infty, l_1)$, (l_1, l_2), …, (l_{M_t}, ∞) where $l_i - l_{i-1} = \Delta l$ is fixed. We denote the probabilities given by analyst j by $P_t^j = (p_{1,t}^j, \ldots, p_{M_t+1,t}^j)$, where $p_{1,t}^j$ is the probability that GDP growth is lower than l_1, $p_{i,t}^j$ (for $i = 2, \ldots, M_t$) is the probability that it falls in the interval (l_{i-1}, l_i), and $p_{M_t+1,t}^j$ is the probability that it is above l_{M_t}. At each point in time t we calculate analyst j's assessment of the trend growth rate $x_{j,t}$ and growth volatility $\sigma_{j,t}^2$ by considering the parameters of that normal distribution whose density function is closest to the analyst's interval forecast. More precisely, we estimate $x_{j,t}$ and $\sigma_{j,t}^2$ via maximum likelihood, i.e. we maximize

$$\log L(x_{j,t}, \sigma_{j,t}^2; P_t^j) = \sum_{i=1}^{M_t+1} \left(p_{i,t}^j \log \left[\Phi\left(\frac{l_i - x_{j,t}}{\sigma_{j,t}}\right) - \Phi\left(\frac{l_{i-1} - x_{j,t}}{\sigma_{j,t}}\right) \right] \right), \qquad (1)$$

where $l_0 = -\infty$, $l_{M_t+1} = \infty$, and Φ denotes the cdf of the normal distribution. We then use information inherent in the cross-section of analysts, that is disagreement in the analysts' assessments of x_t and σ_t^2 as a proxy for uncertainty about trend growth and volatility. More precisely, we define

$$Ex_t = \frac{1}{J_t} \sum_{j=1}^{J_t} x_{j,t} \qquad Vx_t = \frac{1}{J_t - 1} \sum_{j=1}^{J_t} (x_{j,t} - Ex_t)^2$$

$$E\sigma_t^2 = \frac{1}{J_t} \sum_{j=1}^{J_t} \sigma_{j,t}^2 \qquad V\sigma_t^2 = \frac{1}{J_t - 1} \sum_{j=1}^{J_t} \left(\sigma_{j,t}^2 - E\sigma_t^2\right)^2$$

The two level measures Ex_t and $E\sigma_t^2$ pin down the time t reference model. They correspond to the average trend growth and variance assessments of the analysts. Vx_t, often simply referred to as *forecast dispersion*, is a widely used proxy for ambiguity in general (see e.g. Bansal and Shaliastovich (2010), Drechsler (2013), and Ulrich (2012)). We distinguish this popular measure from ambiguity about volatility $V\sigma_t^2$. It is defined analogously to Vx_t, such that all arguments for why analysts' disagreement is a reasonable measure for ambiguity apply here. Patton and Timmermann (2010) find that analysts disagree because they use different models for forecasting. We assume that each analyst represents one economic model. Hence, we assume that the set $(\mathcal{N}(x_{j,t}, \sigma_{j,t}^2))_{j\in\{1,\dots,J_t\}}$ is a reasonable approximation of the model set that investors face at time t. Differences in model-implied volatility levels, i.e. ambiguity about volatility, can thus be approximated by the cross-sectional variation in analysts' volatility-assessments as extracted from interval forecasts.

We only consider the first two moments of x_t and σ_t^2. Colacito et al. (2012) also look at the cross-sectional skewness in forecasts of trend consumption growth. Our framework could easily be extended to incorporate skewness on the single analyst level as well as in the cross-section. We leave this for future research.

2.2 Descriptive statistics

Although the first SPF was conducted in 1968:Q4, we use data from 1992:Q1 to 2012:Q4 in our analysis and the following empirical exercise. We omit earlier data for the following reasons: First, from 1968:Q4 to 1981:Q2, analysts reported their assessments of growth in nominal GNP and in real GNP from 1981:Q3 to 1991:Q4. To guarantee a consistent measure we only rely on the recent 21 years. Bansal and Shaliastovich (2010) use inflation forecasts to convert nominal quantities to real ones. However, although the SPF provides interval forecasts of future inflation, it is not possible to compound these with GDP growth forecasts due to possible dependencies between inflation and GDP growth. Second, before 1992:Q1 the quality of the data is rather low. While some surveys asked for growth from the last to the current year, others asked for growth from the current to the next, and for some surveys it is even unclear which growth rate the analysts referred to. This makes it extremely difficult to work with the data, in particular, to do a valid seasonal adjustment. Third, a look at the time series of our proxies between 1968 and 1992 suggests that they are not stationary. While all four measures moved on rather high levels during the 1970's, all declined during the great moderation in the late 1980's. This finding, especially the decrease in the uncertainty measures might well be linked (or rather might well have caused) price dividend ratios to rise steadily during the same period of time. Lettau and Van Nieuwerburgh (2008) investigate price dividend ratios and report a structural break in the early 1990's. Although this link suggests itself, we only rely on data from 1992 on to avoid working with non-stationary time series.[2]

In Figure 1 we plot seasonally adjusted quarterly per capita time series $(Ex_t)_t$, $(E\sigma_t^2)_t$, $(Vx_t)_t$, and $(V\sigma_t^2)_t$. Analysts' forecasts are about annual-average over annual-average output growth. It can hence be suggested that the three uncertainty measures are systematically lower in late quarters of a year. We do a X-12-ARIMA seasonal adjustment of all time series to account for that problem. Moreover, we calculate per capita quantities by dividing all growth rates by the 12-month moving average growth of US population before calculating the measures.

Our proxy Ex_t for trend consumption growth shows a clearly cyclical behavior. During the recessions in 2001 and 2009 analysts predicted much lower consumption growth rates compared to the rest of the sample. At the same time, particularly during the 2009 financial crisis, ambiguity about trend consumption growth $(Vx_t)_t$ spiked. Interestingly, the pure risk measure $E\sigma_t^2$ stayed at low levels during recessions. Ambiguity about volatility $(V\sigma_t^2)_t$ seems to show a rather erratic behavior. However, it spikes in periods of high expected volatility. Just as for $(Ex_t)_t$ and $(Vx_t)_t$ who have a correlation of -55.94%, there is a strong relation between $E\sigma_t^2$ and $V\sigma_t^2$ with a correlation of 65.28%.

[2]Ulrich (2012) sheds light on the connection between the decrease in ambiguity and the simultaneous decrease in dividend yields and interest rates during the great moderation.

Properties of the time series are listed in Table 1. We also report moments of aggregate consumption growth to enable an interpretation of the statistical properties of the extracted time series. Our trend growth measure Ex_t has a mean of 38.8033 basis points (bp) per quarter which is in line with the mean log consumption growth during the same period. The mean expected variance $E\sigma_t^2$ with 0.1597 bp is clearly below the variance of log consumption growth which is 0.5708 bp. A direct comparison is however misleading since Δc also varies due to time variation in x_t. We also find, mainly due to the financial crisis in 2009, that the sample distribution of Ex_t is left-skewed, while Vx_t and $V\sigma_t^2$ are right-skewed, and all are leptokurtic.

3 Time-series regressions

In this section we analyze whether the four measures constructed from the SPF contemporaneously explain or forecast asset pricing quantities. To facilitate the interpretation of the regression coefficients, we standardize the uncertainty measures, i.e. calculate standard deviations instead of variances. We moreover winsorize all time series at the 5%- and 95%-quantile to avoid that possibly spurious outliers drive the results. Appendix B provides an overview of the data. For robustness of all regression results, see Appendix C.

Table 2 reports results of contemporary regressions. The first two lines show that the price dividend ratio is high in periods of high expected consumption growth Ex_t. We do not find a significant relation between innovations in any of the uncertainty measures and the price dividend ratio. However, positive innovations in $E\sigma_t^2$ come along with negative innovations in interest rates. The same seems to be true for ambiguity about volatility $V\sigma_t^2$. Due to the high positive correlation of $E\sigma_t^2$ and $V\sigma_t^2$ it is however difficult to say which of the two measures drives the effect. Intuitively, an increase in expected volatility and/or ambiguity about volatility increases the precautionary savings motive of investors which leads to lower interest rates. As the price dividend ratio, interest rates are not affected by ambiguity about trend consumption Vx_t. As shown in the lower two lines of Table 2, the variance premium is significantly higher in periods of high ambiguity about volatility. This finding suggests a (positive) link between uncertainty about macroeconomic volatility, as measured by $V\sigma_t^2$, and uncertainty about volatility of future stock returns, as quantified by the variance premium. All other variables do not explain variations in the variance premium.

Tables 3 and 4 report results of predictive regressions. In Table 3 we show results of regressions of semiannual consumption and dividend growth after period t. Ex_t predicts consumption and dividend growth. The ambiguity measure Vx_t seems to predict dividend growth. This effect is however likely to be driven by the high negative correlation between Ex_t and Vx_t.

Return predictability of our risk and ambiguity measures is reported in Table 4. We find a high positive premium for ambiguity about volatility, while there is no significant premium for ambiguity about trend growth. This sheds light on the predictive power of the variance premium in predictive return regressions, which is known to be highest for short horizons (see Bollerslev et al. (2009), Drechsler (2013), and the discussion in Section 5). Accordingly, the predictive power of $V\sigma_t^2$ is high for semi-annual returns, deteriorates for horizons of one and two years, and vanishes at the three year horizon. This result shows that investors care for ambiguity about volatility and claim a compensation for holding equity in periods in which it is high. The risk premium, i.e. the coefficient of $E\sigma_t^2$, is negative at best but usually insignificant. The negative coefficient at the semiannual horizon might be caused by the high negative correlation between $E\sigma_t^2$ and $V\sigma_t^2$. As we learn from the lower two lines, an upswing in $E\sigma_t^2$ does however predict a significant increase in the return volatility during the following quarter. This finding identifies a link between (expected) macroeconomic volatility and return volatility.

Summing up, our proxies have properties that seem reasonable: Ex_t predicts cash flows, while $E\sigma_t^2$ predicts return volatility. Moreover, the price dividend ratio covaries with expected trend growth Ex_t, whereas there is a precautionary savings motive related to expected volatility $E\sigma_t^2$. The information content of the two ambiguity measures are diverging: While ambiguity about trend growth does not explain time-series variation in any of the considered quantities, ambiguity about volatility explains the variance premium and predicts excess returns at low horizons.

4 A model with ambiguous volatility

In this section we introduce a model that allows for ambiguity about consumption growth volatility. Our model can easily be extended to also incorporate ambiguity about trend consumption growth. We however omit this feature as a result of the negligible role of the proxy Vx_t, documented in Section 3. We start with a discussion of our representative agent model which features three state variables that describe the properties of the model set that is considered by the agent. We then briefly review smooth ambiguity preferences and its consequences for asset prices through the stochastic discount factor. Our model yields close approximate analytic solutions. We discuss asset pricing quantities in equibrium in the last part of this section.

4.1 Endowment

Assume that a representative investor is endowed with an exogenous stream of a perishable consumption good and prices a claim on all future dividends. Growth rates of aggregate consumption and aggregate dividends are conditionally lognormal:

$$\Delta c_{t+1} = \mu_c + x_t + \tilde{\sigma}_t \varepsilon^c_{t+1},$$
$$\Delta d_{t+1} = \mu_d + \varphi_x x_t + \pi_d \tilde{\sigma}_t \varepsilon^c_{t+1} + \varphi_\sigma \tilde{\sigma}_t \varepsilon^d_{t+1},$$

where $(\varepsilon^c_{t+1}, \varepsilon^d_{t+1})' \sim \mathcal{N}(0, I_2)$. Dividends are levered consumption, i.e. φ_x and $(\pi_d^2 + \varphi_\sigma^2)$ are greater than one, and consumption and dividend growth are locally correlated. To model ambiguity about volatility we assume that $\tilde{\sigma}^2_t$ is not in the information set of the investor at time t. The investor entertains a non-degenerate model set, whose elements can be indexed by the realizations σ_t of the random variable $\tilde{\sigma}^2_t$. Instead of discussing the nature of single models and resulting volatility levels, we make the assumption that $\tilde{\sigma}^2_t$ is conditionally Gaussian:

$$\tilde{\sigma}^2_t = v_t + \sqrt{q_t}\, \varepsilon^\sigma_t,$$

where $\varepsilon^\sigma_t \sim \mathcal{N}(0, I_1)$ and independent from shocks to Δc and Δd. Hence, there is a continuum of models that all yield the same growth rate $\mu_c + x_t$ of consumption but different volatility levels. The most likely model implied volatility level is the reference volatility v_t, which is time-varying. The magnitude of possible deviations from that reference is given by q_t, which quantifies the time-varying ambiguity about consumption growth volatility. To close the model, we define the evolution of the state variables $s_t = (x_t, v_t, q_t)'$ as

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \sqrt{\pi_v v_t + \pi_q q_t}\, \varepsilon^x_{t+1} \\ v_{t+1} &= \bar{v} + \rho_v (v_t - \bar{v}) + \sigma_v\, \varepsilon^v_{t+1} \\ q_{t+1} &= \bar{q} + \rho_q (q_t - \bar{q}) + \sigma_q\, \varepsilon^q_{t+1} \end{aligned}$$

where $(\varepsilon^x_{t+1}, \varepsilon^v_{t+1}, \varepsilon^q_{t+1})' \sim \mathcal{N}(0, I_3)$ and independent from shocks to consumption and dividend growth and the shock ε^σ_t which resolves ambiguity. The long run risks model of Bansal et al. (2012) is the special case of our model in which q_t is constantly zero (i.e. $\bar{q} = \sigma_q = 0$), which means that the investor imperturbably trusts the reference model v_t.

4.2 Preferences

Consider a representative investor with recursive preferences as developed by Epstein and Zin (1989) and Kreps and Porteus (1978). The investor evaluates future consumption plans $C = (C_t)_{t \in \mathbb{N}}$ with respect to the value function

$$V_t(C) = \left[\left(1 - e^{-\delta}\right) C_t^{1-\rho} + e^{-\delta}(\mathcal{R}_t(V_{t+1}(C)))^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where δ denotes her subjective time discount rate and ρ the reciprocal of her elasticity of intertemporal substitution (EIS). The uncertainty aggregator $\mathcal{R}$ is utilized to account for risk and ambiguity in the continuation value $V_{t+1}(C)$ of future consumption. We use the specification of Klibanoff et al. (2009)

$$\mathcal{R}_t(z) = v^{-1}\left(\mathbb{E}_{s_t}\left[v\left(u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)\right)\right]\right),$$

where u and v are utility functions and we use the notation $\mathbb{E}_{\sigma_t}[\,\cdot\,] := \mathbb{E}[\,\cdot\,|\tilde{\sigma}_t^2]$ for expectations conditional on $\tilde{\sigma}_t^2$. If there is no ambiguity about volatility, i.e. $q_t = 0$ and $\tilde{\sigma}_t^2$ trivializes to v_t, then $\mathbb{E}_{\sigma_t}[u(z)]$ is a constant conditional on s_t and $\mathcal{R}_t(z) = u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)$. We then end up with standard Epstein and Zin (1989) recursive preferences. Hence, the curvature of the utility function u characterizes the investor's risk attitude. In general, if $q_t \neq 0$, $u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)$ is the certainty equivalent of z given full information about the distribution of z. As long as the volatility $\tilde{\sigma}_t$ is ambiguous $u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)$ is a random variable and the investor considers expected utility of certainty equivalents conditional on the available information s_t about the model set. Here, large variation amongst expected utilities $\mathbb{E}_{\sigma_t}[u(z)]$ is appreciated by the investor if $v \circ u^{-1}$ is convex while she is anxious about variation if $v \circ u^{-1}$ is concave. Hence, the curvature of $v \circ u^{-1}$ determines the investor's ambiguity attitude. In our application we use power utility functions for u and v:

$$u : x \mapsto \frac{x^{1-\gamma}}{1-\gamma}, \qquad v : x \mapsto \frac{x^{1-\eta}}{1-\eta},$$

with uncertainty attitude parameters γ and η. The investor is risk averse whenever u is concave, i.e. $\gamma > 1$, and ambiguity averse whenever $v \circ u^{-1}$ is concave, i.e. $\eta > \gamma$.

A smooth ambiguity investor prices any claim on a future dividend stream $(D_{i,t})_t$, such that the return $R_{i,t+1}$ on this claim in the next period satisfies

$$1 = \mathbb{E}_{s_t}\left[\xi_{t,t+1} R_{i,t+1}\right], \tag{2}$$

where $\xi_{t,t+1}$ the stochastic discount factor (SDF). As shown by Hayashi and Miao (2011) the SDF of a smooth ambiguity investor is

$$\xi_{t,t+1} = e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1} \left(\mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1}\right]\right)^{\theta_2 - 1} \tag{3}$$

where $\theta_1 = \frac{1-\gamma}{1-\rho}$, $\theta_2 = \frac{1-\eta}{1-\gamma}$, and $R_{w,t+1}$ denotes the return on the claim on aggregate consumption. A detailed derivation of the pricing kernel can be found in Thimme and Völkert (2012) in a more general framework. We consider two special cases: First, if there is no ambiguity, i.e. $q_t = 0$, Equation (2) simplifies to

$$1 = \mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1} R_{i,t+1}\right] \left(\mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1}\right]\right)^{\theta_2 - 1}$$

where the second term on the right hand side is equal to 1 since it is the Euler equation of the return on the consumption claim. Hence, the SDF facilitates to

$$\xi_{t,t+1} = e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1}.$$

Second, if the investor is ambiguity neutral, i.e. $\gamma = \eta$, we end up with the same SDF, just by introducing $\theta_2 = 1$ into Equation (3). In both cases, the resulting SDF is equal to the SDF of an Epstein-Zin-investor.

4.3 Model solution

As in Bansal and Yaron (2004), we use the return approximation of Campbell and Shiller (1988) and impose affine linear guesses for the valuation ratios of the consumption and dividend claim to find close approximate solutions for the asset pricing quantities of interest.[3] The log wealth-consumption ratio z and the log price-dividend ratio z_d of the dividend claim are given by

$$z_t = A + B' s_t \quad \text{and} \quad z_{d,t} = A_d + B_d' s_t. \tag{4}$$

The coefficients depend on the preference parameters of the investor and are given in Appendix A. Let $r_{d,t+1}$ denote the log return on the claim on aggregate dividends in the period from time t to $t+1$, and $r_{f,t}$ the log return on a risk-free bond over the same period. The latter is given by

$$\begin{aligned} r_t^f \; = \; & \delta - \frac{1}{2}(1-\theta_1\theta_2)k_1^2 \left(B_2^2 \sigma_v^2 + B_3^2 \sigma_q^2 \right) + \rho(\mu_c + x_t) \\ & - \frac{1}{2}\Big((1-\theta_1\theta_2) k_1^2 \pi_v B_1^2 + \rho(\gamma - 1) + \gamma \Big) v_t \\ & - \frac{1}{2}\Big((1-\theta_1\theta_2) k_1^2 \pi_q B_1^2 + \frac{1}{4}\left((\gamma - \eta + \gamma\eta)^2 - (\rho - \eta)(1-\gamma)^2(1-\eta) \right) \Big) q_t. \end{aligned} \tag{5}$$

[3] The solution technique is demonstrated in detail by Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011).

As known from the CCAPM of Lucas (1978) and the model of Bansal and Yaron (2004), interest rates are related to consumption growth x_t via the inverse EIS ρ and there are a short run and long run precautionary savings motives proportional to the volatility level v_t. Apart from that, there is a precautionary savings motive due to ambiguity about volatility. The magnitude of the reaction of the interest rate to innovations in q_t is increasing in γ and η.

The conditional equity premium is given by

$$\begin{aligned}
\mathbb{E}_{s_t}[r_{d,t+1}] - r_{f,t} = \quad & (1-\theta_1\theta_2)k_1 k_{1,d} B_2 B_{d,2}\sigma_v^2 - \frac{1}{2}k_{1,d}^2 B_d^2 \sigma_v^2 \\
+ \; & (1-\theta_1\theta_2)k_1 k_{1,d} B_3 B_{d,3}\sigma_q^2 - \frac{1}{2}k_{1,d}^2 B_d^3 \sigma_q^2 \\
+ & \Big[(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_v - \frac{1}{2}k_{1,d}^2 B_{d,1}^2 \pi_v + \gamma\pi_d - \frac{\pi_d^2+\varphi_\sigma^2}{2}\Big] v_t \\
+ & \Big[(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_q - \frac{1}{2}k_{1,d}^2 B_{d,1}^2 \pi_q \\
& \quad + \frac{1}{2}\big(\eta(1-\gamma)-\gamma\big)\big(\gamma\pi_d - \frac{\pi_d^2+\varphi_\sigma^2}{2}\big) - \frac{1}{2}\big(\gamma\pi_d - \frac{\pi_d^2+\varphi_\sigma^2}{2}\big)^2\Big] q_t.
\end{aligned} \tag{6}$$

The similar looking terms in lines 1-4 are long run risk premia for fluctuations in the state variables (long run risk premia). They are either constant (as the variances of v_t and q_t are constant), or proportional to v_t and q_t (as is the variance of x_t). The $-\frac{1}{2}$-terms are Jensen-variance corrections. Apart from that, the investor claims a compensation for taking risk, which depends on γ, and a compensation for taking ambiguity about volatility, which depends on γ and η. Interestingly, the latter is proportional to the (short run) risk premium. Since the coefficient is negative whenever $\eta \geq \gamma \geq 1$, this "short run ambiguity premium" can only be positive if the "short run risk premium", including the Jensen-correction, is negative.

The local return variance is given by

$$\mathbb{V}_{s_t}[r_{d,t+1}] = k_{1,d}^2 B_{d,2}^2 \sigma_v^2 + k_{1,d}^2 B_{d,3}^2 \sigma_q^2 + \big(k_{1,d}^2 B_{d,1}^2 \pi_v + \pi_d^2 + \varphi_\sigma^2\big) v_t + \big(k_{1,d}^2 B_{d,1}^2 \pi_q\big) q_t, \tag{7}$$

It is affine in v_t and also in q_t if the variance of x_{t+1} in linked to q_t.

The variance premium is the difference between the risk neutral and physical expectations of the local return variance. Hence, whenever σ_t^2 is in the investor's information set at time t, the variance premium should be zero since the local return variance is known. However, Bollerslev et al. (2009) calculate a variance premium in a model with long run risks. They consider the difference between risk neutral and physical expectation of the return variance in the period between $t+1$ and $t+2$ and find that it is approximately proportional to the variance of the conditional variance. This is reasonable as long as the decision interval is shorter than the maturity of the theoretical variance contract. In our model this term is given by

$$\begin{aligned}\left(\mathbb{E}_{s_t}^Q - \mathbb{E}_{s_t}\right)\left[\mathbb{V}_{s_{t+1}}(r_{d,t+2})\right] = \quad & (\theta_1\theta_2 - 1)k_1 B_2\left(k_{1,d}^2 B_{d,1}^2 \pi_v + \pi_d^2 + \varphi_\sigma^2\right)\sigma_v^2 \\ & + (\theta_1\theta_2 - 1)k_1 B_3\left(k_{1,d}^2 B_{d,1}^2 \pi_q\right)\sigma_q^2\end{aligned}$$

We have to add the further term $\left(\mathbb{E}_{s_t}^Q - \mathbb{E}_{s_t}\right)[\mathbb{V}_{\sigma_t}(r_{d,t+1})]$ since the return variance between t and $t+1$ is uncertain in our framework. Following the approximation proposed by Bollerslev et al. (2009), this term is given by

$$\left(\mathbb{E}_{s_t}^Q - \mathbb{E}_{s_t}\right)[\mathbb{V}_{\sigma_t}(r_{d,t+1})] = \frac{1}{2}(\eta(\gamma-1)+\gamma)(\pi_d^2 + \varphi_\sigma^2)q_t, \tag{8}$$

i.e. it is proportional to the amount of ambiguity about volatility. At time t, the investor faces a variety of possible realizations of $\tilde{\sigma}_t^2$, and thus a variety of corresponding return variances. The more these return variance differ from each other, the higher is the variance premium that the investor claims. The premium rises in the investor's risk and ambiguity aversion.

5 A numerical example

In this section, we bring the model introduced in Section 4 to the data by estimating its parameters with GMM. We only use cash flow data and the approximate state variables as constructed in Section 2 to do so. We then look at the consequences for asset pricing quantities within the model. Besides unconditional moments, we put special emphasis on the predictability of returns by the variance premium and the price dividend ratio.

5.1 Estimation

We estimate the 15 model parameters inherent in the equations

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \pi_c \tilde{\sigma}_t \varepsilon^c_{t+1}, \\
\Delta d_{t+1} &= \mu_d + \varphi_x x_t + \pi_d \tilde{\sigma}_t \varepsilon^c_{t+1} + \varphi_\sigma \tilde{\sigma}_t \varepsilon^d_{t+1}, \\
x_{t+1} &= \rho_x x_t + \sqrt{\pi_v v_t + \pi_q q_t} \varepsilon^x_{t+1} \\
v_{t+1} &= \bar{v} + \rho_v (v_t - \bar{v}) + \sigma_v \varepsilon^v_{t+1} \\
q_{t+1} &= \bar{q} + \rho_q (q_t - \bar{q}) + \sigma_q \varepsilon^q_{t+1}
\end{aligned} \tag{9}$$

with GMM. For this purpose, we exclusively rely on cash flow data[4] and the time series Ex_t, $E\sigma_t^2$, and $V\sigma_t^2$ constructed in Section 2 that are supposed to approximate the model's state variables x_t, v_t, and q_t. In the estimation, we do not use asset pricing quantities such as moments of returns or valuation ratios to avoid the concern that the parameter estimates are engineered to fit asset pricing moments. We investigate the model's ability to explain these moments in the following sections and using some of it to estimate parameters would dilute the rigor of the argument.[5]

We demean Ex_t and separately estimate the unconditional mean growth rate

[4]A detailed description of the consumption and dividend data can be found in Appendix B.
[5]This is also pointed out by Nakamura et al. (2012).

μ_c. In addition, we estimate a leverage parameter π_c of consumption growth volatility to account for the low level of our $E\sigma_t^2$ estimates. The parameters π_v and π_q cannot be identified jointly, so we run two estimations in which π_q (respectively π_v) is constrained to zero. The moment conditions we use are the four conditional expectations[6] and five conditional variances arising from Equations (9), the covariance between consumption and dividend growth, and the autocovariance of dividend growth and the three state variables. To weight the moment conditions we use the efficient matrix, as proposed by Hansen (1982).

Point estimates, along with t-statistics, are reported in Table 5. The parameter π_v (π_q) is estimated given that π_q (π_v) is set to zero. Both parameters are identified solely by the variance of Ex_t such that all other parameters are estimated identically. We also report the parameters chosen by Bansal et al. (2012) (BKY, henceforth), transformed to quarterly values. Differences in parameters may point to important differences in mechanisms by which the models generate asset pricing moments. Note however that BKY calibrate the model to match empirical asset pricing moments between 1930 and 2008, while we look at the period between 1992 and 2012. The scaling factor π_c is estimated around 1.8 which indicates that our volatility measure $E\sigma_t^2$, extracted from the SPF, is downward biased.

Compared to BKY, we find a much lower level of volatility $\bar{v}$. Even after multiplication with π_c this results in a lower local volatility of consumption and dividend growth. At the same time, the variance of the predictable component x_t is much higher in our case. The first order autocorrelation of x_t is 84% on a quarterly basis and hence in the same order of magnitude as the value in BKY. For the uncertainty measures, we find autocorrelations significantly different from zero. As pointed out by Constantinides and Ghosh (2011), it is difficult to find empirical motivation for a predictable component in macroeconomic uncertainty. BKY assume

[6] Ex_t is demeaned, so we do not use the conditional mean of x_t.

a coefficient of 0.997 which is however far away from the values we estimate from SPF data. Note however, that the process $(\sigma_t^2)_t$ in BKY describes a cycle which has a half-life of several decades. Such long-lasting changes in macro-economic uncertainty are documented by Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (1999). Obviously, our sample size of 21 years is too short to detect such a component in the data.

5.2 Unconditional asset pricing moments

We use the point estimates obtained in Section 5.1 to calibrate our model and generate asset pricing moments. For that purpose, we follow the literature and assume that the investor's decision interval is monthly, i.e. we convert our quarterly point estimates to monthly parameters. Solely looking at cash flows does not allow to draw inference on preference parameters in an endowment economy. We set the preference parameters regarding intertemporal fluctuations as in BKY: The investor is impatient ($\delta = -\log(0.9989)$) and has an EIS above one ($\rho = \frac{1}{1.5}$). We look at two different specifications for the uncertainty attitude parameters: In the first, the investor is assumed to be ambiguity neutral and risk averse ($\gamma = \eta = 10$) as in BKY. In the second, she is mildly risk averse ($\gamma = 2$) and ambiguity averse ($\eta = 24$). These values are in line with the estimates of Thimme and Völkert (2014), who estimate both parameters given various levels of the EIS.

After solving for the coefficients in Equations (4), we sample from the calibrated model and construct empirical distributions for the asset pricing quantities of interest. We draw 10,000 paths with 121×12 periods and always discard the first 100×12 periods. The monthly data are then aggregated to an annual frequency. Tables 6 and 7 report medians of the monte carlo distributions, along with 90% confidence bounds for the case of an ambiguity neutral investor and an ambiguity averse investor, respectively. In the first columns, we also report the empirical counterparts of the asset pricing moments as observed in the data. Obviously, these moments differ from the values that most asset pricing studies refer to, since we only rely on data from 1992 to 2012. In particular, interest rates are low, while price dividend ratios are high and less volatile compared to earlier samples.

In the second columns of both tables we present asset pricing moments produced by a model without ambiguity about volatility. This model is specified exactly as the model of BKY. The third columns list results if there is ambiguity about volatility but it does not impact uncertainty about future growth rates. In columns four and five, results are reported if uncertainty about x_{t+1} is affine in both v_t and q_t, and in q_t alone, respectively.[7] Varying these parameters allows to trade a long run risk premium for a long run ambiguity premium.

Comparing columns two and three shows that an introduction of ambiguous volatility leads to a decrease of the equity premium if the investor is ambiguity neutral and an increase if she is ambiguity averse. As explained in Section 4.3, the short run premium for ambiguity about volatility is negative if risk-aversion is high. This leads to a decrease in excess returns in Table 6 if ambiguous volatility is introduced. If risk aversion is low, the short run ambiguity premium is positive and increasing in the investor's ambiguity attitude parameter η. This leads to an increase in excess returns if ambiguity about volatility comes into play. If $\pi_q = 0$, there is no long run ambiguity premium, i.e. a compensation for fluctuations in stock prices due to fluctuations in the growth rate x_t. Opening this channel, i.e. letting $\pi_q \neq 0$, increases the equity premium considerably, as can be seen in columns four and five. In case of an ambiguity averse investor, this leads to a sizable premium for ambiguity about volatility which lets the model-implied equity premium match or even overshoot the one observed in the data. If $\pi_v = 0$ the long run risk

[7] Column 4 labeled $\pi_v \bar{v} = \pi_q \bar{q}$ comprehends results if we set $\pi_v = \frac{1}{2}\tilde{\pi}_v$ and $\pi_q = \frac{1}{2}\tilde{\pi}_q$ to one half of the estimated values. This implies that the unconditional volatility of x_t is the same for all four investigated cases.

premium vanishes and, given ambiguity aversion, the short run risk premium is slightly negative which is in line with the findings from Section 3. Due to the high volatility of x_t the return volatility is high in all considered specifications.

Interest rates are not affected by the introduction of ambiguous volatility, as long as $\pi_q = 0$. In general, they are low and volatile, especially if the investor is ambiguity averse. In this case, the decrease in autocorrelation shows that the risk-free rate is more exposed to q_t given $\pi_v = 0$, as to v_t given $\pi_q = 0$.[8] This in turn means that there is a large precautionary savings motive due to ambiguity about volatility, which exceeds the precautionary savings motive due to expected volatility. If uncertainty about the growth rate is linked to ambiguity about volatility, this leads to an interest rate of only 0.55%, given ambiguity aversion.

Just as interest rates, price dividend ratios are unaffected by the introduction of ambiguous volatility, as long as $\pi_q = 0$. Setting $\pi_v = 0$ instead leads to a large decrease in price dividend ratios, which is worth noting, because the unconditional variance of x_t is equal in both cases by construction. Innovations to ambiguity about volatility are more severe to the investor than innovations in expected volatility. A one standard deviation shock in v_t leads to a decrease of only 0.0164 in the log price dividend ratio if $\pi_q = 0$, whereas a one standard deviation shock in q_t leads to a decrease of 0.0644, if $\pi_v = 0$.[9] In equilibrium, this difference leads to much lower price dividend ratios in case of $\pi_v = 0$, especially if the investor is ambiguity averse.

A one standard deviation shock in x_t lets the log price dividend ratio increase by 0.097 if $\pi_q = 0$ and by 0.085 if $\pi_v = 0$. In both cases, innovations in x_t are the driving force for innovations in the log price dividend ratio, which is in line with our findings in Section 3. In all investigated cases the volatility of the price dividend ratio is rather high due to the volatile growth rate x_t. Beeler and Campbell (2012)

[8] In terms of Equation (5), this means that the unconditional mean of line 2 given $\pi_q = 0$ is lower than the unconditional mean of line 3, given $\pi_v = 0$.

[9] These numbers are calculated given an ambiguity averse investor and expressed in monthly terms.

emphasize that the low volatility of the price dividend ratio is a major shortcoming of the BKY model.

If ambiguity about volatility is neglected, the variance premium is constant. If there is ambiguity, the magnitude of the time-varying part only depends on the preference parameters and is generally very small. This leads to a volatility of the variance premium which is virtually zero and an autocorrelation coefficient which is equal to the autocorrelation coefficient of q_t. The variance premium could easily be made more volatile by allowing for time-varying volatility levels σ_v and σ_q of v_t and q_t, for example by introducing square root processes (see e.g. Zhou and Zhu (2014)). The level of the variance premium is the higher the closer the link between q_t and uncertainty about the growth rate x_t, i.e. the larger the impact of ambiguity about volatility on the economy.

The values in Table 7 show that it is possible to match the magnitude of the variance premium in an affine model, i.e. without introducing jumps as suggested by Todorov (2010) and Drechsler and Yaron (2011). Overall, even if there is not the single calibration that matches all moments perfectly, we find that our model is readily able to explain asset pricing moments observed in the data. It is important to keep in mind that we estimate all cash flow parameters from observable cash flow and state variable time series and chose the preference parameters as estimated in the literature. Ambiguity about volatility, especially in combination with ambiguity aversion, leads to high and volatile excess returns, low interest rates, and a sizable variance premium. These findings are in line with the stylized facts in Section 3.

5.3 Return predictability

In this section we analyze the covariation of different asset pricing quantities across time. More specifically, we investigate if the model-implied price dividend ratio and variance premium predict excess returns. In the data we observe that price dividend ratios predict excess returns over long horizons of several years. The rationale provided by the long run risks model of Bansal and Yaron (2004) is that the price dividend ratio decreases with positive innovations in volatility σ_t. At the same time, risk premia are proportional to σ_t^2. The key to explain predictability is that σ_t^2 is very persistent such that once risk premia are on a high level, they stay there for several years. In the model of BKY, a shock to σ_t^2 has a half-life of 57.7 years. As mentioned in Section 5.1, our model does not feature such a highly persistent uncertainty measure. Hence, it is not capable of explaining predictability of returns over long horizons. Using the simulated samples from Section 5.2, we investigate the model-implied predictability of returns over several horizons h by the price dividend ratio, i.e. we perform regressions

$$r_{d,t+h} - r_{f,t} = \alpha(h) + \beta(h)(p_t - d_t) + \varepsilon_{t+h}.$$

Our results closely match the data, i.e. the predictive power is low at short horizons and increases with h (not reported). This finding however relies on the brevity of the drawn samples which are fitted to the observed sample length of 21 years, and are due to "overfitting". Increasing the length of the sample corroborates that the model-implied price dividend ratio does not predict excess returns over long horizons. Moreover, it also does not even predict returns over short horizons. The price dividend ratio mainly fluctuates due to innovations in x_t, such that in our model, at least if calibrated as in Section 5.2 with low ρ_v and ρ_q, it only predicts cash-flows but not returns.

The uncertainty measures v_t and q_t have a very short half-life and may thus be suited to explain return predictability over short horizons. As documented by Bollerslev et al. (2009), Bollerslev et al. (2011), and Drechsler and Yaron (2011), the predictive power of the variance premium is highest for short horizons. We analyze the model-implied predictability of the variance premium by performing regressions

$$r_{d,t+h} - r_{f,t} = \alpha(h) + \beta(h)\ vp_t + \varepsilon_{t+h}, \tag{10}$$

where vp_t denotes the variance premium. The results are listed in Table 8. They are robust to increases in sample length. The first column quotes the regression coefficients $\beta(h)$, the corresponding t-statistics (Newey and West (1987) corrected), and the R^2 of regression equation (10) for several horizons of h months, as observed in the data. As described above, the predictability is highest for short horizons, it peaks at the 3-months horizon, and declines in h afterwards. There is no significant predictability for horizons of two years and more.

In columns 2-5 we analyze if the model-implied variance premium predicts model-implied excess returns. In all four cases, the variance premium is affine in q_t. As shown in Section 5.2, the variance of the model-implied variance premium is too low, such that the regression coefficients cannot match those listed in column 1. It is however the time-series behavior we want to explain. Choosing a square root specification for q_t would arguably lift the variance while leaving the time-series behavior untouched. As we learn from columns 2 and 4, the model-implied variance premium does not predict returns if $v_q = 0$. In this case, there is only a short run ambiguity premium which depends on q_t. This premium is small in magnitude compared to the high risk premium which depends on v_t. Column 3 shows that allowing for a long run ambiguity premium alone does not solve this problem. If the investor is ambiguity neutral, the equity premium comprises a rather large premium for short run cash flow risk which is proportional to σ_t^2 and, hence, to v_t unconditionally. This component is considerably smaller if the investor is only mildly risk averse, as in our ambiguity averse calibration in which $\gamma = 2$. Setting $\eta = 24$ moreover increases the long run ambiguity premium. In this calibration, a large part of the equity premium is a premium for ambiguity about volatility, which in turn is proportional to q_t. This prediction of the model is in line with the empirical findings from Section 3.

Column 5 shows that assuming ambiguity aversion and thereby lifting the ambiguity premium leads to a realistic predictability pattern: The predictive power of the variance premium is highest for the 3 month horizon with an R^2 of about 11% and deteriorates for longer horizons. At the three year horizon, the regression coefficient looses significance. This finding, in combination with the empirical results from Section 3, strongly suggests that there is a large positive premium for ambiguity about macroeconomic volatility.

6 Conclusion

We have defined an intuitive measure that captures time-variation in the amount of ambiguity about macroeconomic volatility. This measure predicts excess returns and explains time-variation in the variance premium. These findings can be explained within a general equilibrium asset pricing model with long run risks. Our model is able to generate sizable equity and variance premia with the help of two key ingredients:

1. The investor has to be ambiguity averse.

2. There has to be a long run ambiguity premium, i.e. uncertainty about the growth rate x_t has to be tied to ambiguity about volatility.

The latter assumption is the channel by which high premia can be generated in long run risks models. It can hence be interpreted as lifting the ambiguity premium as against the risk premium. This mechanism is crucial to generate a realistic return predictability pattern by the variance premium. The implications of this finding is in line with the predictive power of our uncertainty measures. We find a large positive premium for ambiguity about volatility, while the risk premium is at best zero.

Interestingly, our proxy for ambiguity about trend growth does not explain or forecast any asset pricing quantity. This measure is common in the literature as a proxy for ambiguity in general. It is however apparent that ambiguity about trend growth distinguishes from ambiguity about volatility. Based on their economic relevance, our results indicate that future research should attach the same importance to ambiguous volatility as to ambiguous growth. One direction is to study the impact of ambiguous volatility in general equilibrium asset pricing models. Our model could be a starting point in the context of the long run risks literature. Different, possibly richer, dynamics of the uncertainty processes may yield further insights about the importance of ambiguous volatility. We hypothesized that a square-root specification of the ambiguity process may generate a more realistic variance of the variance premium. But more sophisticated dynamics should also be considered. The properties of our ambiguity about volatility-measure suggest a non-Gaussian distribution. It is e.g. interesting to analyze the impact of large positive innovations (jumps) to ambiguous volatility in the course of extreme events such as the recent financial crisis.

Ambiguity about macroeconomic- or stock return volatility should also be analyzed more exhaustively by empirical studies. Unfortunately, our analysis is restricted to a rather short sample. Future research should come up with other empirical proxies that measure ambiguity about volatility and analyze their relation with stock returns and other asset pricing quantities.

A Model solution

We impose an affine guess for the log wealth-consumption ratio

$$z_t = A + B' s_t = A + B_1 x_t + B_2 v_t + B_3 q_t, \tag{11}$$

and approximate the log return on the claim to aggregate consumption with the linearization of Campbell and Shiller (1988)

$$r_{w,t+1} = k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{12}$$

where k_0 and k_1 are linearizing constants. It holds that $k_0 = \log(1 - e^{\bar{z}}) - k_1 \bar{z}$ and $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$, where $\bar{z}$ is the long run mean of the log wealth-consumption ratio. Introducing Equations (11) and (12) into the Euler equation $1 = \mathbb{E}_{s_t}[\xi_{t,t+1} R_{w,t+1}]$ and solving for the coefficients yields

$$\begin{aligned}
A &= \frac{1}{1-k_1}\left(-\delta + k_0 + (1-\rho)\mu_c + k_1 B_2 (1-\rho_v)\bar{v} + k_1 B_3 (1-\rho_q)\bar{q} + \frac{1}{2}\theta_1\theta_2 k_1^2 (B_2^2 \sigma_v^2 + B_3^2 \sigma_q^2)\right), \\
B_1 &= \frac{1-\rho}{1-k_1\rho_x} \\
B_2 &= \frac{1}{2(1-k_1\rho_v)}\big(\theta_1\theta_2 k_1^2 B_1^2 \pi_v + (1-\rho)(1-\gamma)\big) \\
B_3 &= \frac{1}{2(1-k_1\rho_q)}\big(\theta_1\theta_2 k_1^2 B_1^2 \pi_q + \frac{1}{4}(1-\rho)(1-\gamma)^2(1-\eta)\big)
\end{aligned}$$

With these coefficients at hand we calculate coefficients for the log risk-free interest rate $r_{f,t} = -\log \mathbb{E}_{Y_t}\mathbb{E}_{X_t}[\xi_{t,t+1}]$ which are given in Equations (5). The price dividend ratio $z_{d,t} = A_d + B_d' Y_t$ can be calculated similar to the wealth-consumption ratio. Its constant coefficient is

$$\begin{aligned}
A_d = &\frac{1}{1-k_{1,d}}\Big(-\delta + k_{0,d} + \mu_d - \rho\mu_c + k_{1,d} B_{d,2}(1-\rho_v)\bar{v} + k_{1,d} B_{d,3}(1-\rho_q)\bar{q} \\
&+ \frac{1}{2}(1-\theta_1\theta_2)k_1^2(B_2^2\sigma_v^2 + B_3^2\sigma_q^2) - (1-\theta_1\theta_2)k_1 k_{1,d}(B_2 B_{d,2}\sigma_v^2 + B_3 B_{d,3}\sigma_q^2) + \frac{1}{2}k_{1,d}^2(B_{d,2}^2\sigma_v^2 + B_{d,3}^2\sigma_q^2)\Big)
\end{aligned}$$

while B_d is given by

$$\begin{aligned}
B_{d,1} =& \frac{\varphi_x - \rho}{1 - k_{1,d}\rho_x} \\
B_{d,2} =& \frac{1}{2(1-k_{1,d}\rho_v)}\Big((1-\theta_1\theta_2)k_1^2 B_1^2 \pi_v - 2(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_v + k_{1,d}^2 B_{d,1}^2 \pi_v \\
&+ (\pi_d - \gamma)^2 + (\gamma - \rho)(1-\gamma) + \varphi_\sigma^2\Big) \\
B_{d,3} =& \frac{1}{2(1-k_{1,d}\rho_q)}\Big((1-\theta_1\theta_2)k_1^2 B_1^2 \pi_q - 2(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_q + k_{1,d}^2 B_{d,1}^2 \pi_q \\
&+ \frac{1}{4}\big(\big[(\pi_d-\gamma)^2 + (\gamma-\eta)(1-\gamma) + \varphi_\sigma^2\big]^2 + (\eta-\rho)(1-\gamma)^2(1-\eta)\big)\Big)
\end{aligned}$$

Introducing these coefficients into Equation (12) yields a representation of the return on the dividend claim from which the conditional equity premium and the return volatility can easily be calculated. The formulae can be found in Equations (6) and (7).

B Data

We use quarterly data from the first quarter of 1992 to the fourth quarter of 2012.

Consumption: We use data from NIPA Table 2.3.5 released by the Bureau of Economic Analysis (`www.bea.gov/iTable/index_nipa.cfm`). The data is seasonally adjusted at annual rates. We only use nondurables and services and transform to 2012 U.S. dollars by adjusting with the Consumer Price Index (CPI). We obtain the CPI from the Bureau of Labor Statistics (`www.bls.gov/cpi`). We divide by a one year moving average of U.S. population to calculate real per capita consumption. We use a one-year moving average due to the strong seasonality in U.S. population growth. Data about U.S. population is from NIPA Table 7.1. For the predictive regressions in Section 3 we use consumption growth in the quarters that followed the quarter in which the respective surveys were published.

Dividends: Dividends are taken from the homepage of Robert Shiller at Yale (`www.econ.yale.edu/~shiller/data.htm`). It comprehends real dividends of all firms that are listed in the S&P Composite Index. To calculate real growth rates we subtract log dividends of the preceding month from log dividends of the current month. For the predictive regressions in Section 3 we add log growth rates of the six months following the month in which the respective survey was published.

Price-dividend ratio: We use the price dividend ratio from the same table as the dividends, i.e. from Robert Shiller's homepage (`www.econ.yale.edu/~shiller/data.htm`). For the regressions in Section 3 we use log price dividend ratios from February, May, August, and November, i.e. the months in which the respective surveys were published.

Risk-free rate: The 3-month secondary market Treasury bill rate is taken from the H.15 release of the Federal Reserve Board of Governors (`http://www.federalreserve.gov/releases/h15/data.htm`) as risk-free rate. To calculate real rates, we proceed as Beeler and Campbell (2012) and Constantinides and Ghosh (2011), i.e. we regress the ex post real yield on a 3-month Treasury bill on the three-month nominal yield and the realized growth in the CPI and use the fitted value as ex ante real rate. For the regressions in Sections 3 we use the rates from February, May, August, and November, i.e. the months in which the surveys were published.

Variance premium: We use data from the personal homepage of Hao Zhou (`https://sites.google.com/site/haozhouspersonalhomepage/`). We use the difference between risk-neutral and physical expectation, where the latter is calculated with the help of a time-series model as explained in Bollerslev et al. (2009). For the regressions in Section 3 we use variance premia from February, May, August, and November, i.e. the months in which the surveys were published.

Stock returns: Monthly excess returns are taken from Kenneth French's homepage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`), which is based on the CRSP value-weighted stock return index. For the predictive regressions in Sections 3 we add log excess returns of the twelve (respectively six, 24, or 60) months following the month in which the respective survey was published.

Return volatility: We use the daily excess return from Kenneth French's homepage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`), which is based on the CRSP value-weighted stock return index. To calculate realized variance, we follow Anderson et al. (2009) and employ the formula

$$\hat{\sigma_t^2} = \sqrt{\frac{1}{n-1}\left(\sum_{t=1}^{n}(r_t - \bar{r})^2 + 2\sum_{t=2}^{n}(r_{t-1} - \bar{r})(r_t - \bar{r})\right)},$$

i.e. we allow for the effect of serial correlation in daily returns. $\bar{r}$ denotes the mean of the n daily returns considered. For the regressions in Section 3 we use daily returns from the 66 trading days after the end of the month in which the respective surveys were published.

C Robustness

To encounter possible endogeneity problems in the contemporary regressions we run two stage least squares regressions and include lagged cross-sectional moments as instruments. The results remain widely unchanged: The price dividend ratio covaries positively with Ex_t and interest rates covary negatively with $E\sigma_t^2$ and $V\sigma_t^2$.

In the return regressions, we add the price dividend ratio and the variance premium as control variables and find that the coefficients of $V\sigma_t^2$ stays significant. Without $E\sigma_t^2$ and $V\sigma_t^2$, the price dividend ratio and the variance premium explain 22.09% of the variation in annual stock returns. Including both variables yields an R^2 of 32.88%.

One concern about our proxies might be that there is time variation due to a varying number of analysts featured in the different surveys. We add the number of analysts as a further control variable in all regressions and find that it does not change our results in any case. There is no considerable correlation between the time series of the number of analysts and any of the cross-sectional moments.

To avoid a strong impact of extreme outliers we winsorize all time series. Omitting this step does not change our results perceivably. We also use variances instead of standard deviations in all regressions. The signs of the coefficients usually remain unchanged. While the results of the predictive regressions are similar to those with standard deviations, the uncertainty measures gain significance in explaining the price dividend ratio while they loose significance in explaining interest rates and the variance premium. Results are available upon request.

References

Andersen, E., L. Hansen, and T. Sargent (2000): "Robustness, Detection and the Price of Risk," Working Paper, University of Chicago.

Anderson, E., E. Ghysels, and J. Juergens (2009): "The Impact of Risk and Uncertainty on Expected Returns," *Journal of Financial Economics*, 94, 233–263.

Bansal, R., D. Kiku, and A. Yaron (2012): "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *Critical Finance Review*, 1, 183–221.

Bansal, R. and I. Shaliastovich (2010): "Confidence Risk and Asset Prices," *American Economic Review, Papers and Proceedings*, 100, 537–541.

Bansal, R. and A. Yaron (2004): "Risks for the Long-Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Beeler, J. and J. Campbell (2012): "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1, 141–182.

Bollerslev, T., M. Gibson, and H. Zhou (2011): "Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-implied and Realized Volatilities," *Journal of Econometrics*, 160, 235–245.

Bollerslev, T., N. Sizova, and G. Tauchen (2012): "Volatility in Equilibrium: Asymmetries and Dynamic Dependencies," *Review of Finance*, 16, 31–80.

Bollerslev, T., G. Tauchen, and H. Zhou (2009): "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22, 4463–4492.

Campbell, J. and R. Shiller (1988): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–228.

Carr, P. and R. Lee (2009): "Volatility Derivatives," *Annual Review of Financial Economics*, 1, 319–339.

Colacito, R., E. Ghysels, and J. Meng (2012): "Skewness in Expected Macro Fundamentals and the Predictability of Equity Returns: Evidence and Theory," Working Paper.

Collard, F., S. Mukerji, K. Sheppard, and J. Tallon (2011): "Ambiguity and the Historical Equity Premium," Working Paper.

Constantinides, G. and A. Ghosh (2011): "Asset Pricing Tests with Long Run Risks in Consumption Growth," *Review of Asset Pricing Studies*, 1, 96–136.

Drechsler, I. (2013): "Uncertainty, Time-Varying Fear, and Asset Prices," *Journal of Finance*, 68, 1843–1889.

Drechsler, I. and A. Yaron (2011): "What's Vol Got to Do With It," *Review of Financial Studies*, 24, 1–45.

Epstein, L. and S. Ji (2013): "Ambiguous Volatility and Asset Pricing in Continuous Time," *Review of Financial Studies*, 26, 1740–1786.

Epstein, L. and M. Schneider (2010): "Ambiguity and Asset Markets," *Annual Review of Financial Economics*, 2, 315–346.

Epstein, L. and S. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behaviour of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Eraker, B. and I. Shaliastovich (2008): "An Equilibrium Guide to Designing Affine Pricing Models," *Mathematical Finance*, 18, 519–543.

Etner, J., M. Jeleva, and J. Tallon (2012): "Decision Theory under Ambiguity," *Journal of Economic Surveys*, 26, 234–270.

Gilboa, I. and D. Schmeidler (1989): "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, 18, 141–153.

Guidolin, M. and F. Rinaldi (2013): "Ambiguity in Asset Pricing and Portfolio Choice: A Review of the Literature," *Theory and Decision*, 74, 183–217.

Hansen, L. (1982): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029–1084.

Hayashi, T. and J. Miao (2011): "Intertemporal Substitution and Recursive Smooth Ambiguity Preferences," *Theoretical Economics*, 6, 423–472.

Jahan-Parvar, M. and H. Liu (2014): "Ambiguity Aversion and Asset Prices in Production Economies," *Review of Financial Studies*, forthcoming.

Jin, J. (2013): "Jump-Diffusion Long-Run Risks Models, Variance Risk Premium and Volatility Dynamics," Working Paper.

Ju, N. and J. Miao (2012): "Ambiguity, Learning, and Asset Returns," *Econometrica*, 80, 559–591.

Kim, C. and C. Nelson (1999): "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-switching Model pf the Business Cycle," *Review of Economics and Statistics*, 81, 608–616.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005): "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, 76, 1849–1892.

——— (2009): "Recursive Smooth Ambiguity Preferences," *Journal of Economic Theory*, 144, 930–976.

Kreps, D. and E. Porteus (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46, 185–200.

Lettau, M. and S. Van Nieuwerburgh (2008): "Reconciling the Return Predictability Evidence," *Review of Financial Studies*, 21, 1607–1652.

Lucas, R. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 48, 1149–1168.

McConnell, M. and G. Perez-Quiros (2000): "Output Fluctuations in the United States: What has Changed since the Early 1980's?" *American Economic Review*, 90, 1464–1476.

Miao, J., B. Wei, and H. Zhou (2012): "Ambiguity Aversion and Variance Premium," Working Paper.

Nakamura, E., D. Sergeyev, and J. Steinsson (2012): "Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence," Working Paper.

Newey, W. and K. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–708.

Patton, A. and A. Timmermann (2010): "Why do Forecasters Disagree? Lessons from the Term Structure of Cross-Sectional Dispersion," *Journal of Monetary Economics*, 57, 803–820.

Stock, J. and M. Watson (1999): "Has the Business Cycle Changed and Why?" *NBER Macroeconomics Annual 2002*, 17, 159–230.

Thimme, J. and C. Völkert (2012): "High Order Smooth Ambiguity Preferences and Asset Prices," Working Paper, University of Müenster.

——— (2014): "Ambiguity in the Cross-Section of Expected Returns: An Empirical Assessment," *Journal of Business and Economic Statistics*, forthcoming.

Todorov, V. (2010): "Variance Risk Premium Dynamics: The Role of Jumps," *Review of Financial Studies*, 23, 345–383.

Ulrich, M. (2011): "Observable Long-Run Ambiguity and Long-Run Risk," Working Paper, Columbia Business School.

——— (2012): "The Real Risk-Free Rate and the Price of Equity: What Role does Macroeceonomic Uncertainty Play?" Working Paper, Columbia Business School.

Zarnowitz, V. and P. Braun (1993): "Twenty-two Years of the NBER-ASA Quarterly Economic Outlook Surveys: Aspects and Comparisons of Forecasting Performance," in *Business Cycles, Indicators, and Forecasting, NBER Studies in Business Cycles*, ed. by J. Stock and M. Watson, Chicago: University of Chicago Press, vol. 28.

Zhou, G. and Y. Zhu (2014): "Macroeconomic Volatilities and Long-run Risks of Asset Prices," Working Paper.

	mean	std	AC(1)	skew	kurt
Ex_t	1.55×10^{-2}	3.20×10^{-3}	0.8272	-1.6312	4.2243
$E\sigma_t^2$	5.72×10^{-5}	2.92×10^{-6}	0.2256	0.3541	1.2607
Vx_t	2.16×10^{-5}	1.14×10^{-6}	0.6186	3.0978	13.2306
$V\sigma_t^2$	6.42×10^{-9}	3.00×10^{-10}	0.3427	2.4945	10.2119
Δc_{t+1}	1.53×10^{-2}	7.56×10^{-3}	-0.1345	-0.5389	1.5224

Table 1: **Descriptive statistics of the extracted time series**

This table shows descriptive statistics of the time series $(Ex_t)_t$, $(E\sigma_t^2)_t$, $(Vx_t)_t$, $(V\sigma_t^2)_t$, and log consumption growth. The means are annualized, all other moments are on a quarterly basis. The sample period is from the first quarter of 1992 to the fourth quarter of 2012. *AC(1)* denotes first order autocorrelation. *skew* and *kurt* denote skewness and excess kurtosis. The data on consumption growth is described in Appendix B.

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	R^2	$\overline{R^2}$
$p_t - d_t$	**60.00**	150.39	113.10	-97.30	23.46	19.58
	[3.68]	[1.13]	[1.33]	[-0.62]		
$p_t - d_t$	**50.52**				20.57	19.60
	[2.87]					
$r_{f,t}$	0.27	**-5.00**	0.69	-1.77	20.23	16.19
	[1.10]	[-2.60]	[0.32]	[-0.92]		
$r_{f,t}$		**-6.15**			17.22	16.21
		[-3.65]				
$r_{f,t}$				**-3.60**	6.16	5.01
				[-1.99]		
vp_t	-0.52	10.37	-6.42	**10.81**	5.41	0.62
	[-0.64]	[1.61]	[-1.48]	[1.73]		
vp_t				**13.62**	3.20	2.02
				[2.27]		

Table 2: **Contemporaneous regression results 1**

This table presents results of regressions of the log price dividend ratio, the log real risk-free rate, and the variance premium on Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. A detailed description of the data can be found in Appendix B. Columns 2-5 report estimated coefficients together with Newey and West (1987) t-statistics in brackets. Columns 6 and 7 report R^2 and adjusted R^2.

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	R^2	$\overline{R^2}$
Δc_{t+1}	**1.13**	-4.16	3.49	-1.41	9.04	4.31
	[2.21]	[-1.13]	[0.84]	[-0.28]		
Δc_{t+1}	**1.05**				6.95	5.79
	[2.50]					
Δd_{t+1}	4.55	14.17	**-69.11**	9.82	38.00	34.87
	[1.32]	[1.07]	[-3.11]	[0.57]		
Δd_{t+1}	**7.66**				20.28	19.31
	[1.73]					
Δd_{t+1}			**-81.17**		30.91	30.07
			[-3.08]			

Table 3: **Predictive regression of cash flows**

This table presents results of regressions of log consumption growth and log dividend growth on Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. A detailed description of the data can be found in Appendix B. Columns 2-5 report estimated coefficients together with Newey and West (1987) t-statistics in brackets. Columns 6 and 7 report R^2 and adjusted R^2.

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	R^2	$\overline{R^2}$
$r_{d,t+3} - r_{f,t}$		-28.53	-19.67	50.94	2.47	-1.19
		[-0.69]	[-0.66]	[1.36]		
$r_{d,t+6} - r_{f,t}$		**-110.23**	-63.67	**171.07**	13.12	9.86
		[-1.68]	[-1.23]	[2.58]		
$r_{d,t+6} - r_{f,t}$		-53.91			1.25	0.04
		[-1.01]				
$r_{d,t+6} - r_{f,t}$				**114.32**	5.86	4.71
				[2.41]		
$r_{d,t+12} - r_{f,t}$		-141.67	-82.96	**176.40**	8.69	5.17
		[-1.34]	[-0.93]	[2.51]		
$r_{d,t+24} - r_{f,t}$		-182.88	54.71	**203.91**	5.07	1.22
		[-1.42]	[0.37]	[2.00]		
$r_{d,t+36} - r_{f,t}$		-93.86	167.30	-7.79	2.18	-2.01
		[-0.57]	[0.98]	[-0.05]		
$\sigma(r_{d,t+3})$	-0.28	**6.07**	2.34	0.73	15.39	11.11
	[-0.75]	[2.07]	[1.24]	[0.30]		
$\sigma(r_{d,t+3})$		**7.54**			11.68	10.60
		[2.66]				

Table 4: **Predictive regression of returns and return volatility**

This table presents results of regressions of excess returns on the and the return volatility on Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. A detailed description of the data can be found in Appendix B. Columns 2-5 report estimated coefficients together with Newey and West (1987) t-statistics in brackets. Columns 6 and 7 report R^2 and adjusted R^2.

Aggregate consumption growth				
Parameter	μ_c		π_c	
Estimate	3.76×10^{-3}		1.80	
	$[7.07\times10^{-4}]$		[0.23]	
BKY	4.5×10^{-3}			
Aggregate dividend growth				
Parameter	μ_d	ϕ_x	π_d	φ_σ
Estimate	5.19×10^{-3}	6.43	0.51	3.15
	$[4.73\times10^{-3}]$	[0.73]	[0.90]	[0.44]
BKY	4.5×10^{-3}	2.5	2.6	5.96
Trend consumption growth				
Parameter		ρ_x	π_v	π_q
Estimate		0.84	0.22	7.64×10^{3}
		[0.06]	[0.12]	$[4.10\times10^{3}]$
BKY		0.93	1.44×10^{-3}	
Expected consumption growth variance				
Parameter	$\bar{v}$	ρ_v	σ_v	
Estimate	1.43×10^{-5}	0.23	2.84×10^{-6}	
	$[4.40\times10^{-7}]$	[0.08]	$[2.24\times10^{-7}]$	
BKY	1.56×10^{-4}	0.997	4.85×10^{-6}	
Ambiguity about consumption growth variance				
Parameter	$\bar{q}$	ρ_q	σ_q	
Estimate	4.04×10^{-10}	0.34	2.81×10^{-10}	
	$[4.50\times10^{-11}]$	[0.07]	$[5.98\times10^{-11}]$	

Table 5: **Estimated model parameters**

This table shows GMM estimates of the parameters of the model of BKY. It quotes point estimates together with HAC standard errors are in parenthesis. The values of the respective parameters as chosen by BKY are given below, as long as their model features the respective parameter. The data on consumption and dividend growth is described in Appendix B.

Moment	**Data**	$\bar{q} = \sigma_q = \pi_q = 0$	$\pi_q = 0$	$\pi_v \bar{v} = \pi_q \bar{q}$	$\pi_v = 0$
Excess return on equity					
mean	5.38	1.42	1.38	1.55	2.07
		[-13.21,14.51]	[-13.16,14.48]	[-14.02,15.80]	[-15.11,17.50]
std	22.16	33.94	33.99	34.95	35.98
		[23.53,45.64]	[23.25,45.50]	[24.33,47.99]	[24.47,50.56]
$AC1$	-0.07	-0.06	-0.06	-0.05	-0.06
		[-0.43,0.36]	[-0.43,0.36]	[-0.42,0.36]	[-0.43,0.35]
Risk-free interest rate					
mean	0.13	1.66	1.66	1.61	1.58
		[0.02,3.13]	[0.02,3.13]	[-0.18,3.21]	[-0.36,3.28]
std	1.53	1.95	1.95	2.02	2.09
		[1.26,3.03]	[1.26,3.03]	[1.30,3.10]	[1.33,3.21]
$AC1$	0.71	0.51	0.51	0.50	0.50
		[0.13,0.77]	[0.13,0.77]	[0.13,0.77]	[0.12,0.77]
Price dividend ratio					
mean	3.96	4.79	4.78	4.67	4.35
		[4.50,5.04]	[4.50,5.04]	[4.37,4.94]	[4.03,4.66]
std	0.29	0.35	0.35	0.36	0.37
		[0.23,0.51]	[0.23,0.51]	[0.24,0.54]	[0.23,0.55]
$AC1$	0.68	0.42	0.42	0.42	0.42
		[-0.02,0.72]	[-0.02,0.72]	[-0.02,0.72]	[0.01,0.72]
Variance premium					
mean	18.29	0.13	0.13	0.52	1.92
		[0.13,0.13]	[0.13,0.13]	[0.52,0.52]	[1.92,1.92]
std	22.58	0.00	0.00	0.00	0.00
		[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
$AC1$	0.28	-	0.64	0.64	0.64
			[0.52,0.73]	[0.52,0.73]	[0.52,0.73]

Table 6: **Unconditional asset pricing moments given ambiguity neutrality**

This table presents asset pricing moments from the data between 1992:Q1 and 2012:Q4 (column 1) and as a result of simulations of the calibrated model (columns 2-5), given an investor with $\gamma = \eta = 10$. The data is described in Appendix B. The median values and 90% confidence intervals (in brackets) reported in columns 2-5 are from 10,000 simulation runs of equivalent length to the data.

Moment	**Data**	$\bar{q} = \sigma_q = \pi_q = 0$	$\pi_q = 0$	$\pi_v \bar{v} = \pi_q \bar{q}$	$\pi_v = 0$
Excess return on equity					
mean	5.38	3.54	3.63	6.54	15.42
		[-10.60,16.29]	[-10.65,16.27]	[-8.18,20.12]	[1.03,28.70]
std	22.16	33.30	33.45	33.55	32.49
		[23.17,44.48]	[23.02,44.60]	[23.31,45.98]	[22.25,45.44]
$AC1$	-0.07	-0.06	-0.06	-0.07	-0.09
		[-0.43,0.36]	[-0.44,0.37]	[-0.45,0.36]	[-0.48,0.32]
Risk-free interest rate					
mean	0.13	0.99	0.99	0.82	0.55
		[-0.66,2.47]	[-0.66,2.47]	[-1.02,2.42]	[-1.37,3.28]
std	1.53	1.96	1.96	2.04	2.16
		[1.27,3.02]	[1.27,3.02]	[1.34,3.12]	[1.40,3.28]
$AC1$	0.71	0.50	0.50	0.49	0.47
		[0.13,0.77]	[0.13,0.77]	[0.12,0.77]	[0.08,0.75]
Price dividend ratio					
mean	3.96	3.77	3.77	2.97	1.99
		[3.49,4.01]	[3.48,4.02]	[2.68,3.23]	[1.70,2.26]
std	0.29	0.34	0.34	0.35	0.34
		[0.23,0.51]	[0.23,0.51]	[0.23,0.52]	[0.21,0.51]
$AC1$	0.68	0.42	0.42	0.41	0.40
		[-0.02,0.72]	[-0.03,0.72]	[-0.02,0.72]	[0.00,0.71]
Variance premium					
mean	18.29	0.73	0.73	2.87	8.66
		[0.73,0.73]	[0.73,0.73]	[2.87,2.87]	[8.66,8.66]
std	22.58	0.00	0.00	0.00	0.00
		[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
$AC1$	0.28	-	0.64	0.64	0.64
			[0.52,0.73]	[0.52,0.73]	[0.52,0.73]

Table 7: **Unconditional asset pricing moments given ambiguity aversion**

This table presents asset pricing moments from the data between 1992:Q1 and 2012:Q4 (column 1) and as a result of simulations of the calibrated model (columns 2-5), given an investor with $\gamma = 2$ and $\eta = 24$. The data is described in Appendix B. The median values and 90% confidence intervals (in brackets) reported in columns 2-5 are from 10,000 simulation runs of equivalent length to the data.

		ambiguity neutral		ambiguity averse	
	Data	$\pi_q = 0$	$\pi_v = 0$	$\pi_q = 0$	$\pi_v = 0$
$\beta(1)$	5.43×10^{-4}	-0.08	56.47	3.37	593.14
		[-64.89,60.05]	[-21.46,139.42]	[-243.77,288.97]	[299.15,909.10]
t-stat	5.41	-0.00	1.44	0.03	4.10
		[-2.13,1.90]	[-0.58,3.84]	[-2.04,1.85]	[1.86,7.05]
$R^2(1)$	6.33	0.17	0.99	0.16	6.89
		[0.00,1.83]	[0.00,5.42]	[0.00,1.77]	[1.94,13.87]
$\beta(3)$	1.32×10^{-3}	-0.42	111.75	-2.32	1203.65
		[-149.39,157.59]	[-68.17,316.31]	[-580.58,611.07]	[550.46,1889.01]
t-stat	6.98	-0.00	1.27	-0.01	3.86
		[-2.13,2.17]	[-0.87,3.81]	[-2.10,2.11]	[1.50,7.07
$R^2(3)$	11.07	0.37	1.34	0.35	10.81
		[0.00,3.46]	[0.00,9.86]	[0.00,3.52]	[2.40,23.70]
$\beta(6)$	1.64×10^{-3}	-0.89	140.50	0.28	1525.01
		[-260.19,267.65]	[-168.58,457.19]	[-1000.83,997.87]	[500.43,2555.60]
t-stat	4.16	-0.01	1.01	0.00	3.31
		[-2.33,2.30]	[-1.11,3.69]	[-2.36,2.28]	[1.00,6.70]
$R^2(6)$	7.74	0.56	1.22	0.60	9.47
		[0.00,5.36]	[0.00,10.38]	[0.00,5.34]	[1.12,25.62]
$\beta(12)$	1.49×10^{-3}	2.64	169.67	13.15	1694.96
		[-426.24,452.92]	[-316.71,655.49]	[-1629.18,1698.70]	[54.64,3285.03]
t-stat	2.03	0.02	0.87	0.02	2.60
		[-2.41,2.65]	[-1.68,3.48]	[-2.42,2.65]	[0.08,5.50]
$R^2(12)$	3.04	0.73	1.23	0.75	6.57
		[0.00,7.72]	[0.01,11.59]	[0.00,7.70]	[0.06,23.56]
$\beta(24)$	1.87×10^{-3}	14.04	181.55	61.45	1716.17
		[-667.74,674.77]	[-574.97,858.23]	[-2500.68,2558.00]	[-705.73,4004.93]
t-stat	1.47	0.05	0.68	0.07	1.94
		[-2.60,2.62]	[-1.93,3.19]	[-2.66,2.64]	[-0.67,4.65]
$R^2(24)$	2.11	0.85	1.11	0.84	4.00
		[0.00,10.07]	[0.00,10.52]	[0.00,10.25]	[0.01,18.91]
$\beta(36)$	6.60×10^{-4}	-29.93	135.97	-98.53	1559.52
		[-858.42,858.13]	[-793.84,1123.44]	[-3226.67,3224.09]	[-1472.24,4877.66]
t-stat	0.47	-0.10	0.41	-0.09	1.49
		[-2.87,2.81]	[-2.40,3.21]	[-1.28,4.36]	[24.47,50.56]
$R^2(36)$	0.18	0.89	0.97	0.90	2.74
		[0.00,10.51]	[0.00,11.49]	[0.00,10.43]	[0.01,18.44]

Table 8: **Variance premium return regressions**

This table presents results of regressions as specified in Equation (10) performed on stock market data between 1992:Q1 and 2012:Q4 (column 1) and performed on simulated data from the calibrated model (columns 2-5). It reports regression coefficients $\beta(h)$, Newey and West (1987) t-statistics, and R^2s for return horizons of 1, 3, 6, 12, 24, and 36 months. The median values and 90% confidence intervals (in brackets) reported in columns 2-5 are from 10,000 simulation runs of equivalent length to the data.

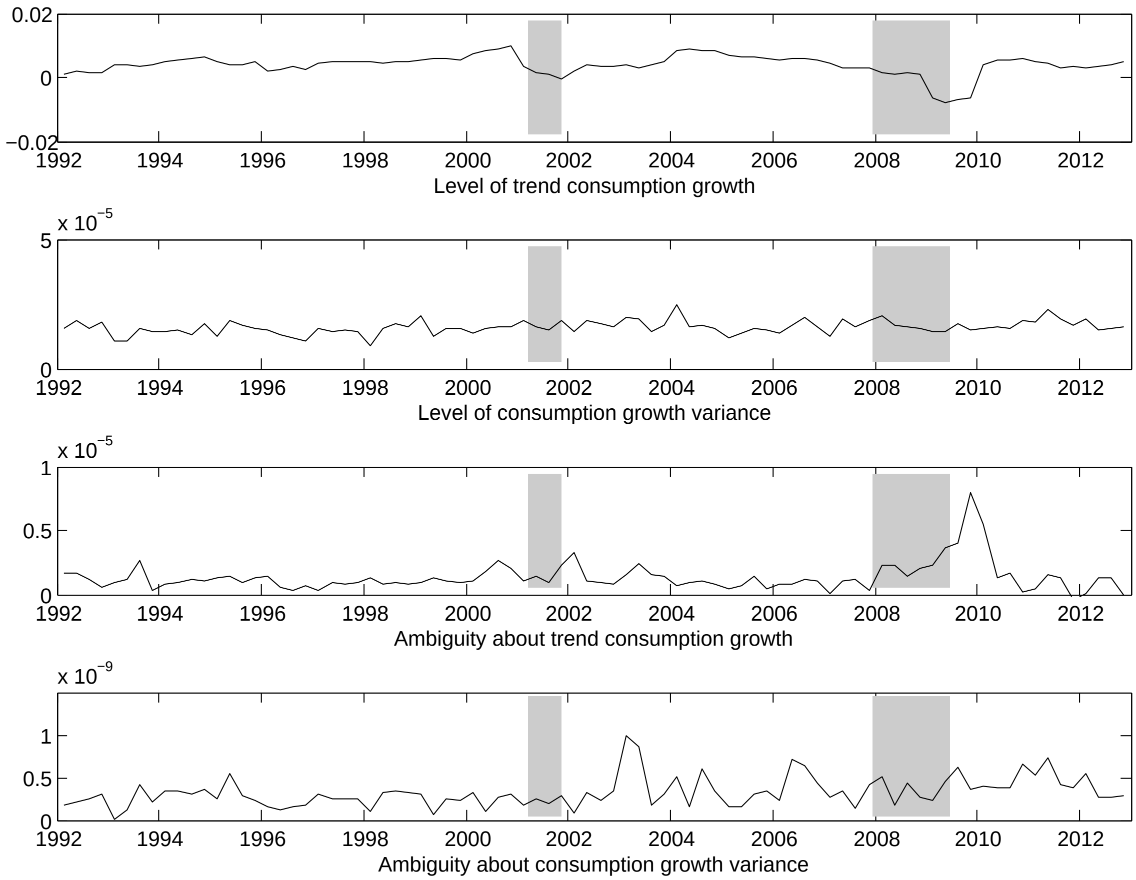

Figure 1: **Time series of growth moments**

This figure shows time series of the extracted state variables as defined in Section 2.1. The first plot shows the time series $(Ex_t)_t$, the second $(E\sigma_t^2)_t$, the third $(Vx_t)_t$, and the fourth $(V\sigma_t^2)_t$ between 1992:Q1 and 2012:Q4. The shaded areas represent NBER recessions.