High Order Smooth Ambiguity Preferences and Asset Prices

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First version: December 20, 2011

This version: February 13, 2012

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Abstract

This paper extends the recursive smooth ambiguity decision model developed in Klibanoff, Marinacci, and Mukerji (2005, 2008) by relaxing the uniformity imposed on higher order acts. This generalization permits a separation of intertemporal substitution, risk aversion, and ambiguity aversion towards different sources of uncertainty. We apply our preference specification to a consumption-based asset pricing model with long-run risks and assess the impact of ambiguity on important asset pricing moments and predictability patterns. We find that modeling attitudes towards uncertainty and risk through high order smooth ambiguity preferences has important implications for asset prices. Our model significantly improves upon the special cases of Epstein and Zin utility and standard smooth ambiguity preferences.

Keywords: Ambiguity aversion, asset pricing, long-run risks

JEL: D81, E44, G12

1 Introduction

The subjective expected utility theory (SEU) of Savage (1954) postulates that a decision maker (DM) only cares about *risk*. According to Knight (1921), risk refers to a situation where information is described by a probability distribution. If there are two or more distinct probability measures which the DM deems possible, uncertainty about the true probability measure is considered irrelevant. However, the experiments of Ellsberg (1961) and Halevy (2007) demonstrate that the SEU approach is inconsistent with reasonable decision making. Subjects usually prefer situations where uncertainty concerning the true probability measure is low. This type of uncertainty is commonly called ambiguity.

Several models have been developed, that account for Ellsberg type behavior. In the smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2005) (KMM), the DM believes that several probability measures are possible and calculates a certainty equivalent for each of these. She then uses expected utility to arrive at a single value. Formally, their smooth ambiguity model has the following representation:

$$\int_{S_2} u_2\left(u_1^{-1}\left[\int_{S_1} u_1 \circ f \; d\mu_1\right]\right) \; d\mu_2^*(\mu_1).$$

For each possible measure μ_1, the term in parentheses is the corresponding certainty equivalent, u_1 is the usual utility function which displays attitudes towards risk, while u_2 is a utility function which characterizes attitudes towards ambiguity. Similar to the SEU approach, the ultimate probability measure (now μ_2^* on the space S_2 of probability measures) needs to be specified. This paper relaxes this assumption and allows for high order ambiguity.

To get the intuition behind high order ambiguity, we consider a thought experiment similar to Halevy (2007), who extended the experiments in Ellsberg (1961). In Halevy's experiment subjects won a price if they picked a red ball from 4 distinct urns presented to them. Among them, urn 1 contained 5 red balls and 5 black balls.

For urn 3, the number of red and black balls was determined by drawing one ticket from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number on the drawn ticket determined the number of red balls in urn 3. Halevy (2007) finds that ambiguity averse subjects (tested with the usual Ellsberg (1961) experiment) do not compound lotteries, i.e. urn 1 is preferred to urn 3. This result is consistent with the KMM approach. Now imagine a game with an additional stage: Subjects can choose between Halevy's urn 1 and urn 3. If the ball drawn is red, subjects have to pick a ball from a "final" urn containing 2 red balls and 1 black ball. If the ball from urn 1 or urn 3 is black, the "final" urn contains 2 black balls and 1 red ball. The KMM model suggests that subjects are indifferent between both alternatives.[1] Halevy's findings indicate that urn 3 is perceived as ambiguous by subjects. Why should ambiguity about the composition of urn 3 vanish if it is used as a "construction urn"? We believe that subjects care about ambiguity concerning the composition of construction urns and thus it should be incorporated into the decision model. More formally, uncertainty about the true measure μ_2 in the above KMM representation of smooth ambiguity preferences should be considered. In our high order smooth ambiguity model this type of uncertainty is accounted for and modeled as in KMM for uncertainty regarding μ_1.

In economics, especially in asset pricing theory, the DM usually faces a variety of sources of uncertainty. In existing models of ambiguity, studying the implications of ambiguity aversion requires a classification of these sources into two categories (usually called *risk* and *ambiguity*). One may, for example, consider consumption to

[1]This is also true for other models of ambiguity, e.g. the multiple priors model of Gilboa and Schmeidler (1989). Klibanoff, Marinacci, and Mukerji (2011) point out that it is important to carefully specify the relevant state space. Following their line of argument the state space of Halevy's urn 3 contains all possible pairs of tickets and balls. Ambiguity exists only "in the mind of the subjects". However, the composition of the bag containing the tickets is known. Thus, subjects should be indifferent between the two urns. More consistent with Halevy's findings, Epstein (2010) considers a state space that would not include the outcome of the draw from the bag of tickets.

have a stochastic growth rate as well as a stochastic volatility. Is the investor ambiguous about the drift, the volatility or possibly both, and does she treat these sources of uncertainty equally? The usual procedure is to consider diffusive consumption uncertainty as risk and all other sources of uncertainty as ambiguity.[2] If we abandon the rule that an investor evaluates different sources of uncertainty equally, it does not seem to be a sensible assumption that she categorizes them into exactly two classes about whose elements she has homogeneous tastes. Our high order smooth ambiguity model allows for a more flexible specification of attitudes towards uncertainty. In addition to the separation of intertemporal substitution, risk aversion, and ambiguity aversion, our model differentiates between several sources of uncertainty and assigns each kind of uncertainty an individual ambiguity parameter.

Ju and Miao (2011) use the standard smooth ambiguity model of KMM to calculate asset prices in an endowment economy where the investor has to learn about a latent factor that drives consumption growth. They choose a hidden Markov regime-switching model, while Collard, Mukerji, Sheppard, and Tallon (2011) use an $AR(1)$ specification for the expected growth rate of consumption. Among other findings, both papers discover that ambiguity aversion helps to generate a sizable equity premium, while keeping the risk aversion parameter at low values. In contrast to these papers, we omit learning and use our high order smooth ambiguity model to characterize preferences. We follow Bansal and Yaron (2004) and use autoregressive processes to characterize the level and the volatility of consumption growth. Extensions of their long-run risks (LRR) model introduce additional state variables and jump components.[3] In contrast to these studies, we use a similar endowment process and focus on the preference specification.[4] The parameters in Bansal and

[2] See e.g. Collard, Mukerji, Sheppard, and Tallon (2011) and Ju and Miao (2011).

[3] See e.g. Benzoni, Collin-Dufresne, and Goldstein (2011), Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), and Eraker and Shaliastovich (2008).

[4] Bonomo, Garcia, Meddahi, and Tedongap (2011) also explore the model of Bansal and Yaron (2004) using exotic preferences. They consider the generalized disappointment aversion (GDA) of

Yaron (2004) and Bansal, Kiku, and Yaron (2011) were calibrated to match important cash-flow and asset pricing moments. However, Beeler and Campbell (2011) and Constantinides and Ghosh (2011) identify several shortcomings of the LRR model. Two serious problems are the volatility of the log price-dividend ratio and the predictability of cash-flows and asset returns. We find that our model can significantly improve upon the LRR model. We match several important unconditional asset pricing moments, including the high standard deviation of the log price-dividend ratio, and bring the predictive power of the price-dividend ratio for cash-flows and asset returns in line with the values found in the data.

The KMM approach is just one of many models that can account for Ellsberg type behavior. Another prominent example is the multiple priors (or maxmin expected utility) model of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003). In contrast to KMM, where the attitude towards ambiguity is introduced by relaxing the reduction of first and second order probabilities, the multiple priors model takes the minimum expected utility with respect to a set of priors. This model has been extensively applied in the asset pricing context, among others by Epstein and Wang (1994), Epstein and Schneider (2008), and Drechsler (2011). A survey of the literature can be found in Epstein and Schneider (2010). From a technical perspective the smooth ambiguity model is easier to work with compared to the multiple priors model as it avoids the non-differentiability of the min operator. As our model shares the "smoothness" of the KMM model, we are able to derive approximate analytic solutions.

The remainder of this paper is organized as follows. In Section 2 we introduce high order smooth ambiguity preferences. We derive the pricing kernel in Section 3. In Section 4 we apply our decision model to an endowment process with a persistent long-run growth rate and a time-varying conditional volatility. Section 5 concludes.

Routledge and Zin (2010) and find that their model can improve upon the benchmark LRR model. However, working with GDA preferences is difficult due to the kink in the utility function.

2 High Order Smooth Ambiguity Preferences

In this section we introduce high order smooth ambiguity preferences. We start with a static setting and generalize to a recursive model of preference in the manner of Epstein and Zin (1989) and Kreps and Porteus (1978). Our approach extends the decision model of Klibanoff, Marinacci, and Mukerji (2005) which was put in a dynamic setting by Klibanoff, Marinacci, and Mukerji (2008) and Ju and Miao (2011) and axiomatized by Hayashi and Miao (2011).

2.1 The static setting

Let S_1 be a state space, equipped with a sigma-algebra Σ_1. A *first order act* is a usual Savage act, i.e. a map $f\colon S_1 \longrightarrow \mathcal{C}$ to a set $\mathcal{C}$ of consequences. We assume that $\mathcal{C}$ is a convex subset of $\mathbb{R}$.[5] $\mathcal{A}_1$ denotes the set of all Σ_1-measurable bounded first order acts. The DM's preferences are given by a binary relation $\preceq_1$ on $\mathcal{A}_1$. It is a well-known fact that if we agree with the validity of certain axioms[6], there is a utility function $u_1 : \mathcal{A}_1 \longrightarrow \mathbb{R}$ such that the DM prefers an act f to an act g, i.e. $f \succeq_1 g$, if and only if $F_1(f) \geq F_1(g)$, where F_1 is the functional

$$\begin{aligned} F_1\colon \mathcal{A}_1 &\longrightarrow \mathbb{R}, \\ f &\longmapsto \int_{S_1} u_1 \circ f \; d\mu_1^*. \end{aligned}$$

Here, μ_1^* denotes a probability measure on the measure space (S_1, Σ_1), which is assumed to be known to the DM. Another interpretation might be, that she is uncertain about the true probability measure, but not averse against this kind of uncertainty. In this case, she simply aggregates the different measures she considers possible to the single measure μ_1^*.

[5] More generally, one may assume that $\mathcal{C}$ is a connected separable topological space, see Ghirardato and Marinacci (2003).

[6] See von Neumann and Morgenstern (1944).

If the DM is averse against ambiguity, her preferences do not permit such an aggregation. Let S_2 denote the set of probability measures[7] on the measure space (S_1, Σ_1). We equip S_2 with the vague topology and consider the corresponding Borel-sigma-algebra Σ_2 on S_2. A *second order act* is a map $f\colon S_2 \longrightarrow \mathcal{C}$ and the set of all Σ_2-measurable bounded second order acts is denoted by $\mathcal{A}_2$. We assume that the investor entertains a preference relation $\preceq_2$ on $\mathcal{A}_2$.

The KMM model assumes that the DM knows the probability measure on (S_2, Σ_2) or is (just as before) not averse against uncertainty about which probability measure is present. She therefore aggregates measures to a single one, called μ_2^* for the moment. KMM show that a DM that accepts the validity of certain axioms will prefer the first order act $f \in \mathcal{A}_1$ to the act $g \in \mathcal{A}_1$, i.e. $f \succeq_1 g$, if and only if $F_2(f) \geq F_2(g)$, where F_2 denotes the functional

$$\begin{aligned} F_2\colon \mathcal{A}_1 &\longrightarrow \mathbb{R}, \\ f &\longmapsto \int_{S_2} u_2\left(u_1^{-1}\left[\int_{S_1} u_1 \circ f \; d\mu_1\right]\right) \; d\mu_2^*(\mu_1), \end{aligned}$$

and $u_2\colon \mathcal{C} \longrightarrow \mathbb{R}$ denotes a further utility function. Intuitively, the DM calculates the certainty equivalent for each possible measure μ_1 and then considers her expected utility over the space S_2. KMM show that the DM is smooth ambiguity *averse* if u_2 is a concave transform of u_1, meaning that $(u_2 \circ u_1^{-1})\colon \mathbb{R} \supseteq u_1(\mathcal{C}) \longrightarrow \mathbb{R}$ is a concave function.[8] If the function $(u_2 \circ u_1^{-1})$ is linear, the DM is ambiguity neutral and her preferences coincide with SEU preferences.

For the KMM model it is crucial that the DM knows the measure μ_2^* on S_2 or

[7]Our approach roots in the theory of Klibanoff, Marinacci, and Mukerji (2005) who consider the state space $S_1 = \Omega \times (0, 1]$. They define S_2 as the set of all countably additive product probability measures with the Lebesgue measure on the Borel-sigma-algebra on $(0, 1]$. Our treatment does not require such a specification.

[8]For this expression to be well defined, u_1 needs to be injective. We assume throughout the paper, that all utility functions exhibit this characteristic. In applications, utility functions are considered to be increasing if the DM is not saturated.

is at least not averse against ambiguity concerning the true measure. This implies that second order acts are evaluated using SEU. There is no obvious reason why this has to be the case. In contrast, we treat second order acts in the same fashion as first order acts, meaning that the DM is not neutral to uncertainty about the measure μ_2. Let S_3 be the set of probability measures on (S_2, Σ_2). As before, we equip S_3 with the vague topology and the corresponding Borel-sigma-algebra, which we call Σ_3. In analogy to the preceding approaches, we assume that the DM entertains utility functions u_1, u_2, and u_3 such that her preferences concerning first order acts can be described in terms of the functional

$$\begin{aligned} F_3\colon \mathcal{A}_1 &\longrightarrow \mathbb{R}, \\ f &\longmapsto \int_{S_3} u_3 \left(u_2^{-1} \left[\int_{S_2} u_2 \left(u_1^{-1} \left[\int_{S_1} u_1 \circ f \; d\mu_1 \right] \right) \; d\mu_2(\mu_1) \right] \right) \; d\mu_3^*(\mu_2), \end{aligned}$$

meaning that $f \succeq_1 g$ if and only if $F_3(f) \geq F_3(g)$. In case the DM is ambiguous about the true measure μ_3^* on S_3, we might consider a further state space S_4 that contains all probability measures on (S_3, Σ_3).

Recursively, we define S_n as the set of probability measures on (S_{n-1}, Σ_{n-1}) and equip it with the Borel-sigma-algebra of the vague topology, which we call Σ_n. An n-th order act is a map $f : S_n \longrightarrow \mathcal{C}$ and $\mathcal{A}_n$ denotes the set of all Σ_n-measurable bounded n-th order acts, on which the DM entertains a preference relation $\succeq_n$. We assume that there is a number $N \in \mathbb{N}$ such that the DM knows the true probability measure μ_N^* on S_N or that she is neutral concerning uncertainty about the true measure on S_N. We call that specific N the DM's *level of abstraction*. SEU is the special case with $N = 1$, the KMM model refers to $N = 2$. Assume that the DM prefers $f \in \mathcal{A}_1$ to $g \in \mathcal{A}_1$ if and only if $F_N(f) \geq F_N(g)$, where F_N denotes the functional

$$\begin{aligned} F_N\colon \mathcal{A}_1 &\longrightarrow \mathbb{R}, \\ f &\longmapsto \int_{S_N} u_N \left(u_{N-1}^{-1} \left[\int_{S_{N-1}} \cdots \int_{S_1} u_1 \circ f \; d\mu_1 \ldots d\mu_{N-1}(\mu_{N-2}) \right] \right) d\mu_N^*(\mu_{N-1}), \end{aligned}$$

for utility functions $u_1, \dots, u_N : \mathcal{C} \longrightarrow \mathbb{R}$.

For $n \in \{0, \dots, N\}$ define the n-th order certainty equivalent recursively via

$$\begin{aligned} CE_0(x) &:= x, \qquad \text{and} \\ CE_n(x) &:= u_n^{-1}\left(\mathbb{E}_{\mu_n}\left[u_n(CE_{n-1}(x))\right]\right). \end{aligned}$$

Note that for all $0 < n < N$ the n-th order certainty equivalent CE_n depends on the chosen measure μ_n. Consequently, for each first order act $f \in \mathcal{A}_1$, $CE_n(f)$ denotes an $(n+1)$-th order act $CE_n(f) : S_{n+1} \longrightarrow \mathcal{C}$, since each measure $\mu_n \in S_{n+1}$ leads to a definite certainty equivalent. However, there is one particular measure μ_N^* on S_N, thus CE_N denotes a scalar value. We call the DM n-th order ambiguity averse if $(u_n \circ u_{n-1}^{-1})$ is concave, n-th order ambiguity loving if it is convex, and n-th order ambiguity neutral if it is linear. The intuition of our representation extends that of the KMM model: For an order $n \in \{1, \dots, N-1\}$, the DM assesses one certainty equivalent for each measure on S_n she considers possible. She then calculates expected utility of these certainty equivalents with respect to a measure μ_{n+1}. If she is uncertain, which measure μ_{n+1} is the true one, she repeats the procedure until she reaches her level of abstraction. If the DM is n-th order ambiguity averse, she will reject mean-preserving spreads in expected utilities of $(n-2)$-th order certainty equivalents for $n \geq 2$. Risk aversion corresponds to first order ambiguity aversion in our setup. Second order ambiguity aversion is the *ambiguity aversion* as defined in KMM.

If the DM's level of abstraction equals N, she cannot be N-th order ambiguity neutral, because otherwise her level of abstraction would have been less than N. Following this argument, we could also say that the DM never knows the true probability measure and takes infinitely many integrals into consideration. However, it is reasonable to believe that there is a certain point where she is neutral concerning ambiguity of higher orders. This fits to our intuition of reasonable decision making in the extended urn model. Consider urn 3 in Halevy (2007) and add further construction urns determining the composition of the subsequent urns. It is sensible to assume that there is a certain point, where the DM is indifferent between the "higher order alternatives", i.e. if her level of abstraction is 5 she will be indifferent between the presence of 5 or more construction urns.

Above, we used the notation $d\mu_n^*(\mu_{n-1})$ which can be interpreted in two different ways. It can either mean that for $n \in \{2, \dots, N\}$ the distribution of μ_{n-1} is fully characterized by (a realization of) μ_n. Thus, there is a strictly ascending order of measures. In the urn model, this would imply that the composition of an urn only depends on the outcome of the draw from the preceding urn. Alternatively, it can be understood as an abbreviation of $d\mu_n^*(\mu_{n-1}, \dots, \mu_1)$, meaning that for example μ_3^* might affect μ_1 immediately and not only through μ_2.[9] The first interpretation yields that a DM's preferences are fully characterized by a unique series of utility functions $(u_1, \dots, u_N)$. Irrespective of the context, all decision problems are evaluated in line with the functional F_N corresponding to that series. The other interpretation does not have this property. In applications the ordering of sources of uncertainty is often not strictly ascending. If one does not want to impose uniformity of preferences concerning uncertainty of a certain order, one may follow the second interpretation and assign different attitudes towards these sources of uncertainty.

2.2 The dynamic setting

We now integrate the model of Section 2.1 into a dynamic setting. Let $C = (C_t)_{t\in\mathbb{N}}$ denote the DM's consumption plan, which is a series of random variables such that C_t is time t-measurable for all $t \in \mathbb{N}$. Inspired by Kreps and Porteus (1978) and Epstein and Zin (1989), we assume that the DM's time t value function is given recursively by

$$V_t(C) = W(C_t, \mathcal{R}_t(V_{t+1}(C))),$$

[9] μ_1 might e.g. induce a $\mathcal{N}(\alpha, \beta)$-distribution as pushforward measure on $\mathcal{C}$, where the distribution of α is described by μ_2 and that of β by μ_3^*.

where W denotes the time aggregator and $\mathcal{R}_t$ the uncertainty aggregator. The latter is specified as the N-th order certainty equivalent, known from the static case

$$\mathcal{R}_t(x) = u_N^{-1}\left(\mathbb{E}_{\mu^*_{N,t}}\left[u_N \circ u_{N-1}^{-1}\left(\mathbb{E}_{\mu_{N-1,t}} \dots u_2 \circ u_1^{-1}\left(\mathbb{E}_{\mu_{1,t}}\left[u_1(x)\right]\right)\dots\right)\right]\right).$$

We adopt the notation from Section 2.1 with the little difference that we write $\mu_{n,t}$ since for each $n \in \{1, \dots, N\}$ the measure μ_n might depend on time. Imagine e.g. a state variable whose realization influences the distribution of future outcomes. The distribution of future states usually depends on the current state and is therefore time-varying. For the sake of brevity, we omit the time index in the following.

There is a huge variety of possible specifications for the functions $(u_n)_{n\in 1,\dots,N}$ and W. A popular choice for W is the constant elasticity time aggregator

$$W(x, y) = \left[(1 - e^{-\delta})x^{1-\rho} + e^{-\delta}y^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where δ denotes the DM's subjective time discount rate and ρ the reciprocal of the intertemporal elasticity of substitution (IES). We choose utility functions of the power utility type, thus

$$u_n\colon \mathcal{C} \longrightarrow \mathbb{R}, \quad c \longmapsto \frac{c^{1-\gamma_n}}{1-\gamma_n}, \quad \gamma_n > 0, \neq 1,$$

for all $n \in \{1, \dots, N\}$. We then have

$$V_t(C) = \left[(1 - e^{-\delta})C_t^{1-\rho} + e^{-\delta}\left\{\mathcal{R}_t(V_{t+1}(C))\right\}^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where

$$\mathcal{R}_t(V_{t+1}(C)) = \left\{\mathbb{E}_{\mu^*_N}\left[\left(\mathbb{E}_{\mu_{N-1}} \cdots \left(\mathbb{E}_{\mu_2}\left[\left(\mathbb{E}_{\mu_1}\left[V_{t+1}^{1-\gamma_1}(C)\right]\right)^{\frac{1-\gamma_2}{1-\gamma_1}}\right]\right)^{\frac{1-\gamma_3}{1-\gamma_2}} \cdots\right)^{\frac{1-\gamma_N}{1-\gamma_{N-1}}}\right]\right\}^{\frac{1}{1-\gamma_N}}.$$

With this specification, the DM is n-th order ambiguity averse if $\gamma_n > \gamma_{n-1}$, n-th order ambiguity loving if $\gamma_n < \gamma_{n-1}$, and n-th order ambiguity neutral if $\gamma_n = \gamma_{n-1}$.

There are a lot of prominent special cases of our dynamic preference model. If the investor is n-th order ambiguity neutral for all $n \geq 2$, we end up with the well-known Epstein and Zin utility, i.e. with a level of abstraction of 1, the DM's value function is given by

$$V_t(C) = \left[(1 - e^{-\delta}) C_t^{1-\rho} + e^{-\delta} \left\{ \mathbb{E}_{\mu_1^*} (V_{t+1}^{1-\gamma_1}(C)) \right\}^{\frac{1-\rho}{1-\gamma_1}} \right]^{\frac{1}{1-\rho}} .$$

If the DM's preferences display smooth ambiguity in the sense of Ju and Miao (2011), i.e. the DM's level of abstraction is 2, we have

$$V_t(C) = \left[(1 - e^{-\delta}) C_t^{1-\rho} + e^{-\delta} \left\{ \mathbb{E}_{\mu_2^*} \left[\left(\mathbb{E}_{\mu_1} \left[(V_{t+1}^{1-\gamma_1}(C)) \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}} \right] \right\}^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1}{1-\rho}} .$$

Ju and Miao (2011) discuss further limiting and special cases of value functions of a DM with a level of abstraction of 2.

3 The Pricing Kernel

The link between preferences and asset prices is the pricing kernel. The price P_t of a claim on future payoffs D can be determined using the relation

$$P_t = \mathbb{E}_t \left[\xi_{t,t+1} (P_{t+1} + D_{t+1}) \right] ,$$

where we use $\mathbb{E}_t$ as shorthand notation for $\mathbb{E}_{\mu_N,t} \dots \mathbb{E}_{\mu_1,t}$. Under complete markets, the pricing kernel $\xi_{t,t+1}$ is a unique random variable. It is commonly expressed either in terms of continuation values or in terms of market returns.

Following Duffie and Skiadas (1994) and Hansen, Heaton, Lee, and Roussanov (2007) the pricing kernel in terms of continuation values satisfies[10]

$$\begin{aligned} \xi_{t,t+1} \quad &= \quad e^{-\delta} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} (V_{t+1})^{\rho-\gamma_1} \left(\mathcal{R}_t(V_{t+1}) \right)^{\gamma_N - \rho} \times \dots \\ &\prod_{n=1}^{N-1} \left(\mathbb{E}_{\mu_n} \left[\left(\mathbb{E}_{\mu_{n-1}} \cdots \left(\mathbb{E}_{\mu_2} \left[\left(\mathbb{E}_{\mu_1} \left[V_{t+1}^{1-\gamma_1}(C) \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}} \right] \right)^{\frac{1-\gamma_3}{1-\gamma_2}} \cdots \right)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}} \right] \right)^{\frac{1-\gamma_{n+1}}{1-\gamma_n} - 1} . \end{aligned}$$

[10] A detailed derivation can be found in Appendix A.1.

Remember that

$$\mathcal{R}_t(V_{t+1}) = \left\{ \mathbb{E}_{\mu_N^*}\left[\left(\mathbb{E}_{\mu_{N-1}} \cdots \left(\mathbb{E}_{\mu_2}\left[\left(\mathbb{E}_{\mu_1}\left[V_{t+1}^{1-\gamma_1}(C) \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}} \right] \right)^{\frac{1-\gamma_3}{1-\gamma_2}} \cdots \right)^{\frac{1-\gamma_N}{1-\gamma_{N-1}}} \right] \right\}^{\frac{1}{1-\gamma_N}},$$

thus, we can interpret the continuation value as the N-th factor of the product with exponent $\frac{1-\rho}{1-\gamma_N} - 1$ instead of $\frac{1-\gamma_{N+1}}{1-\gamma_N} - 1$. In the power utility case, the n-th order certainty equivalent for $n \in \{0, \dots, N\}$ is

$$\begin{aligned} CE_0(x) &= x, \qquad \text{and} \\ CE_n(x) &= \left(\mathbb{E}_{\mu_n}\left[\left(CE_{n-1}(x) \right)^{1-\gamma_n} \right] \right)^{\frac{1}{1-\gamma_n}}. \end{aligned}$$

Since $CE_N(V_{t+1}) = \mathcal{R}_t(V_{t+1})$, the pricing kernel is given by

$$\xi_{t,t+1} = e^{-\delta} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\rho-\gamma_1} \prod_{n=1}^{N-1} \left(\frac{CE_n(V_{t+1})}{\mathcal{R}_t(V_{t+1})} \right)^{\gamma_n-\gamma_{n+1}}.$$

The first two parts correspond to the Epstein and Zin pricing kernel, which collapses to the usual CRRA pricing kernel for $\gamma_1 = \rho$. For an ambiguity averse investor with a level of abstraction of N there are $N-1$ multiplicative factors in the pricing kernel that reflect ambiguity aversion. If the DM is ambiguity averse of order $n+1$, the exponent $\gamma_n - \gamma_{n+1}$ is negative. Therefore, the investor puts more weight on bad states of the nature. Those are characterized by probability measures on S_n which imply a low n-th order certainty equivalent relative to the continuation value. Intuitively, this means that the DM is pessimistic and down-weights favorable outcomes. Note that in the case $N = 2$ of usual smooth ambiguity aversion the pricing kernel is

$$\begin{aligned} \xi_{t,t+1} &= e^{-\delta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} (V_{t+1})^{\rho-\gamma_1} \left(\mathbb{E}_{\mu_1}\left[V_{t+1}^{1-\gamma_1} \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}-1} \left(\mathbb{E}_{\mu_2^*}\left[\left(\mathbb{E}_{\mu_1}\left[V_{t+1}^{1-\gamma_1} \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}} \right] \right)^{\frac{1-\rho}{1-\gamma_2}-1} \\ &= e^{-\delta} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\rho-\gamma_1} \left(\frac{\left(\mathbb{E}_{\mu_1}\left[V_{t+1}^{1-\gamma_1} \right] \right)^{\frac{1}{1-\gamma_1}}}{\mathcal{R}_t(V_{t+1})} \right)^{\gamma_1-\gamma_2}, \end{aligned}$$

as reported in Hayashi and Miao (2011), Proposition 8.

In applications, it is usually more convenient to work with the pricing kernel in terms of the market return[11]

$$\begin{aligned}\xi_{t,t+1} = \; & e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta} R_{t+1}^{\theta-1} \times \ldots \\ & \prod_{n=1}^{N-1}\left(\mathbb{E}_{\mu_n}\left[\left(\mathbb{E}_{\mu_{n-1}}\cdots\left(\mathbb{E}_{\mu_1}\left[e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta}R_{t+1}^{\theta}\right]\right)^{\frac{1-\gamma_2}{1-\gamma_1}}\cdots\right)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}\right]\right)^{\frac{1-\gamma_{n+1}}{1-\gamma_n}-1},\end{aligned} \tag{1}$$

where $\theta = \frac{1-\gamma_1}{1-\rho}$ and R_{t+1} denotes the return on the wealth portfolio, i.e. the claim on aggregate consumption. If the investor is ambiguity neutral for $n \geq 2$ and thus $\gamma_1 = \gamma_2 = \ldots = \gamma_N$ Equation (1) yields the usual Epstein and Zin pricing kernel. Ambiguity aversion distorts the pricing kernel, giving less weight to favorable outcomes. The second line of Equation (1) shows that it is the difference between the γ's that describes the investors attitudes towards ambiguity.

4 Asset Pricing with Long-Run Risks

Bansal and Yaron (2004) introduce a general equilibrium asset pricing model with persistent variations in both the level and the volatility of consumption growth. We use a similar endowment process and assume that the investor is ambiguous about the state variables that drive consumption and dividend growth in the long-run. To characterize the investors preferences concerning the different sources of uncertainty, we employ our high order smooth ambiguity decision model.

4.1 The economy and the investor

Similar to Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011), we assume that the dynamics of log consumption growth $\Delta c_{t+1} = \log C_{t+1} - \log C_t$, log dividend

[11] A detailed derivation can be found in Appendix A.2.

growth $\Delta d_{t+1} = \log D_{t+1} - \log D_t$, and of the state variables are given by

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_{t+1} + \sigma_{t+1} w^c_{t+1}, \\
\Delta d_{t+1} &= \mu_d + \lambda x_{t+1} + \sigma_{t+1} \phi_{d,\sigma} \left(\rho_{cd} w^c_{t+1} + \sqrt{1-\rho_{cd}^2} w^d_{t+1} \right), \\
x_{t+1} &= \varphi_x x_t + \phi_x \sigma_{t+1} w^x_{t+1}, \\
\sigma^2_{t+1} &= \sigma^2 + \varphi_\sigma (\sigma^2_t - \sigma^2) + \phi_\sigma w^\sigma_{t+1},
\end{aligned} \tag{2}$$

where $w^c_{t+1}, w^d_{t+1}, w^x_{t+1}, w^\sigma_{t+1} \sim i.i.d.\ \mathcal{N}(0,1)$. Both consumption and dividends contain a persistent long-run growth component x_{t+1} and the conditional volatilities are driven by a time-varying uncertainty factor σ_{t+1}. Note that in Bansal and Yaron (2004) consumption growth depends on x_t and σ_t and therefore at time t the investor knows that $\Delta c_{t+1} \sim \mathcal{N}(\mu_c + x_t, \sigma^2_t)$. Consequently, she cannot be ambiguous about the distribution. We follow Collard, Mukerji, Sheppard, and Tallon (2011) and Ju and Miao (2011) and assume that consumption growth depends on x_{t+1} and σ_{t+1}.[12] Dividends are levered as compared to consumption, so that the dividend leverage λ and the dividend volatility multiple $\phi_{d,\sigma}$ are both greater than one. Furthermore, dividends and consumption are locally correlated with coefficient ρ_{cd}. The long-run growth factor and the current volatility of consumption growth are mean-reverting. Both processes are assumed to be very persistent and thus shocks in the state variables have a long-lasting impact on future consumption and dividends. As in Bansal and Yaron (2004), the volatility factor also determines the variation of the long-run growth component. We again assume that the distribution depends on σ_{t+1} and is therefore perceived as ambiguous by the investor at time t.

The representative investor is assumed to have high order smooth ambiguity preferences. She does not impose uniformity of preferences concerning innovations in the endowment process and adopts different attitudes towards different sources

[12] However, it is instructive to look at the original Bansal, Kiku, and Yaron (2011) endowment process. We sketch the solution for their model in Appendix C. Due to the lagged structure of this endowment process the ordering of sources of uncertainty is not ascending.

of uncertainty. As usual, the investor treats short-run consumption and dividend uncertainty as risk and evaluates it with the risk aversion coefficient γ_1. She assigns uncertainty about the long-run growth factor the ambiguity parameter γ_2, and γ_3 characterizes ambiguity about the conditional volatility of consumption growth.

In terms of our decision model log consumption growth Δc_{t+1} is a first order act. The induced pushforward measure on the set $\mathcal{C}$ of consequences is a $\mathcal{N}(\mu_c + x_{t+1}, \sigma^2_{t+1})$ distribution, and therefore depends on the realizations x_{t+1} and σ^2_{t+1}. Each realization of σ^2_{t+1} corresponds to a probability distribution of x_{t+1} and Δc_{t+1}. A second order act is a certainty equivalent conditional on the realization σ^2_{t+1}. Intuitively, comparing second order acts means comparing the implications of different volatility levels. Knowing the distribution of σ^2_{t+1} pins down the distribution of consumption growth, meaning that a third order act is simply the unconditional expected consumption growth.

Using the general expression for the pricing kernel in terms of the market return in Equation (1) and setting $N = 3$ yields

$$\begin{aligned}\xi_{t,t+1} = \; & e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta} R_{t+1}^{\theta-1} \left(\mathbb{E}_{\mu_1}\left[e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta} R_{t+1}^{\theta}\right]\right)^{\frac{1-\gamma_2}{1-\gamma_1}-1} \times \ldots \\ & \left(\mathbb{E}_{\mu_2}\left\{\left(\mathbb{E}_{\mu_1}\left[e^{-\delta\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta} R_{t+1}^{\theta}\right]\right)^{\frac{1-\gamma_2}{1-\gamma_1}}\right\}\right)^{\frac{1-\gamma_3}{1-\gamma_2}-1}.\end{aligned} \tag{3}$$

In the following we work with the log pricing kernel $m_{t,t+1} = \log \xi_{t,t+1}$.

4.2 Model solution

We solve the model in the same manner as Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2011), and Beeler and Campbell (2011) using analytical approximations. We assume that the log wealth-consumption ratio z is affine in the state variables

$$z_t = A + B_x x_t + B_\sigma(\sigma_t^2 - \sigma^2).$$

For the log return on the consumption claim $r_{c,t} = \log R_t$ we use the log-linear return approximation of Campbell and Shiller (1988)

$$r_{c,t+1} = k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1},$$

where k_0 and k_1 are linearizing constants. It holds that $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$ and $k_0 = \log(1 + e^{\bar{z}}) - k_1 \bar{z}$, where $\bar{z}$ is the long-run mean of the log wealth-consumption ratio. Using the Euler equation $\mathbb{E}_t \left[e^{m_{t,t+1} + r_{c,t+1}} \right] = 1$ yields the following coefficients for the wealth-consumption ratio

$$\begin{aligned} A &= \frac{1}{1-k_1} \left(-\delta + k_0 + (1-\rho)\mu_c + (1 - k_1 \varphi_\sigma) \frac{B_\sigma}{\varphi_\sigma} \sigma^2 + \frac{1-\gamma_3}{2(1-\rho)} \left(\frac{B_\sigma}{\varphi_\sigma} \phi_\sigma \right)^2 \right), \\ B_x &= \frac{1-\rho}{1-k_1\varphi_x} \varphi_x, \\ B_\sigma &= \frac{(1-\gamma_1)(1-\rho)}{2(1-k_1\varphi_\sigma)} \left(1 + \frac{1-\gamma_2}{1-\gamma_1} \left(\frac{\phi_x}{1-k_1\varphi_x} \right)^2 \right) \varphi_\sigma. \end{aligned} \tag{4}$$

The coefficients depend on the preference parameters, the parameters that describe the consumption growth rate, and the linearization constants. Note that the loading B_σ does not depend on the associated ambiguity aversion parameter γ_3. This is due to the $AR(1)$ variance specification.[13] For $\gamma_i > 1$, $i = 1, 2$, and $\rho < 1$, it holds that $B_x > 0$, and $B_\sigma < 0$, i.e. the wealth-consumption ratio is increasing in the expected growth rate and decreasing in volatility. It is generally accepted that the coefficient of relative risk aversion lies somewhere in the range between 1 and 10 (see Mehra and Prescott (1985)). Furthermore, experimental work has confirmed that subjects display ambiguity aversion, which implies $\gamma_2 > \gamma_1$. The value of the IES is more controversial.[14] We follow Bansal and Yaron (2004) and assume that the IES is greater than one, which is crucial for procyclical variation in the wealth-consumption ratio.

[13] In Tauchen (2005) the conditional variance of σ^2_{t+1} is proportional to σ^2_t. Using such a square-root specification implies that B_σ depends on γ_3.

[14] See the discussion in Bansal and Yaron (2004), Beeler and Campbell (2011), and Constantinides and Ghosh (2011).

We can proceed in a similar fashion to solve for the price-dividend ratio $z_{d,t} = A_d + B_{d,x} x_t + B_{d,\sigma}(\sigma_t^2 - \sigma^2)$.[15] The coefficients $B_{d,x}$ and $B_{d,\sigma}$ have the same sign as those of the log wealth-consumption ratio, but are an order of magnitude higher.

By substituting the return on the consumption claim into Equation (3) we get an expression for the log pricing kernel in terms of the state variables

$$m_{t,t+1} = s_0 + s_x x_t + s_\sigma(\sigma_t^2 - \sigma^2) - \Lambda_c \sigma_{t+1} w_{t+1}^c - \Lambda_x \phi_x \sigma_{t+1} w_{t+1}^x - \Lambda_\sigma \phi_\sigma w_{t+1}^\sigma,$$

with the drift characterized by the coefficients

$$\begin{aligned}
s_0 &= -\delta - \rho\mu_c - \frac{(1-\gamma_3)(\rho-\gamma_3)}{2(1-\rho)^2}\left(\frac{B_\sigma}{\varphi_\sigma}\phi_\sigma\right)^2 + \frac{s_\sigma}{\varphi_\sigma}\sigma^2, \\
s_x &= -\rho\varphi_x, \\
s_\sigma &= \frac{1}{2}(1-\gamma_1)(\gamma_1-\rho)\varphi_\sigma + \frac{1}{2}(1-\gamma_2)(\gamma_2-\rho)\left(\frac{\phi_x}{1-k_1\varphi_x}\right)^2\varphi_\sigma.
\end{aligned}$$

The coefficients Λ_c, Λ_x, and Λ_σ determine the market prices of risk in consumption, expected consumption growth, and volatility

$$\begin{aligned}
\Lambda_c &= \gamma_1, \\
\Lambda_x &= \frac{\gamma_2-\rho}{1-\rho}\,k_1\,B_x + \gamma_2, \\
\Lambda_\sigma &= \frac{\gamma_3-\rho}{1-\rho}\,k_1\,B_\sigma + \frac{1}{2}(\gamma_3-\gamma_1)(1-\gamma_1) + \frac{1}{2}(\gamma_3-\gamma_2)(1-\gamma_2)\left(\frac{\phi_x}{1-k_1\varphi_x}\right)^2.
\end{aligned}$$

As usual, the market price of consumption risk is the risk-aversion coefficient. The market prices of risk induced by the state variables are proportional to the respective ambiguity aversion parameters. Λ_x can be decomposed into the usual long-run component $\frac{\gamma_2-\rho}{1-\rho}\,k_1\,B_x$ and a short-run component γ_2. The latter appears because x_{t+1} and not x_t enters the consumption growth rate. A similar decomposition holds for Λ_σ. The first short-run component appears since σ_{t+1} enters the consumption process, the second one since σ_{t+1} drives the volatility of x_{t+1}. The additional short-run components depend on the differences between the respective γ's and γ_3. It is

[15] See Appendix B.

this difference that describes the investor's attitudes towards ambiguity. Note that for Epstein and Zin preferences used in Bansal and Yaron (2004) all sources of uncertainty are treated in the same fashion, implying that all market prices of risk directly depend on the relative risk-aversion coefficient γ_1. This is not the case in our model, which disentangles the different sources of uncertainty.

Given the log pricing kernel, the continuously compounded risk-free rate can be easily calculated as

$$\begin{aligned} r_{f,t} &= -\log \mathbb{E}_t \left(e^{m_{t,t+1}} \right) \\ &= r_{f,0} + r_{f,x} x_t + r_{f,\sigma} (\sigma_t^2 - \sigma^2), \end{aligned} \tag{5}$$

with

$$\begin{aligned} r_{f,0} &= -s_0 - \frac{1}{2} \left(\Lambda_\sigma - \frac{1}{2} \left(\Lambda_c^2 + \Lambda_x^2 \phi_x^2 \right) \right)^2 \phi_\sigma^2 - \frac{1}{2} \left(\Lambda_c^2 + \Lambda_x^2 \phi_x^2 \right) \sigma^2, \\ r_{f,x} &= -s_x, \\ r_{f,\sigma} &= -s_\sigma - \frac{1}{2} \left(\Lambda_c^2 + \Lambda_x^2 \phi_x^2 \right) \varphi_\sigma. \end{aligned}$$

A higher conditional mean of the pricing kernel (due to ambiguity aversion) leads to lower interest rates. Intuitively, a pessimistic agent invests less in risky assets, which induces a lower equilibrium risk-free rate.

The conditional expected return on the dividend claim and its conditional variance are given by

$$\begin{aligned} \mathbb{E}_t \left[r_{d,t+1} \right] &= k_{0,d} + (k_{1,d} - 1) A_d + \mu_d \\ &\quad + ((k_{1,d} \varphi_x - 1) B_{d,x} + \lambda \varphi_x) x_t + (k_{1,d} \varphi_\sigma - 1) B_{d,\sigma} (\sigma_t^2 - \sigma^2), \end{aligned} \tag{6}$$

$$\begin{aligned} Var_t \left[r_{d,t+1} \right] &= (k_{1,d} B_{d,\sigma})^2 \phi_\sigma^2 + \left((k_{1,d} B_{d,x} + \lambda)^2 \phi_x^2 + \phi_{d,\sigma}^2 \right) \sigma^2 \\ &\quad + \left((k_{1,d} B_{d,x} + \lambda)^2 \phi_x^2 + \phi_{d,\sigma}^2 \right) \varphi_\sigma (\sigma_t^2 - \sigma^2). \end{aligned} \tag{7}$$

The equity risk premium follows by subtracting the risk-free rate from the expected return on the dividend claim

$$\begin{aligned}\mathbb{E}_t\left[r_{d,t+1}\right]-r_{f,t}=&\frac{1}{2}\left(\left(\Lambda_\sigma-\frac{1}{2}(\Lambda_c^2+\Lambda_x^2\phi_x^2)\right)^2-\left(\Lambda_\sigma-\frac{B_{d,\sigma}-s_\sigma}{\varphi_\sigma}\right)^2\right)\phi_\sigma^2 \qquad (8)\\ &+\left(\Lambda_x\left(k_{1,d}B_{d,x}+\lambda\right)\phi_x^2-\frac{1}{2}\left(k_{1,d}B_{d,x}+\lambda\right)^2\phi_x^2+\Lambda_c\phi_{d,\sigma}\rho_{cd}-\frac{1}{2}\phi_{d,\sigma}^2\right)\sigma^2\\ &+\left(\Lambda_x\left(k_{1,d}B_{d,x}+\lambda\right)\phi_x^2-\frac{1}{2}\left(k_{1,d}B_{d,x}+\lambda\right)^2\phi_x^2+\Lambda_c\phi_{d,\sigma}\rho_{cd}-\frac{1}{2}\phi_{d,\sigma}^2\right)\varphi_\sigma(\sigma_t^2-\sigma^2).\end{aligned}$$

An ambiguity averse investor is more conservative and demands additional compensation (ambiguity premium). This is reflected in the higher market prices of risk for expected consumption growth Λ_x and volatility Λ_σ. Thus, the pricing kernel is more volatile. Since consumption and dividends share the same variance process, this effect magnifies the negative correlation between the pricing kernel and returns. Consequently, the equity premium increases. Note that the equity premium is time-varying. It rises in periods of high economic uncertainty (large σ_t^2).

4.3 Quantitative results

We fix the model parameters at the same values as Bansal, Kiku, and Yaron (2011) and Beeler and Campbell (2011).[16] The mean consumption growth rate is 1.8% per annum and the volatility of consumption growth is 2.5% per annum. Consumption and dividend shocks are positively correlated ($\rho_{cd} \approx 0.4$). The values for the mean-reversion parameters imply half-lives of expected consumption growth and volatility shocks around 2.28 and 57.73 years, respectively. Concerning the preference parameters, we follow Bansal, Kiku, and Yaron (2011) and set the time-discount factor δ to $-\log(0.9989)$ and the IES to 1.5. We also consider an IES of 2.0, as e.g. used in Drechsler and Yaron (2011). If the IES is greater than the inverse of the coefficient

[16]The major differences between the parametrization in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) are the greater persistence of volatility shocks and the positive correlation between dividend shocks and consumption innovations in the latter paper.

of relative risk aversion, the agent prefers early resolution of uncertainty. The model is calibrated on a monthly basis. All parameter values are summarized in Table 1.

4.3.1 Basic asset pricing implications

In order to understand the basic mechanism of our decision model, we vary the ambiguity parameters γ_2 and γ_3 while holding all other parameters constant. The risk aversion coefficient is set to 5, the ambiguity parameters γ_2 and γ_3 range from 5 to 50, and the IES is fixed at 1.5. The differences between the γ's characterize ambiguity aversion. The main diagonal ($\gamma_2 = \gamma_3$) refers to the KMM model. Note that increasing γ_2 and γ_3, while keeping γ_1 constant, does not only introduce ambiguity aversion, but also raises the overall level of aversion against uncertainty.

The coefficients of the wealth-consumption and price-dividend ratio are shown in Figure 1. B_x and B_σ are the loadings of the wealth-consumption ratio for expected consumption growth and volatility, while A is the mean log wealth-consumption ratio. For higher levels of the ambiguity parameters, the investor is more pessimistic about future economic growth perspectives. Thus, she decreases her position in the risky asset and the wealth-consumption ratio decreases. Good news about future economic growth cause moderate price increases if the investor is pessimistic. Consequently, B_x is decreasing in both ambiguity parameters. The effect is indirect via the linearization constant k_1, which is smaller for high values of the ambiguity parameters.[17] As k_1 depends only on the mean log wealth-consumption ratio, a combined increase of γ_2 and γ_3 has the greatest effect. In the lower left part of Figure 1 we observe that in the special case of the KMM model the impact of news about future economic uncertainty remains almost unchanged if the agent is more

[17] The linearization constants k_0 and k_1 depend on the average wealth-consumption ratio. We solve for the linearization constants by numerically iterating on $\bar{z} = A(\bar{z})$ until reaching a fixed point for $\bar{z}$. The mean log wealth-consumption ratio decreases for higher ambiguity parameters, consequently k_0 increases and k_1 decreases in γ_i, $i = 1, 2, 3$.

conservative. To change the sensitivity of the wealth-consumption ratio to volatility shocks, the investor needs to be non-neutral concerning third order ambiguity. For the Bansal and Yaron (2004) endowment process, B_σ is decreasing (in absolute terms) in the amount of third order ambiguity aversion, i.e. $(\gamma_3 - \gamma_2)$. While the influence of γ_2 is direct, γ_3 does not appear in Equation (4). An increase in γ_3 decreases the mean wealth-consumption ratio and thus also decreases the linearization constant k_1, which results in a lower magnitude of B_σ. From Figure 1 it is obvious that B_σ is largest in absolute terms if γ_2 is high and γ_3 is small, i.e. the investor is third order ambiguity loving. For the loadings of the price-dividend ratio we observe the same behavior as for the wealth-consumption ratio.

Figure 2 shows the risk-free rate, the mean equity premium, and the mean and conditional variance of the return on the dividend claim. For higher values of the ambiguity parameters, holding claims on equity is less attractive, and the investor demands additional compensation. Thus, the risk-free rate in Equation (5) decreases and the return on the dividend claim in Equation (6) increases in the ambiguity parameters. As for the wealth-consumption ratio, the combined working of both ambiguity parameters is essential. The behavior of the conditional return variance in Equation (7) follows directly from the coefficient B_σ. For high values of γ_2 and low values of γ_3 the loading B_σ is largest in absolute terms, consequently the conditional return variance is substantial.

Similar to Ju and Miao (2011) and Collard, Mukerji, Sheppard, and Tallon (2011), we find that the equity premium is increasing in ambiguity aversion. In Figure 3 we decompose the conditional equity premium given in Equation (8). The upper left part of the figure shows the compensation for diffusive consumption risk, which is proportional to the risk aversion coefficient γ_1. Due to the low value of γ_1 this part of the equity premium is negligible. The upper right corner displays the premium for second order ambiguity, i.e. uncertainty about the mean consumption growth rate. It is clearly driven by the ambiguity parameter for this type of uncertainty, while γ_3 has no significant impact. We label the first line of Equation (8) third order equity premium. It is the compensation for shocks in the state variable σ_{t+1}, which influence the log consumption growth rate Δc_{t+1} directly and indirectly through the mean growth rate x_{t+1}. Consequently, both γ_2 and γ_3 are relevant for this part of the equity premium. According to Equation (8), the time-varying component of the equity premium results from adding up the first and the second order premia. For the Bansal, Kiku, and Yaron (2011) parametrization, a large proportion of the equity premium is a compensation for uncertainty in the conditional volatility of consumption growth. In their LRR model it is not possible to change the proportions of the premia paid for different kinds of uncertainty. However, for a given level of the equity premium, increasing the time-varying part leads to a more pronounced countercyclical behavior of the equity premium, which is crucial to reconcile the predictability patterns in the data.

4.3.2 Unconditional asset pricing moments

We simulate the model 100,000 times at a monthly frequency with a sample size equivalent to the actual data (79 years) to construct finite-sample statistics. We start the simulation at the unconditional means of the state variables and discard the first 10 years of each simulated path (burn-in period). The data is aggregated to an annual frequency as in Beeler and Campbell (2011).[18] For each variable (consumption growth, dividend growth, return on the dividend claim, risk free rate, and price-dividend ratio), we consider the mean, the standard deviation, and the first order autocorrelation. Tables 2 and 4 show median values and 95% confidence

[18]Consumption and dividend growth rates are calculated by adding 12 monthly consumption and dividend levels and then taking the growth rate. For log market returns and risk-free rates we sum up the monthly values. The price-dividend ratio is the end-of-year price divided by the trailing sum of 12 month dividends.

intervals for two values of the IES, 1.5 and 2, respectively.

As benchmark, we use the LRR model with Epstein and Zin preferences. The coefficient of relative risk aversion is set to the same value as in Bansal and Yaron (2004), i.e. for the benchmark we set $\gamma_1 = \gamma_2 = \gamma_3 = 10$. Solely increasing γ_2 and γ_3 while leaving $\gamma_1 = 10$ increases the overall level of uncertainty. To put the models on an equal footing we have to fix the overall uncertainty at about the same level. Thus, when increasing γ_2 and γ_3 we decrease the risk aversion coefficient γ_1. More specifically, we choose the γ's so that they sum up to the same value as in the benchmark model and consider 3 cases of our ambiguity model. In the following, we use $\gamma_1 = 2$, the same value considered in Ju and Miao (2011). To compare our model with the standard smooth ambiguity model of KMM, we set $2 = \gamma_1 < \gamma_2 = \gamma_3 = 14$ and denote this specification Case 1. Case 2 refers to $\gamma_1 = 2$, $\gamma_2 = 10$, $\gamma_3 = 18$, i.e. strictly ascending ambiguity parameters. The findings in Section 4.3.1 suggest to consider a third case with a relatively high value of γ_2. For Case 3 we fix γ_1 and γ_3 at 2 and choose $\gamma_2 = 26$.

Table 2 shows several unconditional asset pricing moments for an IES of 1.5. The empirical values in the second column are taken from Beeler and Campbell (2011) and are based on annual data from 1930 to 2008. The third column shows the results for the benchmark LRR model. The other columns refer to the three cases explained above. We use the same seed value for all simulations and thus the differences between the models are solely due to the differences between the γ's.

The results in Table 2 show that the benchmark model already matches most asset pricing moments very well. A notable shortcoming of the LRR model is that it is not able to generate a high standard deviation of the price-dividend ratio. The results of Case 1 are very similar to those of the Epstein and Zin specification. However, the unconditional moments are matched with a more moderate value for the relative risk aversion coefficient ($\gamma_1 = 2$). In Case 2, the investor puts more weight on volatility shocks, compared to innovations in x_t. We clarified in Section 4.3.1 that, for the endowment process in Equation (2), this leads to a relatively low return on the dividend claim and a high risk-free rate. Furthermore, Case 2 cannot solve the problems regarding the unconditional moments of the price-dividend ratio. In contrast, Case 3 fits the data much better, especially concerning the standard deviation and first order autocorrelation of the price-dividend ratio. For large values of γ_2 and low values of γ_3 the magnitude of the coefficient $B_{d,\sigma}$ outweighs the decrease in $B_{d,x}$. Consequently, the standard deviation of the price-dividend ratio is much higher, and close to what we observe in the data.

Table 4 shows unconditional moments for an IES of 2. For a larger value of the IES the investor is more willing to substitute consumption over time. This lowers her demand for precautionary savings and thus decreases the risk-free interest rate. A higher IES also implies more time-variation in the price-dividend ratio and thus in the return on the dividend claim. The lower risk-free rate and the higher volatility of the price-consumption ratio are in line with the values found in the data. The unconditional moments of the price-dividend ratio fall within the 95% confidence intervals for Case 3. The empirically observed high standard deviation of the price-dividend ratio is difficult to match even for extended versions of the LRR model, e.g. Drechsler and Yaron (2011).

4.3.3 Predictability

We also assess the predictability of excess stock returns, consumption growth, and dividend growth by the price-dividend ratio. To do so, we regress these variables, measured over horizons (h) of 1, 3, or 5 years, onto the log price-dividend ratio

$$\sum_{j=1}^{h} y_{t+j} = \alpha(h) + \beta(h)\ z_{d,t} + \varepsilon_{t+h},$$

where y denotes either log excess stock returns, log consumption growth, or log dividend growth. Tables 3 and 5 show the predictive R^2's and the slope coefficients.

In the LRR model consumption growth is driven by a persistent long-run growth component. This factor is also apparent in the dynamics of valuation ratios. Thus, price-dividend ratios contain information about future consumption and dividend growth. However, price-dividend ratios do not only fluctuate on news about future economic growth but also on news about future economic uncertainty. Price fluctuations coming from time-varying volatility reduce the information content and thus the predictive power of the price-dividend ratio for future consumption and dividend growth. An increase in γ_2 clearly lowers the positive correlation between cash-flows and the price-dividend ratio. For γ_3 the effect is not so obvious. On the one hand a low value of γ_3 implies a high value of $B_{d,x}$, on the other hand $B_{d,\sigma}$ has a larger magnitude for low values of γ_3 (in combination with a high value of γ_2). The parametrization of Bansal, Kiku, and Yaron (2011) implies a high weight on the volatility component and the second effect dominates the first. Thus, the variability of the price-dividend ratio depends to a smaller extend on news about future economic growth and the predictive power of the price-dividend ratio for cash-flows decreases for high values of γ_2 and low values of γ_3.

Excess returns are predictable due to the time-variation of risk premia. More specifically, it follows directly from Equation (8) that a persistent volatility process leads to predictable excess returns. If we increase the ambiguity parameter γ_2, the time-varying parts of the equity premium are larger. In contrast, they are only indirectly influenced through the linearizing constants by γ_3. For the log price-dividend ratio, an increase of γ_2 implies a slightly lower value of $B_{d,x}$ and a large increase (in absolute terms) in $B_{d,\sigma}$. This adds to the negative correlation between excess returns and the price-dividend ratio. Due to the high persistence of the volatility process, the R^2 increases. Furthermore, low values of the ambiguity parameter γ_3 increase the predictive power of the price-dividend ratio as $B_{d,\sigma}$ is largest (in absolute terms) for low values of γ_3. Consequently, a combination of a high ambiguity parameter for uncertainty in the long-run growth factor and a low one for volatility gives the best match to the data. This explains the good results of Case 3 in Table 3. Compared to these, for an IES of 2 Table 5 shows that the predictive power is magnified since the loadings of the price-dividend ratio are larger in absolute terms.

Summing up, if γ_2 is large while γ_3 is low the predictive power of the price-dividend ratio is much closer to the data compared with the standard LRR model. The results for Case 3 do not only outperform the benchmark model, but also improve upon the long-horizon predictability results of Drechsler and Yaron (2011). Note that they use an extended version of the Bansal and Yaron (2004) model, with an additional long-run volatility factor and multiple jump components.

5 Conclusion

In Klibanoff, Marinacci, and Mukerji (2005, 2008) the resolution of uncertainty is described in two stages (first and second order acts). This implies that higher order acts have a uniform degree of ambiguity. We extend their model to higher orders and explicitly allow for different degrees of ambiguity. This generalization permits a separation of intertemporal substitution, risk aversion, and ambiguity aversion towards different sources of uncertainty.

We apply our preference specification to a consumption-based asset pricing model with persistent state variables. In our model the investor perceives and evaluates shocks in future economic growth in a different way compared to volatility shocks. We find that assigning different ambiguity parameters to these sources of uncertainty has important implications for asset prices. Our model generates unconditional asset pricing moments and predictability patterns in line with the data.

There is a wide array of possible applications for the proposed decision model. It is especially suited for situations where subjects may treat several kinds of uncertainty in different manners, as it provides a device to model this behavior. It might be interesting to apply our decision model to other endowment processes or evaluate the implications for portfolio planning.

It is common to quantify the magnitude of ambiguity aversion using Ellsberg style experiments. While second order ambiguity aversion has been extensively tested and it is commonly accepted that individuals share this general pattern in decision-making, there is no experimental evidence for high order ambiguity aversion. An extended version of the experiments in Halevy (2007) might be necessary to establish a sound basis for our decision model.

A Pricing Kernel

A.1 The pricing kernel in terms of continuation values

According to Hansen, Heaton, Lee, and Roussanov (2007) we calculate the pricing kernel as the marginal rate of substitution, thus

$$\xi_{t,t+1} = \frac{MV_{t+1}MC_{t+1}}{MC_t}, \tag{9}$$

where MC_t denotes the time t marginal utility of consumption and MV_{t+1} the marginal utility of next periods continuation value. Due to the homogeneity of the value function V_t, we can apply Euler's homogenous function theorem which yields

$$V_t = (MC_t)\, C_t + \mathbb{E}_{\mu_N,t} \ldots \mathbb{E}_{\mu_1,t}\left[(MV_{t+1})\, V_{t+1}\right].$$

Comparing coefficients with $V_t^{1-\rho} = (1-e^{-\delta})C_t^{1-\rho} + e^{-\delta}\{\mathcal{R}_t(V_{t+1}(C))\}^{1-\rho}$ gives

$$\begin{aligned} MC_t &= (1-e^{-\delta})\, C_t^{-\rho}\, V_t^{\rho}, \text{ and} \\ MV_{t+1} &= e^{-\delta}\, V_t^{\rho}\, V_{t+1}^{-\gamma_1} (\mathcal{R}_t(V_{t+1}))^{\gamma_N-\rho} \prod_{n=1}^{N-1} \left(\mathbb{E}_{\mu_n}\left[\left(\ldots (\mathbb{E}_{\mu_1}[V_{t+1}^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \ldots \right)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}} \right] \right)^{\frac{1-\gamma_{n+1}}{1-\gamma_n}-1}. \end{aligned}$$

Substituting this into Equation (9) yields the proposed pricing kernel.

A.2 The pricing kernel in terms of the market return

Following Epstein and Zin (1989) we express the pricing kernel in terms of the market return. We assume that there are I assets with returns $R_{i,t+1}$ and weights $\pi_{i,t}$ for $i \in \{1, \ldots, I\}$. Let J_t denote the maximum value of the DM's value function V_t and $a := 1 - e^{-\delta}$. The maximization problem is

$$J_t = \sup_{C_t,\pi_t} \left\{ \left[aC_t^{1-\rho} + e^{-\delta} \left\{ \mathbb{E}_{\mu_n}[(\ldots \mathbb{E}_{\mu_2}[(\mathbb{E}_{\mu_1}[J_{t+1}^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}}] \ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}] \right\}^{\frac{1-\rho}{1-\gamma_n}} \right]^{\frac{1}{1-\rho}} \right\}. \tag{10}$$

We substitute the budget constraint $W_{t+1} = (W_t - C_t)\; R_{t+1}$ and conjecture $J_t = h_t W_t$, which yields

$$\begin{aligned} h_t W_t &= \sup_{C_t,\pi_t} \left\{ \left[aC_t^{1-\rho} + e^{-\delta} \left\{ \mathbb{E}_{\mu_n}[(\ldots \mathbb{E}_{\mu_2}[(\mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1} W_{t+1}^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}}] \ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}] \right\}^{\frac{1-\rho}{1-\gamma_n}} \right]^{\frac{1}{1-\rho}} \right\} \\ &= \sup_{C_t,\pi_t} \left\{ [aC_t^{1-\rho} + e^{-\delta}(W_t - C_t)^{1-\rho} \{\mathbb{E}_{\mu_n}[(\ldots (\mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}]\}^{\frac{1-\rho}{1-\gamma_n}}]^{\frac{1}{1-\rho}} \right\}. \end{aligned}$$

The first order condition with respect to C_t is

$$\begin{aligned} 0 &= (1-\rho) \left\{ aC_t^{-\rho} - e^{-\delta}(W_t - C_t)^{-\rho} \{\mathbb{E}_{\mu_n}[(\ldots (\mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}]\}^{\frac{1-\rho}{1-\gamma_n}} \right\} \\ &\overset{\rho\neq 1}{\Longleftrightarrow} a \left(\frac{W_t - C_t}{C_t} \right)^{\rho} = e^{-\delta} \{\mathbb{E}_{\mu_n}[(\ldots (\mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}]\}^{\frac{1-\rho}{1-\gamma_n}}. \end{aligned}$$

Substituting this gives

$$
\begin{aligned}
h_t W_t &= \left\{ a C_t^{1-\rho} + (W_t - C_t)^{1-\rho} a \left(\frac{W_t - C_t}{C_t} \right)^{\rho} \right\}^{\frac{1}{1-\rho}} \\
\implies \quad h_t &= a^{\frac{1}{1-\rho}} \left(\frac{W_t}{C_t} \right)^{\frac{1}{1-\rho} - 1}.
\end{aligned}
$$

We substitute this expression for h_t and the budget constraint into the first order condition

$$
\begin{aligned}
\left(\frac{W_t - C_t}{C_t} \right)^{\rho} &= e^{-\delta} \left\{ \mathbb{E}_{\mu_n} [(\dots (\mathbb{E}_{\mu_1} [\left(\frac{W_t - C_t}{C_t} \right)^{\rho \frac{1-\gamma_1}{1-\rho}} \left(\frac{C_t}{C_{t+1}} \right)^{\rho \frac{1-\gamma_1}{1-\rho}} (R_{t+1})^{\frac{1-\gamma_1}{1-\rho}}])^{\frac{1-\gamma_2}{1-\gamma_1}} \dots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}] \right\}^{\frac{1-\rho}{1-\gamma_n}} \\
\iff 1 &= \mathbb{E}_{\mu_n} \left[\left(\dots \left(\mathbb{E}_{\mu_1} \left[e^{-\delta \frac{1-\gamma_1}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\gamma_1}{1-\rho}} (R_{t+1})^{\frac{1-\gamma_1}{1-\rho}} \right] \right)^{\frac{1-\gamma_2}{1-\gamma_1}} \dots \right)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}} \right].
\end{aligned}
$$

The return on the portfolio $\pi_t = (\pi_{1,t}, \dots, \pi_{I,t})$ yields a return of $R^{\pi}_{t+1} = R_{1,t+1} + \sum_{i=2}^{n} \pi_{i,t}(R_{i,t+1} - R_{1,t+1})$. Substituting this into Equation (10) gives

$$
\begin{aligned}
h_t W_t = \sup_{C_t, \pi_t} \Bigg\{ \Big[a C_t^{1-\rho} + e^{-\delta} (W_t - C_t)^{1-\rho} \{ \mathbb{E}_{\mu_n} [(\dots (\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R_{1,t+1} + \dots \\
\sum_{i=2}^{n} \pi_{i,t} (R_{i,t+1} - R_{1,t+1}))^{1-\gamma_1}]^{\frac{1-\gamma_2}{1-\gamma_1}}) \dots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}} \}^{\frac{1-\rho}{1-\gamma_n}} \Big]^{\frac{1}{1-\rho}} \Bigg\}.
\end{aligned}
$$

The first order condition with respect to $\pi_{i,t}$ is

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \pi_i} \{ \mathbb{E}_{\mu_n} [(\dots (\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R_{1,t+1} + \sum_{i=2}^{n} \pi_{i,t} (R_{i,t+1} - R_{1,t+1}))^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \dots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}] \} \\
\iff 0 &= \mathbb{E}_{\mu_n} [\frac{\partial}{\partial \pi_i} \{ \mathbb{E}_{\mu_n - 1} [(\dots (\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R_{1,t+1} + \sum_{i=2}^{n} \pi_{i,t} (R_{i,t+1} - R_{1,t+1}))^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \dots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}] \}] \\
&= \mathbb{E}_{\mu_n} \Bigg[(\mathbb{E}_{\mu_{n-1}} [(\dots (\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R_{1,t+1} + \sum_{i=2}^{n} \pi_{i,t} (R_{i,t+1} - R_{1,t+1}))^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \dots)^{\frac{1-\gamma_{n-1}}{1-\gamma_{n-2}}}])^{\frac{1-\gamma_n}{1-\gamma_{n-1}} - 1} \times \dots \\
&\quad \mathbb{E}_{\mu_{n-1}} [\frac{\partial}{\partial \pi_i} \{ (\mathbb{E}_{\mu_{n-2}} [(\dots (\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R_{1,t+1} + \sum_{i=2}^{n} \pi_{i,t} (R_{i,t+1} - R_{1,t+1}))^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \dots)^{\frac{1-\gamma_{n-2}}{1-\gamma_{n-3}}}])^{\frac{1-\gamma_{n-1}}{1-\gamma_{n-2}}} \}] \Bigg] \\
&= \dots \\
&= \mathbb{E}_{\mu_n} \Bigg[(\mathbb{E}_{\mu_{n-1}} [\dots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}} - 1} \times \mathbb{E}_{\mu_{n-1}} \Bigg[(\mathbb{E}_{\mu_{n-2}} [\dots])^{\frac{1-\gamma_{n-1}}{1-\gamma_{n-2}} - 1} \times \dots \\
&\quad \dots \times \mathbb{E}_{\mu_2} \left[(\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R^{\pi}_{t+1})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1} - 1} \times \mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R^{\pi}_{t+1})^{-\gamma_1} (1-\gamma) (R_{i,t+1} - R_{1,t+1})] \right] \dots \Bigg] \Bigg] \\
\iff 0 &= \mathbb{E}_{\mu_n} \Bigg[(\mathbb{E}_{\mu_{n-1}} [\dots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}} \times \dots \\
&\quad \dots \times \mathbb{E}_{\mu_2} \left[(\mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R^{\pi}_{t+1})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}} \times \mathbb{E}_{\mu_1} [h_{t+1}^{1-\gamma_1} (R^{\pi}_{t+1})^{-\gamma_1} (R_{i,t+1} - R_{1,t+1})] \right] \dots \Bigg].
\end{aligned}
$$

Multiplication with $\pi_{i,t}$ and summing up yields

$$
\begin{aligned}
0 &= \mathbb{E}_{\mu_n}\left[(\mathbb{E}_{\mu_{n-1}}[\ldots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}-1} \times \mathbb{E}_{\mu_{n-1}}\left[\ldots \times \mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1}^{\pi})^{-\gamma_1}(R_{t+1}^{\pi} - R_{1,t+1})]\ldots\right]\right] \\
\Longleftrightarrow \mathbb{E}_{\mu_n} &\left[(\mathbb{E}_{\mu_{n-1}}[\ldots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}-1} \times \mathbb{E}_{\mu_{n-1}}\left[\ldots \times \mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1}^{\pi})^{-\gamma_1}R_{1,t+1}]\ldots\right]\right] \\
&= \mathbb{E}_{\mu_n}\left[(\mathbb{E}_{\mu_{n-1}}[\ldots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}-1} \times \mathbb{E}_{\mu_{n-1}}\left[\ldots \times \mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1}^{\pi})^{1-\gamma_1}]\ldots\right]\right] \\
&= \mathbb{E}_{n}\left[(\ldots(\mathbb{E}_{\mu_1}[h_{t+1}^{1-\gamma_1}(R_{t+1}^{\pi})^{1-\gamma_1}])^{\frac{1-\gamma_2}{1-\gamma_1}}\ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}\right].
\end{aligned}
$$

Substituting h_t and the budget constraint gives

$$
\begin{aligned}
&\mathbb{E}_{\mu_n}\left[(\mathbb{E}_{\mu_{n-1}}[\ldots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}-1} \times \mathbb{E}_{\mu_{n-1}}\left[\ldots \times \mathbb{E}_{\mu_1}[a^{\frac{1-\gamma_1}{1-\rho}}\left(\frac{W_t - C_t}{C_{t+1}}\right)^{\rho\frac{1-\gamma_1}{1-\rho}}(R_{t+1}^{\pi})^{\frac{1-\gamma_1}{1-\rho}-1}R_{1,t+1}]\ldots\right]\right] \\
&= \mathbb{E}_{\mu_n}\left[(\ldots(\mathbb{E}_{\mu_1}[a^{\frac{1-\gamma_1}{1-\rho}}\left(\frac{W_t - C_t}{C_{t+1}}\right)^{\rho\frac{1-\gamma_1}{1-\rho}}(R_{t+1}^{\pi})^{\frac{1-\gamma_1}{1-\rho}}])^{\frac{1-\gamma_2}{1-\gamma_1}}\ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}\right].
\end{aligned}
$$

Multiplication with the time t-measurable constant $\left(\frac{e^{-\delta}C_t^{\rho}}{a(W_t - C_t)^{\rho}}\right)^{\frac{1-\gamma_1}{1-\rho}}$ yields

$$
\begin{aligned}
1 &= \mathbb{E}_{\mu_n}\left[(\ldots(\mathbb{E}_{\mu_1}[e^{-\delta\frac{1-\gamma_1}{1-\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\frac{1-\gamma_1}{1-\rho}}(R_{t+1})^{\frac{1-\gamma_1}{1-\rho}}])^{\frac{1-\gamma_2}{1-\gamma_1}}\ldots)^{\frac{1-\gamma_n}{1-\gamma_{n-1}}}\right] \\
&= \mathbb{E}_{\mu_n}\Bigg[(\mathbb{E}_{\mu_{n-1}}[\ldots])^{\frac{1-\gamma_n}{1-\gamma_{n-1}}-1} \times \mathbb{E}_{\mu_{n-1}}\Bigg[(\mathbb{E}_{n-2}[\ldots])^{\frac{1-\gamma_{n-1}}{1-\gamma_{n-2}}-1} \times \mathbb{E}_{\mu_{n-2}}\Bigg[\ldots \\
&\qquad \times \mathbb{E}_{\mu_2}\Bigg[(\mathbb{E}_{\mu_1}[e^{-\rho\frac{1-\gamma_1}{1-\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\frac{1-\gamma_1}{1-\rho}}(R_{t+1}^{\pi})^{\frac{1-\gamma_1}{1-\rho}}])^{\frac{1-\gamma_2}{1-\gamma_1}-1} \\
&\qquad\qquad \mathbb{E}_{\mu_1}[e^{-\rho\frac{1-\gamma_1}{1-\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\frac{1-\gamma_1}{1-\rho}}(R_{t+1}^{\pi})^{\frac{1-\gamma_1}{1-\rho}-1}R_{1,t+1}]\Bigg]\ldots\Bigg]\Bigg]\Bigg].
\end{aligned}
$$

Since $R_{1,t+1}$ denotes the return on an arbitrary financial claim, the pricing kernel has the proposed form.

B Price-Dividend Ratio

We rely on the log-linear approximation for the log return on the dividend claim

$$
r_{d,t+1} = k_{0,d} + k_{1,d}z_{d,t+1} - z_{d,t} + \Delta d_{t+1},
$$

where $k_{1,d} = \frac{e^{\bar{z}_d}}{1+e^{\bar{z}_d}}$ and $k_{0,d} = \log(1 + e^{\bar{z}_d}) - k_{1,d}\bar{z}_d$, with $\bar{z}_d$ denoting the long-run mean of the log price-dividend ratio. We conjecture that the log price-dividend ratio z_d is affine in the state variables

$$
z_{d,t} = A_d + B_{d,x}x_t + B_{d,\sigma}(\sigma_t^2 - \sigma^2).
$$

The coefficients of the log price-dividend ratio follow by applying the Euler equation to the log return on the dividend claim

$$
\begin{aligned}
A_d &= \frac{s_0 + k_{0,d} + \mu_d + \frac{1}{2}\left(\frac{B_{d,\sigma}}{\varphi_\sigma} - \frac{s_\sigma}{\varphi_\sigma} - \Lambda_\sigma\right)^2 \phi_\sigma^2 + \left((1-k_{1,d}\varphi_\sigma)\frac{B_{d,\sigma}}{\varphi_\sigma} - \frac{s_\sigma}{\varphi_\sigma}\right)\sigma^2}{(1-k_{1,d})}, \\
B_{d,x} &= \frac{\lambda - \rho}{1 - k_{1,d}\varphi_x}\,\varphi_x, \\
B_{d,\sigma} &= \frac{s_\sigma + \frac{1}{2}\left((k_{1,d}B_{d,x} + \lambda - \Lambda_x)^2\phi_x^2 + \gamma_1^2 + \phi_{d,\sigma}^2 - 2\gamma_1\rho_{cd}\phi_{d,\sigma}\right)\varphi_\sigma}{1 - k_{1,d}\varphi_\sigma}.
\end{aligned}
$$

C Bansal and Yaron (2004) Endowment Process

In Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) the dynamics of log consumption growth $\Delta c_{t+1} = \log C_{t+1} - \log C_t$, log dividend growth $\Delta d_{t+1} = \log D_{t+1} - \log D_t$, and of the state variables are given by

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t w_{t+1}^c, \\
\Delta d_{t+1} &= \mu_d + \lambda x_t + \sigma_t \phi_{d,\sigma}\left(\rho_{cd} w_{t+1}^c + \sqrt{1-\rho_{cd}^2}\, w_{t+1}^d\right), \\
x_{t+1} &= \varphi_x x_t + \phi_x \sigma_t w_{t+1}^x, \\
\sigma_{t+1}^2 &= \sigma^2 + \varphi_\sigma(\sigma_t^2 - \sigma^2) + \phi_\sigma w_{t+1}^\sigma,
\end{aligned}
$$

where $w_{t+1}^c, w_{t+1}^d, w_{t+1}^x, w_{t+1}^\sigma \sim$ *i.i.d.* $\mathcal{N}(0,1)$. Different from Equations (2), the distributions of time $(t+1)$-measurable random variables in this model depend on time t-realizations of the state variables and are therefore unambiguous at time t. Nevertheless, it is interesting to compare the following formulas with the results in Bansal and Yaron (2004), since it can be inferred how the ambiguity parameters γ_2 and γ_3 influence asset prices. It is important to keep in mind that the formulas derived in this section differ from those in Section 4.2.

We solve the model in the same manner as Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2011), and Beeler and Campbell (2011) using analytical approximations. Assume that the log wealth-consumption ratio z is affine in the state variables

$$z_t = A + B_x x_t + B_\sigma(\sigma_t^2 - \sigma^2).$$

For the log return on the consumption claim $r_{c,t} = \log R_t$ we use the log-linear return approximation of Campbell and Shiller (1988)

$$r_{c,t+1} \quad = \quad k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1},$$

where k_0 and k_1 are linearizing constants. It holds that $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$ and $k_0 = \log(1+e^{\bar{z}}) - k_1\bar{z}$, where $\bar{z}$ is the long-run mean of the log wealth-consumption ratio. Using the Euler equation $\mathbb{E}_t\left[e^{m_{t,t+1}+r_{c,t+1}}\right] = 1$ yields the following coefficients for the wealth-consumption ratio

$$
\begin{aligned}
A &= \frac{1}{1-k_1}\left(-\delta + k_0 + (1-\rho)\mu_c + (1-k_1\varphi_\sigma)B_\sigma\sigma^2 + \frac{1-\gamma_3}{2(1-\rho)}(k_1 B_\sigma \phi_\sigma)^2\right), \\
B_x &= \frac{1-\rho}{1-k_1\varphi_x}, \\
B_\sigma &= \frac{(1-\gamma_1)(1-\rho)}{2(1-k_1\varphi_\sigma)}\left(1 + \frac{1-\gamma_2}{1-\gamma_1}\left(\frac{k_1\phi_x}{1-k_1\varphi_x}\right)^2\right).
\end{aligned}
$$

By substituting the return on the consumption claim into Equation (3) we get an expression for the log pricing kernel in terms of the state variables

$$m_{t,t+1} = s_0 + s_x x_t + s_\sigma(\sigma_t^2 - \sigma^2) - \Lambda_c \sigma_t w_{t+1}^c - \Lambda_x \phi_x \sigma_t w_{t+1}^x - \Lambda_\sigma \phi_\sigma w_{t+1}^\sigma,$$

with the drift characterized by the coefficients

$$\begin{aligned}
s_0 &= -\delta - \rho\mu_c - \frac{(1-\gamma_3)(\rho-\gamma_3)}{2(1-\rho)^2}(k_1 B_\sigma \phi_\sigma)^2 + s_\sigma \sigma^2, \\
s_x &= -\rho, \\
s_\sigma &= (k_1\varphi_\sigma - 1)B_\sigma\left(\frac{\rho-\gamma_3}{1-\rho}\right) - \frac{1}{2}(\gamma_3-\gamma_1)(1-\gamma_1) - \frac{1}{2}(\gamma_3-\gamma_2)(1-\gamma_2)\left(\frac{k_1\phi_x}{1-k_1\varphi_x}\right)^2 \\
&= \frac{1}{2}(1-\gamma_1)(\gamma_1-\rho) + \frac{1}{2}(1-\gamma_2)(\gamma_2-\rho)\left(\frac{k_1\phi_x}{1-k_1\varphi_x}\right)^2.
\end{aligned}$$

The coefficients Λ_c, Λ_x, and Λ_σ are the market prices of risk

$$\Lambda_c = \gamma_1, \qquad \Lambda_x = \frac{\gamma_2-\rho}{1-\rho}\, k_1\, B_x, \qquad \Lambda_\sigma = \frac{\gamma_3-\rho}{1-\rho}\, k_1\, B_\sigma.$$

We rely on the log-linear approximation for the log return on the dividend claim

$$r_{d,t+1} = k_{0,d} + k_{1,d} z_{d,t+1} - z_{d,t} + \Delta d_{t+1},$$

where $k_{1,d} = \frac{e^{\bar{z}_d}}{1+e^{\bar{z}_d}}$ and $k_{0,d} = \log(1+e^{\bar{z}_d}) - k_{1,d}\bar{z}_d$, with $\bar{z}_d$ denoting the long-run mean of the log price-dividend ratio. We conjecture that the log price-dividend ratio z_d is affine in the state variables

$$z_{d,t} = A_d + B_{d,x} x_t + B_{d,\sigma}(\sigma_t^2 - \sigma^2).$$

The coefficients of the log price-dividend ratio follow by applying the Euler equation to the log return on the dividend claim

$$\begin{aligned}
A_d &= \frac{s_0 + k_{0,d} + \mu_d + \frac{1}{2}\left(k_{1,d}B_{d,\sigma}\phi_\sigma - \Lambda_\sigma\phi_\sigma\right)^2 + \left((1-k_{1,d}\varphi_\sigma)B_{d,\sigma} - s_\sigma\right)\sigma^2}{(1-k_{1,d})}, \\
B_{d,x} &= \frac{\lambda-\rho}{1-k_{1,d}\varphi_x}, \\
B_{d,\sigma} &= \frac{s_\sigma + \frac{1}{2}\left((k_{1,d}B_{d,x}\phi_x - \Lambda_x\phi_x)^2 + \gamma_1^2 + \phi_{d,\sigma}^2 - 2\gamma_1\rho_{cd}\phi_{d,\sigma}\right)}{1-k_{1,d}\varphi_\sigma}.
\end{aligned}$$

Given the log pricing kernel the continuously compounded risk-free rate can be easily calculated as $r_{f,t} = r_{f,0} + r_{f,x} x_t + r_{f,\sigma}(\sigma_t^2 - \sigma^2)$, with

$$\begin{aligned}
r_{f,0} &= -s_0 - \frac{1}{2}\Lambda_\sigma^2\phi_\sigma^2 - \frac{1}{2}\left(\Lambda_c^2 + \Lambda_x^2\phi_x^2\right)\sigma^2, \\
r_{f,x} &= -s_x, \\
r_{f,\sigma} &= -s_\sigma - \frac{1}{2}\left(\Lambda_c^2 + \Lambda_x^2\phi_x^2\right).
\end{aligned}$$

The equity risk premium is equal to the covariance of the pricing kernel with the return on the dividend claim

$$\begin{aligned}
\mathbb{E}_t\left[r_{d,t+1}\right] - r_{f,t} = {} & \Lambda_\sigma k_{1,d} B_{d,\sigma}\phi_\sigma^2 - \frac{1}{2}(k_{1,d}B_{d,\sigma}\phi_\sigma)^2 \\
& + \left(\Lambda_x k_{1,d} B_{d,x}\phi_x^2 - \frac{1}{2}(k_{1,d}B_{d,x}\phi_x)^2 + \Lambda_c\phi_{d,\sigma}\rho_{cd} - \frac{1}{2}\phi_{d,\sigma}^2\right)\sigma_t^2.
\end{aligned}$$

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Preferences

γ_1	γ_2	γ_3	ρ	δ
$2-10$	$2-50$	$2-50$	$0.5-2/3$	$-\log(0.9989)$

Consumption and Dividends

μ_c	μ_d	λ	$\phi_{d,\sigma}$	ρ_{cd}
0.0015	0.0015	2.5	$\sqrt{2.6^2+5.96^2}$	$2.6/\phi_{d,\sigma}$

Long-Run Growth Rate

φ_x	ϕ_x
0.975	0.038

Volatility

φ_σ	ϕ_σ	σ
0.999	0.0000028	0.0072

Table 1: Parameters

The table gives preference and model parameters, expressed in monthly terms.

	Data	**Models**			
		Benchmark	Case 1	Case 2	Case 3
$E(\Delta c)$	1.93	1.80	1.80	1.80	1.80
		[0.69, 2.90]	[0.69, 2.90]	[0.69, 2.90]	[0.69, 2.90]
$\sigma(\Delta c)$	2.16	2.49	2.49	2.49	2.49
		[1.39, 4.03]	[1.39, 4.03]	[1.39, 4.03]	[1.39, 4.03]
AC1(Δc)	0.45	0.40	0.40	0.40	0.40
		[0.15, 0.61]	[0.15, 0.61]	[0.15, 0.61]	[0.15, 0.61]
$E(\Delta d)$	1.15	1.81	1.81	1.81	1.81
		[−2.82, 6.43]	[−2.82, 6.43]	[−2.82, 6.43]	[−2.82, 6.43]
$\sigma(\Delta d)$	11.05	14.05	14.05	14.05	14.05
		[7.87, 22.47]	[7.87, 22.47]	[7.87, 22.47]	[7.87, 22.47]
AC1(Δd)	0.21	0.26	0.26	0.26	0.26
		[0.03, 0.47]	[0.03, 0.47]	[0.03, 0.47]	[0.03, 0.47]
$E(r_d)$	5.47	6.09	6.37	4.79	6.99
		[1.89, 11.22]	[2.18, 11.46]	[0.40, 9.56]	[2.71, 13.55]
$\sigma(r_d)$	20.17	18.70	18.53	18.36	20.92
		[11.07, 29.10]	[10.86, 28.95]	[10.54, 28.89]	[13.76, 30.84]
AC1(r_d)	0.02	0.00	0.00	0.00	0.00
		[−0.24, 0.23]	[−0.24, 0.23]	[−0.24, 0.24]	[−0.23, 0.22]
$E(r_f)$	0.56	1.20	1.39	1.64	1.48
		[−0.10, 1.87]	[0.46, 1.94]	[0.81, 2.17]	[0.22, 2.15]
$\sigma(r_f)$	2.89	0.95	0.91	0.90	0.95
		[0.50, 1.64]	[0.48, 1.56]	[0.47, 1.55]	[0.50, 1.63]
AC1(r_f)	0.65	0.80	0.79	0.78	0.80
		[0.64, 0.90]	[0.63, 0.89]	[0.62, 0.89]	[0.64, 0.90]
$E(z_d)$	3.36	3.13	3.07	3.50	2.92
		[2.73, 3.33]	[2.71, 3.24]	[3.25, 3.62]	[2.07, 3.34]
$\sigma(z_d)$	45.00	18.30	17.23	15.27	29.85
		[9.95, 31.37]	[9.36, 29.06]	[8.31, 24.94]	[16.33, 56.50]
AC1(z_d)	0.87	0.63	0.59	0.50	0.80
		[0.32, 0.83]	[0.28, 0.81]	[0.20, 0.74]	[0.54, 0.93]

Table 2: Consumption, Dividend, and Asset Pricing Moments (IES = 1.5)

The table gives consumption, dividend, and asset pricing moments from the data and the models. Δc denotes the consumption growth rate, Δd the dividend growth rate, r_d the return on equity, and r_f the risk free rate. All returns and growth rates are in logs. z_d is the log price-dividend ratio. The results in the second column are from Beeler and Campbell (2011) and are based on annual data from 1930 to 2008. The median values and the 95% confidence intervals (in brackets) implied by the models are from 100,000 simulation runs of equivalent length to the data.

	Data	Models			
		Benchmark	Case 1	Case 2	Case 3
Excess Returns					
$R^2(1)$	0.044	0.011	0.010	0.008	0.032
		[0.000, 0.101]	[0.000, 0.093]	[0.000, 0.081]	[0.000, 0.152]
$\beta(1)$	−0.091	−0.090	−0.080	−0.055	−0.121
		[−0.371, 0.138]	[−0.371, 0.160]	[−0.375, 0.213]	[−0.343, 0.027]
$R^2(3)$	0.170	0.029	0.024	0.017	0.089
		[0.000, 0.227]	[0.000, 0.205]	[0.000, 0.163]	[0.000, 0.357]
$\beta(3)$	−0.264	−0.260	−0.232	−0.169	−0.344
		[−0.915, 0.337]	[−0.903, 0.389]	[−0.865, 0.505]	[−0.877, 0.072]
$R^2(5)$	0.269	0.044	0.036	0.025	0.136
		[0.000, 0.315]	[0.000, 0.284]	[0.000, 0.222]	[0.001, 0.488]
$\beta(5)$	−0.413	−0.410	−0.367	−0.271	−0.539
		[−1.369, 0.520]	[−1.344, 0.596]	[−1.261, 0.759]	[−1.305, 0.128]
Consumption Growth					
$R^2(1)$	0.060	0.141	0.158	0.211	0.054
		[0.003, 0.393]	[0.006, 0.407]	[0.028, 0.452]	[0.000, 0.294]
$\beta(1)$	0.011	0.050	0.057	0.075	0.018
		[0.004, 0.099]	[0.009, 0.105]	[0.026, 0.121]	[−0.012, 0.056]
$R^2(3)$	0.013	0.104	0.115	0.155	0.051
		[0.000, 0.414]	[0.001, 0.420]	[0.003, 0.443]	[0.000, 0.358]
$\beta(3)$	0.010	0.094	0.106	0.141	0.034
		[−0.036, 0.222]	[−0.027, 0.234]	[0.007, 0.269]	[−0.056, 0.131]
$R^2(5)$	0.000	0.081	0.087	0.115	0.054
		[0.000, 0.424]	[0.000, 0.424]	[0.001, 0.428]	[0.000, 0.405]
$\beta(5)$	−0.001	0.113	0.127	0.168	0.041
		[−0.093, 0.313]	[−0.083, 0.332]	[−0.039, 0.378]	[−0.108, 0.192]
Dividend Growth					
$R^2(1)$	0.092	0.193	0.217	0.281	0.072
		[0.021, 0.420]	[0.032, 0.441]	[0.078, 0.490]	[0.000, 0.283]
$\beta(1)$	0.074	0.332	0.377	0.488	0.120
		[0.093, 0.613]	[0.125, 0.655]	[0.230, 0.739]	[−0.023, 0.342]
$R^2(3)$	0.059	0.080	0.090	0.118	0.039
		[0.000, 0.343]	[0.000, 0.351]	[0.002, 0.370]	[0.000, 0.292]
$\beta(3)$	0.107	0.430	0.486	0.632	0.156
		[−0.158, 1.016]	[−0.121, 1.081]	[0.009, 1.231]	[−0.242, 0.605]
$R^2(5)$	0.039	0.059	0.064	0.080	0.042
		[0.000, 0.360]	[0.000, 0.361]	[0.000, 0.364]	[0.000, 0.345]
$\beta(5)$	0.089	0.464	0.523	0.681	0.169
		[−0.452, 1.357]	[−0.414, 1.439]	[−0.263, 1.631]	[−0.479, 0.841]

Table 3: Predictability (IES = 1.5)

The table gives R^2's and slope coefficients from the predictive regressions of excess returns, consumption growth, and dividend growth, measured over horizons of 1, 3, or 5 years, onto the log price-dividend ratio. The data values in the second column are taken from Beeler and Campbell (2011) and are based on annual data from 1930 to 2008. The median values and the 95% confidence intervals (in brackets) implied by the models are from 100,000 simulation runs of equivalent length to the data.

	Data	Models			
		Benchmark	Case 1	Case 2	Case 3
$E(\Delta c)$	1.93	1.80	1.80	1.80	1.80
		[0.69, 2.90]	[0.69, 2.90]	[0.69, 2.90]	[0.69, 2.90]
$\sigma(\Delta c)$	2.16	2.49	2.49	2.49	2.49
		[1.38, 4.03]	[1.39, 4.03]	[1.39, 4.03]	[1.39, 4.03]
AC1(Δc)	0.45	0.40	0.40	0.40	0.40
		[0.15, 0.61]	[0.15, 0.61]	[0.15, 0.61]	[0.15, 0.61]
$E(\Delta d)$	1.15	1.81	1.81	1.81	1.81
		[−2.82, 6.43]	[−2.82, 6.43]	[−2.82, 6.43]	[−2.82, 6.43]
$\sigma(\Delta d)$	11.05	14.05	14.05	14.05	14.05
		[7.87, 22.47]	[7.87, 22.47]	[7.87, 22.47]	[7.87, 22.47]
AC1(Δd)	0.21	0.26	0.26	0.26	0.26
		[0.03, 0.47]	[0.03, 0.47]	[0.03, 0.47]	[0.03, 0.47]
$E(r_d)$	5.47	5.95	6.21	4.69	6.94
		[1.76, 11.14]	[2.02, 11.34]	[0.31, 9.49]	[2.62, 13.66]
$\sigma(r_d)$	20.17	19.04	18.84	18.64	21.46
		[11.38, 29.50]	[11.14, 29.34]	[10.77, 29.27]	[14.26, 31.39]
AC1(r_d)	0.02	0.00	0.00	0.00	−0.01
		[−0.24, 0.23]	[−0.24, 0.23]	[−0.24, 0.23]	[−0.24, 0.22]
$E(r_f)$	0.56	0.59	0.66	0.98	0.72
		[−0.68, 1.23]	[−0.30, 1.16]	[0.18, 1.43]	[−0.78, 1.47]
$\sigma(r_f)$	2.89	0.76	0.71	0.69	0.81
		[0.40, 1.32]	[0.37, 1.23]	[0.36, 1.19]	[0.42, 1.40]
AC1(r_f)	0.65	0.81	0.80	0.79	0.83
		[0.66, 0.91]	[0.64, 0.90]	[0.63, 0.89]	[0.67, 0.92]
$E(z_d)$	3.36	3.16	3.10	3.53	2.93
		[2.71, 3.38]	[2.70, 3.30]	[3.24, 3.67]	[2.02, 3.37]
$\sigma(z_d)$	45.00	19.73	18.55	16.44	31.75
		[10.72, 34.24]	[10.07, 31.66]	[8.92, 27.08]	[17.36, 60.29]
AC1(z_d)	0.87	0.66	0.63	0.55	0.81
		[0.36, 0.85]	[0.32, 0.83]	[0.25, 0.77]	[0.55, 0.93]

Table 4: Consumption, Dividend, and Asset Pricing Moments (IES = 2.0)

The table gives consumption, dividend, and asset pricing moments from the data and the models. Δc denotes the consumption growth rate, Δd the dividend growth rate, r_d the return on equity, and r_f the risk free rate. All returns and growth rates are in logs. z_d is the log price-dividend ratio. The results in the second column are from Beeler and Campbell (2011) and are based on annual data from 1930 to 2008. The median values and the 95% confidence intervals (in brackets) implied by the models are from 100,000 simulation runs of equivalent length to the data.

	Data	Models			
		Benchmark	Case 1	Case 2	Case 3
Excess Returns					
$R^2(1)$	0.044	0.012	0.011	0.008	0.037
		[0.000, 0.106]	[0.000, 0.098]	[0.000, 0.084]	[0.000, 0.158]
$\beta(1)$	−0.091	−0.094	−0.085	−0.062	−0.125
		[−0.364, 0.121]	[−0.366, 0.141]	[−0.367, 0.190]	[−0.342, 0.019]
$R^2(3)$	0.170	0.032	0.027	0.019	0.100
		[0.000, 0.241]	[0.000, 0.221]	[0.000, 0.176]	[0.001, 0.372]
$\beta(3)$	−0.264	−0.269	−0.246	−0.184	−0.354
		[−0.907, 0.301]	[−0.901, 0.347]	[−0.867, 0.459]	[−0.873, 0.048]
$R^2(5)$	0.269	0.049	0.041	0.028	0.152
		[0.000, 0.336]	[0.000, 0.306]	[0.000, 0.241]	[0.001, 0.506]
$\beta(5)$	−0.413	−0.423	−0.389	−0.294	−0.553
		[−1.355, 0.469]	[−1.339, 0.535]	[−1.268, 0.697]	[−1.297, 0.090]
Consumption Growth					
$R^2(1)$	0.060	0.135	0.152	0.203	0.053
		[0.002, 0.390]	[0.004, 0.404]	[0.022, 0.448]	[0.000, 0.293]
$\beta(1)$	0.011	0.046	0.052	0.068	0.017
		[0.002, 0.092]	[0.006, 0.098]	[0.021, 0.113]	[−0.012, 0.053]
$R^2(3)$	0.013	0.102	0.113	0.152	0.051
		[0.000, 0.418]	[0.001, 0.425]	[0.002, 0.450]	[0.000, 0.360]
$\beta(3)$	0.010	0.087	0.098	0.129	0.032
		[−0.036, 0.207]	[−0.030, 0.220]	[0.000, 0.253]	[−0.053, 0.124]
$R^2(5)$	0.000	0.081	0.087	0.114	0.054
		[0.000, 0.430]	[0.000, 0.432]	[0.001, 0.439]	[0.000, 0.408]
$\beta(5)$	−0.001	0.104	0.117	0.155	0.038
		[−0.092, 0.294]	[−0.084, 0.312]	[−0.047, 0.354]	[−0.102, 0.182]
Dividend Growth					
$R^2(1)$	0.092	0.173	0.195	0.253	0.066
		[0.013, 0.401]	[0.022, 0.421]	[0.058, 0.468]	[0.000, 0.273]
$\beta(1)$	0.074	0.291	0.331	0.430	0.108
		[0.069, 0.561]	[0.096, 0.601]	[0.181, 0.679]	[−0.025, 0.315]
$R^2(3)$	0.059	0.075	0.084	0.110	0.038
		[0.000, 0.340]	[0.000, 0.348]	[0.001, 0.367]	[0.000, 0.290]
$\beta(3)$	0.107	0.382	0.434	0.566	0.142
		[−0.174, 0.944]	[−0.144, 1.006]	[−0.037, 1.145]	[−0.234, 0.568]
$R^2(5)$	0.039	0.057	0.061	0.076	0.042
		[0.000, 0.363]	[0.000, 0.364]	[0.000, 0.368]	[0.000, 0.345]
$\beta(5)$	0.089	0.416	0.470	0.614	0.155
		[−0.455, 1.267]	[−0.429, 1.343]	[−0.306, 1.519]	[−0.458, 0.792]

Table 5: Predictability (IES = 2.0)

The table gives R^2's and slope coefficients from the predictive regressions of excess returns, consumption growth, and dividend growth, measured over horizons of 1, 3, or 5 years, onto the log price-dividend ratio. The data values in the second column are taken from Beeler and Campbell (2011) and are based on annual data from 1930 to 2008. The median values and the 95% confidence intervals (in brackets) implied by the models are from 100,000 simulation runs of equivalent length to the data.

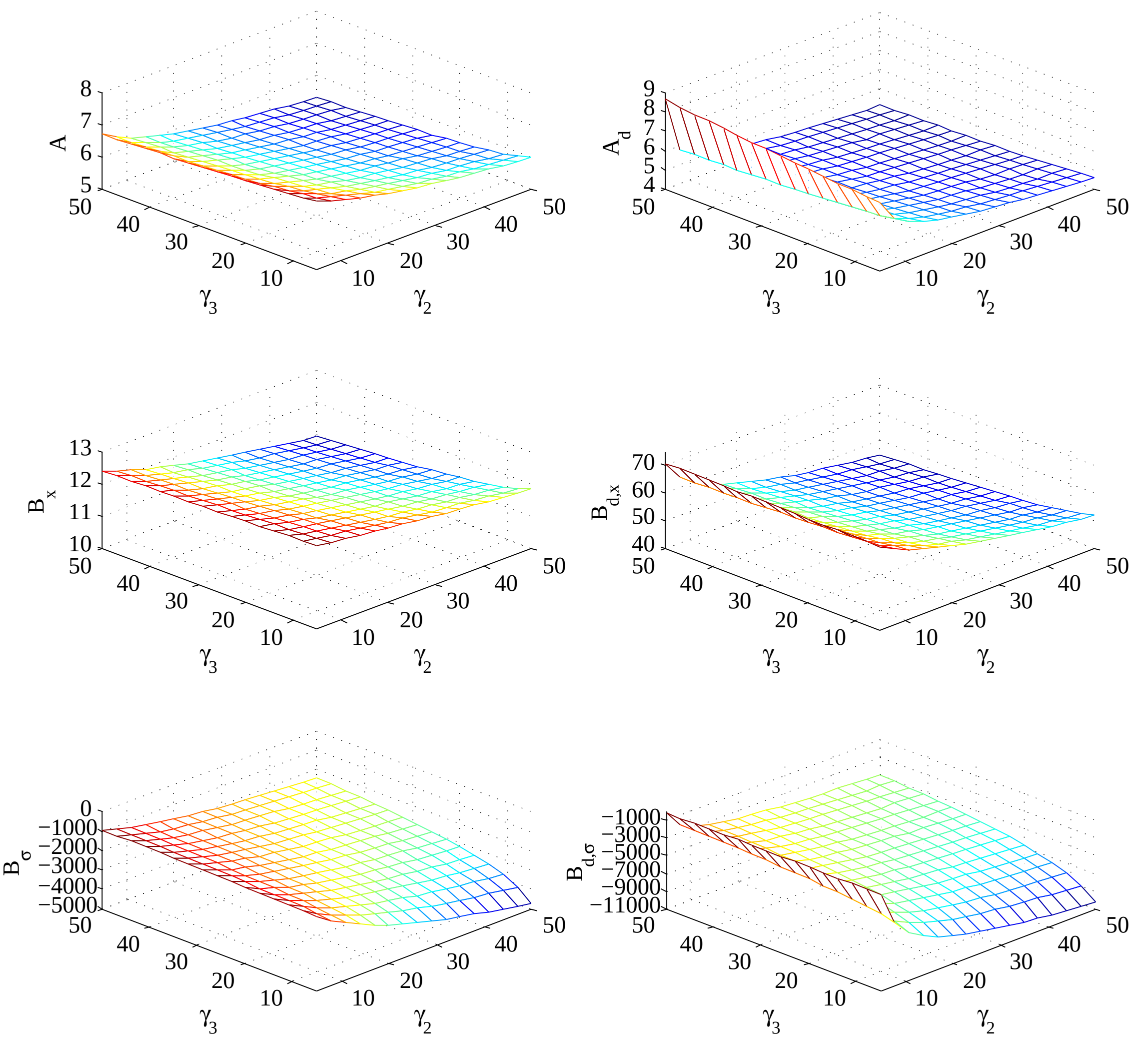

Figure 1: Wealth-Consumption and Price-Dividend Ratio

The figure shows the coefficients of the log wealth-consumption ratio and the log price-dividend ratio. A and A_d are the average of the log wealth-consumption and log price-dividend ratio, respectively. State variables are fixed at their unconditional means.

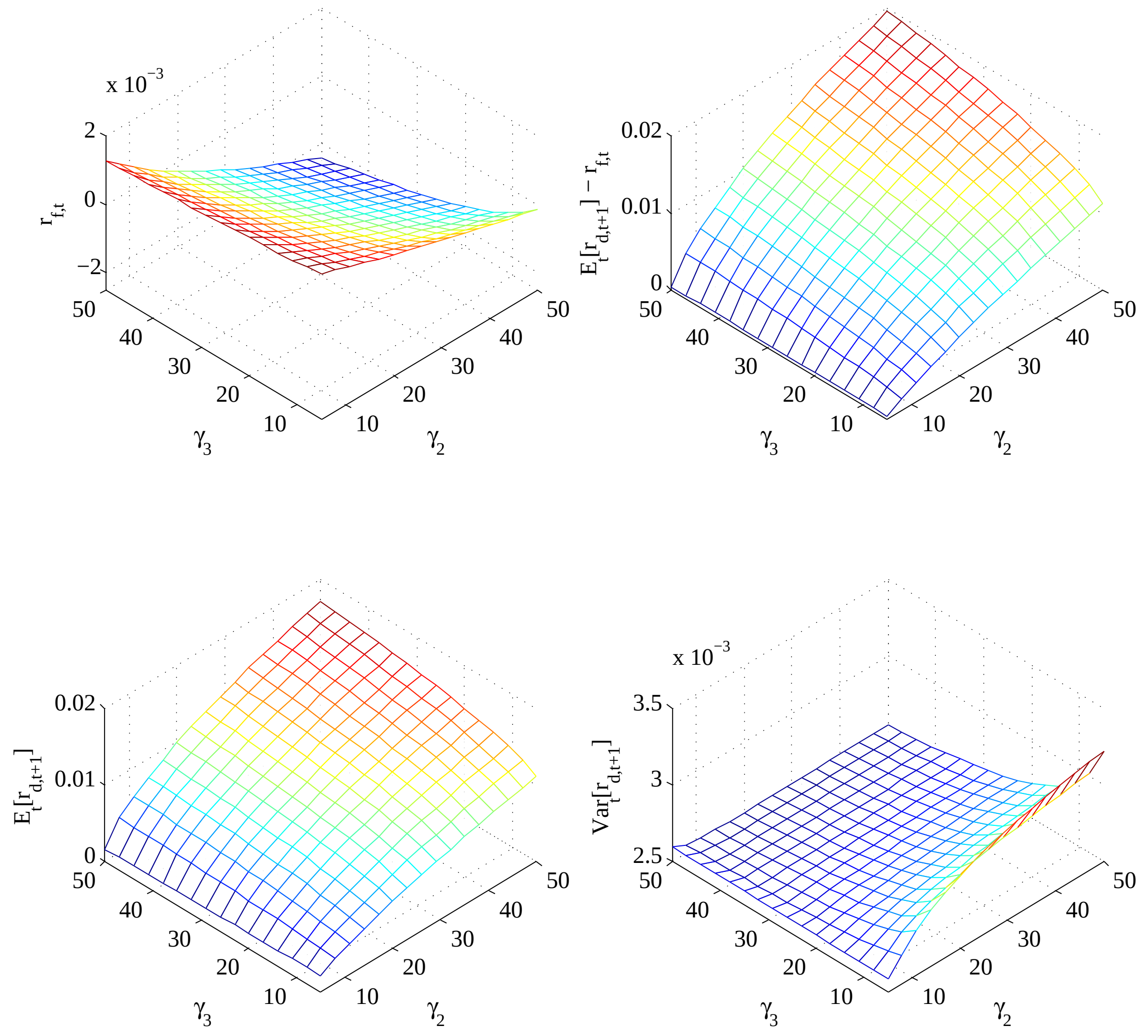

Figure 2: Risk-Free Rate and Return on Dividend Claim

The figure shows the mean risk-free rate, the equity premium, and the return on the dividend claim. The lower right corner of the figure displays the conditional return variance. State variables are fixed at their unconditional means.

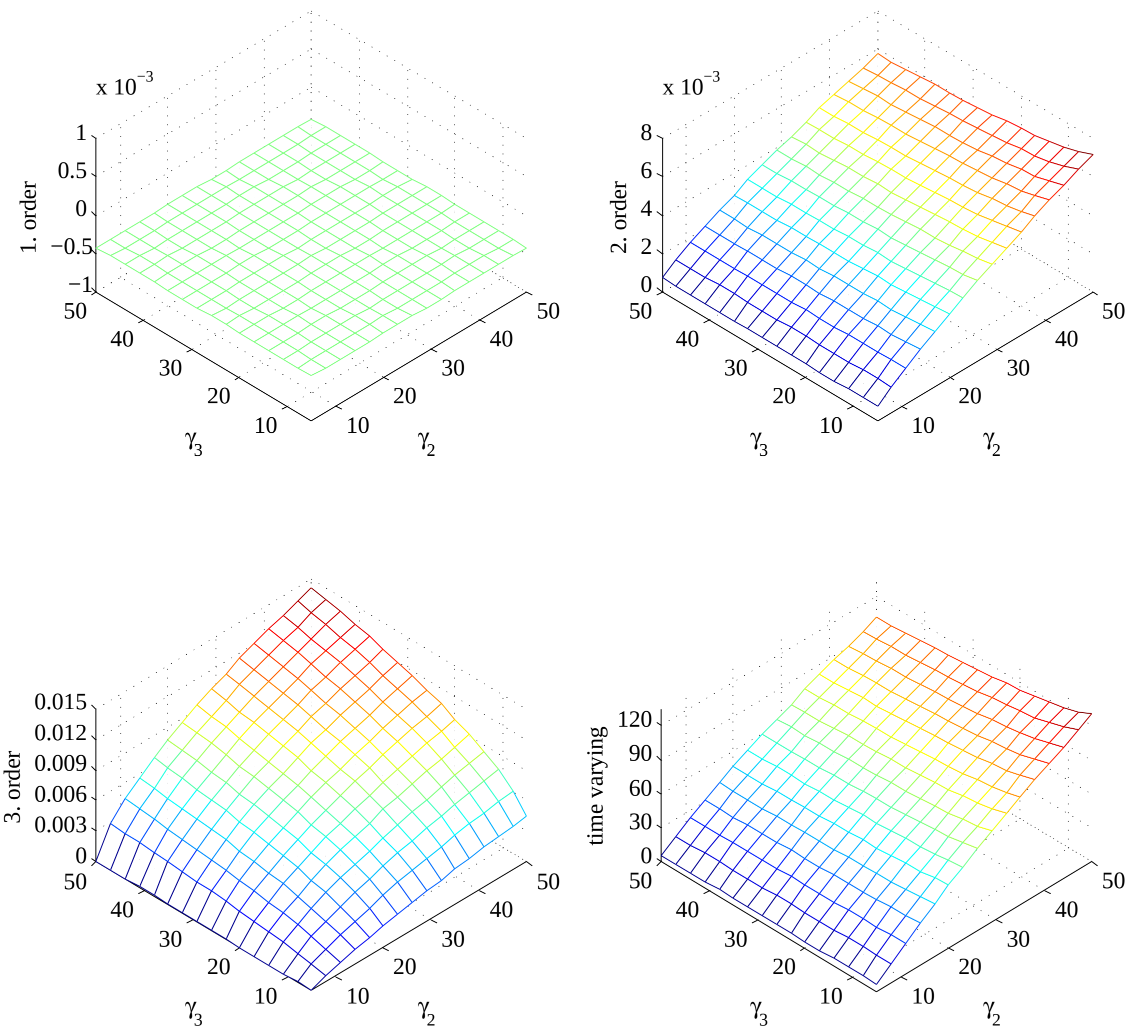

Figure 3: Equity Premium

The figure decomposes the equity premium in Equation (8) into its components.