Partial Information about Contagion Risk and Portfolio Choice

Nicole Branger* Holger Kraft** Christoph Meinerding***

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*Finance Center Münster, Westfälische Wilhelms-Universität Münster, Universitätsstr. 14-16, 48143 Münster, Germany. E-mail: nicole.branger@wiwi.uni-muenster.de.

**Department of Finance, Goethe University, 60323 Frankfurt am Main, Germany. E-mail: holgerkraft@finance.uni-frankfurt.de.

***Department of Finance, Goethe University, 60323 Frankfurt am Main, Germany. E-mail: christoph.meinerding@hof.uni-frankfurt.de.

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Abstract

The immanent threat of contagion effects in financial markets has been documented again during the recent financial crisis. This paper provides a realistic model for contagion effects that are triggered by certain crashes in asset prices. Market participants cannot distinguish between idiosyncratic crashes and crashes that cause the economy to slip into contagion. Therefore, the investor filters the probability of being in the contagion state from price observations. We relate our model to frameworks where contagion is captured by self-exciting processes and show that it induces a particular non-standard self-exciting model. We then study the effect of incomplete information about contagion risk on portfolio decisions in an incomplete market. We find that both contagion and learning have significant effects on the optimal portfolio strategy and, in particular, on the portfolio adjustments after jumps. Partially informed investors overreact to idiosyncratic jumps and underreact to jumps that increase the overall level of risk in the economy. This underreaction pattern is particularly pronounced if the investor filters optimally taking information from jump and diffusive risk into account. If the investor uses information from jumps only, the overreaction pattern dominates and the investor implements a more aggressive strategy. We also find that information about the state of the economy deduced from the observation of crashes is indeed valuable. Surprisingly, however, the additional utility gain provided by diffusive information is negligibly small.

Keywords: Asset Allocation, Jumps, Contagion, Nonlinear Filtering, Hidden State, Self-exciting Processes

JEL: G01, G11

1 Introduction and Motivation

> *"Fear is very contagious. You can get fearful in five minutes, but you don't get confident in five minutes."* Warren Buffett, March 09, 2009

The notion of contagion in financial markets refers to a situation where losses in one asset, one asset class, or one country increase the risk of subsequent losses in other assets, other asset classes, or other countries. Contagion may arise due to firm-specific relations, e.g. dependency on one main customer or producer, due to macroeconomic risk factors, e.g. interest rates or business cycle variables, or due to psychological reasons, e.g. bank runs. The most recent example is the US subprime crisis which started in the financial industry in 2007 and spread to the real economy in the subsequent two years.

Contagion is characterized by several stylized facts. Firstly, contagion describes a period of increased risk in the market where the probability of severe losses in assets is significantly larger than in 'normal' times. Secondly, the economy usually slips into a contagion state due to a key event that affects or is triggered by asset prices. For instance, there may be very bad news about a major company (e.g. Lehman Brothers), a sector (e.g. the financial industry) or a country (e.g. Thailand at the end of the 90's) that induce losses in the corresponding assets and, subsequently, lead to a period of higher risk in the whole economy. Alternatively, an external event (e.g. a natural disaster) might increase the overall level of risk and, at the same time, lead to a large loss in particular assets (e.g. utility stocks).[1] Thirdly, the question whether the economy is in a contagion state cannot be answered easily. Instead, market participants usually need some time to learn about the true level of risk in the economy.

Our paper focuses on the question of how the risk of contagion affects the portfolio decisions of an individual who has only imperfect information about the state of the economy and, consequently, tries to learn about this state using price information. We solve for his optimal portfolio and analyze his reaction to news about market prices. In particular, we study how the uncertainty about contagion changes his trading behavior. In this context, we compare optimal learning using all available information (i.e. from diffusion and event risk) and pure jump learning that only takes event risk into account. Furthermore, we explain how the dynamic trading behavior depends on asset characteristics. In particular, we compare assets that are very likely to trigger contagion (e.g. system-relevant banks) with assets that are heavily influenced by the risk of contagion.

[1] In the following, we will – also for ease of notation – usually apply the first interpretation and think of a situation where a jump in an asset price 'induces' contagion.

Our paper makes contributions in several dimensions. Firstly, we find that an investor with imperfect information – despite behaving optimally – initially either over- or underreacts to price jumps. The reason is that *each* price jump makes him reduce his estimated probability of being in the calm state and adjust his portfolio holdings. In contrast, perfectly informed investors would not adjust their portfolios at all if the jump has not induced contagion. Partial information thus leads to an overreaction to idiosyncratic jumps. Similarly, a fully informed investor would adjust his portfolio much more if the jump indeed triggered contagion. With incomplete information, we thus see an underreaction to events that induce contagion, since the investor waits for subsequent crashes to confirm that the economy really has entered the contagion state. Secondly, we analyze a market where some assets are more likely to induce contagion, while other assets are more heavily affected by contagion. The investor's reaction to price jumps and the trading volume in the assets induced by jumps differ significantly across these assets. Since portfolio adjustments are triggered by updates in the probability of being in the calm state, the investor reacts more heavily to jumps in the asset which is more likely to induce contagion. The trading volume itself, however, is larger in the asset which is considerably influenced by contagion since the impact of the state on this asset is much more pronounced. Thirdly, we analyze the value of information stemming from diffusive and event risk. Investors who filter by taking all information into account (diffusive and event risk) do not perform significantly better than investors ignoring the diffusive information. On the other hand, investors who do not filter at all perform significantly worse than investors filtering from event risk.

Our paper is related to the literature on continuous-time portfolio choice starting with Merton (1969, 1971). Early models with jump-diffusion processes have been developed by Aase (1984) and Jeanblanc-Picqué and Pontier (1990). Liu, Longstaff, and Pan (2003) consider a setup with jumps in stock prices and volatilities and solve for the optimal portfolio in an incomplete market. Liu and Pan (2003) and Branger, Schlag, and Schneider (2008) study related problems with derivatives. Wu (2003) focuses on a stochastic, but predictable investment opportunity set.

There are several ways to take contagion risk into account. One strand of the literature models contagion as simultaneous Poisson jumps in all assets, e.g. Das and Uppal (2004). Kraft and Steffensen (2008) extend this approach to bond markets and default risk. Ait-Sahalia, Cacho-Diaz, and Hurd (2009) consider a setting with several assets. All these papers abstract from the time dimension of contagion. In particular, the probability of subsequent crashes remains the same after a joint jump. The second strand of literature are so-called regime-switching models which were introduced by Hamilton (1989). Ang and Bekaert (2002) apply this approach to a discrete-time asset allocation problem whereas Honda (2003) focuses on a continuous-time framework. Recent studies with different interpretations, parametrizations, and calibrations of the regimes include Kole, Koedijk, and Verbeek (2006) and Guidolin and Timmermann (2007, 2008). Although a regime-switching model can capture the time dimension of contagion, regime shifts are still triggered by a process that is not linked to a particular crash in some assets. Apart from these two main ideas of modeling contagion, other approaches have been developed. Buraschi, Porchia, and Trojani (2010), e.g., focus on the impact of stochastic correlation on an optimal portfolio and suggest contagion risk as one application of their method.

Some recent papers model contagion effects more explicitly. In this respect, our paper is mostly related to Branger, Kraft, and Meinerding (2009). They focus on model risk and show that an investor modeling contagion using joint jumps can suffer severe utility losses once he is confronted with a Markov regime-switching framework. Kraft and Steffensen (2009) develop a similar model and apply it to the bond market, but focus on a complete market only. In contrast to our paper, Branger, Kraft, and Meinerding (2009) and Kraft and Steffensen (2009) assume that investors can observe the state of the economy perfectly. Ding, Giesecke, and Tomecek (2009) and Ait-Sahalia, Cacho-Diaz, and Laeven (2010) propose a different class of stochastic processes to model contagion effects, so-called self-exciting processes (Hawkes processes). They find that these can generate the empirically observed amount of default clustering. Complementarily to their studies, our paper provides a model-endogenous explanation of the exogenously given price dynamics of Ding, Giesecke, and Tomecek (2009) and Ait-Sahalia, Cacho-Diaz, and Laeven (2010) in the sense that the filtered jump intensities in our model follow stochastic processes which are similar to self-exciting processes.

Methodologically, our paper also builds on the large amount of literature on learning and incomplete information. The seminal studies of Detemple (1986) and Dothan and Feldman (1986) were among the first to apply filtering techniques in order to deal with asset pricing and asset allocation under partial information. They decompose these kinds of problems and show that the investor firstly solves a filtering problem, i.e. he estimates the current value of the state variable. Secondly, he decides on his optimal portfolio conditional on the estimated state variable. A portfolio problem with a hidden Markov chain controlling the jump intensity of the risky asset is studied by Bäuerle and Rieder (2007). However, they do not address contagion effects and consider an economy with one risky asset only.

The dynamics of the filtered probability in our model can be obtained as a special case of the results of Frey and Runggaldier (2010). If the investor filters from the observation of jumps only, the filter equation can also be deduced from Brémaud (1981).

The remainder of this paper is structured as follows. In Section 2, we present the exact model, the filtering equations and the link between self-exciting processes and our framework. Section 3 formulates the asset allocation problem and analyzes the solution with the optimal and the pure jump filter as well as with full information. In Section 4, we provide some numerical results in order to show the impact of contagion risk and filtering on an investor's portfolio choice in more detail. Section 5 concludes. All proofs can be found in the Appendix.

2 Model Setup

2.1 Main Idea

The asset prices in our economy follow jump-diffusion processes with negative constant jump sizes. We model contagion risk via two distinct economic regimes, a calm and a contagion state. The jump intensities of the assets depend on the economic regime: they are low in the calm state and significantly higher in the riskier contagion state. Whenever the economy switches from the calm to the contagion state, there is a downward jump in one of the assets at the same time. Besides these 'contagious' jumps, there are also idiosyncratic downward jumps in asset prices that are not linked to a change in the state. Transitions from the contagion to the calm state do not have a direct impact on prices. A sample path is depicted in the upper graph of Figure 1. Initially, the economy is in the calm state and enters the contagion state at time 5. At the same point in time, the price of asset A drops. Until the economy jumps back into the calm state around time 6, the jump intensities are much higher for both asset A (which has triggered contagion) and asset B (which has not caused contagion, but is affected as well).

We focus on the impact of information and learning on the investor's behavior. In particular, we assume that the investor cannot observe the true state of the economy (partial information), but learns about it from the history of prices. Technically, asset price jumps are driven by a Poisson hidden Markov model (PHMM). Assuming Bayesian learning, the investor applies filtering techniques to continuously update the probability of being in the calm state. For our example, the dynamics of the filtered probability are depicted in the lower graph of Figure 1. The probability drops significantly if the investor sees a jump while it moves back to 1 continuously as long as no jumps are observed. For the investor's portfolio problem, the 'estimated probability' is a state variable that follows a jump-diffusion process. Besides this optimal filter, we also analyze the case of pure jump filtering where the investor learns from the observation of jumps only and neglects the information from the diffusion part of the asset price history. Conditional on the state variable 'estimated probability of the calm state', the CRRA investor maximizes the expected utility of terminal wealth. He can trade in all risky assets and in the money market account.

2.2 Economy

The uncertainty in our economy is described by the complete filtered probability space $(\Omega, \mathcal{P}, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T^*]})$ and $\mathcal{F} = \mathcal{F}_{T^*}$. Investors can borrow and lend using a money market account M with dynamics

$$dM_t = rM_t dt, \quad M_0 = 1,$$

where, for simplicity, the interest rate r is assumed to be constant. Besides, there are two risky assets A and B with jump-diffusion-like dynamics. Our model is however more general than ordinary jump-diffusion models since event risk is driven by an n-state Markov chain $Z(t)$ which (loosely speaking) captures the economic conditions.[2] We define an n-dimensional counting process $N = (N^K)_{K=1,\ldots,n}$ where N^K denotes the number of transitions into state K, i.e.

$$N_t^K = \#\Big\{s|\ s \in (0,t],\ \lim_{\tau \nearrow s} Z(\tau) \neq K,\ Z(s) = K\Big\}.$$

The dynamics of the risky assets are then given by

$$\begin{aligned}\begin{pmatrix} \frac{dS_{A,t}}{S_{A,t}} \\ \frac{dS_{B,t}}{S_{B,t}} \end{pmatrix} &= \begin{pmatrix} \mu_A^{Z(t)} \\ \mu_B^{Z(t)} \end{pmatrix} dt + \begin{pmatrix} v_A^{Z(t)} & 0 \\ \rho^{Z(t)} v_B^{Z(t)} & \sqrt{1-(\rho^{Z(t)})^2} v_B^{Z(t)} \end{pmatrix} \begin{pmatrix} dW_{A,t} \\ dW_{B,t} \end{pmatrix} \\ &\quad - \sum_{K \neq Z(t-)} \begin{pmatrix} L_A^{Z(t-),K} \\ L_B^{Z(t-),K} \end{pmatrix} dN_t^K,\end{aligned}$$

where W_A and W_B are independent Brownian motions that capture diffusive risk. The loss in asset i upon a jump from state J into state K is denoted by $L_i^{J,K}$, and the intensity of these jumps is $\lambda^{J,K}$.

The idea of our model is that there are two states, calm and contagion. From a technical point of view, however, more states are needed to model all possible orders of jumps. The corresponding Markov chain that allows us to capture all contingencies has eight states $\{cont_{A1}, cont_{A2}, cont_{B1}, cont_{B2}, calm_{A1}, calm_{A2}, calm_{B1}, calm_{B2}\}$ and is illustrated in Figure 2. The first subscript denotes the asset in which the most recent jump has happened. The second subscript allows us to model several subsequent jumps in the same asset which do not change the economic state. For instance, if there are several jumps of

[2] The process Z is a right-continuous process with left limits (RCLL).

asset A in the calm state that do not trigger contagion, then the Markov chain jumps back and forth between $calm_{A1}$ and $calm_{A2}$.

Since there are only two economic states, the model parameters are assumed to coincide in all calm and in all contagion states. This implies that all calm states and all contagion states are identical in the sense that optimal portfolios and indirect utilities are the same. Unless otherwise stated, we will thus only refer to two states, 'calm' and 'contagion', where 'calm' refers to any of the states $\{calm_{A1}, calm_{A2}, calm_{B1}, calm_{B2}\}$ and 'contagion' refers to any of the states $\{cont_{A1}, cont_{A2}, cont_{B1}, cont_{B2}\}$. Therefore, the structure of the economy can be described as follows: In the calm state, the intensity of a jump in asset i that does not trigger contagion is $\lambda_i^{calm,calm}$, and the corresponding loss in asset i is $L_i^{calm,calm}$. The intensity of a jump in asset i that does trigger contagion is $\lambda_i^{calm,cont}$ and the loss of asset i for such a jump is $L_i^{calm,cont}$. If the economy is in a contagion state, the intensity for a loss in asset i is $\lambda_i^{cont,cont}$, and the corresponding loss size is $L_i^{cont,cont}$. We assume that $\lambda_i^{cont,cont} \geq \lambda_i^{calm,calm} + \lambda_i^{calm,cont}$. After spending some time in the contagion state, the economy will eventually jump back into the calm state. The intensity for this to happen is $\lambda^{cont,calm}$, and it is assumed that this event does not induce any losses in the assets, i.e. $L_i^{cont,calm} \equiv 0$, $i \in \{A, B\}$.

Concerning the diffusion parameters of the model (v_A, v_B and ρ), we make the standing assumption that they do not depend on the state of the economy, i.e. they are constant over time. We do this in order to keep the model consistent from an informational point of view. We want to model an investor who is not able to identify the correct state of the economy at every point in time, but has to filter the state from observations of the asset prices (see Section 2.3). If the diffusion parameters were state-dependent, the investor could, however, perfectly estimate the state of the economy by observing an infinitesimally small sample of the price paths since we are working in a continuous-time framework. A similar argument applies to the loss size in case of a jump. For simplicity, we thus assume a constant loss size L_i for each asset and for all types of jumps throughout the paper.

Finally, we specify the drift and the risk premia of the assets. The drift of asset i is equal to

$$\mu_i^{Z(t)} = r + \phi_i^{Z(t)} + \sum_{K \neq Z(t-)} L_i^{Z(t-),K} \lambda^{Z(t-),K},$$

where the last term is the compensator of the jump processes. The risk premium is given by

$$\phi_i^{Z(t)} = v_i \eta^{diff,Z(t)} + \sum_{K \neq Z(t-)} L_i^{Z(t-),K} \lambda^{Z(t-),K} \eta^{Z(t-),K},$$

where $\eta^{diff,K}$ is the market price for diffusive risk in state K, and $\eta^{J,K}$ is the market price for jumps from J into K. With our definition of the Markov chain, the risk premia only depend on whether the economy is in one of the calm or in one of the contagion states. Consequently, they can be rewritten as[3]

$$\begin{aligned}\phi_i^{calm} &= v_i\eta^{diff,calm} + L_i\lambda_i^{calm,calm}\eta^{calm,calm} + L_i\lambda_i^{calm,cont}\eta^{calm,cont}\\ \phi_i^{cont} &= v_i\eta^{diff,cont} + L_i\lambda_i^{cont,cont}\eta^{cont,cont}.\end{aligned}$$

2.3 Filtering the State of the Economy

The asset price dynamics depend on the current state of the economy. In the following, we assume that the investor has partial information: Although he knows all model parameters, he cannot observe the state of the economy, but has to infer it from asset prices. For instance, during the subprime crisis it has taken investors some time to realize that the economy is in the worst financial crisis since the Great Depression.

Formally, this is captured by having two filtrations in the model. The 'large' filtration $\mathcal{F}$ includes all information describing the true data-generating process, while the 'small' filtration $\{\mathcal{G}_t\}_{t\in[0,T^*]} \subset \{\mathcal{F}_t\}_{t\in[0,T^*]}$ captures the (partial) information available to the investor when he decides upon his portfolio. The filtration $\{\mathcal{G}_t\}_{t\in[0,T^*]}$ includes the history of both asset prices, but not the history of the underlying hidden Markov chain. The asset prices in our economy are thus determined by a Poisson hidden Markov model (PHMM). Detemple (1986) and Dothan and Feldman (1986) show that the portfolio problem can be solved in two steps. Firstly, the investor solves a filtering problem, i.e. he estimates the current value of the state variable. Secondly, he decides on his optimal portfolio conditional on the just estimated state variable.

Denoting by $p_t \in \{0,1\}$ the indicator variable for being in the calm state at time t (i.e. $p_t = 0$ if the economy is in the contagion state), we define $\widehat{p_t}$ as the estimate for p_t that minimizes the mean-square distance between p_t and all square-integrable and $\mathcal{G}_t$-measurable random variables. In other words, $\widehat{p_t}$ gives the investor's subjective probability of being in the calm state at time t. Elementary results from filter theory state that this estimate is given by the conditional expectation $\widehat{p_t} = \mathrm{E}[p_t|\mathcal{G}_t]$.

The investor can perfectly disentangle jumps from diffusions since we assume a continuous-

[3]We assume that the market prices of diffusion risk are the same for asset A and asset B. Analogously, we assume that the market prices of risk for jump events are the same for the two assets, leaving us with four market prices of risk for jumps of the type calm-calm, calm-cont, cont-cont, and cont-calm.

time model.[4] He observes the total number of jumps $\widehat{N}_A$ and $\widehat{N}_B$ defined by

$$\widehat{N}_i = N_i^{calm,calm} + N_i^{calm,cont} + N_i^{cont,cont}$$

with the obvious meaning of the counting processes on the right hand side.[5] However, he is not able to distinguish between the three different kinds of jumps on the right hand side. Furthermore, he cannot observe jumps back from the contagion state to the calm state since these jumps do not have any impact on the asset prices.

Since we allow for state-dependent drift rates $\mu_i^{Z(t)}$, the diffusion parts of the asset prices contain information about the underlying hidden Markov chain as well. The investor has subjective estimates $\widehat{\mu}_i = \widehat{p}\mu_i^{calm} + (1-\widehat{p})\mu_i^{cont}$ for the drift rates and, therefore, computes 'perceived' Brownian motions, i.e. the Brownian motions which have generated the current asset prices under his filtration. These are given as

$$\begin{pmatrix} d\widehat{W}_{A,t} \\ d\widehat{W}_{B,t} \end{pmatrix} = \begin{pmatrix} dW_{A,t} \\ dW_{B,t} \end{pmatrix} + \begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix}^{-1} \begin{pmatrix} \mu_A^{Z(t)} - \widehat{\mu}_{A,t} \\ \mu_B^{Z(t)} - \widehat{\mu}_{B,t} \end{pmatrix} dt.$$

Altogether, the estimate $\widehat{p}$ follows the dynamics

$$\begin{aligned} d\widehat{p}_t \;=\; & \Big((1-\widehat{p}_t)\lambda^{cont,calm} - \widehat{p}_t(\lambda_A^{calm,cont} + \lambda_B^{calm,cont})\Big)\, dt \\ & +\widehat{p}_t(1-\widehat{p}_t)\left[\frac{\mu_A^{calm} - \mu_A^{cont}}{v_A} d\widehat{W}_{A,t} + \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{calm} - \mu_B^{cont}}{v_B} - \rho\frac{\mu_A^{calm} - \mu_A^{cont}}{v_A}\right) d\widehat{W}_{B,t}\right] \\ & + \left(\frac{\widehat{p}_{t-}\lambda_A^{calm,calm}}{\widehat{\lambda}_A(\widehat{p}_{t-})} - \widehat{p}_{t-}\right)\Big(d\widehat{N}_{A,t} - \widehat{\lambda}_A(\widehat{p}_t)dt\Big) + \left(\frac{\widehat{p}_{t-}\lambda_B^{calm,calm}}{\widehat{\lambda}_B(\widehat{p}_{t-})} - \widehat{p}_{t-}\right)\Big(d\widehat{N}_{B,t} - \widehat{\lambda}_B(\widehat{p}_t)dt\Big) \end{aligned} \tag{1}$$

where the estimated subjective intensity of $\widehat{N}_i$ is

$$\widehat{\lambda}_i(\widehat{p}_t) = \widehat{p}_t\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p}_t)\lambda_i^{cont,cont}. \tag{2}$$

A proof is given in Appendix A.1. Note that the intensity $\widehat{\lambda}_i$ depends on $\widehat{p}_t$. To shorten notations, we will usually omit this dependence.

The first line of the filter equation corresponds to the expected change of $\widehat{p}$. It depends on the probability of jumping back into the calm state (conditional on being in the contagion state) and the probability of leaving the calm state (conditional on being in the calm

[4] This is, at least asymptotically, even possible in discrete-time models, see e.g. Ait-Sahalia (2004) or Johannes, Polson, and Stroud (2009).

[5] We will stick to this notational convention throughout the remainder of the paper. Variables with a 'hat' denote subjective numbers that the investor estimates from his observations. Variables without a 'hat' represent the true numbers in the economy.

state). The second line gives the impact of diffusion risk on $\widehat{p}$. This impact is increasing in the difference $\mu_i^{calm} - \mu_i^{cont}$ since the diffusion processes are more informative if the drift rates in the calm and in the contagion state differ significantly. For the same reason, the impact of diffusion risk on $\widehat{p}$ is decreasing in the diffusion volatilities v_A and v_B since large diffusive noise complicates estimating the current economic state from the observation of historical asset price paths. The third line captures the reaction of $\widehat{p}$ to downward jumps. If there is a jump in asset i, the probability decreases from $\widehat{p}_{t-}$ to $\widehat{p}_{t-} \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i(\widehat{p}_{t-})}$ and will eventually approach 0 if a large number of jumps is observed. As long as there are no jumps, the change in $\widehat{p}$ is given by the expected change, the diffusion terms and the compensators of the jump components. In line with intuition, the sum of all drift terms is positive, reflecting the fact that the subjective probability $\widehat{p}$ of being in the calm state increases and eventually approaches 1 if no jumps are observed. Note that the sum of all drift terms is indeed equal to 0 if and only if $\widehat{p} = 1$.

2.4 Pure Jump Filtering

Besides the nonlinear filtering method above, we consider another filter which will be denoted as 'pure jump filter' in the following. Since estimating the current state of the economy from historical asset prices is involved, we introduce a slightly simplified version taking only the information from jumps into account. One can think of an investor with 'average' skills who does not track the whole asset price paths, but reacts to major events only. The inference about the current state of the economy is then based on the history of $\widehat{N}_A$ and $\widehat{N}_B$ only. Formally, one can think of an investor who optimizes his portfolio using a small filtration $\{\mathcal{H}_t\}_{t\in[0,T^*]} \subset \{\mathcal{G}_t\}_{t\in[0,T^*]} \subset \{\mathcal{F}_t\}_{t\in[0,T^*]}$. The corresponding estimate $\widehat{p}_t^{pjf} = \mathrm{E}[p_t|\mathcal{H}_t]$ can directly be obtained from the optimal filter sketched above:

$$\begin{aligned} d\widehat{p}_t^{pjf} \quad = \quad & \Big((1-\widehat{p}_t^{pjf})\lambda^{cont,calm} - \widehat{p}_t^{pjf}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont})\Big)\, dt \\ & + \left(\frac{\widehat{p}_{t-}^{pjf}\lambda_A^{calm,calm}}{\widehat{\lambda}_A(\widehat{p}_{t-}^{pjf})} - \widehat{p}_{t-}^{pjf}\right)\Big(d\widehat{N}_{A,t} - \widehat{\lambda}_A(\widehat{p}_t^{pjf})dt\Big) \\ & + \left(\frac{\widehat{p}_{t-}^{pjf}\lambda_B^{calm,calm}}{\widehat{\lambda}_B(\widehat{p}_{t-}^{pjf})} - \widehat{p}_{t-}^{pjf}\right)\Big(d\widehat{N}_{B,t} - \widehat{\lambda}_B(\widehat{p}_t^{pjf})dt\Big)\,, \end{aligned} \tag{3}$$

where the estimated subjective intensity of $\widehat{N}_i$ is

$$\widehat{\lambda}_i(\widehat{p}_t^{pjf}) = \widehat{p}_t^{pjf}\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p}_t^{pjf})\lambda_i^{cont,cont}.$$

A proof is given in Appendix A.1. Note that again the intensity $\widehat{\lambda}_i$ depends on $\widehat{p}_t^{pjf}$.

2.5 Relation to Self-exciting Processes

The estimated jump intensities within our model are conceptually linked to self-exciting processes (Hawkes processes) which have recently been proposed as an alternative for the modeling of default clustering and contagion effects. Self-exciting dynamics for jump intensities are usually of the form[6]

$$d\lambda_t = \kappa(\theta - \lambda_t)dt + \sigma_t dW_t + \ell dN_t,$$

where N is a point process with intensity λ and ℓ denotes a positive jump size. The volatility σ_t is either set to zero or is chosen such that λ remains positive (e.g. $\sigma_t = \sigma\sqrt{\lambda_t}$). The constant κ captures the speed with which the process reverts back to θ. Our model endogenously induces a variant of such a process where the mean reversion speed and the jump size are state dependent. To understand this point, we calculate the dynamics of $\widehat{\lambda}_A$ using (1) and (2). For simplicity, we assume that there is only one risky asset, but this is without loss of generality.

Proposition 1 (Endogenous Self-exciting Intensity) *In a setting with one asset, its filtered jump intensity is given by the self-exciting process*

$$d\widehat{\lambda}_{A,t} = \kappa_{A,t}(\theta_A - \widehat{\lambda}_{A,t})dt + \zeta_{A,t}d\widehat{W}_{A,t} + \ell_{A,t}d\widehat{N}_{A,t}, \tag{4}$$

with $\lambda_A^{calm,*} = \lambda_A^{calm,calm} + \lambda_A^{calm,cont}$ *and*

$$\begin{aligned}
\kappa_{A,t} &= \lambda_A^{cont,cont} + \lambda^{cont,calm} - \widehat{\lambda}_{A,t} \\
\theta_A &= \lambda_A^{calm,*} \\
\zeta_{A,t} &= (\widehat{\lambda}_{A,t} - \lambda_A^{calm,*})(\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t})\frac{\mu_A^{cont} - \mu_A^{calm}}{v_A(\lambda_A^{cont,cont} - \lambda_A^{calm,*})} \\
\ell_{A,t} &= (\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t})\frac{\widehat{\lambda}_{A,t} - \lambda_A^{calm,calm}}{\widehat{\lambda}_{A,t}}.
\end{aligned}$$

The proof is straightforward. The dynamics (4) have intuitive interpretations. Firstly, recall that $\widehat{\lambda}_A$ is bounded from below by $\theta_A = \lambda_A^{calm,*}$. Therefore, the process reverts to the constant θ_A if no jumps are observed. The mean reversion speed $\kappa_{A,t}$ is the larger the smaller the filtered intensity $\widehat{\lambda}_{A,t}$ becomes, i.e. the mean reversion speed increases endogenously as long as there are no jumps. Notice that $\kappa_{A,t}$ is always strictly positive if there is a positive probability that the economy jumps back into the calm state ($\lambda^{cont,calm} > 0$). This is because $\widehat{\lambda}_A$ is bounded from above by $\lambda_A^{cont,cont}$ and thus $\kappa_{A,t} \geq \lambda^{cont,calm}$. Only if

[6] See, e.g., Ding, Giesecke, and Tomecek (2009) and Ait-Sahalia, Cacho-Diaz, and Laeven (2010).

the contagion state was an absorbing state ($\lambda^{cont,calm} = 0$), then the mean reversion speed could become zero.

The volatility $\zeta_{A,t}$ consists of two parts: the product $(\widehat{\lambda}_{A,t} - \lambda_A^{calm,*})(\lambda_A^{cont,cont} - \widehat{\lambda}_{A,t})$ and the ratio $\frac{\mu_A^{cont} - \mu_A^{calm}}{v_A(\lambda_A^{cont,cont} - \lambda_A^{calm,*})}$. The product ensures that the volatility becomes 0 as soon as the upper or lower bound of the filtered intensity $\widehat{\lambda}_A$ is reached. The ratio is driven by the precision of the diffusive information. This information is particularly valuable if the two states are very different, i.e. the difference of the drifts is large. In this case, the agent relies more on diffusive information and thus the filtered intensity $\widehat{\lambda}_A$ is more exposed to diffusive risk. The opposite is true if the diffusive information becomes more noisy, i.e. v_A becomes larger. Finally, information from jumps is more valuable if the jump intensities in the states are more distinct, i.e. $\lambda_A^{cont,cont}$ is much larger than $\lambda_A^{calm,*}$. Then the diffusive information becomes relatively less valuable and thus $\lambda_A^{cont,cont} - \lambda_A^{calm,*}$ has the same effect on the volatility $\zeta_{A,t}$ as v_A.

Finally, we analyze the jump size $\ell_{A,t}$. Assume, for the moment, that every jump leads to contagion or happens in the contagion state, i.e. $\lambda_A^{calm,calm} = 0$. In this case, every jump indicates that the economy is in (or just entered) the contagion state. Consequently, $\ell_{A,t} = \lambda_A^{cont,cont} - \widehat{\lambda}_{A,t}$, i.e. the filtered jump intensity is updated to its maximum level $\lambda_A^{cont,cont}$ whenever a jump occurs. If there are also idiosyncratic jumps, i.e. $\lambda_A^{calm,calm} > 0$, the jump size $\ell_{A,t}$ is dampened by the factor $\frac{\widehat{\lambda}_{A,t} - \lambda_A^{calm,calm}}{\widehat{\lambda}_{A,t}} < 1$ which captures the likelihood that a jump is contagious.

3 Optimal Portfolio Choice

3.1 Optimization Problem

We consider an investor with CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where $\gamma > 0$ denotes his relative risk aversion. The planning horizon is denoted by T. The investor maximizes expected utility from terminal wealth X_T. His indirect utility at time t depends on the wealth X_t and the filtered probability of being in the calm state, $\widehat{p}_t$, which, depending on the applied filter, follows the dynamics given in Section 2.3 or 2.4. It is defined as

$$G(t, X_t, \widehat{p}_t) = \max_{\Pi \in \mathcal{A}(t,\widehat{p}_t)} \mathrm{E}\big[u(X_T)|\mathcal{G}_t\big] \qquad \text{or} \qquad G(t, X_t, \widehat{p}_t) = \max_{\Pi \in \mathcal{A}(t,\widehat{p}_t)} \mathrm{E}\big[u(X_T)|\mathcal{H}_t\big]$$

where $\mathcal{A}(t, \widehat{p}_t)$ denotes the set of all admissible trading strategies.

Due to the event risk, the investor faces an incomplete market. In order to choose optimal exposures to the different sources of risk (diffusion and jumps), he can adjust the weights

π_A and π_B of the two risky assets in his portfolio. His budget restriction reads

$$\frac{dX_t}{X_t} = \pi_A(t,\widehat{p}_t)\frac{dS_{A,t}}{S_{A,t}} + \pi_B(t,\widehat{p}_t)\frac{dS_{B,t}}{S_{B,t}} + \left[1 - \pi_A(t,\widehat{p}_t) - \pi_B(t,\widehat{p}_t)\right] r dt.$$

In the remainder of this section, we solve for the indirect utility function and the optimal security demands. We will do this for three cases. Firstly, we consider two portfolio problems in which the investor does not know the state of the economy, but has to filter it. He applies either the optimal or the pure jump filter. Subsequently, we briefly summarize the results if the investor is fully informed and knows the state of the economy perfectly.[7] The latter setting serves as a benchmark case for the setting with partial information and allows us to study the effects of information on the security demands.

For the cases with incomplete information (optimal and pure jump filter), we conjecture that the indirect utility is equal to

$$G(t,x,\widehat{p}) = \frac{x^{1-\gamma}}{1-\gamma} f(t,\widehat{p}), \tag{5}$$

where x denotes current wealth and $\widehat{p}$ the filtered probability. In both cases, the corresponding function f is part of the solution and must be determined either explicitly or numerically. If the investor is fully informed, then we obtain one Bellman equation for each state and thus different functions f^j for the states $j \in \{calm, contagion\}$. Nevertheless, (5) then holds statewise. Notice that, with full information, the indirect utility function does not depend on the filtered probability $\widehat{p}$ since filtering is then redundant.

3.2 Portfolio Choice with Optimal Filter

Firstly, we solve the portfolio problem of an investor who uses the optimal filter (1). The indirect utility function satisfies a Bellman equation that is provided in Appendix A.2. Substituting the conjecture (5) into the Bellman equation yields a system of equations for f and the optimal demands, π_A and π_B. The following proposition summarizes our results.[8]

Proposition 2 (Solution with Optimal Filter) *If the investor uses the optimal filter to estimate the current state of the economy, the optimal portfolio weights satisfy the*

[7] This is the situation that has been analyzed by Branger, Kraft, and Meinerding (2009).
[8] The proof is given in Appendix A.2.

first-order conditions

$$\begin{aligned} f \cdot \left[\widehat{\mu}_A - r - \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B)\right] + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_A^{calm} - \mu_A^{cont}) & \\ -f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A &= 0 \\ f \cdot \left[\widehat{\mu}_B - r - \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A)\right] + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_B^{calm} - \mu_B^{cont}) & \\ -f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B &= 0, \end{aligned}$$

where f solves

$$\begin{aligned} & f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\ & \quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2 \pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\ & +f_p \cdot \Big[(1-\gamma)\widehat{p}(1-\widehat{p})\big(\pi_A(\mu_A^{calm} - \mu_A^{cont}) + \pi_B(\mu_B^{calm} - \mu_B^{cont})\big) \\ & \quad +(1-\widehat{p})\lambda^{cont,calm} - \widehat{p}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\ & +f_{pp} \cdot \frac{0.5\widehat{p}^2(1-\widehat{p})^2}{1-\rho^2}\left[\frac{(\mu_A^{calm} - \mu_A^{cont})^2}{v_A^2} - 2\rho\frac{(\mu_A^{calm} - \mu_A^{cont})(\mu_B^{calm} - \mu_B^{cont})}{v_A v_B} + \frac{(\mu_B^{calm} - \mu_B^{cont})^2}{v_B^2}\right] \\ & +f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A + f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t = 0 \end{aligned} \tag{6}$$

with boundary conditions $f(0,\cdot) = 1$ and $f_p(0,\cdot) = 0$. The subjective drift rate and jump intensity of asset i ($i \in \{A, B\}$) are defined as

$$\begin{aligned} \widehat{\mu}_i &= \widehat{p}\mu_i^{calm} + (1-\widehat{p})\mu_i^{cont} \\ \widehat{\lambda}_i &= \widehat{p}\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p})\lambda_i^{cont,cont}. \end{aligned}$$

As usually in incomplete market problems with jumps, the first-order conditions and the Bellman equation (6) can only be solved simultaneously. The indirect utility function and the optimal portfolio weights π_i depend on the state variable $\widehat{p}$. Since $\widehat{p}$ evolves stochastically following a jump-diffusion process, the optimal portfolio weights do so, too. The optimal portfolio weights are monotonic functions of $\widehat{p}$. As long as no jump is observed, they continuously revert back to the optimal portfolio for $\widehat{p} = 1$, i.e. if the investor is sure to be in the calm state. If a jump occurs, they are adjusted by a discrete amount towards the optimal portfolio for $\widehat{p} = 0$.

To interpret the optimal portfolio strategy, let us assume for simplicity that there is only one asset. Rewriting the first-order condition for asset A yields

$$\pi_A = \frac{\widehat{\mu}_A - r}{\gamma v_A^2} + \widehat{p}(1-\widehat{p})\frac{\mu_A^{calm} - \mu_A^{cont}}{\gamma v_A^2}\frac{f_p}{f} - \frac{f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)}{f}\frac{L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A}{\gamma v_A^2} \tag{7}$$

The optimal portfolio strategy consists of three parts:[9] The first term is the myopic demand that depends on the filtered probability of being in the calm state. This term is a weighted average of the optimal demands in the two states if the state was known with certainty:

$$\frac{\widehat{\mu}_A - r}{\gamma v_A^2} = \widehat{p}\frac{\mu_A^{calm} - r}{\gamma v_A^2} + (1-\widehat{p})\frac{\mu_A^{cont} - r}{\gamma v_A^2}. \tag{8}$$

The second term captures hedging motives stemming from the continuous updating of $\widehat{p}$ due to diffusion. The hedge term is large if there is a lot of uncertainty about the state ($\widehat{p} \approx 0.5$), if the states are heterogenous with respect to the drifts ($|\mu_A^{calm} - \mu_A^{cont}|$ large), if the signal is not too noisy (v_A small), or if the indirect utility is sensitive to changes in the filtered probability ($|f_p|/f$ large). The sign of the term depends on which of the two states is more attractive. This is determined by the risk premia in the states. In fact, the derivative f_p can be both positive or negative. For instance, the first calibration of Table 1 implies relatively high risk premia in the contagion state so that the indirect utility G is higher for $\widehat{p} = 0$ than for $\widehat{p} = 1$, and, consequently, $f_p > 0$. For the second calibration, we obtain $f_p < 0$.

The third term adjusts the portfolio strategy with respect to possible crashes in the asset. In models with event risk only (see, e.g., Liu, Longstaff, and Pan (2003)), similar terms as

$$\frac{L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A}{\gamma v_A^2} \tag{9}$$

are part of the optimal solution. The solution in our model additionally involves the ratio $f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)/f$ which is due to the optimal updating of the filtered probability. If the asset price jumps, then the probability of being in the calm state, $\widehat{p}$, is decreased to $\widehat{p}\lambda_A^{calm,calm}/\widehat{\lambda}_A$ (notice that $\lambda_A^{calm,calm} < \widehat{\lambda}_A$). Again, depending on which state is more attractive, the ratio $f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)/f$ is smaller or larger than one. For instance, if the risk premia in the contagion state are smaller than in the calm state, then $f_p < 0$ and this ratio is greater than one, which puts more weight on (9) so that the hedging demand becomes more negative.

Finally, we wish to remark that the second and third term (the hedging terms for diffusive and jump risk) partly cancel each other out. This is for the following reasons: The third term is always negative. The sign of the second term depends on the signs of f_p and $\mu_A^{calm} - \mu_A^{cont}$. Since, by definition, there are more jumps in the contagion state and μ_A^{cont} involves all the compensators for jump risk as well as the risk premia, it is likely that $\mu_A^{calm} - \mu_A^{cont}$ is negative. Furthermore, the contagion state is perceived as worse than the

[9] In the special case of a pure regime switching model ($L_A = 0$), we recover the result of Honda (2003).

calm state if f_p is negative and $f\big(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\big)/f$ is larger than one. Therefore, the second term is positive if the third term is very negative. Vice versa, f_p is positive and the second term is negative if the third term is only slightly negative. This effect can also be seen in Figures 3 and 4: Both terms cancel each other out and the optimal portfolio strategy is almost linear in the filtered probability. This is because the myopic term (8) is linear in this probability. In the next subsection, we will consider an investor who disregards diffusive information. Not surprisingly, the corresponding first-order conditions do not involve diffusive hedge terms (the second term). In line with this result, Figures 3 and 4 show that, in this case, the portfolio strategy, as a function of the filtered probability, has more curvature. An additional discussion of the figures can be found in Section 4.

3.3 Portfolio Choice with Pure Jump Filter

If the investor ignores diffusive information and thus uses the filter that is related to the filtration $\mathcal{H}$, his portfolio problem simplifies. The indirect utility is still of the form (5).

Proposition 3 (Solution with Pure Jump Filter) *If the investor uses the pure jump filter to estimate the current state of the economy, the optimal portfolio weights satisfy the first-order conditions*

$$f \cdot \left[\widehat{\mu}_A - r - \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B)\right] - f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A = 0,$$

$$f \cdot \left[\widehat{\mu}_B - r - \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A)\right] - f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B = 0,$$

where

$$\begin{aligned}
& f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
& \qquad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2\pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\
& + f_p \cdot \Big[(1-\widehat{p}^{pjf})\lambda^{cont,calm} - \widehat{p}^{pjf}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}^{pjf}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
& + f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A \\
& + f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t = 0
\end{aligned}$$

with boundary conditions $f(0,\cdot) = 1$ and $f_p(0,\cdot) = 0$. The subjective drift rate and jump *intensity of asset i ($i \in \{A, B\}$) are defined as*

$$
\begin{aligned}
\widehat{\mu}_i &= \widehat{p}^{pjf} \mu_i^{calm} + (1 - \widehat{p}^{pjf}) \mu_i^{cont} \\
\widehat{\lambda}_i &= \widehat{p}^{pjf} \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont} \right) + (1 - \widehat{p}^{pjf}) \lambda_i^{cont,cont}.
\end{aligned}
$$

The proof is given in Appendix A.2.

3.4 Portfolio Choice with Full information

If the investor has full information and knows the true state of the economy at every point in time, the indirect utility function satisfies one Bellman equation for each state. Therefore, we obtain two ordinary differential equations. The following proposition summarizes results from Branger, Kraft, and Meinerding (2009).

Proposition 4 (Solution with Full Information) *The optimal portfolio weights π_i^j ($i = A, B$, $j = calm, cont$) of a fully informed investor solve the following system of equations*

$$
\begin{aligned}
0 &= \mu_A^{calm} - r - \gamma (v_A^{calm})^2 \pi_A^{calm} - \gamma \pi_B^{calm} v_A^{calm} v_B^{calm} \rho^{calm} \\
&\quad - \lambda_A^{calm,cont} L_A (1 - \pi_A^{calm} L_A)^{-\gamma} \frac{f^{cont}}{f^{calm}} - \lambda_A^{calm,calm} L_A (1 - \pi_A^{calm} L_A)^{-\gamma} \\
0 &= \mu_B^{calm} - r - \gamma (v_B^{calm})^2 \pi_B^{calm} - \gamma \pi_A^{calm} v_A^{calm} v_B^{calm} \rho^{calm} \\
&\quad - \lambda_B^{calm,cont} L_B (1 - \pi_B^{calm} L_B)^{-\gamma} \frac{f^{cont}}{f^{calm}} - \lambda_B^{calm,calm} L_B (1 - \pi_B^{calm} L_B)^{-\gamma} \\
0 &= \mu_A^{cont} - r - \gamma (v_A^{cont})^2 \pi_A^{cont} - \gamma \pi_B^{cont} v_A^{cont} v_B^{cont} \rho^{cont} - \lambda_A^{cont,cont} L_A (1 - \pi_A^{cont} L_A)^{-\gamma} \\
0 &= \mu_B^{cont} - r - \gamma (v_B^{cont})^2 \pi_B^{cont} - \gamma \pi_A^{cont} v_A^{cont} v_B^{cont} \rho^{cont} - \lambda_B^{cont,cont} L_B (1 - \pi_B^{cont} L_B)^{-\gamma}
\end{aligned}
$$

where the indirect utility functions $G^j(t, x) = \frac{x^{1-\gamma}}{1-\gamma} f^j(t)$ are given as the solution of the following ordinary differential equations:

$$
\begin{aligned}
0 &= f_t^{calm} + (1 - \gamma) \left(r + \pi_A^{calm} (\mu_A^{calm} - r) + \pi_B^{calm} (\mu_B^{calm} - r) \right) f^{calm} \\
&\quad - 0.5 \gamma (1 - \gamma) \left((\pi_A^{calm} v_A^{calm})^2 + (\pi_B^{calm} v_B^{calm})^2 + 2 \pi_A^{calm} \pi_B^{calm} v_A^{calm} v_B^{calm} \rho^{calm} \right) f^{calm} \\
&\quad + \lambda_A^{calm,cont} \left((1 - \pi_A^{calm} L_A)^{1-\gamma} f^{cont} - f^{calm} \right) + \lambda_B^{calm,cont} \left((1 - \pi_B^{calm} L_B)^{1-\gamma} f^{cont} - f^{calm} \right) \\
&\quad + \lambda_A^{calm,calm} \left((1 - \pi_A^{calm} L_A)^{1-\gamma} - 1 \right) f^{calm} + \lambda_B^{calm,calm} \left((1 - \pi_B^{calm} L_B)^{1-\gamma} - 1 \right) f^{calm} \\
0 &= f_t^{cont} + (1 - \gamma) \left(r + \pi_A^{cont} (\mu_A^{cont} - r) + \pi_B^{cont} (\mu_B^{cont} - r) \right) f^{cont} \\
&\quad - 0.5 \gamma (1 - \gamma) \left((\pi_A^{cont} v_A^{cont})^2 + (\pi_B^{cont} v_B^{cont})^2 + 2 \pi_A^{cont} \pi_B^{cont} v_A^{cont} v_B^{cont} \rho^{cont} \right) f^{cont} \\
&\quad + \lambda_A^{cont,cont} \left((1 - \pi_A^{cont} L_A)^{1-\gamma} - 1 \right) f^{cont} + \lambda_B^{cont,cont} \left((1 - \pi_B^{cont} L_B)^{1-\gamma} - 1 \right) f^{cont} \\
&\quad + \lambda^{cont,calm} (f^{calm} - f^{cont}).
\end{aligned}
$$

with boundary conditions $f^{calm}(0) = f^{cont}(0) = 1$.

A proof can be found in Branger, Kraft, and Meinerding (2009) and is also available upon request. Note that the optimal portfolio of a fully informed investor in the calm or contagion state is not equal to the limits of the optimal portfolio with partial information for $\widehat{p} \to 0$ or $\widehat{p} \to 1$. The reason is the hedging demand of the investor. In the case with partial information, a jump leads to a reduction of $\widehat{p}$ by a certain percentage, whereas, in a setup with full information, the indicator variable p can only take the two values 0 or 1. Therefore, the hedging demand against jump risk is different in the partial and in the full information case. Note, however, that due to market incompleteness, the overall hedging demand is rather small and the described effect is not very pronounced.

4 Numerical Results

4.1 Parametrization and Calibration

We consider a CRRA investor with a relative risk aversion of $\gamma = 5$ and a planning horizon of 10 years. The riskless interest rate is set to $r = 0.01$. In the first case, the risky assets are assumed to follow identical stochastic processes. Furthermore, we assume that only the jump intensities and the drift rates differ between the calm and the contagion state, while the diffusion parameters, the loss size and all market prices of risk do not depend on the current state. We choose representative parameters for our model that are roughly in line with Eraker, Johannes, and Polson (2003) who estimate the parameters of a jump-diffusion model under the true physical measure from S&P500 and Nasdaq 100 index returns.

The diffusion volatility σ is set to 0.15, and the two Wiener processes are correlated with $\rho = 0.3$. The constant jump size is assumed to be -5%, i.e. the loss size L_i equals 0.05. The total jump intensity in the calm state equals 0.5. The difference between the jump intensities in the calm and the contagion state is captured by the multiple $\xi_i \geq 1$:

$$\lambda_i^{cont,cont} = \xi_i \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont} \right), \quad i \in \{A, B\},$$

where we set ξ_i equal to 5. The jump intensity of both assets is thus multiplied by 5 as soon as the economy enters the contagion state. The conditional probability that a loss in one of the assets actually triggers contagion is given by α_i:

$$\lambda_i^{calm,cont} = \alpha_i \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont} \right), \quad i \in \{A, B\}.$$

We set $\alpha_i = 0.2$ so that, on average, every fifth jump in the calm state triggers contagion. The average time the economy stays in the contagion state is captured by ψ:

$$\lambda^{cont,calm} = \psi \left(\lambda_A^{cont,cont} + \lambda_B^{cont,cont} \right).$$

We start with $\psi = 0.2$. With $\lambda_i^{calm,calm} = 0.4$, the other jump intensities then equal $\lambda_i^{calm,cont} = 0.1$, $\lambda_i^{cont,cont} = 2.5$ and $\lambda^{cont,calm} = 1$. Consequently, the average time the economy stays in the calm state is five years, and the average duration of the contagion regime is one year.

We set the market price of jump risk $\eta^{J,K}$ equal to 0.5 for all jumps that induce a loss in one of the assets while the market price $\eta^{cont,calm}$ is 0. The total jump risk premium in the calm state equals 0.0125 and the premium for jumps that do not trigger contagion equals 0.0025 for each asset. Consequently, the total jump risk premium in the contagion state amounts to 0.0625 for each asset. The diffusion risk premium for both assets is set to 0.0425 in both states so that the total equity premium for both assets switches between 0.055 in the calm state and 0.105 in the contagion state. The first column of Table 1 gives an overview over this parametrization.

Besides, we also consider a different specification for the market prices of risk which is given in the second column of Table 1. In this parametrization, the market prices of jump risk are chosen such that the risk premia are constant across states. Multiplying the jump intensities of both assets by $\xi_i = 5$ upon a switch to the contagion state, the market prices of jump risk must then decrease to one fifth of their value in the calm state. We choose the market prices of risk such that the total equity risk premium is constant at a level of 7%.[10]

Furthermore, we also consider a situation where the assets are heterogenous. In particular, we study a situation where asset A is more severely affected by contagion than asset B, while asset B is more likely to trigger contagion. For instance, asset A could represent the index of a developing country or the stock of a small sub-supplier depending on one main customer and asset B the index of a developed country or the stock of a large company. With respect to the subprime crisis, the assets might also represent the financial and non-financial sector of the economy. The third and fourth column of Table 1 provide parametrizations for the heterogenous case. We reduce the multiple ξ_B to 2, i.e. the jump intensity of asset B is multiplied with 2 as soon as the economy enters the contagion state, whereas ξ_A remains equal to 5. The parameter α_B is increased from 0.2 to 0.5 so that now

[10]The question whether the risk premia are higher in the calm or contagion state can only be answered empirically. This is beyond the scope of this paper, but we discuss some benchmark calibrations in this section.

every second jump in asset B triggers contagion (as opposed to every fifth jump in asset A). The overall jump intensity in the calm state remains equal to 0.5 for both assets. The market prices of jump risk are again equal to 0.5 for all types of jumps which affect the asset prices so that the jump risk premium of asset B in the contagion state reduces to 0.025 (instead of 0.0625 in the case with identical assets).

4.2 Optimal Portfolios with Identical Assets

Figure 3 depicts the optimal portfolio weights for identical assets and constant market prices of risk as a function of the state variable $\widehat{p}$. Since both assets have identical parameters, the portfolio weights for asset A and asset B are equal. It can be seen that modeling contagion as an unobserved state of the economy has a significant impact on the overall optimal portfolio. Since the investor has to learn about the true state of the economy, his portfolio weights depend on the estimated probability $\widehat{p}$ of being in the calm state. In our numerical example the optimal portfolio weights vary between about 30%, if the investor is sure to be in the calm state, and 65%, if the investor is sure to be in the contagion state. Notice that the dependence of the weights on $\widehat{p}$ is monotonous and nonlinear.

The figure illustrates some dynamic implications of learning. Assume that we start from $\widehat{p} = 1$, i.e. the investor is completely sure to be in the calm state. In this situation, the investor adjusts the probability to 0.8 after one jump and to 0.3556 after two jumps, no matter which of the two assets has jumped. The probability update in case of a jump in one of the assets is also depicted in the left panel of Figure 6. The impact on the portfolio weights can be seen in Figure 3 where the red crosses mark the updated pairs of subjective probabilities $\widehat{p}$ and portfolio weights after one and two jumps.

The results show that the investor underreacts to jumps that induce contagion and overreacts to idiosyncratic jumps. If a jump in one of the assets triggers contagion, a fully informed investor should switch to the optimal portfolio in the contagion state in one single step. However, the reaction of an investor with incomplete information is too small, and it takes several subsequent jumps for the investor to gradually adjust his portfolio towards the portfolio that is optimal in the contagion state (contagion portfolio). If, on the other hand, an idiosyncratic, non-contagious jump occurs, then the investor overreacts to this event by adjusting the weights towards the contagion portfolio while a fully informed investor would have kept the weights constant. If no subsequent jumps are observed, the partially informed investor will then continuously readjust his portfolio back to the optimal portfolio in the calm state.

Furthermore, notice that the portfolio adjustment after the first jump, which may have induced contagion, is relatively small in absolute terms while the reaction to a subsequent jump is larger since it becomes likelier that the economy is indeed in the contagion state. This pattern is particularly pronounced for an investor using the optimal filter. To get the intuition, recall that an investor using the optimal filter receives some information about the current state of the economy also in the aftermath of a jump by observing and evaluating the diffusion parts of the two asset prices. The investor using the pure jump filter, on the other hand, only reacts to relatively rare jump events. Therefore, the investor who filters optimally can react to jumps more conservatively, and further adjusts his portfolio after the jump, whereas the investor filtering only from jumps changes his portfolio strategy more aggressively upon a jump. Analytically, the decomposition (7) of the optimal portfolio weights has shown that the investor filtering optimally implements a hedging demand against both sources of state variable risk, diffusion and jumps. Numerically, these hedging terms nearly cancel each other out. On the other hand, an investor using the pure jump filter does not implement a hedging demand against diffusive state variable risk so that the function of his optimal portfolio weights shows up a higher curvature.

The first parametrization described so far assumes constant market prices of risk across states. This implies that the assets' excess returns increase sharply upon a switch to the contagion state. As a result, the investor behaves anti-cyclically and increases his portfolio weights in reaction to each jump. This result changes if we keep the risk premia constant across states and assume that the market prices of risk are smaller in the contagion state. The optimal portfolio weights in this case for which the parameters are given in the second column of Table 1 can be seen in Figure 4. If the market prices of risk decrease upon a switch to the contagion state, the investor behaves pro-cyclically and reduces the portfolio weights of the risky assets in reaction to any jump in the asset prices. However, the over- and underreaction patterns and the higher curvature for the pure jump filter can also be found under this calibration and do not depend on the specification of the risk premia.

4.3 Optimal Portfolios with Heterogenous Assets

The optimal portfolio weights in the case with heterogenous assets are given in Figure 5. Again, we assume that the market prices of jump risk are constant across states. Therefore, the risk premium of asset A increases upon a transition to the contagion state since the jump intensity increases. Consequently, the optimal portfolio weight of asset A is more sensitive to changes in the estimated probability $\widehat{p}$ than the weight of asset B.

On the other hand, the right graph in Figure 6 shows the different information conveyed by jumps in asset A and asset B. If $\widehat{p}$ is close to 1, then a jump in asset B (which is more likely to trigger contagion) induces a larger adjustment in the probability than a jump in asset A. For a small $\widehat{p}$, however, the reaction to a jump in asset A (which is more heavily affected by contagion) is slightly larger than that to a jump in asset B. Put differently, a jump in asset B is generally a strong indicator for the economy having switched to the contagion state while subsequent jumps in asset A provide a stronger confirmation of really being in the contagion state. The probability and portfolio updating as a response to jumps is again depicted by the red and blue crosses in Figure 5. Since adjustments in $\widehat{p}$ are linked to portfolio adjustments, a jump in asset B has much larger overall portfolio implications than a jump in asset A.

Taking both effects together, the optimal portfolio weights show that the investor takes into account that crash risk might 'spill over' from asset B to asset A and reacts accordingly. Jumps in asset B trigger large adjustments of the portfolio that however mainly take place in asset A. This effect is particularly pronounced in the case where the investor filters from jumps only. In this situation, the investor again reacts more aggressively to jumps in both assets (especially in the contagion-triggering asset B), whereas an investor using diffusive and event information behaves more conservatively.

4.4 Value of Information

We now turn to another relevant aspect of our numerical results. We compare the performances of three different investment strategies: (a) the optimal strategy of an investor using the optimal filter, (b) the optimal strategy of an investor using the pure jump filter, and (c) the optimal strategy with constant portfolio weights. To assess the performance, we calculate the relative utility losses compared to the optimal strategy (a). These are defined as the percentage decreases δ_ω in initial wealth that are necessary to reduce the expected utility of (a) to the expected utilities of the other strategies, i.e. $G_a(x(1-\delta_\omega)) = G_\omega(x)$ for $\omega \in \{b, c\}$.[11] The form (5) of the indirect utility functions yields

$$\delta_\omega = 1 - \left(\frac{f_\omega}{f_a}\right)^{\frac{1}{1-\gamma}}.$$

Table 2 summarizes the results for an investor with relative risk aversion $\gamma = 5$ and $\gamma = 2$ which have been computed by running Monte Carlo simulations.[12] We report the results for the parametrization with equal market prices of risk in both states in this section. The

[11] We have omitted the dependence of the indirect utilities on time and the filtered probability.

[12] The overall small numbers are due to the fact that we use relatively moderate parameters and the market is incomplete.

numbers change only slightly if we choose the second parametrization with constant risk premia. Technically, we simulate 500,000 paths of the economy under the full filtration and compute the perceived Brownian motions and Poisson processes defined in Section 2.3 and 2.4. Then, we implement the portfolio strategies according to the resulting sample paths of $\widehat{p}$ and $\widehat{p}^{pjf}$.[13] For the strategy with constant portfolio weights, we choose the constant weights that ex-ante yield the highest indirect utility.

We find several relevant results: firstly, a comparison of the second and the third column of Table 2 reveals that filtering from jumps indeed improves the performance of the investor's strategy. For a moderate risk aversion $\gamma = 5$, the decrease in initial wealth that is required to bring the expected utility with filtering down to the expected utility with constant portfolio weights exceeds 1%.[14] For a less risk-averse investor with $\gamma = 2$, these numbers become even more substantial: the relative utility loss is about 3%. Overall, we conclude that information about the true state of the economy deduced from the observation of jumps is valuable for the investor.

Secondly, the simulation study allows us to assess whether including diffusive information into the filtering procedure is profitable for the investor. The numbers in the second column of Table 2 show that, surprisingly, the losses of disregarding diffusive information are negligible. In all cases, the relative utility losses are below 0.06% (of the initial wealth). Therefore, our results suggest that adding diffusive information to the filtering process does not improve the performances of the investment policies. Stated differently, filtering from time series of crashes is of first-order importance, whereas incorporating diffusive information is only of second-order importance. Finally, we wish to remark that the utility losses of strategy (a) compared to the strategy of an omniscient investor who knows the actual state are about 0.6% for the case with $\gamma = 5$ and 1.5% for $\gamma = 2$.

4.5 Robustness Checks

We will now give a brief overview over the results of extensive robustness checks that we did. Firstly, we have varied the jump and diffusion parameters (jump sizes, jump

[13]For each sample path, we assume an initial value of 1 for both state variables as well as for the true indicator variable p_0. This implies that, at the beginning of each sample path, the economy is in the calm state and the investor is perfectly informed about this fact. Various robustness checks have shown that this assumption is not crucial. In fact, our results hardly depend on the initial values of $\widehat{p}_0$, $\widehat{p}_0^{pjf}$ and p_0 in the simulation. In very extreme cases, the relative utility loss due to pure jump filtering can increase up to 0.2% of the initial wealth.

[14]In life-cycle portfolio problems, losses of above 1% are usually considered as substantial. See, e.g., Cocco, Gomes, and Maenhout (2005).

intensities, diffusion volatilities, diffusion correlations) in the initial parametrization. In general, none of these parameters affects our qualitative results. However, one remarkable observation seems to be very persistent. Whenever we increase the ratio of jump risk to diffusion risk, which drives a larger wedge between the return distributions of the calm and contagion state, the investor using the pure jump filter becomes even more aggressive in the sense that his overreaction to jumps becomes more pronounced compared to the benchmark calibration. In contrast, the investor using the optimal filter reacts nearly as hesitatingly to jumps as under the benchmark calibration. This effect is also reflected in the relative utility loss of an investor filtering from jumps only as compared to an investor filtering optimally. This loss can now exceed 0.1% of the initial wealth which is, however, still surprisingly small. The result that filtering from diffusions hardly enhances the performance of the investor is extremely robust against various parameterizations of the model.

The average time spent in the contagion regime is controlled via the parameter ψ. The calibrations above assume $\psi = 0.2$ (with identical assets) or $\psi = 0.28$ (with heterogenous assets). Given the other jump intensities, the average duration of the contagion regime is then one year. Varying ψ between 0.05 and 0.8, we find that this average duration has only marginal effects. The numerical results including the findings about the expected utility are hardly changed at all implying that it is not the duration of contagion which matters. Instead, the mere fact that there is a threat of a contagion state drives our main findings.

Additionally, we have varied the investment horizon and the relative risk aversion. The utility losses increase almost linearly in the investment horizon, i.e. the losses are about twice as large for a horizon of 20 years. Besides, the utility losses hardly depend on $\widehat{p}_0$ and $\widehat{p}_0^{pjf}$ for investment horizons of 20 years and longer: in the long run, it does not matter whether the world is in the calm or contagion state today. Changing the investor's relative risk aversion between values of 1.5 and 10 does not yield any further insights for the portfolio weights.

5 Conclusion

This paper provides a realistic model for contagion effects that are triggered by certain crashes in asset prices. Since the individual cannot distinguish between idiosyncratic crashes and crashes that let the economy slip into contagion, he filters the probability of being in the contagion state from price observations. We relate our model to frameworks using self-exciting processes and study the optimal asset allocation of a CRRA investor.

Our numerical results show that the risk of contagion and the partial information about the current state of the economy can have a substantial effect on an investor's optimal portfolio choice and on his trading volume. Since the investor only learns gradually about whether the economy has entered the contagion state, he gradually adjusts his portfolio towards the portfolio that would be optimal in the (unobservable) contagion state. This causes him to underreact to jumps that induce contagion and to overreact to idiosyncratic jumps. The underreaction is particularly pronounced for an investor behaving optimally in the sense that he uses all available information (diffusive and jump). An investor who uses the information from jump observations only invests more aggressively and shows a large overreaction to idiosyncratic jumps. Allowing for cross-sectional differences between the assets, we find that the investor reacts most heavily to jumps in the asset which is more likely to induce contagion since portfolio adjustments are triggered by updates in the probability of being in the calm state. On the other hand, the trading volume itself is largest in those assets that are most affected by contagion.

In an extensive simulation study, we evaluate the performance of several investment strategies. We find that information about the state of the economy deduced from the observation of crashes is indeed valuable. The decrease in initial wealth which is required to reduce the expected utility with filtering to the expected utility for a strategy with constant portfolio weights can reach 3%, depending on the investor's risk aversion. On the other hand, the extra information in the diffusion processes does not add much to the expected utility. An investor who takes the information from jumps and diffusion into account hardly outperforms an investor filtering from jumps only. Several robustness checks provide evidence that our results are not sensitive to variations of the calibration.

There are several directions for future research. Our numerical results suggest that the absolute and relative sizes of the market prices of risk have significant effects on optimal portfolio choices. It would thus be interesting to compute these market prices endogenously in a general equilibrium framework. In particular, one could study the equilibrium market prices of contagion risk and analyze the differences for assets that induce contagion and other assets that are particularly affected by contagion.

A Proofs

A.1 Filter Equation

Under the full filtration $\mathcal{F}$, the asset prices follow

$$\begin{aligned}\frac{dS_{A,t}}{S_{A,t}} &= \mu_A^{Z(t)}dt + v_A dW_t^A - \sum_{K \neq Z(t-)} L_A dN_t^K \\ \frac{dS_{B,t}}{S_{B,t}} &= \mu_B^{Z(t)}dt + v_B\left(\rho dW_{A,t} + \sqrt{1-\rho^2}dW_{B,t}\right) - \sum_{K \neq Z(t-)} L_B dN_t^K\end{aligned}$$

where $Z(t)$ denotes the state of the Markov chain at time t. Under the smaller filtration $\mathcal{G}$, their dynamics are

$$\begin{aligned}\frac{dS_{A,t}}{S_{A,t}} &= \widehat{\mu}_A dt + v_A d\widehat{W}_{A,t} - L_A d\widehat{N}_{A,t} \\ \frac{dS_{B,t}}{S_{B,t}} &= \widehat{\mu}_B dt + v_B\left(\rho d\widehat{W}_{A,t} + \sqrt{1-\rho^2}d\widehat{W}_{B,t}\right) - L_B d\widehat{N}_{B,t}\end{aligned}$$

where the drift and the jump intensity of asset $i \in \{A, B\}$ are defined as

$$\begin{aligned}\widehat{\mu}_i &= \widehat{p}_t \mu_i^{calm} + (1-\widehat{p}_t)\mu_i^{cont} \\ \widehat{\lambda}_i &= \widehat{p}_t\left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1-\widehat{p}_t)\lambda_i^{cont,cont}\end{aligned}$$

and $\widehat{p}_t$ denotes the subjective probability of being in the calm state at time t. Note that the diffusion volatilities and correlations do not depend on the state of the economy and are known to the investor. The Brownian motions under the investor's filtration $\mathcal{G}$ are given as

$$\begin{pmatrix} d\widehat{W}_{A,t} \\ d\widehat{W}_{B,t} \end{pmatrix} = \begin{pmatrix} dW_{A,t} \\ dW_{B,t} \end{pmatrix} + \begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix}^{-1} \begin{pmatrix} \mu_A^{Z(t)} - \widehat{\mu}_A \\ \mu_B^{Z(t)} - \widehat{\mu}_B \end{pmatrix} dt$$

and the observable jumps are driven by the processes

$$\widehat{N}_i = N_i^{calm,calm} + N_i^{calm,cont} + N_i^{cont,cont} \qquad (i \in \{A, B\}).$$

In order to deduce the filter equation, we rely on the results of Frey and Runggaldier (2010). Our model can be viewed as a special case of theirs. The subjective probability of being in the calm state, $\widehat{p}$, can be written as

$$\widehat{p} = \frac{\sigma^{calm}}{\sigma^{cont} + \sigma^{calm}} .$$

Applying section 4 of Frey and Runggaldier (2010) and translating all equations into the terms of our model, the Zakai equations for σ^{calm} and σ^{cont} read

$$\begin{aligned}
d\sigma_t^{calm} &= -\lambda^{calm,*}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt + \sigma_t^{calm}\Big(\mu_A^{calm}, \mu_B^{calm}\Big)\begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} d\Psi_t \\
&\quad + \left(\frac{\lambda_A^{calm,calm}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right) d\widehat{N}_{A,t} \\
&\quad + \left(\frac{\lambda_B^{calm,calm}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right) d\widehat{N}_{B,t} \qquad (10) \\
d\sigma_t^{cont} &= -\lambda^{cont,*}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt + \sigma_t^{cont}\Big(\mu_A^{cont}, \mu_B^{cont}\Big)\begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} d\Psi_t \\
&\quad + \left(\frac{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + \lambda_A^{calm,cont}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right) d\widehat{N}_{A,t} \\
&\quad + \left(\frac{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + \lambda_B^{calm,cont}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right) d\widehat{N}_{B,t} \qquad (11)
\end{aligned}$$

where we abbreviate

$$\begin{aligned}
\lambda^{calm,*} &= \lambda_A^{calm,calm} + \lambda_B^{calm,calm} + \lambda_A^{calm,cont} + \lambda_B^{calm,cont} \\
\lambda^{cont,*} &= \lambda_A^{cont,cont} + \lambda_B^{cont,cont}.
\end{aligned}$$

Here, $d\Psi_t$ denotes the diffusion part of the asset price. Under the full filtration $\mathcal{F}$, this diffusion part reads

$$d\Psi_t = \begin{pmatrix} \mu_A^{Z(t)} \\ \mu_B^{Z(t)} \end{pmatrix} dt + \begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix}\begin{pmatrix} dW_{A,t} \\ dW_{B,t} \end{pmatrix}.$$

Under the investor filtration $\mathcal{G}$, this diffusion part reads

$$d\Psi_t = \begin{pmatrix} \widehat{\mu}_A \\ \widehat{\mu}_B \end{pmatrix} dt + \begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix}\begin{pmatrix} d\widehat{W}_{A,t} \\ d\widehat{W}_{B,t} \end{pmatrix}. \qquad (12)$$

Plugging (12) into (11) and (10) gives

$$
\begin{aligned}
&d\sigma_t^{calm} = \\
&-\lambda^{calm,*}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt \\
&+\sigma_t^{calm}\Big(\mu_A^{calm}, \mu_B^{calm}\Big) \begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} \left[\frac{\sigma_t^{calm}}{\sigma_t^{cont} + \sigma_t^{calm}} \begin{pmatrix} \mu_A^{calm} \\ \mu_B^{calm} \end{pmatrix} + \frac{\sigma_t^{cont}}{\sigma_t^{cont} + \sigma_t^{calm}} \begin{pmatrix} \mu_A^{cont} \\ \mu_B^{cont} \end{pmatrix} \right] dt \\
&+\sigma_t^{calm}\Big(\mu_A^{calm}, \mu_B^{calm}\Big) \begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} \left[\begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix} \begin{pmatrix} d\widehat{W}_{A,t} \\ d\widehat{W}_{B,t} \end{pmatrix} \right] \\
&+ \left(\frac{\lambda_A^{calm,calm}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm} \right) d\widehat{N}_{A,t} \\
&+ \left(\frac{\lambda_B^{calm,calm}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm} \right) d\widehat{N}_{B,t}
\end{aligned}
$$

and

$$
\begin{aligned}
&d\sigma_t^{cont} = \\
&-\lambda^{cont,*}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt \\
&+\sigma_t^{cont}\Big(\mu_A^{cont}, \mu_B^{cont}\Big) \begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} \left[\frac{\sigma_t^{calm}}{\sigma_t^{cont} + \sigma_t^{calm}} \begin{pmatrix} \mu_A^{calm} \\ \mu_B^{calm} \end{pmatrix} + \frac{\sigma_t^{cont}}{\sigma_t^{cont} + \sigma_t^{calm}} \begin{pmatrix} \mu_A^{cont} \\ \mu_B^{cont} \end{pmatrix} \right] dt \\
&+\sigma_t^{cont}\Big(\mu_A^{cont}, \mu_B^{cont}\Big) \begin{pmatrix} v_A^2 & \rho v_A v_B \\ \rho v_A v_B & v_B^2 \end{pmatrix}^{-1} \left[\begin{pmatrix} v_A & 0 \\ \rho v_B & \sqrt{1-\rho^2}v_B \end{pmatrix} \begin{pmatrix} d\widehat{W}_{A,t} \\ d\widehat{W}_{B,t} \end{pmatrix} \right] \\
&+ \left(\frac{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + \lambda_A^{calm,cont}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont} \right) d\widehat{N}_{A,t} \\
&+ \left(\frac{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + \lambda_B^{calm,cont}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont} \right) d\widehat{N}_{B,t} \ .
\end{aligned}
$$

These equations simplify to:

$$
\begin{aligned}
d\sigma_t^{calm} \quad = \quad & -\lambda^{calm,*}\sigma_t^{calm}dt + \lambda^{cont,calm}\sigma_t^{cont}dt \\
& + \frac{\sigma_t^{calm}}{1-\rho^2}\Bigg(\widehat{p}_t\left(\frac{(\mu_A^{calm})^2}{v_A^2} + \frac{(\mu_B^{calm})^2}{v_B^2} - 2\rho\frac{\mu_A^{calm}\mu_B^{calm}}{v_A v_B}\right) \\
& \qquad + (1-\widehat{p}_t)\left(\frac{\mu_A^{calm}\mu_A^{cont}}{v_A^2} + \frac{\mu_B^{calm}\mu_B^{cont}}{v_B^2} - \rho\frac{\mu_A^{calm}\mu_B^{cont} + \mu_A^{cont}\mu_B^{calm}}{v_A v_B}\right)\Bigg)dt \\
& + \frac{\mu_A^{calm}}{v_A}\sigma_t^{calm}d\widehat{W}_{A,t} + \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{calm}}{v_B} - \rho\frac{\mu_A^{calm}}{v_A}\right)\sigma_t^{calm}d\widehat{W}_{B,t} \\
& + \left(\frac{\lambda_A^{calm,calm}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right)d\widehat{N}_{A,t} \\
& + \left(\frac{\lambda_B^{calm,calm}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{calm}\right)d\widehat{N}_{B,t}
\end{aligned}
$$

and

$$
\begin{aligned}
d\sigma_t^{cont} \quad = \quad & -\lambda^{cont,*}\sigma_t^{cont}dt - \lambda^{cont,calm}\sigma_t^{cont}dt \\
& + \frac{\sigma_t^{cont}}{1-\rho^2}\Bigg((1-\widehat{p}_t)\left(\frac{(\mu_A^{cont})^2}{v_A^2} + \frac{(\mu_B^{cont})^2}{v_B^2} - 2\rho\frac{\mu_A^{cont}\mu_B^{cont}}{v_A v_B}\right) \\
& \qquad + \widehat{p}_t\left(\frac{\mu_A^{calm}\mu_A^{cont}}{v_A^2} + \frac{\mu_B^{calm}\mu_B^{cont}}{v_B^2} - \rho\frac{\mu_A^{calm}\mu_B^{cont} + \mu_A^{cont}\mu_B^{calm}}{v_A v_B}\right)\Bigg)dt \\
& + \frac{\mu_A^{cont}}{v_A}\sigma_t^{cont}d\widehat{W}_{A,t} + \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{cont}}{v_B} - \rho\frac{\mu_A^{cont}}{v_A}\right)\sigma_t^{cont}d\widehat{W}_{B,t} \\
& + \left(\frac{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + \lambda_A^{calm,cont}\sigma_{t-}^{calm}}{\lambda_A^{cont,cont}\sigma_{t-}^{cont} + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right)d\widehat{N}_{A,t} \\
& + \left(\frac{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + \lambda_B^{calm,cont}\sigma_{t-}^{calm}}{\lambda_B^{cont,cont}\sigma_{t-}^{cont} + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})\sigma_{t-}^{calm}} - \sigma_{t-}^{cont}\right)d\widehat{N}_{B,t} \ .
\end{aligned}
$$

To get the filtering equation at last, we apply Ito's Lemma to

$$
\widehat{p} = \frac{\sigma^{calm}}{\sigma^{cont} + \sigma^{calm}} \ . \tag{13}
$$

This gives

$$
\begin{aligned}
d\widehat{p}_t &= \widehat{p}_t(1-\widehat{p}_t)\left[\lambda_A^{cont,cont}+\lambda_B^{cont,cont}-\lambda_A^{calm,calm}-\lambda_B^{calm,calm}-\lambda_A^{calm,cont}-\lambda_B^{calm,cont}\right]dt \\
&\quad +(1-\widehat{p}_t)^2\lambda^{cont,calm}dt \;+\; \widehat{p}_t(1-\widehat{p}_t)\lambda^{cont,calm}dt \\
&\quad +\frac{\widehat{p}_t(1-\widehat{p}_t)}{1-\rho^2}\Bigg[\widehat{p}_t\left(\frac{(\mu_A^{calm})^2}{v_A^2}+\frac{(\mu_B^{calm})^2}{v_B^2}-2\rho\frac{\mu_A^{calm}\mu_B^{calm}}{v_Av_B}\right) \\
&\qquad\qquad +(1-\widehat{p}_t)\left(\frac{\mu_A^{calm}\mu_A^{cont}}{v_A^2}+\frac{\mu_B^{calm}\mu_B^{cont}}{v_B^2}-\rho\frac{\mu_A^{calm}\mu_B^{cont}+\mu_A^{cont}\mu_B^{calm}}{v_Av_B}\right) \\
&\qquad\qquad -(1-\widehat{p}_t)\left(\frac{(\mu_A^{cont})^2}{v_A^2}+\frac{(\mu_B^{cont})^2}{v_B^2}-2\rho\frac{\mu_A^{cont}\mu_B^{cont}}{v_Av_B}\right) \\
&\qquad\qquad -\widehat{p}_t\left(\frac{\mu_A^{calm}\mu_A^{cont}}{v_A^2}+\frac{\mu_B^{calm}\mu_B^{cont}}{v_B^2}-\rho\frac{\mu_A^{calm}\mu_B^{cont}+\mu_A^{cont}\mu_B^{calm}}{v_Av_B}\right)\Bigg]dt \\
&\quad -\widehat{p}_t^2(1-\widehat{p}_t)\frac{(\mu_A^{calm})^2}{v_A^2}dt+\widehat{p}_t(1-\widehat{p}_t)^2\frac{(\mu_A^{cont})^2}{v_A^2}dt \\
&\quad -\widehat{p}_t^2(1-\widehat{p}_t)\frac{1}{1-\rho^2}\left(\frac{\mu_B^{calm}}{v_B}-\rho\frac{\mu_A^{calm}}{v_A}\right)^2dt+\widehat{p}_t(1-\widehat{p}_t)^2\frac{1}{1-\rho^2}\left(\frac{\mu_B^{cont}}{v_B}-\rho\frac{\mu_A^{cont}}{v_A}\right)^2dt \\
&\quad +(2\widehat{p}_t-1)\widehat{p}_t(1-\widehat{p}_t)\left(\frac{\mu_A^{calm}\mu_A^{cont}}{v_A^2}+\frac{1}{1-\rho^2}\left(\frac{\mu_B^{calm}}{v_B}-\rho\frac{\mu_A^{calm}}{v_A}\right)\left(\frac{\mu_B^{cont}}{v_B}-\rho\frac{\mu_A^{cont}}{v_A}\right)\right)dt \\
&\quad +\widehat{p}_t(1-\widehat{p}_t)\left[\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A}d\widehat{W}_{A,t}+\frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{calm}-\mu_B^{cont}}{v_B}-\rho\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A}\right)d\widehat{W}_{B,t}\right] \\
&\quad +\left(\frac{\lambda_A^{calm,calm}\widehat{p}_{t-}}{\lambda_A^{cont,cont}(1-\widehat{p}_{t-})+(\lambda_A^{calm,calm}+\lambda_A^{calm,cont})\widehat{p}_{t-}}-\widehat{p}_{t-}\right)d\widehat{N}_{A,t} \\
&\quad +\left(\frac{\lambda_B^{calm,calm}\widehat{p}_{t-}}{\lambda_B^{cont,cont}(1-\widehat{p}_{t-})+(\lambda_B^{calm,calm}+\lambda_B^{calm,cont})\widehat{p}_{t-}}-\widehat{p}_{t-}\right)d\widehat{N}_{B,t}\ .
\end{aligned}
$$

This can be simplified to

$$
\begin{aligned}
&d\widehat{p}_t = \\
&\widehat{p}_t(1-\widehat{p}_t)\left[\lambda_A^{cont,cont}+\lambda_B^{cont,cont}-\lambda_A^{calm,calm}-\lambda_B^{calm,calm}-\lambda_A^{calm,cont}-\lambda_B^{calm,cont}\right]dt+(1-\widehat{p}_t)\lambda^{cont,calm}dt \\
&+\widehat{p}_t(1-\widehat{p}_t)\left[\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A}d\widehat{W}_{A,t}+\frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{calm}-\mu_B^{cont}}{v_B}-\rho\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A}\right)d\widehat{W}_{B,t}\right] \\
&+\left(\frac{\lambda_A^{calm,calm}\widehat{p}_{t-}}{\lambda_A^{cont,cont}(1-\widehat{p}_{t-})+(\lambda_A^{calm,calm}+\lambda_A^{calm,cont})\widehat{p}_{t-}}-\widehat{p}_{t-}\right)d\widehat{N}_{A,t} \\
&+\left(\frac{\lambda_B^{calm,calm}\widehat{p}_{t-}}{\lambda_B^{cont,cont}(1-\widehat{p}_{t-})+(\lambda_B^{calm,calm}+\lambda_B^{calm,cont})\widehat{p}_{t-}}-\widehat{p}_{t-}\right)d\widehat{N}_{B,t}
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
d\widehat{p}_t \;=\;& \Big((1-\widehat{p}_t)\lambda^{cont,calm} - \widehat{p}_t(\lambda_A^{calm,cont} + \lambda_B^{calm,cont})\Big)\, dt \qquad (14)\\
&+\widehat{p}_t(1-\widehat{p}_t)\left[\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A} d\widehat{W}_{A,t} + \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\mu_B^{calm}-\mu_B^{cont}}{v_B} - \rho\frac{\mu_A^{calm}-\mu_A^{cont}}{v_A}\right) d\widehat{W}_{B,t}\right]\\
&+\left(\frac{\widehat{p}_{t-}\lambda_A^{calm,calm}}{\widehat{\lambda}_A} - \widehat{p}_{t-}\right)\Big(d\widehat{N}_{A,t} - \widehat{\lambda}_A dt\Big) + \left(\frac{\widehat{p}_{t-}\lambda_B^{calm,calm}}{\widehat{\lambda}_B} - \widehat{p}_{t-}\right)\Big(d\widehat{N}_{B,t} - \widehat{\lambda}_B dt\Big)\ .
\end{aligned}
$$

The subjective probability $\widehat{p}^{pjf}$ under the smaller filtration $\mathcal{H}$ (pure jump filter) can be obtained – informally – by setting $\mu_A^{calm} = \mu_A^{cont}$ and $\mu_B^{calm} = \mu_B^{cont}$, i.e. eliminating the information from the drift and diffusion of the asset prices:

$$
\begin{aligned}
d\widehat{p}_t^{pjf} \;=\;& \Big((1-\widehat{p}_t^{pjf})\lambda^{cont,calm} - \widehat{p}_t^{pjf}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont})\Big)\, dt \qquad (15)\\
&+\left(\frac{\widehat{p}_{t-}^{pjf}\lambda_A^{calm,calm}}{\widehat{\lambda}_A(\widehat{p}_{t-}^{pjf})} - \widehat{p}_{t-}^{pjf}\right)\Big(d\widehat{N}_{A,t} - \widehat{\lambda}_A(\widehat{p}_t^{pjf})dt\Big)\\
&+\left(\frac{\widehat{p}_{t-}^{pjf}\lambda_B^{calm,calm}}{\widehat{\lambda}_B(\widehat{p}_{t-}^{pjf})} - \widehat{p}_{t-}^{pjf}\right)\Big(d\widehat{N}_{B,t} - \widehat{\lambda}_B(\widehat{p}_t^{pjf})dt\Big)\ .
\end{aligned}
$$

A formal proof of the pure jump filter can be deduced from Brémaud (1981), pp. 94ff., and is available from the authors upon request. If the investor filters from the observation of jumps only, the filter problem is virtually equivalent to the problem of determining the current state of a Markov chain from observations of Markov chain transitions only, which is much simpler than the nonlinear filtering computation presented above.

A.2 Portfolio Optimization

The budget equation is given by

$$
\frac{dX_t}{X_t} = r dt + \pi_A\left(\frac{dS_{A,t}}{S_{A,t}} - r dt\right) + \pi_B\left(\frac{dS_{B,t}}{S_{B,t}} - r dt\right) \qquad (16)
$$

and the indirect utility function is denoted by $G(t, x, \widehat{p})$. The Bellman equation reads

$$
\begin{aligned}
\max_{\pi_A,\pi_B}\;\Big[\;& G_t + G_x\cdot[\text{drift from (16)}] + G_p\cdot[\text{drift from (14)}]\\
&+0.5G_{xx}\cdot[\text{squared volatility from (16)}]\\
&+0.5G_{pp}\cdot[\text{squared volatility from (14)}]\\
&+G_{px}\cdot[\text{volatility from (16)}]\cdot[\text{volatility from (14)}]\\
&+\big(G^{A,+}-G\big)\widehat{\lambda}_A + \big(G^{B,+}-G\big)\widehat{\lambda}_B\;\Big] \qquad = 0
\end{aligned}
$$

where subscripts denote partial derivatives. The notation $G^{A,+}$ (and similar notation hereafter) refers to the function G immediately after a jump in asset A. With the usual conjecture

$$G(t,x,\widehat{p}) = \frac{x^{1-\gamma}}{1-\gamma} f(t,\widehat{p}),$$

we get the following differential equation:

$$\begin{aligned}
\max_{\pi_A,\pi_B} \Bigg[\quad & f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
& \quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2\pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\
& + f_p \cdot \Big[(1-\gamma)\widehat{p}(1-\widehat{p})\big(\pi_A(\mu_A^{calm} - \mu_A^{cont}) + \pi_B(\mu_B^{calm} - \mu_B^{cont})\big) \\
& \quad +(1-\widehat{p})\lambda^{cont,calm} - \widehat{p}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
& + f_{pp} \cdot \frac{0.5\widehat{p}^2(1-\widehat{p})^2}{1-\rho^2}\left[\frac{(\mu_A^{calm} - \mu_A^{cont})^2}{v_A^2} - 2\rho\frac{(\mu_A^{calm} - \mu_A^{cont})(\mu_B^{calm} - \mu_B^{cont})}{v_A v_B} + \frac{(\mu_B^{calm} - \mu_B^{cont})^2}{v_B^2}\right] \\
& + f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A + f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t\Bigg] = 0.
\end{aligned}$$

Taking derivatives with respect to π_A and π_B gives the following first-order conditions:

$$\begin{aligned}
f \cdot (\widehat{\mu}_A - r) - f \cdot \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B) + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_A^{calm} - \mu_A^{cont}) & \\
-f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A &= 0 \\
f \cdot (\widehat{\mu}_B - r) - f \cdot \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A) + f_p \cdot \widehat{p}(1-\widehat{p})(\mu_B^{calm} - \mu_B^{cont}) & \\
-f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B &= 0
\end{aligned}$$

Altogether, this yields the system of three nonlinear differential-algebraic equations with boundary conditions $f(0,\cdot) = 1$ and $f_p(0,\cdot) = 0$ given in Proposition 2. A numerical solution can, e.g., be obtained with finite differences.

If the investor uses the pure jump filter instead of the optimal one, the optimal portfolio weights can be computed rather similarly. The budget equation equals the one in the case with optimal filtering. The only change is that the drift and volatility from (14) in the Bellman equation are replaced by the drift and volatility of the suboptimal filter from (15). Since the filter equation (15) contains only drift terms and jump processes, the second-order partial derivatives with respect to $\widehat{p}^{pjf}$ vanish and we end up with the following partial differential equation:

$$
\begin{aligned}
\max_{\pi_A,\pi_B} \Bigg[\quad & f \cdot \Big[(1-\gamma)r + (1-\gamma)\pi_A(\widehat{\mu}_A - r) + (1-\gamma)\pi_B(\widehat{\mu}_B - r) \\
& \quad -0.5\gamma(1-\gamma)\big(v_A^2\pi_A^2 + 2\rho v_A v_B \pi_A \pi_B + v_B^2\pi_B^2\big) - \widehat{\lambda}_A - \widehat{\lambda}_B\Big] \\
& + f_p \cdot \Big[(1-\widehat{p}^{pjf})\lambda^{cont,calm} - \widehat{p}^{pjf}(\lambda_A^{calm,cont} + \lambda_B^{calm,cont}) + \widehat{p}^{pjf}\big(\widehat{\lambda}_A + \widehat{\lambda}_B - \lambda_A^{calm,calm} - \lambda_B^{calm,calm}\big)\Big] \\
& + f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot (1-\pi_A L_A)^{1-\gamma}\widehat{\lambda}_A \\
& + f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot (1-\pi_B L_B)^{1-\gamma}\widehat{\lambda}_B \quad + f_t\Bigg] = 0.
\end{aligned} \tag{17}
$$

Deriving with respect to π_A and π_B gives the first-order conditions

$$
f \cdot (\widehat{\mu}_A - r) - f \cdot \gamma(\pi_A v_A^2 + \rho v_A v_B \pi_B) - f\left(t, \frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A}\widehat{p}^{pjf}\right) \cdot L_A(1-\pi_A L_A)^{-\gamma}\widehat{\lambda}_A = 0
$$

$$
f \cdot (\widehat{\mu}_B - r) - f \cdot \gamma(\pi_B v_B^2 + \rho v_A v_B \pi_A) - f\left(t, \frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B}\widehat{p}^{pjf}\right) \cdot L_B(1-\pi_B L_B)^{-\gamma}\widehat{\lambda}_B = 0
$$

With the same boundary conditions as for the optimal filter, this yields the system of three nonlinear differential-algebraic equations given in Proposition 3. Since the differential equation (17) is of first order, however, the numerical solution using finite differences has to take the existence of characteristic manifolds into account. A potential solution to this challenge are the so-called upwind techniques.

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		Identical assets, constant market prices of risk	Identical assets, constant risk premia	Heterogenous assets, constant market prices of risk: Asset A	Heterogenous assets, constant market prices of risk: Asset B
Data-generating process	σ_i^{calm}	0.15	0.15	0.15	0.15
	σ_i^{cont}	0.15	0.15	0.15	0.15
	ρ^{calm}	0.30	0.30	0.30	0.30
	ρ^{cont}	0.30	0.30	0.30	0.30
	μ_i^{calm}	*0.09*	*0.105*	*0.09*	*0.09*
	μ_i^{cont}	*0.24*	*0.205*	*0.24*	*0.1275*
	$\lambda_i^{calm,calm}$	0.40	0.40	0.40	0.25
	$\lambda_i^{calm,cont}$	*0.10*	*0.10*	*0.10*	*0.25*
	$\lambda_i^{cont,cont}$	*2.50*	*2.50*	*2.50*	*1.00*
	$\lambda^{cont,calm}$	*1.00*	*1.00*	*1.00*	
	$L_i^{calm,calm}$	0.05	0.05	0.05	0.05
	$L_i^{calm,cont}$	0.05	0.05	0.05	0.05
	$L_i^{cont,cont}$	0.05	0.05	0.05	0.05
	$L_i^{cont,calm}$	0.00	0.00	0.00	0.00
	ξ_i	5.00	5.00	5.00	2.00
	α_i	0.20	0.20	0.20	0.50
	ψ	0.20	0.20	0.28	
Market prices of risk	η^{calm}	0.2833	0.2833	0.2833	
	η^{cont}	0.2833	0.2833	0.2833	
	$\eta^{calm,calm}$	0.50	1.1	0.50	
	$\eta^{calm,cont}$	0.50	1.1	0.50	
	$\eta^{cont,cont}$	0.50	0.22	0.50	
	$\eta^{cont,calm}$	0.00	0.00	0.00	
Risk premia	diffusion risk calm state	*0.0425*	*0.0425*	*0.0425*	*0.0425*
	diffusion risk contagion state	*0.0425*	*0.0425*	*0.0425*	*0.0425*
	jump risk calm state	*0.0125*	*0.0275*	*0.0125*	*0.0125*
	jump risk contagion state	*0.0625*	*0.0275*	*0.0625*	*0.025*

Table 1: Parametrizations

The table gives the parametrizations of our model used in Section 4. The first two columns show the parameters in the case of identical assets, with constant market prices of risk or constant risk premia across states. The third and fourth column give the parameters for the case where both assets differ with respect to ξ_i and α_i. Note that we assume constant market prices of risk across states in this setup. The italic numbers denote those parameters which are not chosen freely, but follow from the other ones.

	Optimal filter	Pure jump filter	Constant weights
Identical assets, constant market prices of risk, $\gamma = 5$			
Loss (% of initial wealth)	0.000%	0.042%	1.178%
Heterogenous assets, constant market prices of risk, $\gamma = 5$			
Loss (% of initial wealth)	0.000%	0.014%	1.157%
Identical assets, constant market prices of risk, $\gamma = 2$			
Loss (% of initial wealth)	0.000%	0.059%	3.056%
Heterogenous assets, constant market prices of risk, $\gamma = 2$			
Loss (% of initial wealth)	0.000%	0.050%	2.772%

Table 2: Relative utility losses for different investment strategies

The table reports the percentage decrease in initial financial wealth which is necessary to reduce the expected utility with optimal filtering (strategy (a)) to the expected utility with (b) pure jump filtering or (c) constant portfolio weights. The parametrization for each panel is given in Table 1. All results have been computed in a Monte Carlo simulation with 500,000 sample paths for each case and a planning horizon of 10 years.

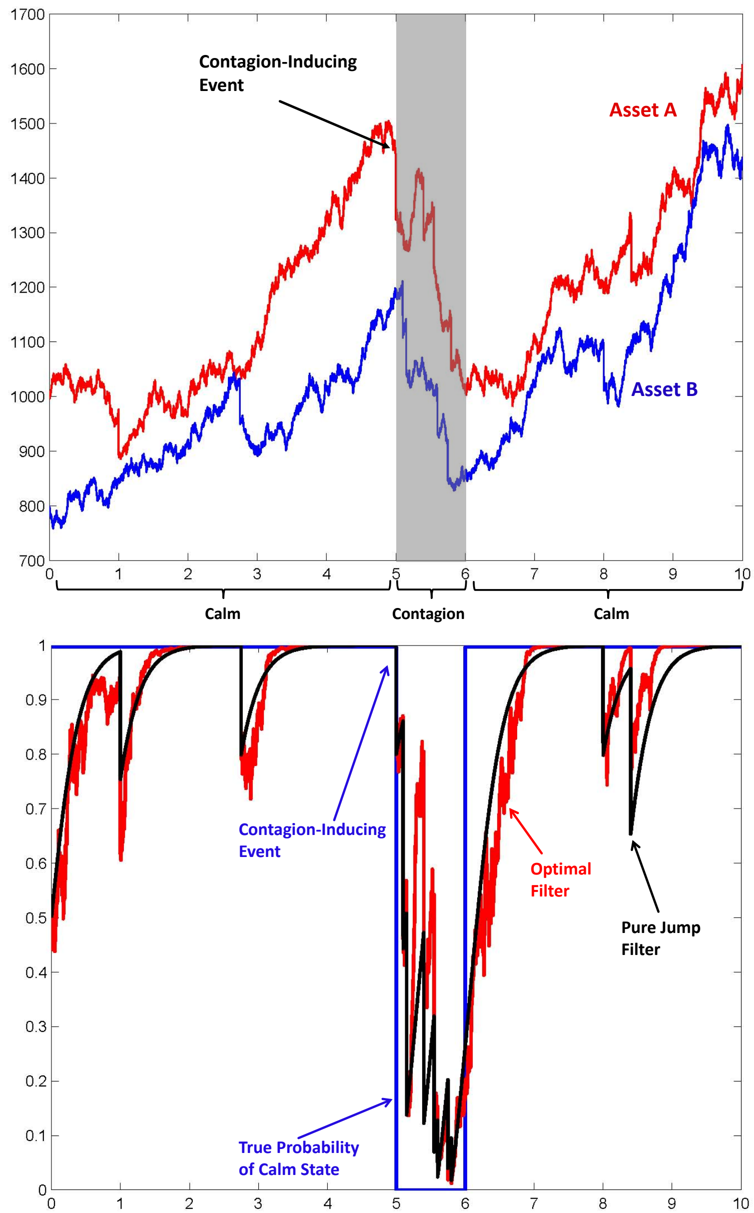

Figure 1: Typical sample paths

The figure shows typical sample paths in our model. The upper panel gives the asset prices while the lower panel indicates the according state variable dynamics. Both assets (A and B) follow jump-diffusion processes and are subject to the risk of downward jumps. In the example, the downward jump in asset A at time 5 triggers contagion. The jump probabilities for both assets are significantly larger until the economy leaves the contagion state again at time 6. While there is a loss in one asset (here: asset A) as the economy enters the contagion state, jumps back to the calm state have no direct impact on the asset prices. The filtered probabilities of being in the calm state are adjusted downwards at every jump and increase afterwards as long as no further jump occurs. While the pure jump filter comprises drift and jump terms only, the optimal filter shows up a diffusion part as well.

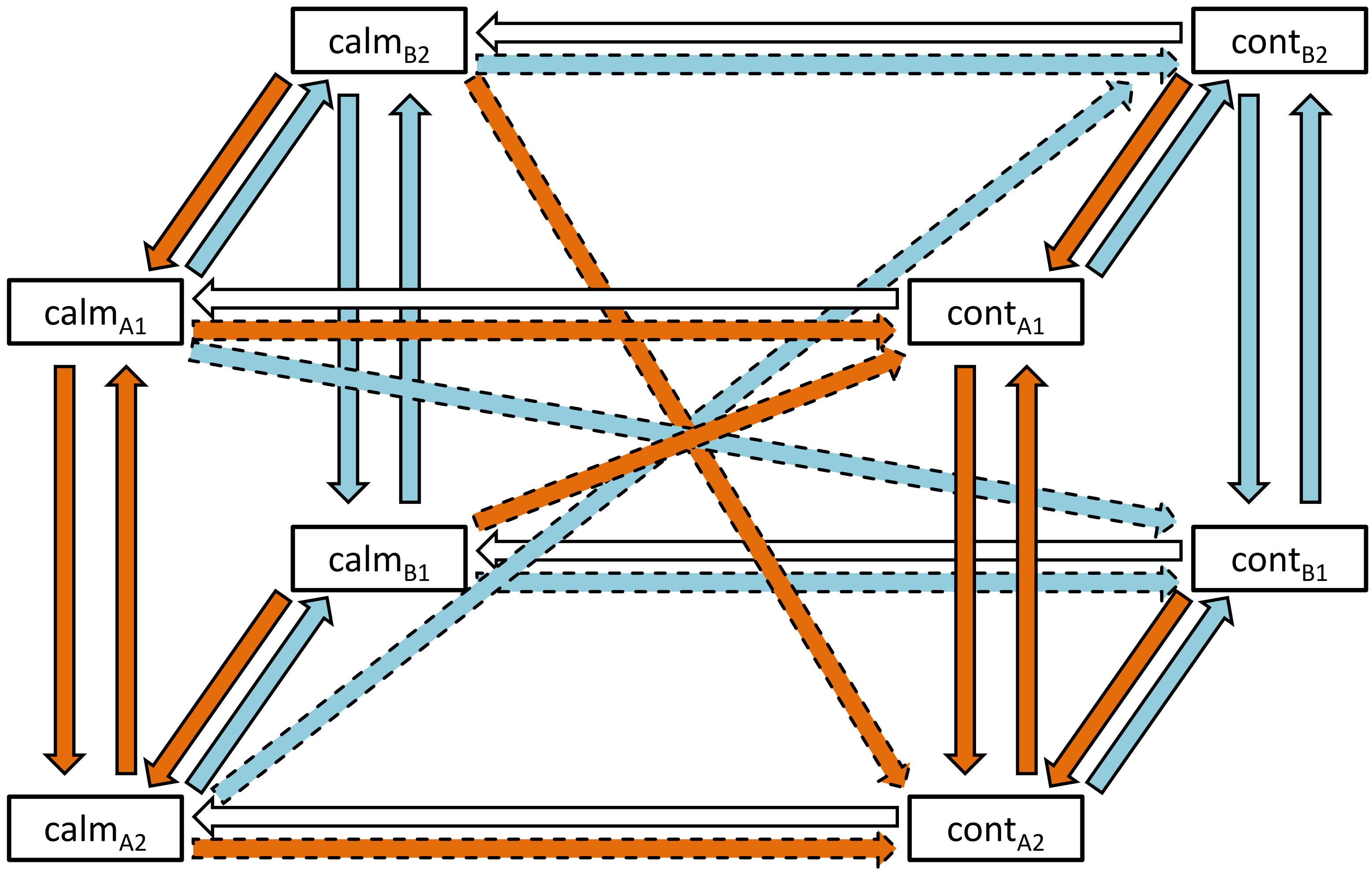

Figure 2: Hidden Markov chain

The figure depicts the hidden Markov chain which drives the economy. The states on the left hand side denote the calm states of the economy, while the ones on the right hand side denote those states in which the assets are affected by contagion. The solid arrows indicate an idiosyncratic jump event leading to a loss in asset A (orange arrows) or asset B (blue arrows). The dashed orange and blue arrows indicate jumps in asset A and asset B, respectively, which trigger contagion. The white arrows denote a change of state without any impact on the asset prices, i.e. a jump from the contagion state back to the calm state. Missing arrows between two states indicate that the corresponding jump intensities are zero.

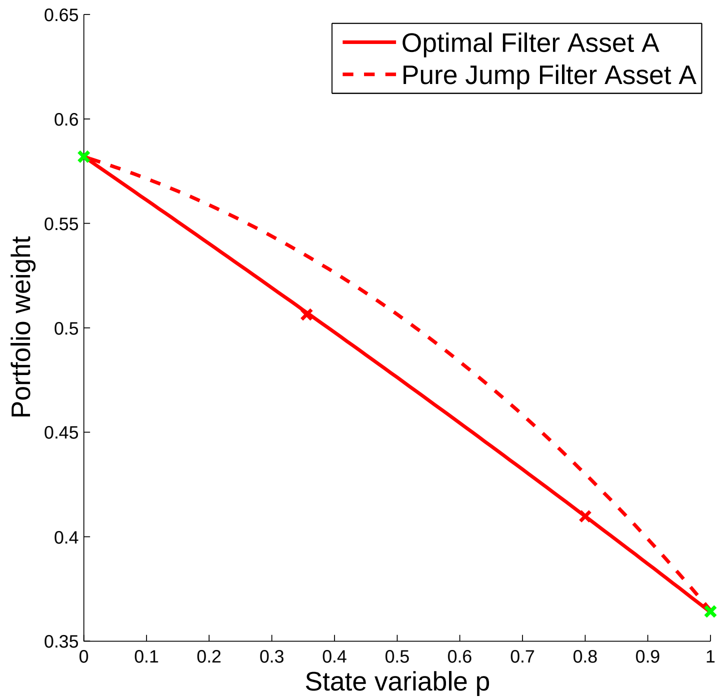

Figure 3: Optimal portfolio weights for identical assets and constant market prices of risk

The figure gives the optimal portfolio weights in the case with identical assets and constant market prices of risk across states for which the parameters can be found in the first column of Table 1. The solid line gives the portfolio weights of an investor filtering optimally while the dashed line gives the portfolio weights of an investor using the pure jump filter who takes only the information from jumps into account. Note that the optimal portfolio weights of both assets equal since the assets are identically parameterized. The red crosses in the graph mark the updated subjective probability $\widehat{p}_i^+$ after one and two jumps assuming that the initial probability $\widehat{p}$ is equal to 1. These updated probabilities can also be seen in Figure 6. The green crosses give the optimal portfolio weights of an investor with full information in the calm and in the contagion state.

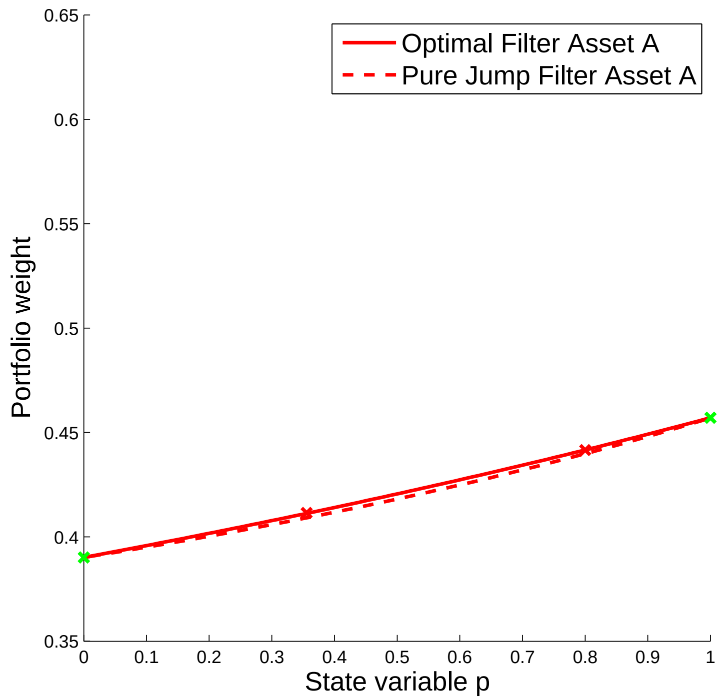

Figure 4: Optimal portfolio weights for identical assets and constant risk premia

The figure gives the optimal portfolio weights in the case with identical assets and constant risk premia across states for which the parameters can be found in the second column of Table 1. The solid line gives the portfolio weights of an investor filtering optimally while the dashed line gives the portfolio weights of an investor using the pure jump filter who takes only the information from jumps into account. The red crosses again mark the updated subjective probability $\widehat{p}_i^+$ after one and two jumps assuming that the initial probability $\widehat{p}$ is equal to 1. The green crosses give the optimal portfolio weights of an investor with full information in the calm and in the contagion state.

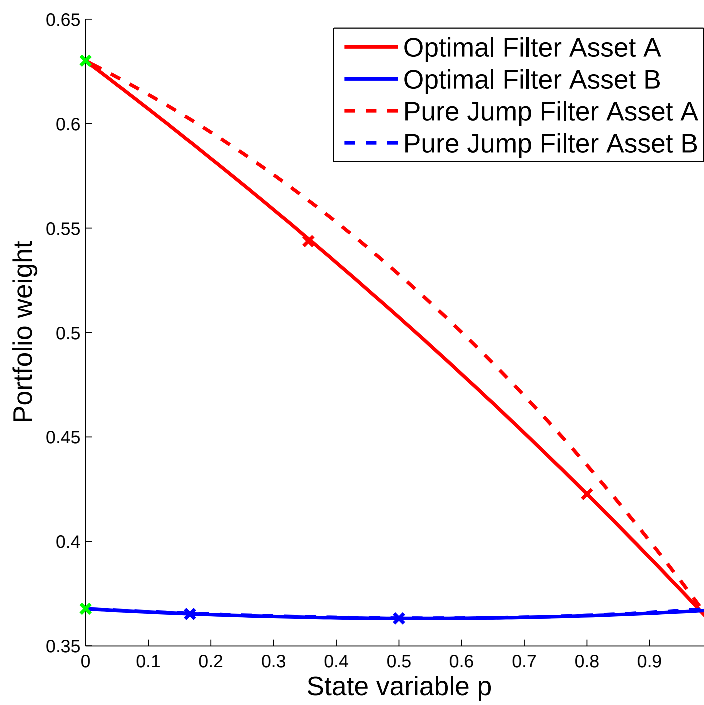

Figure 5: Optimal portfolio weights for heterogenous assets and constant market prices of risk

The figure gives the optimal portfolio weights in the case with heterogenous assets and constant risk premia across states for which the parameters can be found in the third and fourth column of Table 1. The solid lines give the portfolio weights of an investor filtering optimally while the dashed lines give the portfolio weights of an investor using the pure jump filter who takes only the information from jumps into account. The portfolio weights of asset A are marked in red, those of asset B are given in blue. The red and blue crosses in the graph have the same meaning as in Figure 3 except that we distinguish between the effect of jumps in asset A (red crosses) and jumps in asset B (blue crosses) on the estimated probability which can also be seen in Figure 6. The green crosses again give the optimal portfolio weights of an investor with full information in the calm and in the contagion state.

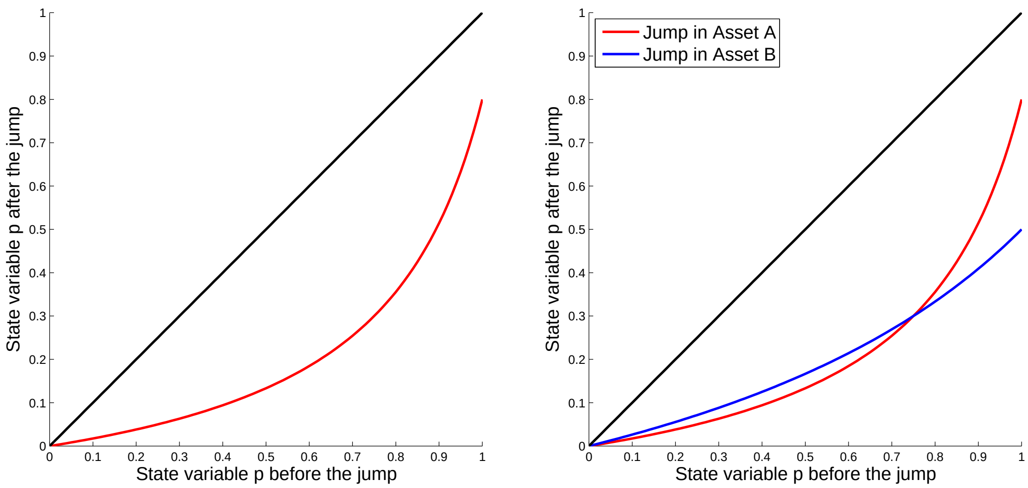

Figure 6: Probability update upon a jump

The figure depicts the updated probability $\widehat{p}_i^+$ after a jump as a function of the respective probability before that jump. For comparison, the black line gives the bisecting line. The left panel gives the probability update in the case with identical assets where the updates due to jumps in asset A and jumps in asset B equal (red line). The right panel gives the update in the case with heterogenous assets where the update due to jumps in asset A is marked by the red line while the update due to asset B is given by the blue line. The red and blue crosses in Figure 3, 4 and 5 refer to these graphs.