Risk-Based Commodity Investing*

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ABSTRACT

Pursuing risk-based allocation across a universe of commodity assets, we find two alternative notions of risk parity to provide convincing results, diversified risk parity (DRP) and principal risk parity (PRP). DRP strives for maximum diversification along the uncorrelated risk sources embedded in the underlying commodities, while PRP budgets risk proportional to the risk source's relevance in terms of their variance. These strategies are characterized by concentrated allocations that are actively adjusted to changes in the underlying risk structure. We also document competing risk-based allocation techniques to be rather similar to the $1/N$-strategy or market indices in picking on concentrated market risk. Finally, we demonstrate how to enhance given risk-based allocation strategies by means of common commodity anomalies while preserving a meaningful degree of diversification.

Keywords: Commodity Strategies, Risk-Based Portfolio Construction, Risk Parity, Diversification

JEL Classification: G11; D81

1. Introduction

Commodity investing is often advocated for diversifying traditional stock-bond portfolios. While there is plenty of evidence[1] of commodities enhancing a given portfolio's risk-return profile, there is less research on the diversification potential inherent within the universe of commodity assets. From a portfolio optimization point of view, commodities are especially appealing because of their high returns and risk at relatively low pairwise correlations.[2] Moreover, commodity investments can exhibit negative correlation to stocks during stock market downturns, rendering commodities a perfect hedging instrument (see Bodie and Rosansky (2000)).

Interestingly, Erb and Harvey (2006) document that static trading of commodities has not been profitable because of the inherent heterogeneity within this asset class paired with high volatility and excess kurtosis. On the other hand, the authors document abnormal returns for specific combinations of commodities that focus on commodity futures that exhibit a forward curve with attractive term structure characteristics. Hence, to derive profitable commodity trading strategies, one should typically resort to momentum or commodity term structure signals, see Fuertes, Miffre, and Rallis (2010). Recently, Fuertes, Fernandez-Perez, and Miffre (2011) document abnormal returns when trading long low-idiosyncratic volatility positions versus high ones, thus evidencing an inverse risk-return relationship as prevailing in equities, see Ang, Hodrick, Xing, and Zhang (2006). However, while a trading strategy can generate abnormal returns within specific periods of time, their characteristics need to be carefully analyzed to understand, if the underlying commodity effects are permanent or transitory, see Basu and Miffre (2013b).

We contribute to the literature on commodity investing by exploring a different route. Despite the heterogeneity in commodity markets, the classic mean-variance approach of Markowitz (1952) to optimally trade off assets' risk and return will most likely be confounded by the associated estimation risk. Even more so, there is a large literature starting with Haugen and Baker (1991) that, for equity markets, demonstrates minimum-variance strategies to be far more efficient than capitalization-weighted benchmarks. At the same time, alternative risk-based allocation techniques increased in popularity, as they are claimed to promise superior historical performance. Qian (2006, 2011) and Maillard, Roncalli, and Teiletche (2010) advocate the risk

[1] See among others Kat and Oomen (2007) for an overview.

[2] See, e.g., Vrugt, Bauer, Molenaar, and Steenkamp (2007).

parity approach that allocates capital such that all assets contribute equally to portfolio risk. The common rationale of the classic Markowitz approach or the more ad-hoc risk parity strategy is diversification. Still, diversification is a rather elusive concept which is hardly made explicit in portfolio optimization studies.

A notable exception is Meucci (2009). Striving for diversification, he pursues principal component analysis of the portfolio assets to extract principal portfolios representing the uncorrelated risk sources inherent in the underlying assets. For a portfolio to be well-diversified, its overall risk should be evenly distributed across these principal portfolios. Recently, Lohre, Opfer, and Ország (2012) adopt this framework to determine maximum diversification portfolios in a multi-asset allocation study. This strategy coincides with a risk parity strategy that is allocating risk by principal portfolios rather than the underlying assets. The authors demonstrate this diversified risk parity strategy to provide convincing risk-adjusted performance together with superior diversification properties when benchmarked against other competing risk-based investment strategies.

We translate the idea of diversified risk parity to the commodities space. Unlike in the multi-asset domain, this task is significantly more challenging, because the number of relevant principal portfolios is most likely smaller than the number of underlying commodities. Hence, one has to decide what principal portfolios to cut off. We resort to statistical information criteria as provided by Bai and Ng (2002) that determine the optimal number of principal portfolios based on a trade-off between the number of factors and the explained variation of the asset universe. Obviously, this procedure introduces some undesirable degrees of freedom into the analysis. Therefore, we additionally consider a second alternative risk parity strategy that is related to the concept of Meucci (2009). Instead of equally budgeting risk across a subset of principal portfolios, we additionally devise a strategy that budgets risk along all principal portfolios proportional to their contribution to overall variance. This strategy coincides with equally weighting the principal portfolios. Thus, we dub this approach principal risk parity.

In our empirical analysis of the Standard and Poor's Goldman Sachs Commodity Index (GSCI) universe, we find the following results. While both diversified risk parity and principal risk parity strategies provide superior risk-adjusted performance in a 30-year-backtest, they also differ from the prevailing risk-based allocation schemes like $1/N$, minimum-variance, maximum diversification or traditional risk parity. The strategies are characterized by concentrated allocations that are altered actively whenever a significant change in risk structure calls for adjusting the risk exposure. In addition, x-raying the risk structure of competing alternatives, we find the traditional risk parity strategy to be similar to the $1/N$-strategy or the market indices in picking on concentrated risk. When it comes to diversification, minimum-variance strategies typically prove to be rather concentrated in low-volatility assets. In the equity domain, this observation resonates with the finding of Scherer (2011) that minimum-variance strategies implicitly capture risk-based pricing anomalies inherent in the cross-section of stock returns, especially the low-volatility and low-beta anomalies. In this vein, a commodity factor model accounting for common commodity risk factors is a prerequisite for explaining a given commodity strategy's return. Moreover, we demonstrate that the superior performance of the diversified and principal risk parity strategies is not just a common commodity anomaly in disguise.

The paper is structured as follows. Section 2 describes the methodology of the risk-based asset allocation techniques. In Section 3, we provide a preliminary analysis of our commodity data. Section 4 studies in detail the empirical implementation of risk-based strategies in the classic commodities universe. Section 5 concludes.

2. Methodology

In this section, we present the theoretical underpinnings of different approaches to find diversified portfolios, which will be used in our empirical study.

2.1. Managing Diversification

We consider a portfolio consisting of N commodities with weight and return vectors $\mathbf{w}$ and $\mathbf{R}$ that give rise to a portfolio return of $R_w = \mathbf{w}'\mathbf{R}$. At the heart of diversification is the search for low-correlated assets. Although commodities are a rather heterogenous asset class, the corresponding correlation figures will hardly be zero. Still, it is possible to construct uncorrelated assets from a given variance-covariance matrix. Along these lines, Meucci (2009) extracts uncorrelated risk sources by applying a principal component analysis (PCA) to the variance-covariance matrix $\mathbf{\Sigma}$ of the portfolio assets. From the spectral decomposition theorem it follows that

$$\mathbf{\Sigma} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}', \tag{1}$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, ..., \lambda_N)$ is a diagonal matrix consisting of $\mathbf{\Sigma}$'s eigenvalues that are assembled in descending order, $\lambda_1 \geq ... \geq \lambda_N$. The columns of matrix $\mathbf{E}$ represent the eigenvectors of $\mathbf{\Sigma}$ that define a set of N uncorrelated principal portfolios with variance λ_i for $i = 1, ..., N$ and returns $\tilde{R} = \mathbf{E}^{'}\mathbf{R}$.[3] Hence, one can think of a given portfolio either in terms of its weights $\mathbf{w}$ in the original assets or in terms of its weights $\tilde{\mathbf{w}} = \mathbf{E}^{'}\mathbf{w}$ in the principal portfolios. Because principal portfolios are uncorrelated by design, the total portfolio variance emerges from simply computing a weighted average over the principal portfolios' variances λ_i using weights $\tilde{w}_i^2$:

$$Var(R_w) = \sum_{i=1}^{N} \tilde{w}_i^2 \lambda_i. \tag{2}$$

Normalizing the principal portfolios' contributions by the portfolio variance then yields the diversification distribution:

$$p_i = \frac{\tilde{w}_i^2 \lambda_i}{Var(R_w)}, \qquad i = 1, ..., N. \tag{3}$$

Note that the diversification distribution is always positive and that all p_i sum to one.

Building on the above concept, Meucci (2009) conceives a portfolio to be well-diversified when the principal portfolio's contributions p_i are "approximately equal and the diversification distribution is close to uniform". Conversely, portfolios mainly loading on a single principal portfolio display a peaked diversification distribution. Aggregating the diversification distribution, Meucci (2009) chooses the exponential of its entropy for evaluating a portfolio's degree of diversification:

$$\mathcal{N}_{Ent} = \exp\left(-\sum_{i=1}^{N} p_i \ln p_i\right). \tag{4}$$

Intuitively, we can interpret $\mathcal{N}_{Ent}$ as the number of uncorrelated bets. For instance, a completely concentrated portfolio is characterized by $p_i = 1$ for one i and $p_j = 0$ for $i \neq j$ resulting in an

[3] Partovi and Caputo (2004) have coined the term principal portfolios while recasting the efficient frontier in terms of these principal portfolios.

entropy of 0, which implies $\mathcal{N}_{Ent} = 1$. Conversely, $\mathcal{N}_{Ent} = N$ obtains for a portfolio that is completely homogenous in terms of uncorrelated risk sources. In this case, $p_i = p_j = 1/N$ holds for all i, j, implying an entropy equal to $\ln(N)$ and $\mathcal{N}_{Ent} = N$. In the spirit of Markowitz (1952), this framework allows for determining a mean-diversification frontier that trades off expected return against a certain degree of diversification.

2.2. Diversified Risk Parity

Lohre, Opfer, and Ország (2012) implement the above definition of a well-diversified portfolio by constructing an allocation strategy, which allocates equal risk budgets to every uncorrelated principal portfolio. We obtain the weights $\mathbf{w}_{DRP}$ of this strategy, dubbed diversified risk parity (DRP), by solving

$$\mathbf{w}_{DRP} = \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \, \mathcal{N}_{Ent}(w), \tag{5}$$

i.e., by maximizing the entropy measure, where the weights $\mathbf{w}$ may possibly be restricted according to a set of constraints $\mathcal{C}$.

In theory, there are several portfolios maximizing objective function (5). Obviously, an inverse volatility strategy along the principal portfolios is a strategy maximizing the objective function (5). However, buying or selling a certain amount of a given principal portfolio gives rise to the same ex ante risk exposure. As a result, there are several optimal solutions. For K principal portfolios, there exist 2^K solutions, all of which are inverse volatility strategies reflecting all possible variations of long and short principal portfolios. Typically, most of these portfolios tend to be hard to implement in practice because of rather infeasible portfolio weights. In fact, Maillard, Roncalli, and Teiletche (2010) show that imposing positive asset weights guarantees a unique solution in case of the traditional risk parity strategy. Conversely, positivity of asset weights is not a sufficient condition for determining a unique DRP strategy.

In that regard, Bruder and Roncalli (2012) and Roncalli and Weisang (2012) investigate general risk budgeting strategies and demonstrate that uniqueness obtains when imposing positivity constraints with respect to the underlying risk factors. In our case, this requirement translates into imposing sign constraints with respect to the principal portfolios representing uncorrelated risk factors.

Restricting the sign of a given principal portfolio is equivalent to expressing a view with respect to the according risk premium. While one could resort to elaborate forecasting techniques to derive these views we pursue a quite simple approach. We equalize the desired sign of the principal portfolio with the one of its corresponding historical risk premium. Thus, we intend to design a strategy that is geared at exploiting long-term risk premia. The respective historical risk premia result from multiplying the current principal portfolio weights with historical asset returns using an expanding window.

While the intuition of diversified risk parity is straightforward, one might feel uneasy about allocating any risk budget to rather marginal principal portfolios, i.e., to those explaining rather minuscule fractions of overall variance. In this vein, Lohre, Neugebauer, and Zimmer (2012) cut off rather irrelevant principal portfolios resorting to statistical information criteria advocated by Bai and Ng (2002). Using a consistent statistical methodology for factor selection avoids overfitting and bias issues embedded in an arbitrary choice.

2.3. Principal Risk Parity

In addition to the DRP strategy, we suggest an alternative strategy that invests into the main uncorrelated risk sources in a more natural fashion. Instead of equally budgeting risk across principal portfolios, we propose to budget risk proportional to the principal portfolios' contribution to total variance. As a result, we end up allocating the lion share of capital to the most relevant uncorrelated risk sources carrying risk premia with higher probability, basically neglecting most of the less relevant principal portfolios.

Technically, we are simply allocating equal weights to the principal portfolios, which is why we label this strategy principal risk parity (PRP). To obtain the strategy weights $\mathbf{w}_{PRP}$, we need to solve

$$\mathbf{w}_{DRP} = \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \, \mathcal{M}_{Ent}(w), \tag{6}$$

where $\mathcal{M}_{Ent}$ is defined as

$$\mathcal{M}_{Ent} = \exp\left(- \sum_{i=1}^{N} q_i \ln q_i \right), \tag{7}$$

and

$$q_i = \frac{\tilde{w}_i^2}{\sum_{i=1}^{N} \tilde{w}_i^2}, \qquad i = 1, ..., N. \tag{8}$$

Also, we impose the same sign constraints with respect to the principal portfolios as in the optimization of the DRP strategy. That is, we seek to capture long-term risk premia associated with the principal portfolios.

2.4. Benchmark Strategies

For benchmarking the diversified and principal risk parity strategy, we consider four alternative risk-based asset allocation strategies: $1/N$, minimum-variance, risk parity, and the most-diversified portfolio of Choueifaty and Coignard (2008).

First, we implement the $1/N$-strategy that rebalances monthly to an equally weighted allocation scheme. Hence, the portfolio weights $\mathbf{w}_{1/N}$ are simply

$$\mathbf{w}_{1/N} = \frac{\mathbf{1}}{N}. \tag{9}$$

Second, we compute the minimum-variance (MV) portfolio building on a rolling 252-days window for covariance-matrix estimation. We derive the corresponding weights $\mathbf{w}_{MV}$ from

$$\mathbf{w}_{MV} = \operatorname*{argmin}_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}, \tag{10}$$

subject to the full investment and positivity constraints, $\mathbf{w}'\mathbf{1} = 1$ and $\mathbf{w} \geq \mathbf{0}$.

Third, we construct the original risk parity (RP) strategy by allocating capital such that the asset classes' risk budgets contribute equally to overall portfolio risk. Note that these risk budgets also depend on a rolling window estimation. Since there are no closed-form solutions available, we follow Maillard, Roncalli, and Teiletche (2010) to obtain $\mathbf{w}_{RP}$ numerically via

$$\mathbf{w}_{RP} = \operatorname*{argmin}_{\mathbf{w}} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(w_i(\boldsymbol{\Sigma}\mathbf{w})_i - w_j(\boldsymbol{\Sigma}\mathbf{w})_j \right)^2, \tag{11}$$

which essentially minimizes the variance of the risk contributions. Again, the above full investment and positivity constraints apply.

Fourth, we use the approach of Choueifaty and Coignard (2008) to build maximum diversification portfolios. To this end, the authors define a portfolio diversification ratio $D(\mathbf{w})$:

$$D(\mathbf{w}) = \frac{\mathbf{w}' \cdot \boldsymbol{\sigma}}{\sqrt{\mathbf{w}' \boldsymbol{\Sigma} \mathbf{w}}}, \tag{12}$$

where $\boldsymbol{\sigma}$ is the vector of portfolio assets' volatilities. Thus, the most-diversified portfolio (MDP) simply maximizes the ratio of two distinct definitions of portfolio volatility, i.e., the ratio of the average portfolio assets' volatility and the total portfolio volatility. We obtain MDP's weights vector $\mathbf{w}_{MDP}$ by numerically computing

$$\mathbf{w}_{MDP} = \underset{\mathbf{w}}{\operatorname{argmax}}\, D(\mathbf{w}). \tag{13}$$

As before, we enforce the full investment and positivity constraints.

3. Preliminary Data Analysis

We start with a brief discussion of some descriptive statistics of commodity markets.

3.1. Descriptive statistics of the commodity market

We investigate risk-based commodity strategies using the 24 constituents of the S&P Goldman Sachs Commodity Index (GSCI) over the time period January 1983 to July 2012, see Table 1 for the complete list of the underlying commodities. Compared to other major commodity indices like the Dow Jones UBS Commodity Index or the UBS Bloomberg CMCI, the GSCI puts a high weight on oil and gas.[4]

[Table 1 about here.]

The annualized return of the GSCI excess return amounts to 5.9% at a volatility of 23.0% which implies a Sharpe Ratio of 0.25. Among the constituent commodities there are many time series with more sizable volatility figures. The range is from 10.3% (sugar) to 60.1% (nickel).

[4]The actual commodity allocation varies with market prices. These deviations from target weights were not found to be relevant from a pure diversification point of view and are therefore neglected.

Likewise, the range of annualized return is quite large.[5] Nickel has returned the most (24.7%) and natural gas is the only commodity with a negative return (-2.6%). Across the board, one has to note that investing in single commodities entails significant downside risk. The maximum loss within the three decades ranges from -25.1% (feeder cattle) to -79.9% (natural gas).

[Figure 1 about here.]

Figure 1 depicts the average correlation structure of the 24 single commodities using the maximum amount of data available for any pair within the sample period from 1983 to 2012. The commodities are sorted and grouped according to the corresponding commodity sector. A block structure emerges when examining the near-diagonal correlations. Commodities are strongly correlated within the same sector but less so between sectors, confirming the findings of Vrugt, Bauer, Molenaar, and Steenkamp (2007).

Table 2 reduces the full correlation matrix to a sector correlation matrix giving the average within-sector and between-sector correlations of the eight main commodity sectors corresponding to the 24 commodities. The within-sector correlations are calculated by averaging the pairwise correlations among all commodity futures in each sector over each of the 30 annual periods from 1983 to 2012.

[Table 2 about here.]

The between-sector correlations for any pair of groups is obtained by averaging the correlations between individual futures in both groups over the individual 30 years of history. Paraphrasing the above block structure we note that while all of the within-sector correlations are generally high, the between-sector correlations most often are not. On the one hand, the most heterogenous commodity sectors are softs, grains, and livestocks given within-correlations around 0.5. On the other hand, the energy and metals sectors prove to be more in sync given within-correlations close to one. Between sectors, livestocks are hardly correlated to anything else. Their correlation to precious metals is even slightly negative. Most of the remaining between-sector correlations are around 0.5 with the exception of industrial versus precious metals (0.74) and crude oil versus

[5] As for commodities' returns we rely on the generic commodity futures returns as compiled by Bloomberg. These generic time series impose a roll of the future 3 days prior maturity. The corresponding Bloomberg Tickers can be retrieved from Table 1.

refined oil (0.98). These results suggest that there is ample room for diversification embedded in the universe of commodities.

3.2. How many risk sources are embedded in the S&P GSCI?

In theory, one can construct as many principal portfolios as assets entering the PCA decomposition. However, it is well-known that already a few number of principal portfolios are sufficient for explaining most of the assets' variance. We compute the 24 principal portfolios pertaining to the GSCI by performing a PCA-reestimation over a rolling 1-year window of 252 trading days with a holding period of one month. In Figure 2, we assess the relevance of the principal portfolios over time. We observe principal portfolio 1 (PP1) to typically account for around 30% of total variability.

[Figure 2 about here.]

Principal portfolio 2 (PP2) captures 16% on average, PP3 averages to 11% thus leaving only single-digit fractions for the subsequent principal portfolios PP4 to PP24. Of course, with their relevance quickly dying off, it seems hardly reasonable to allocate any risk budget to higher principal portfolios. Hence, it is crucial to determine an adequate threshold for cutting off rather irrelevant principal portfolios.

There are different possibilities to define an appropriate cut-off level. We could fix a fraction of total variance to be explained by the most relevant factors. Alternatively, we rely on the PC_{p1} and PC_{p2} information criteria of Bai and Ng (2002) for determining a meaningful number of principal portfolios. These two criteria allow for consistent estimation of the optimal number of factors explaining the variability of a given dataset. Both criteria are found to be superior to standard information criteria used in statistical regression methods like AIC or BIC.[6]

Applying these criteria to the GSCI constituents, we detect between four and eight relevant principal portfolios over time depending on the respective underlying risk structure, see Figure

[6]Given a linear k-factor model $F = \sum_{j=1}^{k} \lambda_k F^k + \epsilon$, one seeks to optimize

$$V(k, F) := \min_{\lambda_1 \dots \lambda_k} \frac{1}{NT} \sum_{n=1}^{N} \sum_{t=1}^{T} (X_{nt} - \lambda_k^n F_t^k)^2. \tag{14}$$

3. The number of relevant principal portfolios averages around seven for the period 1984-2004. However, this number decreases until 2007-2009 when no more than four principal portfolio were deemed to be relevant. Still, the average number of principal portfolios amounts to 6.35. This finding resonates with the significant degree of heterogeneity within commodities. At a given date, our implementation of the diversified risk parity strategy is restricted to the then prevailing number of relevant principal portfolios. Conversely, the principal risk parity strategy is including all of the principal portfolios, but assigns lesser weight to higher principal portfolios by design.

[Figure 3 about here.]

4. Commodity Investing with Principal Portfolios

In this section, we bring the previously defined diversification strategies to the data.

4.1. Rationalizing Principal Portfolios

Investigating the uncorrelated risk sources inherent in the underlying assets, we first analyze the (static) principal portfolios arising from a PCA over the whole sample period from January 1983 to July 2012. For the ease of interpretation, we disentangle the eigenvectors representing the principal portfolios' weights in the underlying assets. By construction, these weights are standardized to lie within the [-1,1]-interval. We illustrate these weights by means of bi-plots in Figure 4.

[Figure 4 about here.]

The PC_{p1} and PC_{p2} information criteria of Bai and Ng (2002) simply entail an extension of this model. To the function V, which measures the squared deviations of the sample data from the linear approximation given by the factor model, a penalty function $g(N,T)$ times the number of factors, k, is added:

$$PC_{pi}(k) = V(k,F) + kg_i(N,T), \quad i = 1,2. \tag{15}$$

The penalty functions for PC_{p1} and PC_{p2} result in minimizing the following functions

$$\begin{aligned} PC_{p1}(k) &:= V(k,F) + k\sigma^2 \left(\frac{N+T}{NT}\right) \ln\left(\frac{NT}{N+T}\right), \\ PC_{p2}(k) &:= V(k,F) + k\sigma^2 \left(\frac{N+T}{NT}\right) \ln\left(\min(N,T)\right), \end{aligned}$$

where $\sigma^2 = (NT)^{-1} \sum_{n=1}^{N} \sum_{t=1}^{T} E(\epsilon_{nt})^2$.

Given that we find up to eight relevant principal portfolios, it is conceivable that their economic rationale is closely tied to the eight commodity sectors. Obviously, principal portfolio 1 (PP1) qualifies for a common commodity risk factor as evidenced by the outright positive weights for all constituent commodity assets. Unsurprisingly, this common factor loads more heavily towards highly volatile commodity sectors like energy and metals rather than livestocks. Conversely, the subsequent principal portfolios are characterized by both positive and negative weights. Still, the implicit relative bets are fairly intuitive. For instance, principal portfolio 2 (PP2) is basically short energy and agriculture and long in precious and industrial metals, whereas principal portfolio 3 (PP3) is long agricultures and short energy. Note that the direction of these positions is of second-order importance because principal portfolios are uncorrelated by design. Hence, one is solely interested in the relative positioning when it comes to judge the diversification potential. Turning to the remaining principal portfolios, we find principal portfolio 4 (PP4) to be dominated by a single commodity, namely natural gas, reflecting the very low correlation with other commodity sectors evidenced in Table 2. In particular, PP4 plays natural gas against oil and refined products.

Principal portfolio 5 (PP5) is basically mimicking the spread between precious and industrial metals, whereas principal portfolio 6 (PP6) is reflecting diversity within the agriculture sector by playing softs against grains. Admittedly, principal portfolios 7 and 8 (PP7 and PP8) are less straightforward to interpret. This observation is expected, as these principal portfolios are not deemed to be relevant at all dates. Also, these principal portfolios can be better assessed on the commodity-level rather than the sector-level. We particularly detect PP7 to be short nickel and long lead. PP8 is long sugar and short cocoa.

A common objection with respect to statistical risk factors derived from principal component analysis is the stability of factors over time. Addressing this objection, we compute principal portfolio weights over time and plot their evolution in Figure 5. We aim to include as many commodities as possible while keeping a sufficient history of data. In particular, we exclude Gasoline RBOB[7] from the universe and compute weights starting January 1998 to July 2012. The results derive from a PCA estimation using an expanding window with initial length of 252 days. We find the resulting principal portfolio weights to be fairly close to the ones of the static

[7] The acronym RBOB stands for *Reformulated Blendstock for Oxygenate Blending*.

PCA, hence, the above interpretation holds true throughout the sample period. Even more so, we do not observe any change in the order of relevance across the principal portfolios.

[Figure 5 about here.]

While the investigation of portfolio weights helps shaping our understanding, we further seek to characterize the principal portfolios by means of time-series regressions against a set of well-known commodity factor portfolios.[8] The work of Miffre and Rallis (2007), Basu and Miffre (2013) and Fuertes, Miffre and Rallis (2010, 2013) identifies four main commodity risk factors: A market risk factor, a momentum factor, a term structure factor, and a risk reversal factor. Thus, we employ a commodity factor model of the form

$$R_{PPi,t} = \alpha + \beta_1 R_{Market,t} + \beta_2 R_{Momentum,t} + \beta_3 R_{TermStructure,t} + \beta_4 R_{RiskReversal,t} + \varepsilon_t, \quad (16)$$

where $R_{PPi,t}$ is the return of one of the principal portfolios PPi, for $i = 1, ..., 8$. The excess return of the GSCI relative to the risk-free rate serves as the market return $R_{Market,t}$.

The remaining factor controls in (16) are long-short portfolios. In particular, we construct the long-short factors' portfolios as follows. At any given rebalancing date, we sort the GSCI constituents according to the factor's defining criterion. For instance, the momentum factor is long the top quartile of the best performing commodities and short the quartile of worst performing commodities, as measured by the commodities' past 12-months return. The momentum risk factor helps weeding out strategies which generate abnormal positive profits by overly investing in low volatility assets during periods of rising commodity markets. As for the term structure factor, we rank commodities according to the steepness of their corresponding term structure, where steepness we simply take the difference between 3M- and 1M-contracts. The term structure factor is long the top quartile of commodities in contango and short the lowest quartile of commodities that have the flattest term structure or are in backwardation. Thus, the term structure factor

[8] While factor models can be considered to be well functioning with strong explanatory power for equities, commodity factor models are more complicated. In fact, single commodities, like gold or oil, are often used as risk factors and as independent variables in factor models and regressions analysis and less so as dependent variables. Attempts to link traditional equity factors to commodities have already been tried with poor success, see Daskalaki, Kostakis, and Skiadopoulos (2012). Also, using traditional equity factors restricted to those companies with a direct relationship or correlation (extraction, production, refinement, trade, selling, etc...) with particular commodities, failed to reach sufficient explanatory power probably due to the absence of a cash flow dimension in the commodity world.

identifies strategies which overweight commodities with more favorable term structures. Finally, the risk reversal factor invests in undervalued commodities that experienced a prolonged period of bad performance. It is based on a similar metric like momentum portfolio, based on a longer in-sample observation period, it goes short the top quartile and long the lowest quartile, shorting overpriced commodities and investing in underpriced ones.

Equipped with the factor structure in (16), we can identify the common factor exposures of the principal portfolios. Table 4 documents that PP1 is indeed loading heavily on market risk given a t-statistic of 23.5. All in all, the chosen factor structure accounts for 74.3% of its time series variation. In unreported results, we find the market factor to already account for 58.2%. The adjusted R^2 in the remaining regressions are considerably smaller in size, because we seek to explain the variation of long-short principal portfolios. PP2 is significantly loading on all four factors with the positive momentum and the negative term structure exposure being most prominent. Ultimately, the factor structure gives rise to an adjusted R^2 of 32.5%. PP3 is positively loading on market, term structure, and risk reversal with an adjusted R^2 of 38.8%.

[Table 4 about here.]

Given that the regressions for the remaining principal portfolios cannot explain more than three quarters of the time series variation we feel that this analysis may be rather limited for higher order principal portfolios. Nevertheless, the non-significance of the market factor reflects the high degree of heterogeneity in commodity markets. Rationalizing these observations, we argue that either the higher order principal portfolios are not meaningful (at least for some periods) or the commodity factor structure in (16) is not rich enough.[9]

4.2. Risk-Based Commodity Strategies

For benchmarking the diversified and principal risk parity strategies, we consider four alternative risk-based commodity strategies: $1/N$, minimum-variance, traditional risk parity, and the MDP. All of the strategies are implemented subject to the full investment constraint $\mathbf{w}'\mathbf{1} = 1$. We

[9] Potentially, the factor structure may be missing important factors related to inventory levels, hedging pressure or similar factors. Their construction is not straightforward because of specific data requirements.

further restrict weights to be positive. Given that the first PCA estimation consumes 252 days of data, the strategy performance can be assessed from January 1984 to July 2012.

Table 5 gives performance and risk statistics of the risk-based commodity strategies. Across the board we find the "classic" strategies to yield quite similar annualized returns. Unsurprisingly, the lowest annualized volatility (11.7%) is achieved by the minimum-variance strategy together with the lowest annualized return (4.4%), which compares to 5.9% for the GSCI. Also, its maximum drawdown (-33.9%) is relatively small when compared to the one of the index (-70.1%). The volatility of $1/N$ is higher (15.9%) and therefore it is still smaller than the energy-induced GSCI volatility (23.0%). Also, the return of the $1/N$ strategy amounts to 6.6%. Hence, the GSCI is slightly outperformed by the 1/N strategy. Reiterating Maillard, Roncalli, and Teiletche (2010), we find the traditional risk parity strategy to be a middle-ground portfolio between $1/N$ and minimum-variance. Its return is 6.2% at a 13.0% volatility thus giving rise to a Sharpe Ratio of 0.43 which compares to 0.41 for $1/N$ and 0.38 for minimum-variance. Also, its maximum drawdown statistics are slightly reduced when compared to the $1/N$-strategy. The MDP fares similar to the minimum-variance strategy giving slightly more return (4.6%) at a higher volatility (13.0%).

[Table 5 about here.]

Having recovered the well-known risk and return characteristics of the classic risk-based strategies we next inspect the diversified and principal risk parity strategies. The annualized return amounts to 8.0% for DRP and to 9.7% for PRP. This performance does not come at the cost of excessive volatility implying the highest Sharpe Ratios (0.57 and 0.54) among all alternatives. Moreover, DRP has the smallest drawdown (-31.2%), followed by minimum-variance (-33.9%) and PRP (-38.9%), indicating that both strategies are less vulnerable to severe market crashes. Note that DRP entails the largest turnover among the risk-based commodity strategies with 37.1% (suggesting that transaction costs may reduce the relative return potential though) while PRP has a turnover of 27.3%, which is comparable to the one of MV (24.1%)

4.3. Risk and Diversification Characteristics

Judging risk-based strategies by their Sharpe Ratios alone is not meaningful given that returns are not entering the respective objective functions, see Lee (2011). In a similar vein, we resort to evaluating the strategies along their risk and diversification characteristics. We first decompose risk by the underlying commodities and second by the according principal portfolios. This approach provides us with a concise picture of the underlying risk structure and number of uncorrelated bets implemented in a given portfolio.

To set the stage, we start by analyzing the GSCI. While we refrain from plotting weights for single commodities, we nevertheless provide some aggregate figures summarizing the characteristics of the weight decomposition on commodity-level, see Panel B of Table 5. We report Gini coefficients for the stock weight decomposition ($Gini_{Weights}$), the risk decomposition by stocks ($Gini_{Risk}$), and the risk decomposition by principal portfolios ($Gini_{PPRisk}$). The Gini coefficient is a measure of concentration which is 0 in case of no concentration (equal weights throughout time) and 1 in case of full concentration (one commodity or principal portfolio attracts all of the weight all of the time). For the GSCI the $Gini_{Weights}$ (0.54) and the $Gini_{Risk}$ (0.74) show the index to be rather concentrated.

In Figure 6, we plot sector weights and risk contributions. The left chart of Figure 6 gives the GSCI's sector weights over time. While the GSCI weights decomposition has been dominated by softs and grains in the middle of the eighties it has slowly evolved into an energy-driven index. Moreover, according to the risk decomposition by sector (middle chart), crude oil absorbs more than half of the risk budget most of the time. Finally, the right chart depicts the risk decomposition with respect to the uncorrelated risk sources. A portfolio that reflects eight uncorrelated bets should thus exhibit a risk parity profile along the principal portfolios, i.e., the decomposition should follow a constant 1/8 risk budget allocation over time. For the GSCI this decomposition is almost exclusively exposed to the single risk factor PP1 which typically accounts for more than 80% of the total risk throughout time.

[Figure 6 about here.]

Interestingly, this verdict seems to apply for the two other major commodities as well. Figure 6 gives the according analysis for the Dow Jones UBS Commodity Index and the UBS Bloomberg CMCI using static weights as per end of 2012 throughout the whole sample period.[10] Even though these indices display a more diverse weights allocation, this observation does not translate into a diverse risk allocation. Conversely, these seemingly diversified indices emerge as one-bet strategies in most recent times.

Figure 7 depicts the results for $1/N$, the minimum-variance strategy, traditional risk parity, MDP, and the alternative risk parity strategies. For $1/N$ this decomposition almost collapses into a blue square, indicating that this strategy is almost exclusively exposed to market risk as represented by PP1.

[Figure 7 about here.]

The weights decomposition of minimum-variance is concentrated in a few assets, because the strategy is collecting the lowest volatility assets. The traditional risk decomposition by assets is likewise concentrated, but is not overly biased towards specific commodities. In terms of commodity sector composition, the minimum-variance strategy is overweighting more defensive sectors like softs and livestocks. Its risk decomposition by principal portfolios is more diverse than the one for $1/N$ or the index. Still, PP1 explains around 60% of the total risk on average. As for the traditional risk parity strategy, the weights decomposition is less concentrated as evidenced by an average Gini of 0.26. However, while its risk decomposition by commodities is not concentrated by design, its risk decomposition by principal portfolios merely indicates 2.5 bets on average. The MDP is similar to MV but slightly more diversified with 3.0 bets on average.

Documenting all of the classical risk-based strategies to heavily load on the common risk factor we are especially interested in testing whether the DRP strategy is providing a more diversified risk profile. When compared to the other strategies the DRP strategy seems actively reallocating across sectors, see Figure 8. More importantly, the common risk factor as reflected by PP1 has lost its dominance on the risk budget which reflects some 5.9 bets on average. The DRP strategy's combination of concentrated positioning together with its active re-positioning over time seems to be key for maintaining a fairly balanced risk decomposition across the uncorrelated risk sources.

[10] We do not have access to a time series of weights for these indices.

Alternatively, the PRP strategy is tracking closely the principal portfolio's variance decomposition over time. Obviously, this characteristic comes at the cost of less bets over time (4.4). However, the strategy's turnover is much smaller than the one of the DRP.

[Figure 8 about here.]

Figure 9 contrasts the strategies' degree of diversification over time. First, note that 1/N is in general dominated by the other strategies. While minimum-variance, MDP, and traditional risk parity represent a slightly higher number of bets one can observe a significant deterioration in diversification over the last decade in the sample period. As a result, 1/N, minimum-variance, MDP, and risk parity are rendered one-bet strategies. Conversely, diversified risk parity is maintaining the highest number of bets over time according to the number of relevant principal portfolios, see the lower chart of Figure 9. Of course, this observation is expected. Interestingly, the principal risk parity strategy is also maintaining a relatively constant number of around five bets over time.

[Figure 9 about here.]

4.4. Dismantling Risk-Based Commodity Strategies

To further characterize the risk-based equity strategies we relate the strategies' returns to common risk factors. To this end, we rely on the same factor structure we have used for characterizing the principal portfolios in Section 4.1 The model thus reads:

$$R_{RBS,t} = \alpha + \beta_1 R_{Market,t} + \beta_2 R_{Momentum,t} + \beta_3 R_{TermStructure,t} + \beta_4 R_{RiskReversal,t} + \varepsilon_t \tag{17}$$

where $R_{RBS,t}$ is the excess return of one of the risk-based strategies relative to the risk-free rate. To set the stage we estimate a reduced version of factor model (17) to assess the factor exposures of the market itself:

$$R_{Market,t} = \alpha + \beta_2 R_{Momentum,t} + \beta_3 R_{TermStructure,t} + \beta_4 R_{RiskReversal,t} + \varepsilon_t \tag{18}$$

Table 6 documents the GSCI to positively load on momentum and risk reversal, which explain more than half of the index' time series variation (57.0%). Regarding the risk-based strategies, the common risk factors sometimes do a good job in explaining the strategies' returns. Unsurprisingly, $1/N$ and RP load positively on the market factor. Also, both load negatively on the momentum factor mirroring a contrarian-like allocation, but they differ in the term structure loadings. As for $1/N$, the common factors explain two thirds of the time series variation. Compared to $1/N$, the RP strategy's exposure to the two factors is considerably less prominent. The strategy is most closely related to the term structure factor as evidenced by a t-stat of 5.4. Taken together, 53.5% of the return variation can be accounted for by the common factor structure. Conversely, the adjusted R^2's for the remaining strategies are considerably smaller.

[Table 6 about here.]

The very similar return pattern of MV strategy and MDP resonates with a similar risk factor decomposition. For both strategies we detect a significant negative exposure to momentum and the term structure factor. Only one quarter of the excess time series variation can be attributed to common factors for MV (26.6%) and MDP (26.1%). Among the alternative risk parity strategies, the highest adjusted R^2 obtains for PRP (32.1%). It has a positive market exposure together with a negative loading to the momentum factor. In contrast, DRP is not significantly exposed to common commodity factors. As a result, the according adjusted R^2 is quite low (9.7%). We conjecture that the DRP might be playing commodity factors more actively than the other strategies making it hard to pinpoint these exposures in a static time series regression.

4.5. Enhancing Risk-Based Commodity Strategies

Having shed light on the diversification and risk characteristics of diversified and principal risk parity strategies, we seek to improve the strategies' risk-return profile. The general idea is to include the return dimension into the optimization procedure by combining a kernel asset allocation (as given by DRP or PRP) with trading signals related to momentum, term structure and risk reversal strategies.

A naive combination is a convex mixture that builds on portfolio allocations generated separately by the kernel and the enhancing strategies. Relative to the original kernel asset allocation, we observe in untabulated results that these simple mixtures hardly yield higher returns, but are considerably less diversified. Avoiding the latter, we follow a quite simple approach highlighted in Daly, Rossi, and Herzog (2012). The authors suggest to run risk parity strategies not on the overall original asset universe, but rather on a subset of it. In this vein, we identify optimal subsets of commodities at a given date according to trading signals derived from momentum, term structure, and risk reversal. Table 7 reports key risk, return, and diversification figures regarding the enhanced DRP and PRP strategies applied to these commodity subsets.

[Table 7 about here.]

In particular, all of the available commodities are ranked according to the respective criteria of the selected trading strategy at a given rebalancing date. For instance, using momentum, we sort commodities with respect to their 12M realized return during a given out of sample period and we exclude the worst percentile from the DRP or PRP optimization. Gradually increasing the fraction of excluded commodities, we initially observe an increase in return, volatility and Sharpe ratio and a decrease in the average number of uncorrelated bets. This observation continues to hold until we have excluded some 25% of the ranked commodities. Going forward, it appears that poor diversification (induced by the reduced number of securities) renders the enhanced asset allocation strategies inferior with respect to the original strategies based on the whole asset universe. While the results are consistent across all three filtering criteria, differences in magnitude are reflective of the underlying trading strategies. Filtering with respect to momentum and term structure is less sensitive with respect to the choice of the parameters. Generally, poorly performing commodities are excluded in the first place. On the other hand, risk reversal is more sensitive with respect to the filtering criteria. The number of uncorrelated bets is quite resilient to filtering the low-ranked commodities. This result is in favor of active DRP and PRP strategies relative to a naive mixture, because it allows to maintain the diversification characteristics of DRP and PRP despite investing in a reduced number of commodities.

5. Conclusion

Given an increased desire for risk control emanating from the most recent financial crisis, there has been considerable interest of investors in risk-based strategies. As noticed in Lintner (1983), commodity futures, portfolios, indices and CTAs provide a huge potential for investors striving for diversification, exploiting the low correlation with respect to stocks and bonds. Our research objective was to exploit these stylized facts when investigating a new way of commodity investing that provides maximum diversification along the uncorrelated risk sources inherent in the commodity assets. Judging by the results from our study, these diversified and principal risk parity strategies are distinct in several aspects from prevailing risk-based portfolio construction paradigms.

Moreover, our research has several practical implications: First, we have extracted the relevant uncorrelated risk sources embedded in a classic commodity universe and foster intuition with respect to their economic nature. Second, the framework is a convenient risk management tool for decomposing the risk of any given strategy by uncorrelated risk sources and for assessing its degree of diversification. Third, the alternative risk parity strategies represent an innovative way of risk-based portfolio construction to generate truly diversified commodity portfolios. Besides the above long-only strategies, the commodity futures market provides a natural environment for implementing long-short self-financing strategies. Allowing for negative weights, there is even more room to exploit the diversification potential via principal and diversified risk parity strategies.

Based on our empirical results, we find the diversified and principal risk parity strategies to follow a rather concentrated allocation which is actively rebalanced at some dates. This behavior allows the diversified risk parity strategy to constantly adapt to changes in risk structure and to maintain a balanced exposure to the then prevailing uncorrelated risk sources.

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Table 1
Descriptive Statistics of the GSCI Commodity Universe

The table lists the descriptive statistics for the 24 constituents of the S&P GSCI. Corresponding Bloomberg tickers and sectors are given together with the beginning date of the underlying time series. Target weights are given for the S&P GSCI, the UBS Bloomberg CMCI, and the Dow Jones UBS Commodity Index. Target weights refer to the end of 2012. The rightmost panel gives performance and risk statistics of the constituent commodities of the S&P GSCI. Annualized average return and volatility figures are reported together with the corresponding Sharpe Ratio. Value at risk (VaR) and expected shortfall (ES) are computed at the 95% confidence interval over 1 year period. For the computation of the maximum drawdown (MDD) the whole sample period is considered. For each commodity all available data has been used. The maximum time window is January 1983 to July 2012.

Commodity	Ticker	Sector	Start	Index Weights			Return	Volatility	SR	VaR	ES	MDD
				CMCI	*GSCI*	*DJ UBS*				95%	95%	
WTI	CL	Crude Oil	Apr '83	12.3%	30.2%	9.7%	20.4%	44.2%	0.46	-41.5%	-54.3%	-70.1%
Brent	CO	Crude Oil	Jul '88	7.7%	17.4%	5.3%	17.3%	45.8%	0.38	-50.1%	-59.8%	-73.2%
Heating Oil	HO	Refined Products	Jul '86	3.5%	4.7%	3.5%	19.7%	46.1%	0.43	-39.8%	-49.9%	-66.5%
Gas Oil	QS	Refined Products	Jul '89	4.3%	7.5%	0.0%	21.1%	47.1%	0.45	-39.9%	-51.8%	-67.5%
Gasoline RBOB	XB	Refined Products	Oct '05	4.2%	4.7%	3.4%	16.6%	38.9%	0.43	-36.0%	-47.0%	-67.8%
Natural Gas	NG	Gas	Apr '90	4.4%	2.6%	10.8%	-2.6%	55.6%	-0.05	-62.4%	-69.4%	-79.9%
Lead	LL	Industrial Metals	Jul '97	1.3%	0.4%	0.0%	16.7%	48.0%	0.35	-26.0%	-43.2%	-69.8%
Zinc	LX	Industrial Metals	Jul '97	2.2%	0.6%	3.1%	8.6%	50.5%	0.17	-35.3%	-45.8%	-63.2%
Aluminium	LA	Industrial Metals	Jul '97	6.7%	2.5%	5.9%	0.1%	21.4%	0.04	-24.8%	-39.5%	-63.4%
Nickel	LN	Industrial Metals	Jul '97	2.3%	0.8%	2.6%	24.7%	60.1%	0.41	-35.3%	-50.7%	-72.5%
Copper	LP	Industrial Metals	Jul '97	12.4%	3.8%	7.1%	18.3%	42.7%	0.43	-24.4%	-38.1%	-62.2%
Gold	GC	Precious Metals	Jan '83	5.0%	2.7%	9.8%	4.1%	15.5%	0.27	-17.3%	-20.9%	-33.9%
Silver	SI	Precious Metals	Jan '83	0.9%	0.6%	2.8%	5.1%	28.5%	0.18	-28.3%	-33.2%	-48.6%
Wheat	W	Grains	Jan '83	3.6%	3.2%	5.0%	1.1%	26.9%	0.04	-34.2%	-42.0%	-63.7%
Kansas Wheat	KW	Grains	Jan '83	0.0%	1.0%	0.0%	8.8%	30.1%	0.28	-31.1%	-38.0%	-59.5%
Soybeans	S	Grains	Jan '83	7.0%	2.5%	7.1%	8.2%	24.9%	0.33	-23.5%	-27.6%	-41.8%
Corn	C	Grains	Jan '83	5.1%	4.6%	6.7%	0.1%	26.2%	0.02	-32.5%	-40.1%	-55.3%
Sugar	SB	Softs	Jan '83	7.4%	2.3%	3.8%	9.6%	10.3%	0.27	-41.1%	-53.6%	-69.1%
Coffee	KC	Softs	Jan '83	1.3%	1.0%	2.6%	5.7%	47.3%	0.12	-45.5%	-50.0%	-62.5%
Cocoa	QC	Softs	Jan '83	0.8%	0.3%	0.0%	0.0%	27.0%	-0.01	-34.9%	-41.2%	-50.4%
Cotton	CT	Softs	Jan '83	1.6%	1.8%	2.0%	7.1%	39.9%	0.18	-38.6%	-45.0%	-61.2%
Feeder Cattle	FC	Livestocks	Jan '83	0.0%	0.4%	0.0%	6.7%	17.1%	0.39	-16.9%	-19.5%	-25.1%
Live Cattle	LC	Livestocks	Jan '83	2.3%	2.5%	3.6%	9.5%	18.3%	0.52	-17.1%	-20.3%	-30.4%
Lean Hogs	LH	Livestocks	Apr '86	1.7%	1.5%	2.1%	6.5%	31.3%	0.21	-41.8%	-48.5%	-63.3%
Others	-	-	-	2.0%	0.0%	3.1%	-	-	-	-	-	-

Table 2
Commodity Sector Correlations

The table summarizes the average within- and between-sector correlations of the eight main commodity sectors consisting of 24 commodities. The within-sector correlations are calculated by averaging the pairwise correlations across all commodity futures in each sector over each of the 30 annual periods. The between-sector correlations are obtained by averaging the correlations between individual futures in the two sectors over the individual 30 years of history.

Commodity Sector	Crude Oil	Ref. Oil	Natural Gas	Industrial Metals	Precious Metals	Grains	Softs	Live Stocks
Crude Oil	0.99							
Refined Oil	0.98	0.95						
Natural Gas	0.36	0.35	1.00					
Industrial Metals	0.44	0.38	0.42	0.71				
Precious Metals	0.61	0.55	0.40	0.74	0.91			
Grains	0.56	0.51	0.54	0.54	0.62	0.63		
Softs	0.51	0.46	0.49	0.61	0.65	0.55	0.41	
Live Stocks	0.32	0.31	0.04	0.02	-0.06	0.09	0.16	0.55

Table 3
Commodity Factor Correlations

The table summarizes the correlations of the commodity factors: Market, Momentum Term Structure and Risk Reversal. Correlations are calculated over 29 years.

Commodity Factor	Market	Momentum	Term Structure	Risk Reversal
Market	1.00			
Momentum	0.13	1.00		
Term Structure	0.01	-0.07	1.00	
Risk Reversal	-0.22	-0.17	0.06	1.00

Table 4
Time Series Regressions of Principal Portfolios

The table gives time series regression results according to factor model (16) for the principal portfolios using the period from January 1984 to July 2012. Coefficients are in bold face when significant on a 5%-level and in italics when significant on a 10%-level.

	PP1	PP2	PP3	PP4	PP5	PP6	PP7	PP8
Panel A: Coefficients								
Alpha	**0.12%**	**-0.21%**	-0.17%	0.20%	-0.12%	0.08%	0.11%	*-0.19%*
Market	**0.58**	*0.06*	**0.49**	0.05	-0.01	0.03	*-0.06*	0.04
Momentum	0.01	**0.25**	0.01	**0.11**	-0.04	**-0.08**	-0.01	-0.04
Term Structure	-0.03	**-0.12**	**0.19**	**0.12**	**0.12**	0.02	**-0.21**	**0.26**
Risk Reversal	**-0.31**	*0.10*	**0.35**	0.09	**-0.21**	0.12	0.06	-0.05
Panel B: t-statistics								
Alpha	2.33	-2.01	-0.74	0.98	-0.72	0.50	0.59	-1.68
Market	23.5	1.76	12.1	0.87	-0.25	0.97	-1.67	1.25
Momentum	0.59	2.93	0.25	1.98	-1.43	-2.50	-0.35	-1.36
Term Structure	-1.25	-2.91	4.21	2.00	3.67	0.44	-5.53	7.76
Risk Reversal	-5.38	1.81	3.70	1.43	-3.07	1.46	0.71	-0.66
Adjusted R^2	74.3%	32.5%	38.8%	21.9%	10.3%	7.9%	10.9%	20.0%

Table 5
Performance and Risk Statistics of Risk-Based Commodity Strategies

The table gives performance and risk statistics of the risk-based commodity strategies from January 1984 to July 2012. Annualized return and volatility figures are reported together with the according Sharpe Ratio. Maximum Drawdown is computed over the whole sample period of 30 years. Value at risk and expected shortfall are computed at the 95% confidence interval over 1 year. Turnover is the portfolios' mean monthly turnover over the whole sample period. Gini coefficients are reported using portfolios' weights ($Gini_{Weights}$) and risk decomposition with respect to the underlying asset classes ($Gini_{Risk}$) or with respect to the principal portfolios ($Gini_{PPRisk}$). The # bets is the exponential of the risk decomposition's entropy when measured against the uncorrelated risk sources.

Statistic	**S&P GSCI**	**Risk-Based Allocations**					
		1/N	*MV*	*RP*	*MDP*	*DRP*	*PRP*
Panel A: Risk and Return Figures							
Return p.a.	5.9%	6.6%	4.4%	6.2%	4.6%	8.0%	9.7%
Volatility p.a.	23.0%	15.9%	11.7%	14.5%	13.0%	14.1%	17.9%
Sharpe Ratio	0.25	0.41	0.38	0.43	0.35	0.57	0.54
VaR 95% 1year	-31.7%	-23.0%	-17.5%	-21.3%	-19.3%	-13.8%	-20.0%
ES 95% 1year	-44.1%	-31.4%	-20.2%	-27.0%	-35.5%	-17.6%	-26.1%
Maximum Drawdown	-70.1%	-49.5%	-33.9%	-41.4%	-40.0%	-31.2%	-38.9%
Panel B: Weights and Risk Decomposition Characteristics							
Turnover	8.4%	0.0%	24.1%	13.8%	20.3%	37.1%	27.3%
$Gini_{Weights}$	0.54	0.00	0.52	0.26	0.46	0.68	0.61
$Gini_{Risk}$	0.74	0.34	0.40	0.01	0.38	0.73	0.64
$Gini_{PPRisk}$	0.85	0.84	0.70	0.75	0.71	0.32	0.63
# bets	1.83	1.95	2.79	2.53	2.97	5.92	4.40

Table 6
Time Series Regressions of Risk-Based Commodity Strategies

The table gives time series regression results according to factor models (17) and ((18) for the GSCI and the risk-based commodity strategies using the period from January 1984 to July 2012. Coefficients are in bold face when significant on a 5%-level and in italics when significant on a 10%-level.

Statistic	**S&P GSCI**	**Risk-Based Allocations**					
		1/N	*MV*	*RP*	*MDP*	*DRP*	*PRP*
Panel A: Coefficients							
Alpha	0.09%	0.07%	0.03%	-0.01%	0.04%	0.05%	0.11%
Market	-	**0.47**	0.05	**0.19**	0.02	0.04	**0.30**
Momentum	**0.11**	**-0.25**	**-0.20**	**-0.20**	**-0.22**	0.07	**-0.23**
Term Structure	0.03	*-0.03*	**0.13**	**0.24**	**0.08**	-0.01	-0.02
Risk Reversal	**0.27**	0.01	-0.07	*0.01*	0.05	0.03	-0.09
Panel B: t-statistics							
Alpha	1.52	1.03	0.95	-0.23	1.29	1.30	1.61
Market	-	12.32	1.39	3.84	1.17	0.50	2.32
Momentum	2.56	-6.71	-5.64	-3.17	-5.91	1.09	-2.55
Term Structure	1.44	-1.71	2.19	5.41	1.97	-0.28	-0.89
Risk Reversal	4.01	1.12	-0.97	1.70	0.23	0.63	-1.46
Adjusted R^2	57.0%	66.9%	26.6%	53.5%	26.1%	9.7%	32.1%

Table 7
Statistics of Enhanced Risk-Based Commodity Strategies

The table gives performance and risk statistics of the enhanced risk-based commodity strategies from January 1984 to July 2012. Annualized return and volatility figures are reported together with the according Sharpe Ratio. The # bets is the exponential of the risk decomposition's entropy when measured against the uncorrelated risk sources. The strategies are labeled with percentages indicating the percentage of available commodities excluded at every rebalancing date according to either momentum, risk reversal or term structure trading signals.

	Return p.a.	Volatility p.a.	Sharpe Ratio	# bets
Panel A: Momentum Trading Signals				
DRP	8.0%	14.1%	0.57	5.9
DRP 10%	9.1%	14.3%	0.62	5.7
DRP 25%	10.5%	15.0%	0.69	5.5
DRP 50%	12.8%	19.3%	0.60	4.8
DRP 75%	15.4%	27.2%	0.55	3.2
PRP	9.7%	17.9%	0.54	4.4
PRP 10%	11.2%	19.5%	0.56	4.2
PRP 25%	13.9%	23.7%	0.63	3.7
PRP 50%	15.9%	29.2%	0.54	3.0
PRP 75%	16.2%	29.6%	0.53	2.2
Panel A: Risk Reversal Trading Signals				
DRP	8.0 %	14.1 %	0.57	5.9
DRP 10%	8.9 %	14.8 %	0.60	5.5
DRP 25%	10.7 %	17.1 %	0.63	5.0
DRP 50%	4.6 %	19.1 %	0.24	4.3
DRP 75%	1.5 %	20.7 %	0.07	3.4
PRP	9.7 %	17.9 %	0.54	4.4
PRP 10%	10.6 %	18.6 %	0.57	4.1
PRP 25%	12.1 %	19.8 %	0.61	3.7
PRP 50%	9.3 %	23.3 %	0.40	2.9
PRP 75%	4.1 %	25.8 %	0.16	2.3
Panel A: Term Structure Trading Signals				
DRP	8.0 %	14.1 %	0.57	5.9
DRP 10%	9.0 %	14.3 %	0.63	5.8
DRP 25%	10.6 %	15.1 %	0.70	5.6
DRP 50%	10.9 %	17.5 %	0.62	4.8
DRP 75%	7.6 %	15.1 %	0.50	3.4
PRP	9.7 %	17.9 %	0.54	4.4
PRP 10%	10.6 %	18.6 %	0.57	4.3
PRP 25%	11.9 %	19.5 %	0.61	3.9
PRP 50%	11.0 %	22.0 %	0.50	3.2
PRP 75%	8.5 %	24.6 %	0.35	2.4

Figure 1. Average Correlation of GSCI Constituents

The figure depicts the average correlation structure of the 24 GSCI commodities using the maximum amount of data available for any pair in the sample period from 1983 to 2012. The commodities are sorted and grouped according to the corresponding commodity sector. The first five commodities are from the energy sector consisting of the Crude Oil and Refined Products and Gas subsectors. The subsequent five commodities are industrial metals, followed by two precious metals. The last three commodities are live stocks, and the remaining are from the agriculture sector with softs, grains and live stocks. Colors range from dark red (correlation of 1) to dark blue (correlation of -0.3).

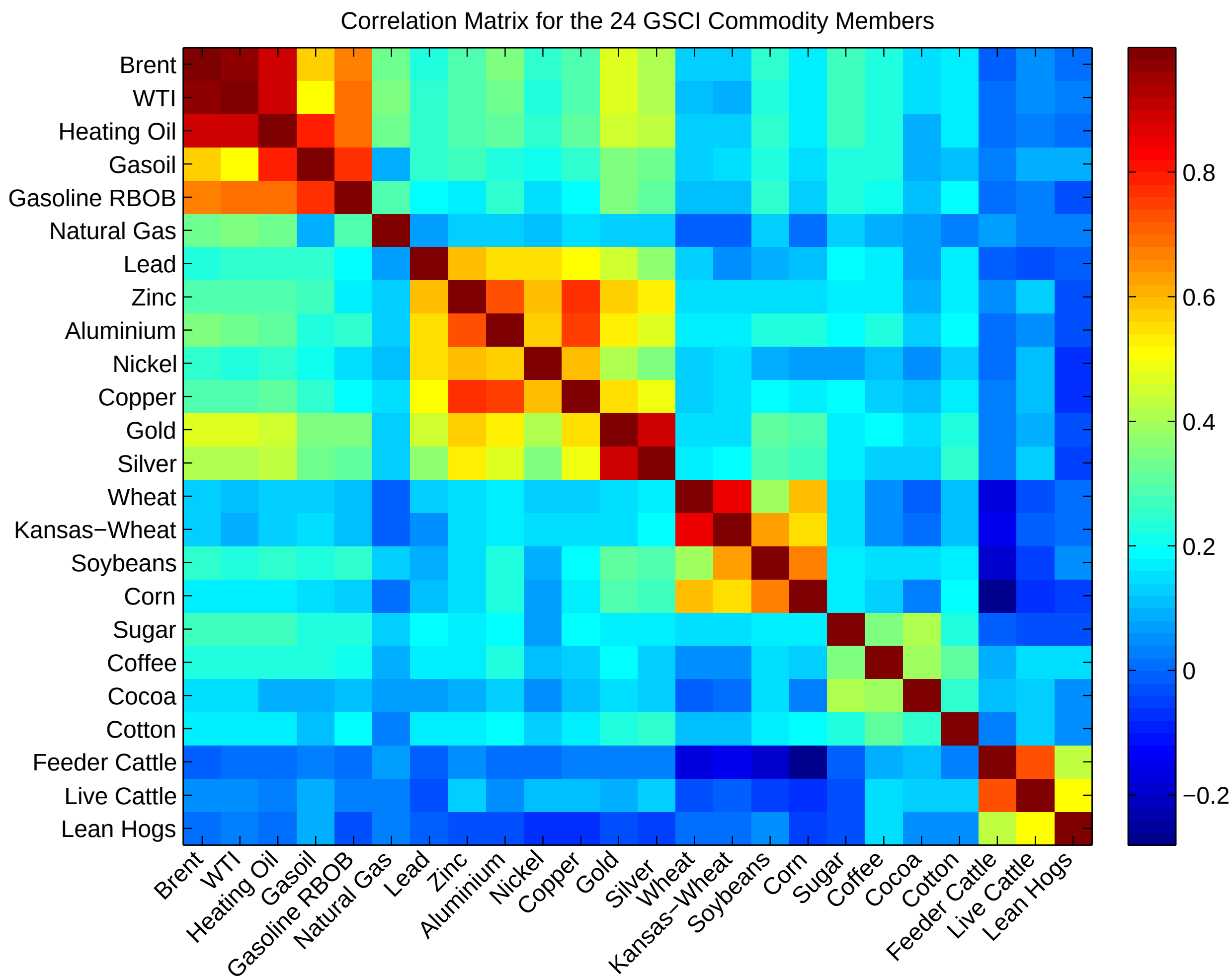
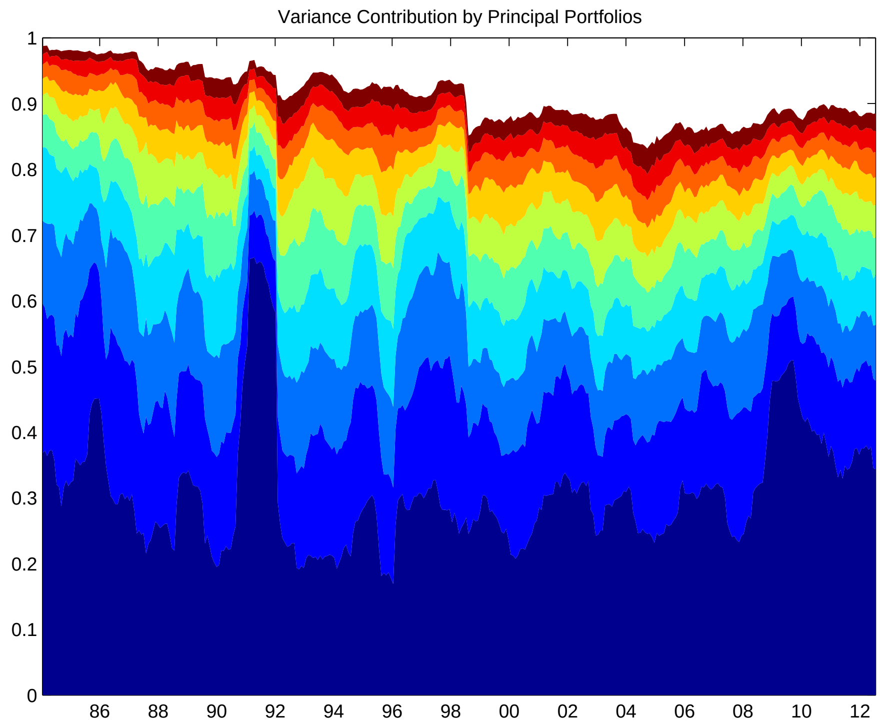


Figure 2. Variance Decomposition of Principal Portfolios' Variance

The upper chart gives the variance of the principal portfolios and its relative decomposition over time. Each month, a PCA is performed to extract the first 10 principal portfolios embeded in the underlying 24 commodities of the GSCI and the corresponding principal portfolio variances are stacked in one bar. The lower charts give the boxplots pertaining to a given principal portfolio's explained fraction of total variance over time. The results are ranging from January 1984 to July 2012.



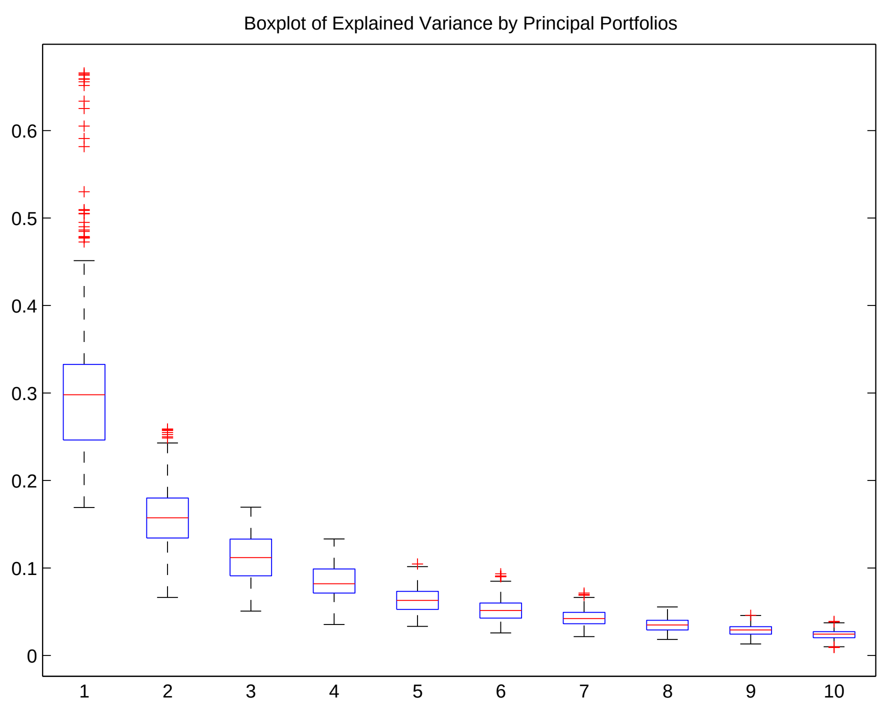


Figure 3. Number of Principal Portfolios

The figure gives the number of principal portfolios over time as suggested by the PC_{p1} and PC_{p2} criteria of Bai and Ng (2002). The results are ranging from January 1984 to July 2012.

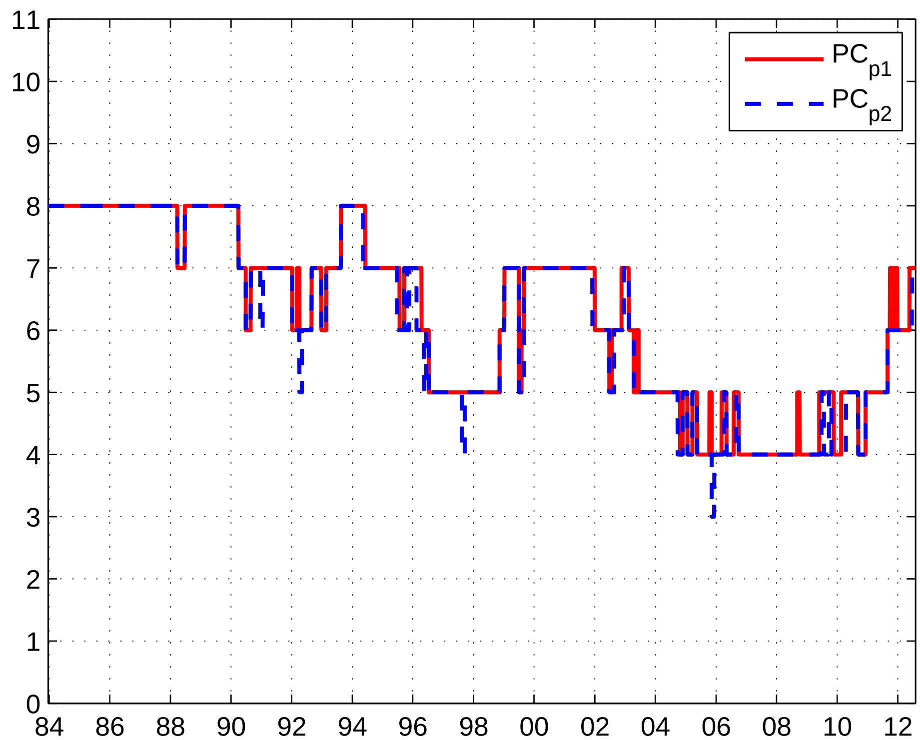


Figure 4. Portfolio Weights of Principal Portfolios

The figure gives the portfolio weights of the principal portfolios arising from a Principal Component Analysis (PCA) where all 24 commodities of the GSCI are available. Lines starting in the origin are added and their color corresponds to the respective commodity's sector classification: Crude Oil is black, Refined Products is dark blue, Gas is light blue, Industrial Metals are green, Precious Metals are orange, Grains are magenta, Softs are yellow, and Livestocks are red. The results are ranging from January 2006 to July 2012.

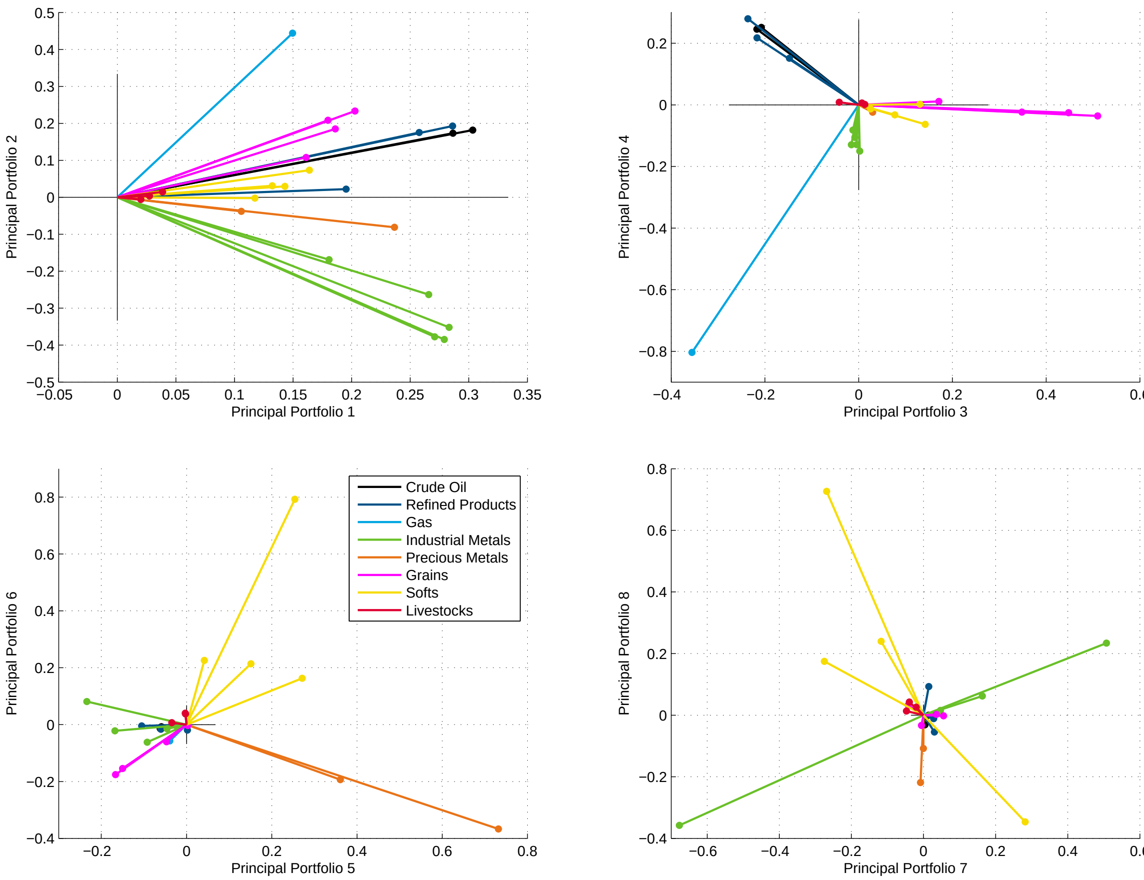


Figure 5. Principal Portfolio Weights over Time

The figure gives the principal portfolios weights over time. The results are obtained using an expanding estimation windows of 252 days. Gasoline RBOB is excluded from the asset universe. The estimation period is restricted to January 1998 to July 2012 in order to have complete time series for all 23 commodities.

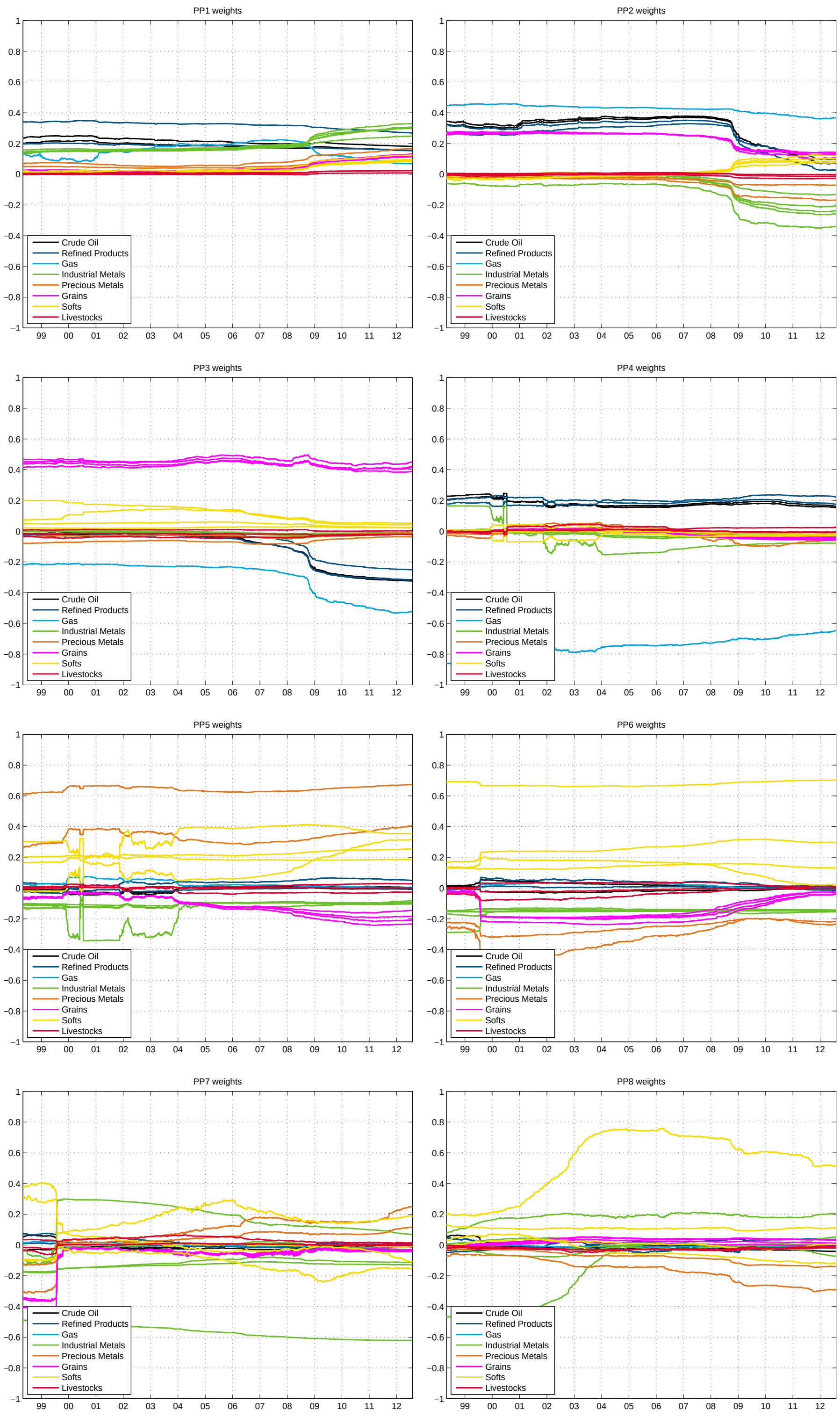


Figure 6. Weights and Risk Decompositions: Major Commodity Indices

The figure gives the decomposition of the three main global commodity indices: the Standard&Poor's Goldman Sachs Commodity index, the UBS Bloomberg Constant Maturity Commodity Index and the Dow Jones UBS Commodity Index in terms of weights and risk. Risk is being decomposed by commodity sectors and by principal portfolios, respectively. Indices are constructed by means of the 2012 target weights. The sample period is from January 1984 to July 2012.

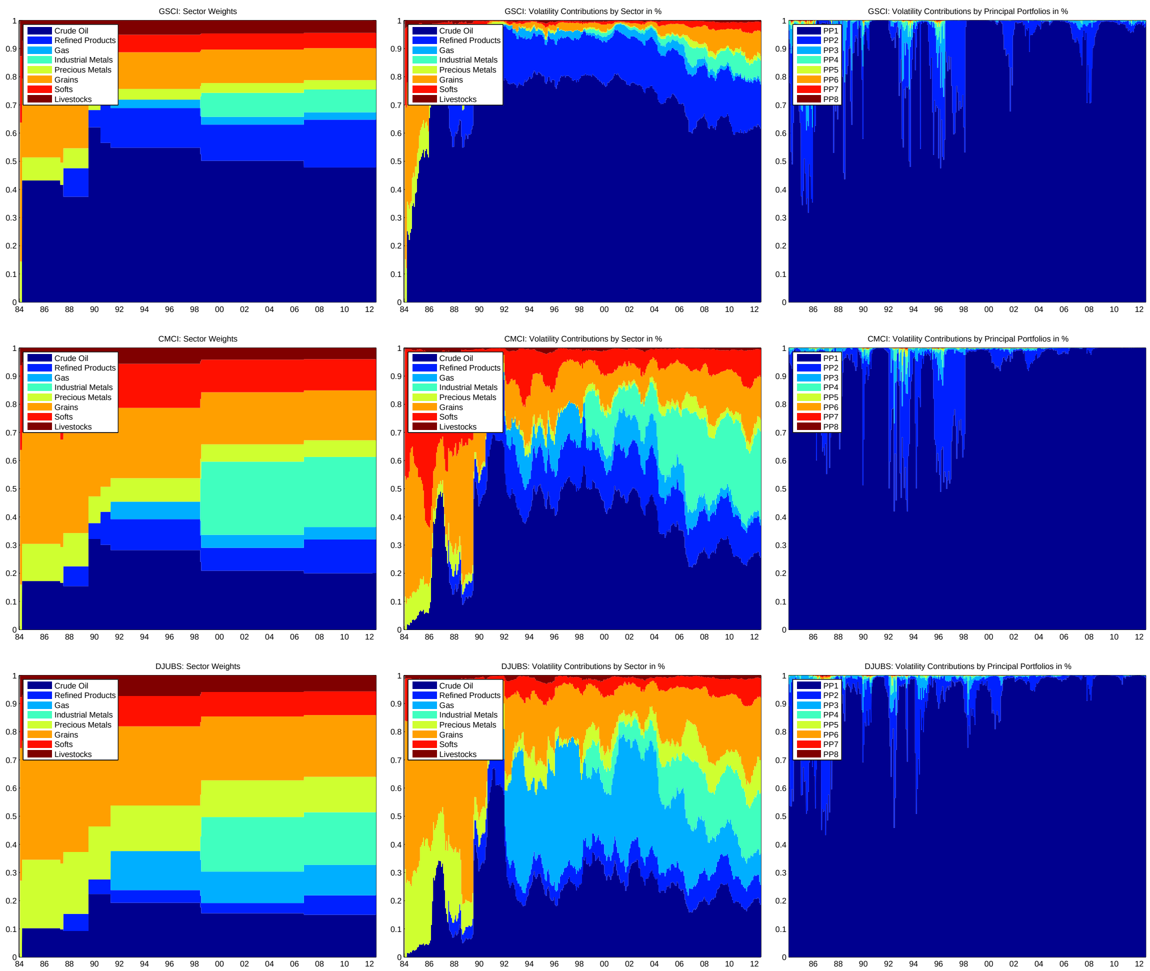


Figure 7. Weights and Risk Decompositions: Risk-Based Commodity Strategies

The figure gives the decomposition of the risk-based commodity strategies in terms of weights and risk. Risk is being decomposed by asset classes and by principal portfolios, respectively. The first row contains the results for the 1/N-strategy, the second row is for minimum-variance, the third row for risk parity, the fourth row for the MDP. The sample period is from September 1984 to July 2012.

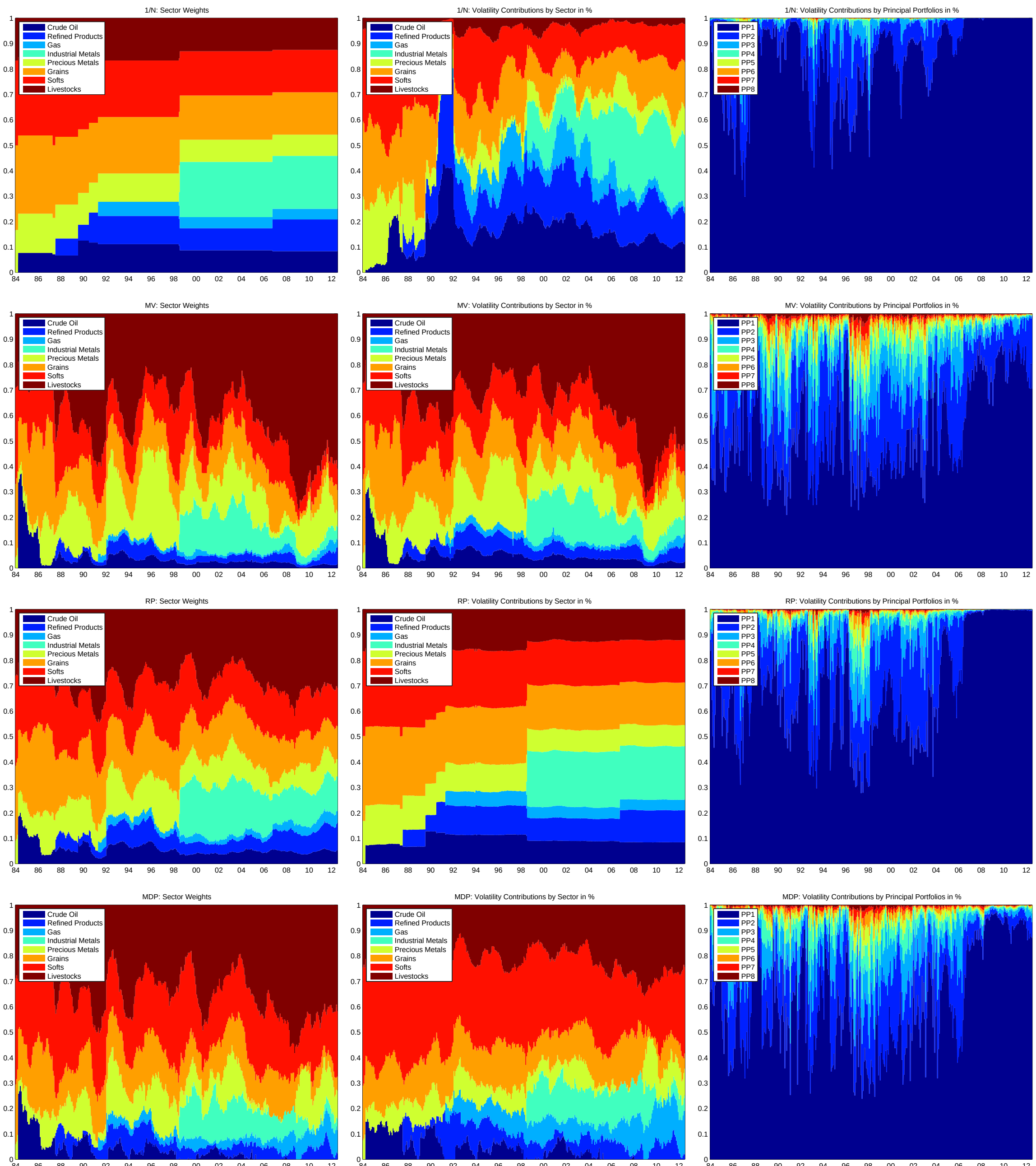


Figure 8. Weights and Risk Decompositions: Alternative Risk Parity Strategies
The figure gives the decomposition of the alternative risk parity strategies. Risk is being decomposed by asset classes and by principal portfolios, respectively. The first row contains the results for the diversified risk parity, and the second row for principal risk parity. The sample period is from September 1984 to July 2012.

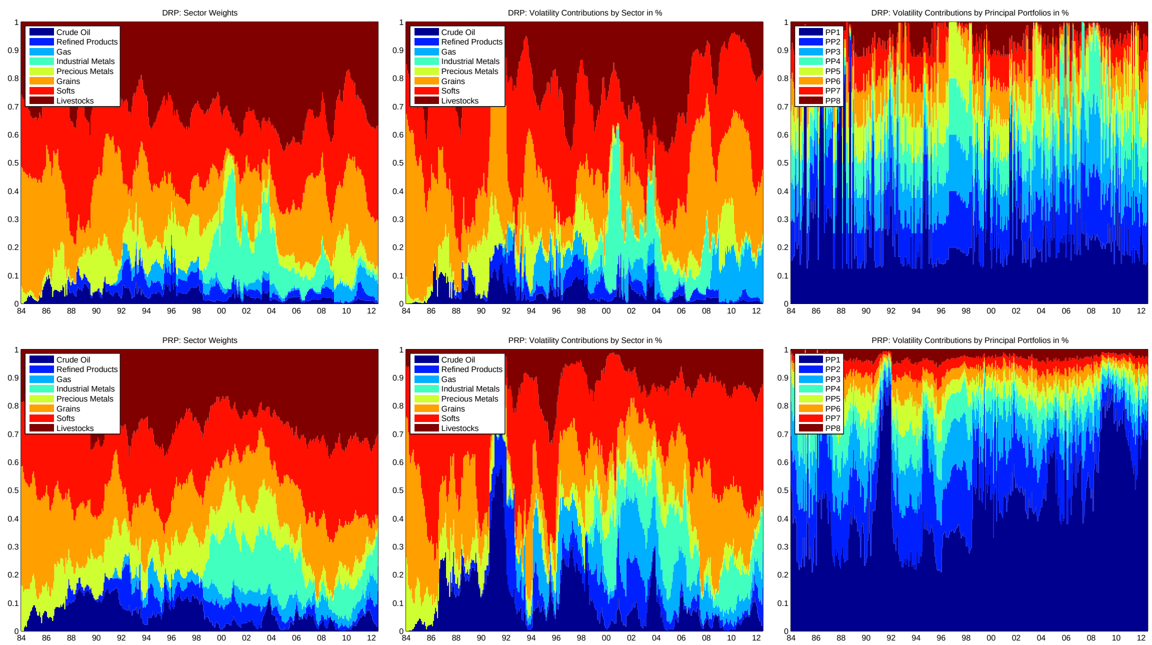


Figure 9. Number of Uncorrelated Bets

We plot the number of uncorrelated bets for the risk-based commodity strategies for the sample period January 1984 to July 2012.